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Cooperative and Consensus-Based Approaches to Formation Control of Autonomous Vehicles

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Abstract:
This paper presents a distributed robust control scheme for the formation flight of autonomous aerial vehicles, which is obtained from combining two existing approaches: cooperative and consensus-based formation control. These schemes are examined using a mixed $\mathcal{H}_\infty/\ell_1$ design approach to provide robustness against arbitrary changes of the communication topology as well as any communication delays. The presented architectures mainly differ in the coupling of the agents, which influences performance and formation maintenance. Furthermore, these approaches are analyzed by means of a simulation study on a group of quad-rotor helicopters. The results indicate significant benefits from combining these existing schemes.

1. INTRODUCTION

Formation control of autonomous vehicles is one of the typical problems addressed in the context of multi-agent systems (MAS). In this paper the formation control problem of a group of N identical agents is considered and exemplified by quad-rotor helicopters.

The agents are assumed to be physically decoupled, but they are able to exchange information via wireless communication links. The communication topology can be modeled as a directed graph (Fax and Murray [2004]). Due to limited ranges of the communication links, possible failures or desired changes of the number of agents during operation, the topology has to be considered as unknown and switching. Furthermore, uncertain time delays caused by the communication links have to be expected.

In Fax and Murray [2004] a cooperative control approach is proposed for such a formation control problem providing robust stability to a formation of N vehicles for an arbitrary known and fixed topology. The associated robust formation stability problem is reduced to a problem on the level of a single agent.

A second formation control scheme is proposed in Fax and Murray [2004], containing an information flow filter (IFF) providing a joint determination of a reference positions to be tracked. A simplifying interpretation of this setup, referred to as consensus-based approach, is presented in Pilz et al. [2011], decomposing it into a consensus algorithm and a local position controller. This allows a separate design of the local position controller and the IFF.

Based on these previous approaches, in Pilz and Werner [2012b], Popov [2012] and Pilz [2013] a general information flow framework with a combined controller is proposed describing both local control of agent dynamics and interaction between agents - respectively their controllers. Within this framework, both cooperative and consensus-based approach can be seen as special cases.

Robust stability of MAS under both unknown and switching topologies as well as unknown communication time delays is examined in Popov and Werner [2012] and a stability condition is expressed as ℓ_1 condition. Based on this, in Pilz and Werner [2012a] an $\mathcal{H}_\infty/\ell_1$ controller design method is proposed, which allows to design a formation controller *a-priori* guaranteeing robust stability. Using this method, a cooperative formation controller for a group of quad-rotor helicopters is synthesized. In Pilz and Werner [2012b] and Pilz [2013] this method is applied to the consensus-based and a combined scheme.

As main contribution of this paper, a modification of the information flow filter scheme is introduced, combining it with the cooperative approach. In contrast to the combined design approach of IFF and local controller presented in Pilz [2013], this scheme still considers separate synthesis of the two parts, but introduces a coupling of the agents to improve the formation maintenance. Furthermore, a comparison of this approach with the underlying ones is given based on a simulation study. Therein, controller synthesis techniques from Pilz and Werner [2012a] based on an $\mathcal{H}_\infty/\ell_1$ condition are used.

The structure of this paper is as follows: In Section 2 a short overview of the graph-theoretical framework of MAS and $\mathcal{H}_\infty/\ell_1$ controller design for such systems is given. The different formation control architectures are presented in Section 3; results of the simulation study on these architectures are given in Section 4. Finally, conclusions are drawn in Section 5.

1.1 Notation

$\mathbb{R}^{p\times q}$ denotes the set of real $p\times q$ matrices. A $q\times q$ identity matrix is denoted by I_q. For the i-th row or the i-th column of a matrix A the shorthand notations $A_{i:}$ and $A_{:i}$ are used. Kronecker extensions of matrices or systems are denoted by $M_{(q)} = M \otimes I_q$ and $\hat{M} = I_N \otimes M$. The ℓ_1 norm of a system $T(z)$ is defined as

$$\|T(z)\|_1 = \sup_{x\neq 0}\frac{\|Tx\|_\infty}{\|x\|_\infty} \quad \text{or} \quad \|\acute{T}\|_1 = \max_i \sum_{j=0}^{q}\sum_{t=0}^{\infty}|\acute{T}_{ij}(t)|,$$

where $\acute{T}_{ij}(t)$ denotes the Markov parameter of the impulse response sequence of $T_{ij}(z)$ at time t.

