

Feedback Control System with Stochastically Deteriorating Actuator: Remaining Useful Life Assessment

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Abstract: Feedback control system performance can be decreasing during its operation because of the components' wear or degradation. To deal with such loss of efficiency, a research aspect of dependability concerns fault-tolerant control (FTC) strategies which give the feedback control system the ability to overcome faults. In this way, the RUL (remaining useful life) becomes valuable information which can be integrated in controller design in efforts to find a satisfactory trade-off between control system performances and components' lifetime. The main aim of this paper is twofold. On one hand, it proposes an integrated model which jointly describes the state of the feedback control system and the actuators degradation. On the other hand, a probabilistic model-based framework is presented in order to assess the RUL of the deteriorating feedback system. No special monitoring device is used to observe the health status of actuator, thus the measurements of closed system response are considered as the only available health information. The RUL is computed by a two-step technique. First, the system state regarding the available observations is estimated on-line by using the Particle Filter method. Then, the reliability of the system is computed with a classical Monte Carlo method. In order to illustrate this approach, a well-known simulated double-tank level control system is used.

Keywords: dynamic systems, closed-loop systems, device degradation, stochastic modelling, stochastic jump process, reliability analysis, stochastic filtering, Remaining Useful Life, Piecewise Deterministic Markov Process

1. INTRODUCTION

Depending on its special structure, the feedback control system (FCS) has the ability to compensate for disturbances in the controlled applications. However, failure or loss of efficiency of system components such as actuators or sensors can lead to the instability of the control loop and reduce the product quality. Therefore, the maintenance optimization is one of the main research aspects to improve the reliability, durability of production systems, and also reduce the cost (Wang (2002); Dieulle et al. (2003)).

Another research aspect of dependability concerns fault-tolerant control (FTC) strategies which give the FCS the ability to overcome faults (Zhang and Jiang (2008)). The key objective of FTC system design may not offer optimal performance in a strict sense for normal operation, but generally it can mitigate effects of system components failures. In this way, the degradation level and/or the remaining useful life (RUL) of the system become a valuable information which could be integrated in controller design with the aim to find a satisfactory trade-off between feedback system performances and its reliability (Langeron et al. (2013); Pereira et al. (2010)).

The main aim of this work is to deal from a dependability point of view with a closed-loop system combining a deterministic part related to the system dynamics and a

stochastic part related to the actuator degradation. Our main contribution in this work is twofold. Firstly, we propose an integrated model which jointly describes the state of the feedback control system and also the stochastic degradation process of the actuator. Secondly, we propose a probabilistic model-based framework with a diagnosis-prognosis approach for dealing with the assessment of the RUL on the basis of the closed system output alone. This means that no additional sensor is devoted to the monitoring of the degradation phenomenon of actuator, only the measurements of the system output are available in order to assess the system health. A diagnosis-prognosis approach composed of two steps is developed which can be justified because the dynamics of the controlled process combined with the degradation process of actuator can be modeled using a Piecewise Deterministic Markov Process. First, the computation of the conditional distribution of the system state regarding the available observations is required (because the actual state of actuator is not observed). Then, the distribution of the remaining lifetime of the system knowing that the current state is distributed according to the density function estimated at first step is computed.

The remainder of this paper is organized as follows. Section 2 is devoted to the description of the system characteristics and the assumption about its degradation process.

Section 3 describes the condition monitoring process and the approach for computing remaining useful life which is relevant to state estimation. In order to estimate the system state, the need of the system health information results in the introduction of excitation signals in the set-point input. To illustrate the methodology, a specific case study is introduced in Section 4. Some numerical results are also discussed here. Finally, conclusions drawn from this work and possible ways for further studies are given.

2. SYSTEM MODELING AND INSPECTION POLICY

2.1 Feedback control system structure

Within a general framework, let consider a dynamical process whose state evolution can be described by the nonlinear differential equation:

$$\dot{x}_t = \mathbf{f}(t, x_t, u_t) \quad (1)$$

where x_t is the state vector of process, u_t denotes control force acting on the process.

