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Direct continuous-time model identification of high-powered light-emitting diodes from rapidly sampled thermal step response data

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Abstract: Transient temperature response measurements of semiconductor devices such as high-powered Light-Emitting-Diodes (LEDs) can be used to detect possible thermal defects. The thermal transient responses of these LEDs appear to be stiff which can be represented by a model with both fast and slow dynamics. It is shown how direct continuous-time model estimation methods, such as the Simplified Refined Instrumental Variable method for Continuous systems (SRIVC), can directly identify with high accuracy a model with both small and large time-constants that can reproduce the thermal effects of the LEDs while conventional discrete-time model identification fails in this stiff response situation.

Keywords: continuous-time model, discrete-time data, instrumental variable, step response data, system identification, stiff systems, light-emitting diodes, time-constant spectrum

1. INTRODUCTION

Systems with both fast and slow dynamics, represented by time-constants that vary over several orders of magnitude are called stiff systems. Identification of such systems requires special care. Indeed, several challenges arise related to the fact that to capture simultaneously both fast and slow dynamics, the system is necessarily sampled very fast and this can result in numerical accuracy issues for the parameter estimation. These factors make in particular the problem very difficult for traditional discrete-time (DT) model identification methods. In contrast to DT model estimation, direct continuous-time (CT) model identification algorithms work well with rapidly sampled data. This is one of the many advantages of identifying CT models directly from sampled data (see e.g. [Garnier and Wang, 2008, Garnier and Young, 2014]). The last decade has witnessed a resurgence of interest in direct CT modelling from sampled data. This is evidenced, for example, by the recent addition of direct CT model identification methods in the latest version of the well known Matlab System Identification (SID) toolbox [Ljung and Singh, 2012].

Direct CT model identification methods can deal very well with stiff systems using data collected at a fast sampling rate. This is the typical situation of thermal transient responses of semiconductor devices, where temperature dynamics might be characterized by a combination of extremely small time constants due to several miniature thermal connection and larger time constants due to large heat sinks. A good example and of importance in modern lighting systems is the thermal response of high-powered Light-Emitting-Diodes (LEDs). These semiconductors are used in applications as diverse as aviation and automotive lighting, advertising, general lighting, and traffic signals. LEDs exhibit a stiff behavior with both fast and slow dynamics due to the presence of distinct physical phenomena as illustrated in [Miller et al., 2013]. The paper highlights how the optimal instrumental variable method for continuous-time model identification (SRIVC) can directly identify with high accuracy a model with small and large time-constants that can reproduce the thermal effects of the LEDs while traditional DT model identification methods fail in this stiff response situation.

The remainder of the paper is organized in the following way. The model of the semiconductor device and the formulation of the parameter estimation problem with the aim of highlighting the difficulties that appear in this rapidly sampled situation are presented in Section 2. An outline of the SRIVC method is recalled in Section 3 where the crucial implementation aspects are detailled . Section 4 presents the identification results of the high-powered LED thermal step response. Finally, Section 5 gives concluding remarks.

2. MODEL OF THE SEMICONDUCTOR DEVICE, DATA SET AND PROBLEM STATEMENT

In this paper, we consider the modelling of a high-powered Cree XLamp XP-E LED based on experimental thermal step response data [Miller et al., 2013].

2.1 Experimental Setup

The junction temperature of the device was measured indirectly as recommended in the Electronic Industries Association specification EIA/JEDEC JESD51-1. In the specification, junction temperature is assumed to be proportional to forward voltage. The scaling between junction temperature and forward voltage, or K-factor, was determined by first driving the forward voltage above the diode cut-in voltage using a measurement current low enough so as not to induce significant self-heating. The K-factor was then be found by adjusting the temperature of the device and measuring the forward voltage. For these experiments, the LED was driven by a Vektrex SpikeSafe current source, voltages were measured with an Agilent 34411A digital multimeter, and temperature was controlled by a prototype Vektrex thermal platform controller.

A 600 mA step was applied to the LED and a total of about 20 seconds of the step response was measured at a very high sampling frequency of 50 kHz. The measured steady-state change in forward voltage was approximately 2.6 V. A plot of the measured voltage after conversion to temperature for $t \in [0, 1.76s]$ can be seen in Figure 1. The step response is plotted in both a linear and logarithmic time scale to reveal the separation between fast and slow thermal phenomena, illustrating the stiff behavior of the system. The full details of the experiment are given in Miller et al. [2013].