2. MULTI-AGENT SYSTEMS

In this section the concept of multi-agent systems and the underlying communication structure is presented.

2.1 Formation Modeling and Communication

The formation of agents is considered here as a set of N identical agents, of which each one receives information from an individual subset N_i of the other agents. Those agents, from which agent i receives data, i.e. the elements of N_i, are called *neighbors* of agent i. The number of neighbors of agent i, meaning the cardinality of N_i, is referred to as the *in-degree* of agent i.

As proposed in Fax and Murray [2004], the communication structure among the agents is modeled as a directed graph, in which the nodes represent the agents and the edges represent existing direct communication links. Such a topology can be described using the adjacency matrix or the graph Laplacian, which in normalized form are defined as in Fax and Murray [2004], Popov and Werner [2012]:

$$\mathcal{A}_{ik} = \begin{cases} -\dfrac{1}{|N_i|} & k \neq i, k \in N_i \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

$$L = \mathcal{A} + I_N. \tag{2}$$

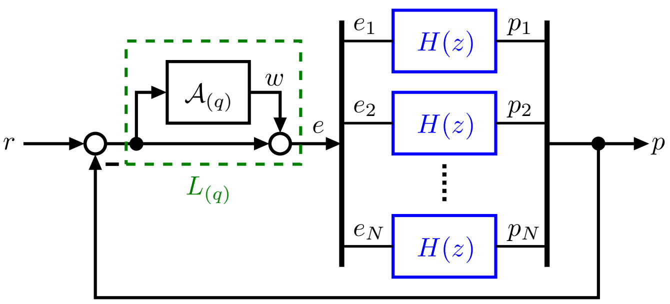


Fig. 1. Global formation control loop (Pilz and Werner [2012a])

Each agent is considered to communicate coordination data $p_i \in \mathbb{R}^q$, which can be interpreted as position values, and to receive the respective data p_k, $k \in N_i$ from its neighbors. From these data a formation control error is defined for every agent as an average of the distance errors for each neighbor:

$$e_i = \frac{1}{|N_i|}\sum_{k\in N_i} e_{ik} \tag{3}$$

$$e_{ik} = \bar{r}_{ik} - \bar{p}_{ik} = (r_i - r_k) - (p_i - p_k), \tag{4}$$

where $\bar{p}_{ik}$ denotes the distance between the agents k and i, $\bar{r}_{ik}$ denotes the corresponding reference distance and r_i the absolute reference for agent i. Using the adjacency and Laplacian matrices, this expression can be rewritten for the whole formation as

$$e_i = r_i - p_i + (\mathcal{A}_{i:})_{(q)}(r-p) \tag{5}$$

$$e = r - p + \mathcal{A}_{(q)}(r-p) = L_{(q)}(r-p) = \bar{r} - L_{(q)}p \tag{6}$$

with $e^T = \begin{bmatrix} e_1^T & \cdots & e_N^T \end{bmatrix}$ and p, r and $\bar{r}$ defined accordingly. The subvectors of $\bar{r}$ denote the average of the reference distances $\bar{r}_{ik}$ from agent i to its neighbors. Using the error defined in (6) as input to the locally controlled agents $H(z)$, they form the global control loop shown in Fig. 1 (Pilz and Werner [2012a], Pilz et al. [2009]).

Note that in general the coordination data p_i do not need to be the physical positions, but they can be interpreted in such a way. Using multi-dimensional signals such as position data to be handled by each agent, the kronecker-extended forms of the adjacency and Laplacian matrices have to be used. Note also that, in the presence of communication delays, the k-indexed terms in (4) are delayed w.r.t. the i-indexed ones.

2.2 Local Agent Architecture

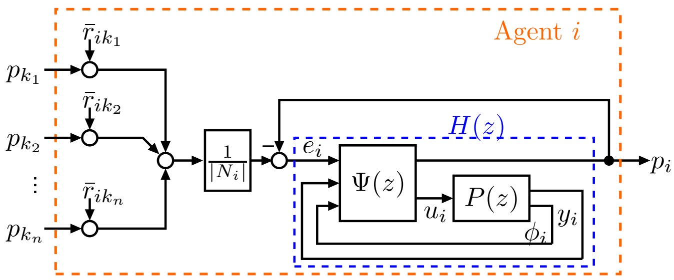


Fig. 2. General local control architecture (Pilz and Werner [2012b], Pilz [2013])

The local control architecture has to fulfill two major tasks: Firstly, it has to stabilize the dynamics of the single agent, secondly it has to evaluate the coordination signals received from the neighbors and to control the agent's position in the formation.