In spite of sophisticated filter structures, noise in measurement process is usually an unavoidable problem. Here, it is assumed that measurement noises $(\epsilon_t)_{t \in \mathbb{R}_+}$ are independent random variables with a probability density g , not necessarily Gaussian, independent of the process state $(x_t)_{t \in \mathbb{R}_+}$. The measurements of controlled process state may be formally expressed as:

$$y_t = \mathbf{h}(t, x_t, u_t) + \epsilon_t \quad (2)$$

In this work, we consider classical Proportional-Integral-Derivative (PID) controllers which are widely used in industrial applications thanks to their simplicity and performance. The PID controller output u_t^c is given by:

$$u_t^c = K_P \left[e_t + \frac{1}{T_I} \int_0^t e_\tau d\tau + T_D \frac{de_t}{dt} \right] \quad (3)$$

where e_t is the error signal defined as $e_t = y_t^{\text{ref}} - y_t$ with y_t^{ref} the desired set-point (reference input), K_P is proportional gain, T_I is the integral time and T_D is the derivative time of the PID controller. Adjustment of these three parameters to the optimum values for the desired control response is extensively studied in control design (Aström and Hägglund (1995)).

2.2 Actuator deterioration modeling

An important element in feedback control is the actual performance level of the actuator because degradation of such a component affects the control action and can lead to a lack of performance of the feedback control system. The real output of the actuator may be defined as the following model:

$$u_t = \mathbf{g}(u_t^c, C_t) \quad (4)$$

where \mathbf{g} is a decreasing function w.r.t. the actual actuator capacity C_t which is related to the degradation process.

In this work, it is assumed that an actuator is subject to shocks that occur randomly in time. Each shock impacts a random quantity of damage to the actuator. Hence, the capacity of the actuator at time t before its failure can be expressed as:

$$C_t = C_0 - \mathcal{D}_t \quad (5)$$

where C_0 is the initial capacity of the actuator, \mathcal{D}_t describes the accumulated deterioration of the actuator at time t (in *capacity* unit)

The most common assumption of this degradation model is that the shocks occur according to a Poisson process with intensity λ . The amount of damage d_j produced by the j -th shock is randomly distributed according to a given probability law G with $G(\delta) = \mathbb{P}(d_j \leq \delta)$. All the $d_j, j \geq 1$ are independent and identically distributed. Let N_t denote the total number of shocks up to time $t \geq 0$. Then the accumulated deterioration of the actuator at time t is:

$$\mathcal{D}_t = \sum_{j=0}^{N_t} d_j \quad (d_0 = 0) \quad (6)$$

Under this modeling assumption, the degradation impacts the actuator only at discrete times. In case of an actuator which gradually deteriorates, other processes should be considered e.g. as the homogeneous Gamma process which can be thought as the accumulation of an infinite number of small shocks during each time interval (Van Noortwijk (2009)).

2.3 Excitation signal and condition monitoring process

No additional sensor is devoted to the monitoring of the actuator degradation. The noisy measurement of system output is considered as the only available health information about the closed system. As a consequence each transient response which occurs in the short period of time immediately after a change of set-point has to be considered to assess the system health.

In this paper a fixed set-point control is considered and the direct use of the system output in the prognostic purpose is not possible because of a lack of transient response. In order to deal with this limit, excitation signals are considered. Short pulses (small changes of the set-point) are emitted in order to allow the observance on how the system responds to such perturbations. The resulting behavior is often called impulse response. This information is then used by the considered prognostic approach in order to make the prediction of the RUL of the system (Figure 1).

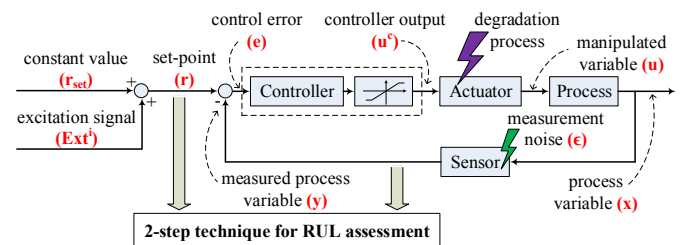


Fig. 1. RUL assessment scheme

The occasional changes of set-point due to short pulses are considered as disturbances for the system. The frequency of the excitation period and the type of excitation signal should be chosen carefully. The simplest kind of pulse is a rectangular pulse. Ext^i denotes the i -th excitation pulse which is characterized by the beginning T_i^b , the excitation duration ΔT_i and the excitation amplitude δ_i