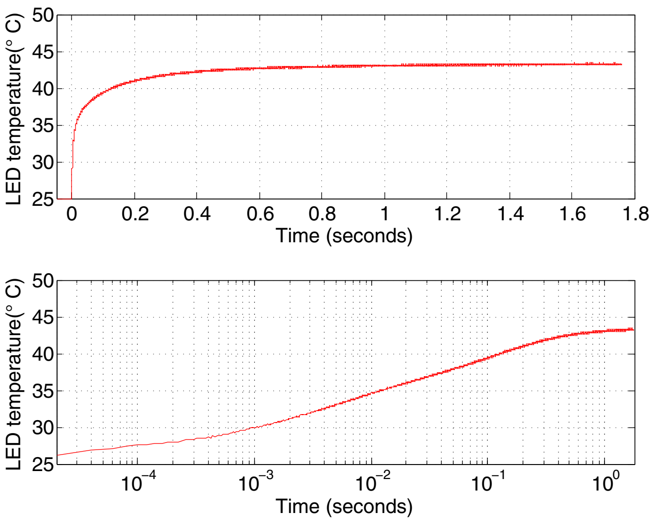

Fig. 1. Step response used for identifying the LED dynamics plotted in both a linear and logarithmic time scale.

2.2 Model of the high-powered LED

From the step response plotted in Figure 1, the LED can be described by a Laplace transfer function of the form

$$G(s) = \sum_{i=1}^{n} \frac{K_i}{(1+\tau_i s)} = \frac{\sum_{i=0}^{n-1} b_{n-i} s^i}{\sum_{i=0}^{n} a_{n-i} s^i}, \quad a_0 = 1 \tag{1}$$

where s is the Laplace variable. It is assumed that the poles are distinct real and that the system is stable

$$\begin{cases} \text{Im}(\tau_i) = 0 \\ \tau_i > 0 \end{cases} \tag{2}$$

2.3 Fast sampled data situation

The developments of data acquisition equipment have open up the possibility to sample very fast. This is required in the case of stiff systems where a very small sampling period is needed in order to capture the fast dynamics resulting in a *fast sampled data situation*. Another consequence of the small sampling period selection is the fact that the number of data available is usually very large, typically several thousand of samples to be processed by the estimation scheme. Unfortunately, in the case of DT models, fast sampled data often gives rise to an ill-conditioned parameter estimation problem. The small sampling period leads to clustering of the poles of the equivalent zero-order or first-order hold DT model around the point $(1, 0)$ in the complex plane, requiring high precision requirements on the coefficients of the DT transfer function to accurately describe both fast and slow dynamics. The high precision requirement of the coefficients is the cause of the numerical issues encountered in DT model identification techniques. And this can deleteriously affect the quality of the estimated model, as we see in the application section.

2.4 Problem statement

The identification problem can be stated as follows: identify first the best model structure (find the order n) from the step response data set; then estimate the parameters (a_i and b_i) that characterize this chosen model structure and deduce the time-constants of the stiff system described by (1) from N fast sampled step measurements of the input and output $Z^N = \{u(t_k); y(t_k)\}_{k=1}^{N}$.

The problem of identifying the CT models from sampled data can be solved in different ways. One possible way is known as the *indirect* approach. It estimates the CT model parameters by first fitting a DT model to the data and then converting this model to a CT model. Another approach, known as the *direct* method, estimate the CT model parameters from the discrete-time data without an intermediate step. The input/output time-derivatives can be estimated using low-pass filtering. A summary of one of the most efficient methods is outlined next.

3. OUTLINE OF THE REFINED IV METHOD FOR CT MODELS

The identification problem can be solved via nonlinear optimization techniques. To avoid the numerical difficulties (such as local minima) related to the use of such techniques, a different strategy based on the instrumental variable (IV) approach is selected. IV methods of parameter estimation have a long history in the statistical and control engineering literature (see e.g. [Söderström and Stoica, 1983, Young, 2011]). Interest in IV methods has been growing in recent years. Some very recent papers include [Wang et al., 2009, Laurain et al., 2010, 2011, Gilson et al., 2011, Han and de Callafon, 2011] and [Young, 2011, Söderström, 2012]. In this section we first recall the main conditions for obtaining optimal (consistent and minimum variance) IV parameter estimates. The SRIVC algorithm which has proven to be one of the most efficient methods for day-to-day use is reviewed in Sub-section 3.3.

3.1 Data-generating system

It is assumed that the input $u(t)$ and the noise-free output $x(t)$ are related by the following transfer function (TF) form

$$x(t) = G_o(p)u(t) = \frac{B_o(p)}{A_o(p)}u(t) \tag{3}$$

with

$$B_o(p) = b_0^o p^{n_b} + b_1^o p^{n_b-1} + \cdots + b_{n_b}^o, \quad (3a)$$
$$A_o(p) = p^{n_a} + a_1^o p^{n_a-1} + \cdots + a_{n_a}^o, \quad n_a \geq n_b \quad (3b)$$

where p is the differential operator, i.e., $p^i x(t) = \frac{d^i x(t)}{dt^i}$; $B_o(p)$ and $A_o(p)$ are assumed to be coprime; and the system is asymptotically stable.