A general agent architecture is shown in Fig. 2 and is based on Pilz and Werner [2012b] and Pilz [2013]. Here a 3-degrees-of-freedom controller $\Psi(z)$ performs both control tasks. Such a multiple-degree-of-freedom formation control scheme was first introduced in Popov [2012]. In the general scheme shown here the controller is considered to have access to the control error e_i, the position y_i of the physical agent model $P(z)$ and a locally measured feedback signal ϕ_i (e.g. measured states for state feedback). Outputs of the controller are the coordination signal p_i transmitted to other agents and the control signal u_i applied to the agents' own dynamics. The control schemes examined in this work can be seen as special cases of this general framework with a certain structure imposed on $\Psi(z)$, see Pilz [2013].

2.3 Stability and Controller Synthesis

A stability condition for a general MAS feedback loop as shown in Fig. 1 is proposed in Popov and Werner [2012] and reduced to an ℓ_1 condition for a single agent:

Theorem 1. (Popov and Werner [2012]) A multi-agent system as shown in Fig. 1 is stable for any number of agents N and arbitrary and switching communication topologies with any time-varying communication delays, if there exists an invertible matrix $D \in \mathbb{R}^{q\times q}$ satisfying $\|DM(z)D^{-1}\|_1 < 1$, with $M(z) = (I_q + H(z))^{-1}H(z)$ denoting the local part of the feedback loop in Fig. 1.

As presented in Pilz and Werner [2012a], a synthesis problem can be derived from this condition by imposing an additional $\mathcal{H}_\infty$ condition for performance optimization:

Problem 1. Find a controller $K(z)$ that satisfies

$$\min \|T_{22}\|_\infty \tag{7}$$

$$\text{subject to } \|DT_{11}D^{-1}\|_1 < 1, \tag{8}$$

with a suitable scaling matrix D and T_{ij} denoting the closed-loop transfer function from reference/disturbance input w_j to performance output z_i of the generalized plant $G(z)$ shown in Fig. 3.

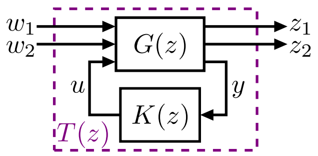


Fig. 3. Generalized plant setup for controller synthesis (Pilz and Werner [2012a])

A method to solve this synthesis problem providing an *a priori* stability guarantee is proposed in Pilz and Werner [2012a], employing a Youla parameter approach discussed in Scherer [2000].

Alternatively, the ℓ_1 condition can be replaced by an $\mathcal{H}_\infty$ condition, such that an $\mathcal{H}_\infty/\mathcal{H}_\infty$ problem has to be solved. This reduces the synthesis effort and leads to a lower order controller, but the ℓ_1 condition has to be checked *a posteriori* to guarantee stability. The controller of the extended consensus-based scheme has so far only been synthesized using this method.

3. FORMATION CONTROL SCHEMES

This section focuses on three formation control schemes mainly differing in the coupling between the agents.

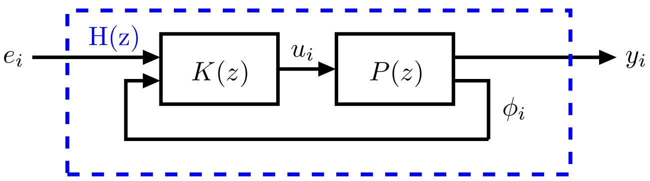


Fig. 4. Local setup of the cooperative architecture (Pilz and Werner [2012a])

3.1 Cooperative Control Scheme

The cooperative control scheme proposed in Popov and Werner [2012] and Pilz and Werner [2012a] considers communication of the agents' physical position, meaning $p_i = y_i$. The controller $\Psi(z)$ is reduced to a 2-degrees-of-freedom controller $K(z)$ and leads to a strong coupling of the agents. The corresponding local control loop is shown in Fig. 4, the synthesis of such a controller $K(z)$ is discussed in Pilz and Werner [2012a].