$$\text{Ext}^i = \begin{cases} 0 & \text{if } t < T_i^b \\ \delta_i & \text{if } T_i^b \leq t < T_i^b + \Delta T_i \\ 0 & \text{if } t \geq T_i^b + \Delta T_i. \end{cases} \quad (7)$$

where the excitation amplitude δ_i is less than a percentage of set-point value (5% or 2%) in order to be able to observe the transient response of the system without affecting the ability of the system to fulfill its task.

The response of the system to this excitation is observed from T_i^b to the instant at which the last observation is recorded, denoted T_i^e . Figure 2 illustrates the inspection process. An observation period is then divided into two phases: excitation and re-establishment phases.

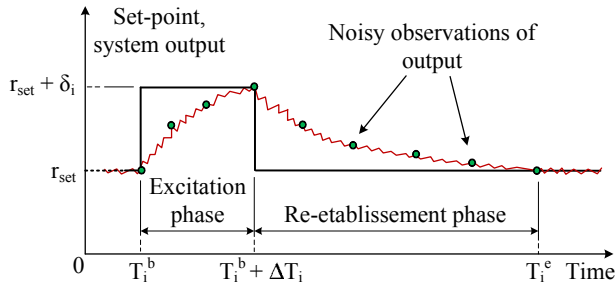


Fig. 2. Rectangular pulse and inspection process

Actually, we take an interest in the observations at some discrete times $T_i^b \leq T_i^1 < \dots < T_i^{n_i} \leq T_i^e$ (n_i observations are recorded). This information on the system state is then modeled by some random variables $Y_i^1, \dots, Y_i^{n_i}$ depending on the actuator random degradation process. The realization y_i^j of Y_i^j can be written from (2):

$$y_i^j = \mathbf{h}(T_i^j, x_{T_i^j}, u_{T_i^j}) + \epsilon_{T_i^j} \quad (8)$$

Let us introduce T_{prog} as the time of the last observation. T_{prog} is also the time at which the estimation of the RUL is expected. Suppose that until this instant m excitation periods are executed. For the simplicity of notation, all the available information until this prediction instant are denoted as Y_1, \dots, Y_n . These random variable are defined at discrete times $0 < T_1 < \dots < T_n = T_{prog}$

3. RUL ASSESSMENT

3.1 Two-step technique for RUL assessment

We adopt a model-based approach that jointly describes the controlled state of system with its degradation. The whole behavior of the deteriorating closed-loop system at time t can be resumed by a random vector as:

$$Z_t = \begin{pmatrix} C_t \\ x_t \\ t \end{pmatrix} \quad (9)$$

with C_t the actual actuator capacity variables related to the degradation, x_t the state variables of controlled process and time t . The time t is useful for the process $(Z_t)_{t \geq 0}$ to be homogeneous in time.

In the context of the feedback control system, the system is considered as failed when it is not able to fulfill its requirements anymore. More concretely, the actual capacity of the actuator has to be greater than a minimal capacity level

which relates to the objectives of control system design. The RUL at time t is defined as the remaining time (from t) before the system can no longer fulfill its mission. More formally if the failure zone \mathcal{F} refers to the set of undesired system states the RUL at time t is defined as:

$$RUL_t = \inf(s \geq t, Z_s \in \mathcal{F}) - t \quad (10)$$

The system state is submitted to random jumps at points in time, but between two jumps its evolution is deterministically governed by a system of differential equations which combine the characteristics of the process dynamics and the PID controller. The system state $(Z_t)_{t \geq 0}$ is a Piecewise Deterministic Markov Process (PDMP). As a consequence see Lorton et al. (2013), the distribution of the RUL of the system at time T_{prog} conditionally to online available information can be computed by a two-step technique representing successively the diagnosis and the prognosis.

$$\begin{aligned} \mathbb{P}(RUL_{T_{prog}} > s | Y_1 = y_1, \dots, Y_n = y_n) \\ = \int R_z(s) \mu_{y_1, \dots, y_n}(dz) \end{aligned} \quad (11)$$

where:

- for diagnosis $\mu_{y_1, \dots, y_n}(dz)$ is the probability law of the system state at time T_{prog} regarding the available observations y_1, \dots, y_n :

$$\mu_{y_1, \dots, y_n} = \mathcal{L}(Z_{T_{prog}} | Y_1 = y_1, \dots, Y_n = y_n) \quad (12)$$

- for prognosis $R_z(s)$ is the reliability of the system at time s starting from the initial state value z :

$$R_z(s) = \mathbb{P}(Z_u \notin \mathcal{F} \quad \forall u \leq s | Z_0 = z) \quad (13)$$

In (11) two characteristic variables have to be considered successively: $\mu_{y_1, \dots, y_n}(dz)$ and $R_z(s)$, leading to the two-steps technique that is detailed in sections 3.2 and 3.2.

3.2 Step 1: Particle filter state estimation

Let Z_{T_0} be the initial state of the μ . The objective is to make inferences on the system state at prognostic instant $Z_{T_{prog}} = Z_{T_n}$ given only realizations $y_{1:n} = y_1, \dots, y_n$ of the observation process $Y_{1:n} = \{Y_i, i = 1, \dots, n\}$ as described in Section 2.3. More specifically, the main task is to estimate the filtering density, $p(z_{T_k} | y_{1:k})$ for any $k \leq n$.

Under nonlinear and non-Gaussian circumstance, particle filtering is used here to allow for numerical computation of the filtering density. The key idea of this sequential Monte Carlo method is to approximate the targeted filtering distribution $p(z_{T_k} | y_{1:k})$ by a cloud of N_s i.i.d. random samples (particles) $\{z_{T_k}^{(i)}, i = 1, \dots, N_s\}$ with associated weights $\{w_{T_k}^{(i)}, i = 1, \dots, N_s\}$, which satisfy $\sum_i w_{T_k}^{(i)} = 1$, so that the target distribution at time T_k can be approximated by

$$p(z_{T_k} | y_{1:k}) \approx \hat{p}(z_{T_k} | y_{1:k}) = \sum_{i=1}^{N_s} w_{T_k}^{(i)} \delta_{z_{T_k}^{(i)}}(dz_{T_k}) \quad (14)$$

where $\delta_{z_{T_k}^{(i)}}(dz_{T_k})$ is the Dirac delta mass located in $z_{T_k}^{(i)}$.

In Daigle et al. (2012) a procedure based on filtering methods is used to predict the RUL of open loop systems. The progression of damage in time is characterized by functions, parameterized by unknown but constant wear

parameters. The damage estimation then reduces to joint state-parameter estimation using for example particle filtering technique for tracking purpose.

The used particle filter is similar to the Generic Particle Filter in (Arulampalam et al. (2002)) with deterministic re-sampling scheme which is one computationally cheaper algorithm (Kitagawa (1996)). Indeed, re-sampling is used to avoid the degeneracy phenomenon that is, avoiding the situation that all but one particle have the negligible importance weight (Arulampalam et al. (2002)). The algorithm uses the prior importance function $p(z_{T_k} | z_{T_{k-1}}^{(i)})$ based on the simulation of the actuator degradation process and the deterministic behavior of the controlled process which is derived from (1) to (7) using a discretized scheme of (1) to (3).

Therefore, the particle filtering system state estimation procedure, given the sequence of measurement $y_{1:k}$ can be summarized by the algorithm in Table 1.

3.3 Step 2: RUL estimation

This step consists in the estimation of the remaining lifetime from time T_{prog} knowing that the system state at T_{prog} is estimated to be distributed according to (14).

Actually, the remaining lifetime is computed by classical Monte Carlo simulation. It means that the simulation of trajectories of the system until its failure is required. The departure point of each trajectory is then randomly selected from the particles set obtained at step 1. Each particle is propagated forward to the failure zone in order to obtain the histogram of RUL. The mean value or quantiles of the RUL can also be derived.