3.2 Consensus-Based Control Scheme

As an alternative to the fully coupled cooperative approach, in Fax and Murray [2004] an information flow scheme is presented. The basic idea of this approach is to let the agents jointly determine a formation reference position to be used for position control. In Pilz et al. [2011] this scheme is simplified and separated into a consensus loop and a local position control loop, as shown in Fig. 5. The loop on the left containing the distributed information flow filter $\hat{F}(z) = I \otimes F(z)$ estimates the absolute reference position for each agent, which is used as coordination data p_i and communicated to the neighbors. Meanwhile, the right part acts as a local position controller to make the agent's physical position y_i follow the estimated reference position p_i.

A major advantage of this setup regarding the controller design is that information flow filter $F(z)$ and position controller $K(z)$ can be designed independently; the position controller is then independent of the formation control context and topology uncertainties and thus can be designed using standard methods.

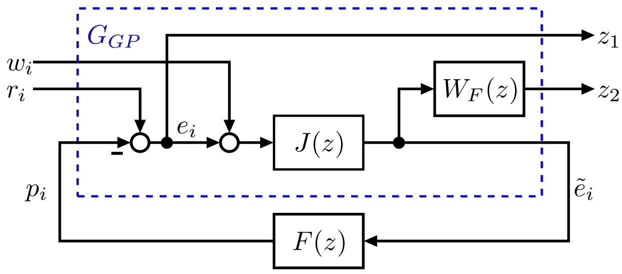


Fig. 6. Generalized plant for information flow filter design (Pilz et al. [2011])

Designing the information flow filter was considered to be done heuristically in Fax and Murray [2004], in Pilz et al. [2011] μ-synthesis techniques are proposed. Here we will adopt the $\mathcal{H}_\infty/\ell_1$ synthesis method already applied to this scheme in Pilz and Werner [2012b] and Pilz [2013]. The relative estimation error e shaped by a shaping filter

$$W_F = \frac{1}{3}\frac{0.01}{(z-1)}I_3 \tag{9}$$

is used as performance output. Using the generalized plant shown in Fig. 6, the following can be stated about stability of the MAS:

Theorem 2. (Pilz [2013]) Assuming that $\hat{K}(z)$ stabilizes $\hat{P}(z)$, a multi-agent system as shown in Fig. 5 is stable for

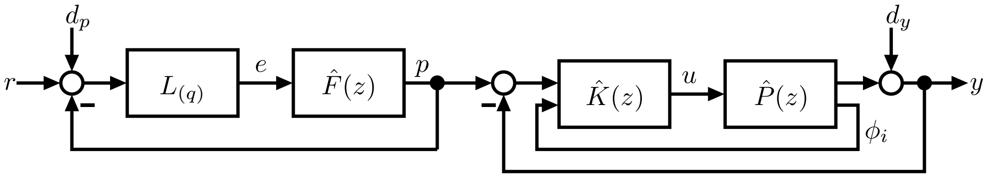


Fig. 5. Consensus-based formation control scheme (Pilz et al. [2011])

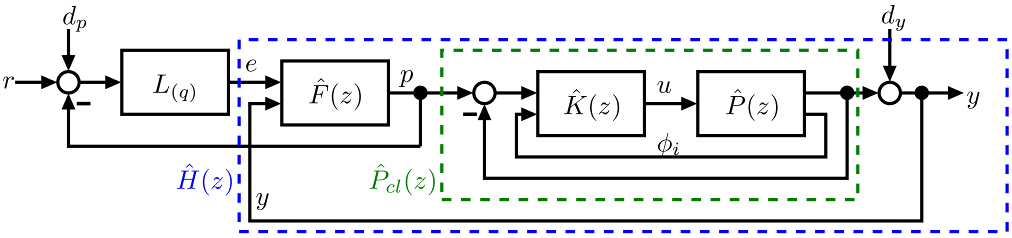


Fig. 7. Extended information flow scheme

any number of agents N and arbitrary and switching communication topologies with any time-varying communication delays, if there exists an invertible matrix $D \in \mathbb{R}^{q\times q}$ such that $\|DT_{z_1w}D^{-1}\|_1 < 1$, where T_{z_1w} denotes the closed-loop transfer function from w_i to z_1 and T_{z_2r} that from r_i to z_2.