4. CASE STUDY: A DOUBLE-TANK LEVEL CONTROL SYSTEM

In the previous section, a methodology to compute the conditional pdf of the RUL of a dynamic system was described. Here, it is illustrated on a well-known feedback control system: a double-tank level control system.

4.1 Description of the case study

Consider a double-tank level system with cross-sectional area of the first tank S_1 and the second one S_2 . Water or other incompressible fluid (i.e. the mass density of fluid ρ is constant) is pumped into the first tank at the top by a pump motor drives. Then, the out flow from the first tank feeds the second tank.

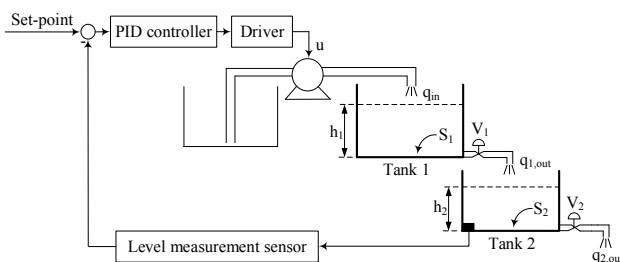


Fig. 3. A double-tank level control system

In order to consider the real response of pump motor, the relation between the inlet flow rate q_{in} and the pump motor control input u is represented as a first order system (Chen and Chen (2008)):

$$\frac{dq_{in}}{dt} = -\frac{1}{\tau_a}q_{in} + \frac{K_a}{\tau_a}u \quad (15)$$

where τ_a is the time constant of pump motor, K_a is the servo amplify gain (with the initial gain $K_{a_{init}}$). The pump saturates at a maximum input u_{max} and it cannot draw water from the tank, so $u \in [0, u_{max}]$.

The fluid leaves out at the bottom of each tank through valves with the flow rates according to the Torricelli rule:

$$q_{j,out} = K_{v_j} \sqrt{2gh_j}, \quad j = 1, 2 \quad (16)$$

where h_j is level of tank j , g is the acceleration of gravity and K_{v_j} is the specified parameter of the valve j .

Using the mass balance equation, the process can be described by following equations:

$$\begin{cases} \frac{dh_1(t)}{dt} = \frac{1}{S_1}q_{in} - \frac{K_{v1}}{S_1}\sqrt{2gh_1(t)} \\ \frac{dh_2(t)}{dt} = \frac{K_{v1}}{S_2}\sqrt{2gh_1(t)} - \frac{K_{v2}}{S_2}\sqrt{2gh_2(t)} \end{cases} \quad (17)$$

The water level of tank 2 is measured by a level measurement sensor and controlled by adjusting the pump motor control input which is calculated by a PID controller. It is assumed that measurement noises are independent gaussian random variables with standard deviation σ and mean equal to zero. The overall tank level control system is shown in Figure 3.

Due to degradation of the pump, its capacity K_a stochastically decreases. The shock instants ξ_i follow a Poisson process with intensity λ . At each time ξ_i the capacity of the pump $K_a(t)$ decreases of a quantity d_i which follows a uniform distribution on $[0; \Delta]$

As mentioned above, the system response (the level of the tank 2) is considered as the only available health information of the system. All excitation signals are the rectangular pulses with the same duration ΔT and the same amplitude δ . At each inspection period, a same finite number of observations are recorded.

Under all these considerations, the behavior of water tank level control system can be summed up using the process $Z = (Z_t)_{t \in \mathbb{R}_+}$, where Z_t is given by:

$$Z_t = (K_a(t), h_1(t), h_2(t), t)^T \quad (18)$$

The current state of the system at time t is then a four-component vector Z_t , which includes the current capacity of the pump, the water levels of two tanks, and the current time t .

According to (15) and (17), the steady states are obtained at instant t_{ss} if

$$u(t_{ss}) = \frac{S_1}{S_2} \frac{K_{v2}}{K_a(t_{ss})} \sqrt{2gh_2(t_{ss})} \quad (19)$$

Since $u(t_{ss}) \leq u_{max}$ so

$$K_a(t_{ss}) \geq \frac{S_1}{S_2} \frac{K_{v2}}{u_{max}} \sqrt{2gh_2(t_{ss})}$$

Table 1. Generic particle filter for system state estimation

Initialization: $\forall i = 1, \dots, N_s$
 Draw particle $z_{T_0}^{(i)}$ according to the initial condition of system and assign corresponding weight $w_{T_0}^{(i)} = \frac{1}{N_s}$

At step k (corresponding to time T_k): Given $\left\{ z_{T_{k-1}}^{(i)}, w_{T_{k-1}}^{(i)} \right\}_{i=1}^{N_s}$, do

(a) Importance sampling: Based on the system description (derived from (1) to (7)), draw particles $z_{T_k}^{(i)} \sim p(z_{T_k} | z_{T_{k-1}}^{(i)})$

(b) Weight update: Based on the likelihoods of the observations y_k collected (Eq. (8)), assign weights $w_{T_k}^{(i)} = w_{T_{k-1}}^{(i)} p(y_k | z_{T_k}^{(i)})$

(c) Weight normalisation: $w_{T_k}^{(i)} = \frac{w_{T_k}^{(i)}}{\sum_{i=1}^{N_s} w_{T_k}^{(i)}}$

(d) Re-sampling decision: If $\hat{N}_{eff} = \frac{1}{\sum_{i=1}^{N_s} (w_{T_k}^{(i)})^2} < N_{thresh}$ then perform re-sampling: $\left\{ z_{T_k}^{(i)}, w_{T_k}^{(i)} \right\}_{i=1}^{N_s} \Rightarrow \left\{ z_{T_k}^{(i)}, \frac{1}{N_s} \right\}_{i=1}^{N_s}$

(e) Distribution: $p(z_{T_k} | y_{1:k}) \approx \sum_{i=1}^{N_s} w_{T_k}^{(i)} \delta_{z_{T_k}^{(i)}}(dz_{T_k})$

Repeat till the prognostic instant T_{prog} is reached

that means the actual capacity of the actuator must be greater than a minimal capacity defined in the control system design phrase. In this case study, the failure zone is defined as:

$$K_a(t) \leq K_{amin} = \frac{S_1}{S_2} \frac{K_{v2}}{u_{max}} \sqrt{2g(r_{set} + \delta)} \quad (20)$$

4.2 Numerical illustrations - Discussions

Numerical values for double-tank level control system are summed up in Table 2

Table 2. Double-tank model

Physical parameters		
$S_1 = 25$	$K_{v1} = 8$	$\tau_a = 1$
$S_2 = 20$	$K_{v2} = 6$	$g = 9.82$
$u_{max} = 100$	$\sigma = 0.01$	
Controller parameters		
$K_P = 4.2519$	$T_I = 18.9817$	$T_D = 1.6182$
Initial condition (t = 0)		
$h_1(0) = 0$	$h_2(0) = 0$	$K_{a_{init}} = 5.0$
Natural degradation		
$\lambda = 0.05$	$\Delta = 0.1$	
Set-point and Excitation signal		
$r_{set} = 10$	$\Delta T = 1$	$\delta = 0.5$

Figure 4 represents one trajectory of the process Z until the complete failure of actuator when no excitation pulse is applied. With the constant set-point, the water level of tank 1 and tank 2 are reflected in Figure 4(b) and Figure 4(c). Figure 4(d) shows the real (unobservable) value of actuator capacity. As depicted in Figure 4, the actuator fails completely (i.e. $K_a = 0$) at 1912.8 time units, but the failure of the system here occurs at 1531.8 time units. One can find that after the system failure date the water level of tank 2 (the controlled variable) cannot track the desired set-point.

The methodology previously described is applied to deduce prognostic about the RUL of the system. It is assumed that 10 excitation signals are introduced in the system at time instants $T_i^b = i.120, i = 1, \dots, 10$. Observations of the system response (measurement of water tank level 2) are then recorded until the prognostic instant $T_{prog} = T_{10}^e = 1209$.