A proof of this theorem is given in Pilz [2013] and Pilz et al. [2012].

To avoid a direct throughput through G_{GP} from control input to measured output, which leads to difficulties in the design procedure, a low-pass filter $J(z)$ is used as pre-filter and chosen to have a sufficiently high bandwidth.

In the same way the stability criterion is applied here, the whole synthesis problem can be formulated for the information flow filter:

Problem 2. Find an information flow filter $F(z)$ such that

$$\min \|T_{z_2r}\|_\infty \tag{10}$$

$$\text{subject to } \|DT_{z_1w}D^{-1}\|_1 < 1. \tag{11}$$

Assuming the signals in the consensus loop to be in Cartesian coordinates, which is usually the case in formation control problems, we can treat the q channels independently and consider $F(z) = I_q \otimes f(z)$. This reduces Problem 2 to the design of a SISO information flow filter $f(z)$ (Pilz et al. [2012]).

3.3 Extended Information Flow Filter Scheme

As a third architecture, here an extension of the consensus-based control scheme is introduced with the goal to combine the performance and design advantages of the consensus-based scheme with the ability of the agents to react on disturbances of other agents. This scheme is shown in Fig. 7. Unlike in the consensus-based scheme, where information flow filter and position controller are only connected in a feed-forward way, here the agent position is fed back locally to an extended information flow filter. Thus the coordination signal p can be influenced by the agent's output, which permits to transport information about an output disturbance d_y acting on the agent and enables the other agents to react on that. Nevertheless, p is used as reference estimation for the position controller.

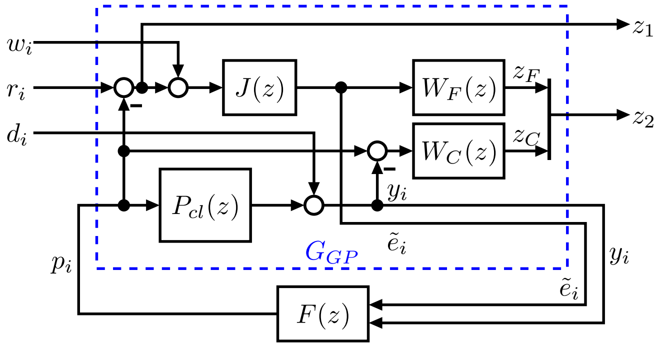


Fig. 8. Generalized plant for combined control scheme

This architecture can as well be seen as a special case of the combined agent setup in Popov [2012] or that in Pilz and Werner [2012b] and Pilz [2013] (shown in Fig. 2), at which the controller is structured as

$$\Psi(z) = \begin{bmatrix} F_1 & F_2 & 0 \\ K_1F_1 & K_1(F_2 - I) & K_2 \end{bmatrix}, \tag{12}$$

where $F(z) = [F_1(z)\ F_2(z)]$ is the information flow filter and $K(z) = [K_1(z)\ K_2(z)]$ the local controller. However, whereas in Pilz and Werner [2012b] and Pilz [2013] the whole combined controller $\Psi(z)$ was synthesized at once, here information flow filter and local controller are synthesized separately.

The local position control part of this architecture is the same as in the consensus-based scheme, thus it can as well be designed independently. For stability analysis and design of the extended IFF, the considerations of the consensus-based scheme are applied to the generalized plant shown in Fig. 8. In particular, here the locally position-controlled plant P_{cl} (see Fig. 7) affected by an output disturbance d_i is included to provide output feedback. Accordingly, in contrast to the consensus-based approach, here the IFF design is not completely independent from the position controller and local dynamics; at least a model of the closed position control loop has to be known. To tune the coupling of the agents, a second performance channel z_C is introduced penalizing the difference between coordination signal p_i and plant output y_i.

A stability condition can be inferred by applying Theorem 1 to the overall structure indicated in Fig. 7:

Theorem 3. Assuming that $\hat{K}(z)$ stabilizes $\hat{P}(z)$, a multi-agent system as shown in Fig. 7 is stable for any number of agents N and arbitrary and switching communication topologies with any time-varying communication delays, if there exists an invertible matrix $D \in \mathbb{R}^{q \times q}$ such that $\|DT_{z_1w}D^{-1}\|_1 < 1$, where T_{z_1w} denotes the closed-loop transfer function from w_i to z_1 and T_{z_2r} that from r_i to z_2 for the setup shown in Fig. 8.