The first step of the method is to compute the pdf of the system state regarding the available observations until the end of observation process. An approximation of the pdfs

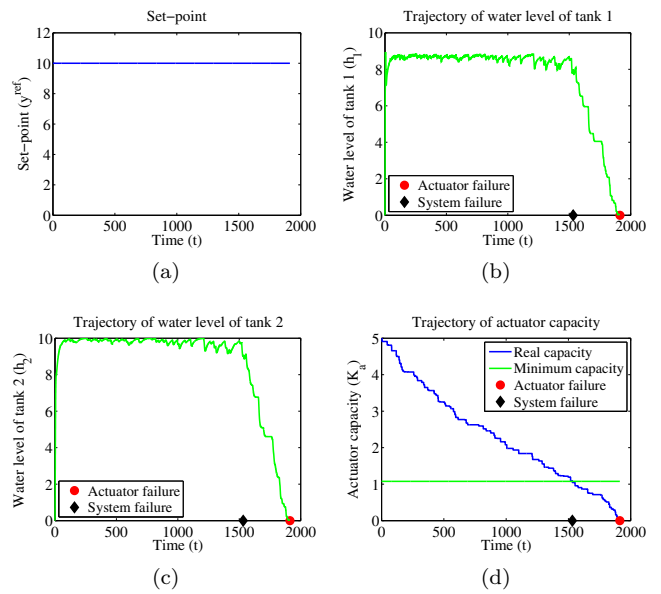


Fig. 4. A trajectory of the water tank level control system until failure of actuator: (a) Set-point, (b) Water level of tank 1, (c) Water level of tank 2 and (d) Actuator capacity

are represented in Figure 5(a) for the water level of tank 1, Figure 5(b) for the water level of tank 2 and Figure 5(c) for the actuator capacity with $N_s = 3000$ particles.

The last step of the method is to compute the distribution of the remaining lifetime of the system starting at T_{prog} knowing the approximated pdf of the system state at that instant. Indeed, the simulation of trajectories of the system until its failure is required. With 3000 simulation trajectories, the obtained pdf of the RUL is shown in Figure 5(d).

The quality of the state estimation hence of the RUL estimation increases with the amount of collected condition information. With the same underlying degradation trajectory of system (as shown in Figure 4(d)) and the parameters of condition monitoring process as described above, several scenarios with different excitation frequency are implemented. The available health information of each scenarios is then different. RUL estimation is done at the same instant $T_{prog} = 1209$.

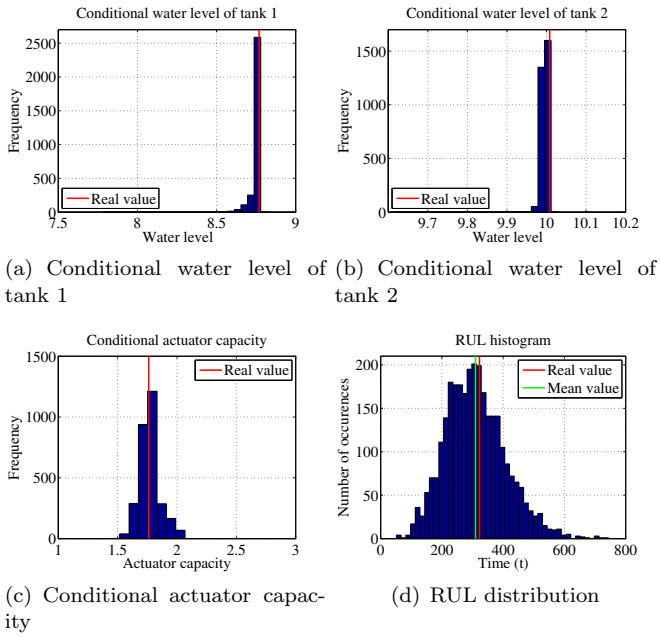


Fig. 5. Conditional distribution of the system state at time $T_{prog} = 1209$ time units according to the noisy observations for $N_s = 3000$ particles and the corresponding distribution of the RUL of the system

Figure 6 illustrates the corresponding RUL predictions. For visualization, the particles populations is then fitted by the kernel density estimation. The probability distributions all cover the true failure time of system, and as frequency progresses, the prediction become significantly more accurate and precise. Note that the black dashed curve presents the density of the RUL with Monte Carlo simulation in the idealistic case when the departure point of each system trajectory is the real value of actuator capacity at instant T_{prog} (it gives a benchmark of the best information one may have on the system).

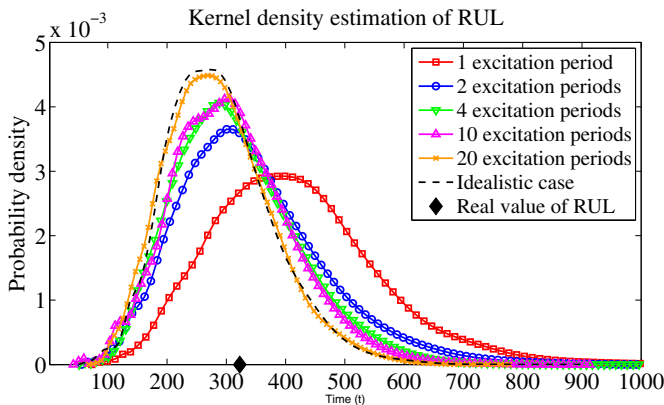


Fig. 6. RUL estimation with different excitation scenarios

5. CONCLUSION

The present paper proposes a modeling framework combining the deterministic behavior of a feedback control system with the stochastic degradation process for the actuator. A perturbation technique is considered on the constant desired output in order to allow the actuator degradation diagnosis and prognosis. Particle filter method

is used to estimate on-line the state of the considered deteriorating system regarding the observation of process response. By using a methodology based on the assumption of Markov property, the Remaining Useful Life can be deduced with Monte Carlo simulation. A simulated double-tank level control system is considered as a case study to illustrate the efficiency of the proposed approach. Future research will focus on the use of the estimation of the system state and the RUL in the design of an adaptive controller.

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REFERENCES

Arulampalam, M.S., Maskell, S., and Gordon, N. (2002). A tutorial on particle filters for online nonlinear/non-gaussian bayesian tracking. *IEEE Transactions on signal processing*, 50, 174–188.

Aström, K.J. and Hägglund, T. (1995). *PID controllers: theory, design and tuning*. Research Triangle Park, 2 edition.

Chen, H.M. and Chen, Z.Y. (2008). Implement of a cascade integral sliding mode controller for a water tank level control system. In *Innovative Computing Information and Control, 2008. ICIC'08. 3rd International Conference on*, 162–162. IEEE.

Daigle, M., Saha, B., and Goebel, K. (2012). A comparison of filter-based approaches for model-based prognostics. In *Aerospace Conference, 2012 IEEE*, 1–10.

Dieulle, L., Bérenguer, C., Grall, A., and Roussinol, M. (2003). Sequential condition-based maintenance scheduling for a deteriorating system. *European Journal of Operational Research*, 150(2), 451 – 461.

Kitagawa, G. (1996). Monte carlo filter and smoother for non-gaussian nonlinear state space models. *Journal of computational and graphical statistics*, 5(1), 1–25.

Langeron, Y., Grall, A., and Barros, A. (2013). Actuator health prognosis for designing lqr control in feedback systems. *Chemical Engineering Transactions*, 33, 979–984.

Lorton, A., Fouladirad, M., and Grall, A. (2013). A methodology for probabilistic model-based prognosis. *European Journal of Operational Research*, 225(3), 443–454.

Pereira, E.B., Galvão, R.K.H., and Yoneyama, T. (2010). Model predictive control using prognosis and health monitoring of actuators. In *Industrial Electronics (ISIE), 2010 IEEE International Symposium on*, 237–243.

Van Noortwijk, J. (2009). A survey of the application of gamma processes in maintenance. *Reliability Engineering & System Safety*, 94(1), 2–21.

Wang, H. (2002). A survey of maintenance policies of deteriorating systems. *European Journal of Operational Research*, 139(3), 469 – 489.

Zhang, Y. and Jiang, J. (2008). Bibliographical review on reconfigurable fault-tolerant control systems. *Annual Reviews in Control*, 32(2), 229–252.