Proof: With $\hat{F}(z)$, $\hat{K}(z)$ and $\hat{P}(z)$ being block-diagonal, the whole local structure $\hat{H}(z)$ shown in Fig. 7 is block-diagonal as well. Thus the global setup has the structure shown in Fig. 1, which allows us to apply Theorem 1, which completes the proof. ∎

Accordingly, the synthesis problem for the extended information flow filter is inferred as follows:

Problem 3. Find an information flow filter $F(z)$ such that

$$\min \|T_{z_2r}\|_\infty \tag{13}$$

$$\text{subject to } \|DT_{z_1w}D^{-1}\|_1 < 1, \tag{14}$$

where D denotes a suitable scaling matrix.

By suitable tuning of the shaping filters W_F and W_C a trade-off between reference consensus and coupling of the agents can be achieved. Here W_C is chosen as a high-pass filter

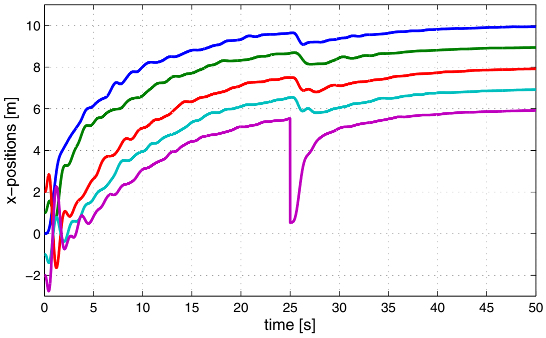
$$W_C(z) = \frac{5z - 4.995}{z} I_3, \tag{15}$$

for W_F the same settings as in (9) are used. This choice is aimed at combining a good accuracy of steady state reference tracking (and thus reference consensus) with a sufficient coupling of the agents' transient behavior.

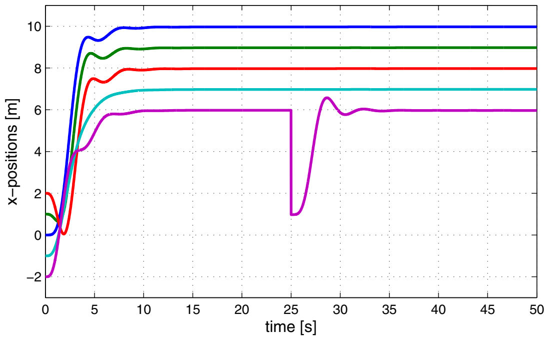
4. SIMULATION RESULTS

The approaches presented in the previous sections were tested in a simulation study, the results are presented in the following. The scenario simulated here is adopted from Pilz and Werner [2012a] and considers a group of five identical quad-rotor helicopters modeled by an unstable and underactuated 12-th order linear time-invariant model proposed in Lara et al. [2006], modeled in discrete time with a sampling time of 10ms. Although the simulation considers a 3D space, here only a change in the x-coordinate is commanded. The agents start at the positions $[x_1(0)\ x_2(0)\ x_3(0)\ x_4(0)\ x_5(0)] = [0m\ 1m\ 2m\ -1m\ -2m]$ and are desired to reach a formation with the positions $[x_{1d}\ x_{2d}\ x_{3d}\ x_{4d}\ x_{5d}] = [10m\ 9m\ 8m\ 7m\ 6m]$. The communication topology is chosen randomly and is changing every 0.1s. Additionally, the communication links are affected by random time delays in the range of 1 to 6 sampling instances. At $t_d = 25s$ agent 5 is affected by a step disturbance $d_y = -5\sigma(t - t_d)$ acting on its output.

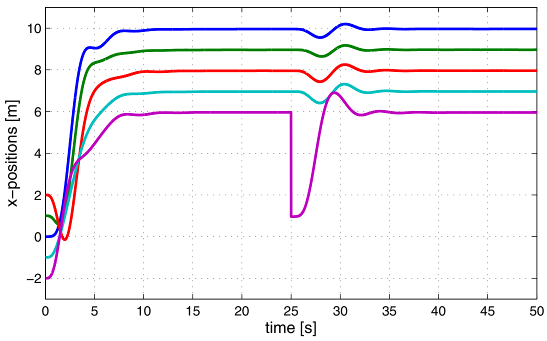
In a formation flight scenario the major objective is to track the desired agent positions within the formation with a reasonable transient behavior and good steady state accuracy. In case of a disturbance acting on one agent, the according objective is to control this agent back to its desired position. Nevertheless, it may also be desirable to let the swarm follow the disturbed agent in order to maintain the formation rather than to maintain correct absolute positions. Besides that, we demand the stability of the system to be robust against the considered disturbing effects. Furthermore, the control effort should remain within certain bounds in order to avoid effects of actuator saturation.



(a) cooperative formation control



(b) consensus-based formation control



(c) extended IFF formation control

Fig. 9. Simulated x-positions of the agents

In case of the consensus-based architecture, no actual position data are exchanged and the position controllers of the agents are completely decoupled. Thus we cannot expect the agents to react on any disturbances acting on other agents.

The results in terms of the agents' positions in x-direction are shown in Fig. 9 for the three examined formation control schemes. Comparing the time to reach the formation, one can see that with the cooperative architecture (Fig. 9a) approx. three times more time is needed than in the other cases. Additionally, here a significant dithering is visible during the whole movement, which can be related to the

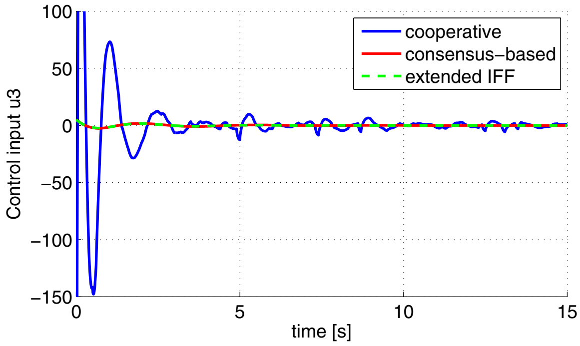


Fig. 10. Simulated control input u_3 at agent 3 for different control architectures

topology changes. In contrast, the results shown in Fig. 9b and 9c are smooth and do not noticeably show such effects.

Rejecting the disturbance on agent 5 at $t = 25s$ is performed rather quickly in all cases, where consensus-based and extended IFF formation control show a significant overshoot. As expected, in the consensus-based case the other agents do not show any reaction on the disturbance. In both other cases a significant attempt to follow the disturbed agent is visible.

In Fig. 10 the control input u_3 representing the torque around the y-axis, which is the control quantity mainly responsible for movements along the x-axis, is shown. As is clearly visible, in case of the cooperative scheme the control signal reaches much higher values than in the other cases and is far beyond any actuator limits. Referring to the output results, in case of the consensus-based and extended IFF schemes a significantly better transient performance is reached with significantly less control effort.

From these observations the consensus-based and the extended IFF control schemes appear to clearly outperform the cooperative scheme. The only major advantage of the cooperative approach over the consensus-based one, which is the ability of the formation to react to disturbances of a single agent, turns out to be achievable with the extended IFF scheme at a significantly lower price in terms of performance. Comparing these results to those of the combined architecture shown in Pilz and Werner [2012b] and Pilz [2013], the extended IFF scheme shows significantly better performance in terms of settling time and formation maintenance.

Further tests have shown that the $\mathcal{H}_\infty/\mathcal{H}_\infty$ controller synthesis method from Pilz and Werner [2012a] is able to generate controllers providing the same performance compared to those generated with the $\mathcal{H}_\infty/\ell_1$ method. Thus, as long as the results fulfill the ℓ_1 condition, the $\mathcal{H}_\infty/\mathcal{H}_\infty$ method can be seen as a preferable alternative.

5. CONCLUSIONS

In this paper a novel combination of two existing architectures for the formation control of autonomous vehicles is presented and attributed to an $\mathcal{H}_\infty/\ell_1$ synthesis method, which guarantees robust stability under switching communication topologies affected by time-varying delays.

This architecture and the preliminary ones are tested in a simulation study for the example of quad-rotor helicopters and their results are compared. Here the combined approach turns out to combine the advantages of both previous approaches regarding performance, synthesis effort and formation maintenance. For the future work these results are to be validated in formation flight experiments.

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