It is further assumed that the deterministic output $x(t)$ is corrupted by an additive, coloured measurement noise $\xi(t)$, so that the complete equation for the data-generating system, denoted by $\mathcal{S}$, can be written in the form,

$$\mathcal{S} : y(t) = G_o(p)u(t) + H_o(p)e_o(t) \quad (4)$$

where $H_o(p)$ is assumed to be stable and stably invertible, while $e_o(t)$ is a zero-mean, stationary CT white noise process.

Of course, in most practical situations, the input and output signals $u(t)$ and $y(t)$ are sampled at a constant sampling interval T_s. The sampled signals are denoted by $u(t_k)$ and $y(t_k)$ and the output observation equation then takes the form,

$$y(t_k) = x(t_k) + v(t_k) \quad k = 1, \cdots, N \quad (5)$$

where $x(t_k)$ is the sampled value of the unobserved, noise-free output $x(t)$ and $v(t_k)$ is a zero-mean stationary colored DT measurement noise.

3.2 Optimal IV estimators

Consider the general class of IV estimators

$$\hat{\boldsymbol{\rho}} = \mathrm{sol}_\rho \frac{1}{N} \sum_{k=1}^{N} \boldsymbol{\zeta}_f(t_k) \left[y_f^{(n_a)}(t_k) - \boldsymbol{\varphi}_f^T(t_k)\boldsymbol{\rho} \right] = 0 \quad (6)$$

where the parameter vector $\boldsymbol{\rho}$ includes the dynamic plant model parameters stacked columnwise as,

$$\boldsymbol{\rho} = [a_1 \; \cdots \; a_{n_a} \; b_0 \; \cdots \; b_{n_b}]^T \in \mathbb{R}^{n_a+n_b+1} \quad (7)$$

and $\boldsymbol{\zeta}_f(t_k) \in \mathbb{R}^{n_a+n_b+1}$ is the filtered version of the instrumental vector $\boldsymbol{\zeta}(t_k)$

$$\boldsymbol{\zeta}_f(t_k) = F(p)\boldsymbol{\zeta}(t_k) \quad (8)$$

while the n_a-th order time-derivative of the output $y^{(n_a)}(t_k)$ and regression vector $\boldsymbol{\varphi}(t_k)$ are also prefiltered by the same filter $F(p)$ with

$$\boldsymbol{\varphi}^T(t_k) = \left[-y^{(n_a-1)}(t_k) \cdots - y(t_k) \; u^{(n_b)}(t_k) \cdots u(t_k) \right] \quad (9)$$

It has been shown that a minimum variance estimator is achieved under the following conditions [Young and Jakeman, 1980] (see also Söderström and Stoica [1983]):

$$\begin{cases} \boldsymbol{\zeta}_f^{\mathrm{opt}}(t_k) = F^{\mathrm{opt}}(p)\mathring{\boldsymbol{\varphi}}(t_k) \\ F^{\mathrm{opt}}(p) = \dfrac{1}{H_o(p)A_o(p)} \end{cases} \quad (10)$$

where $\mathring{\boldsymbol{\varphi}}(t_k)$ is the noise-free version of the regression vector $\boldsymbol{\varphi}(t_k)$ defined as

$$\mathring{\boldsymbol{\varphi}}(t_k) = \left[-x^{(n_a-1)}(t_k) \; \cdots \; - x(t_k) \; u^{(n_b)}(t_k) \; \cdots \; u(t_k) \right]^T \quad (11)$$

It can be noted that both the optimal prefilter and instruments in (10) require the knowledge of the true plant and noise models. This is a usual dilemma encountered in the problem of accuracy optimization. One particularly successful implementation of the optimal IV is known as the *refined Instrumental Variable* method where an adaptive procedure for the estimation of the system model parameters is used. This refined IV method uses an iterative procedure, in which, at each iteration, an auxiliary model is used to generate the instrumental variables and prefilter based on the parameters obtained at the previous iteration.

Refined IV method for Continuous-time systems (RIVC) was first developed in 1980 by Young and Jakeman [1980] and has been used successfully for many years, demonstrating the advantages that this stochastic formulation of the CT estimation problem provides in practical applications (see, *e.g.*, some recent such examples in Garnier et al. [2007, 2008], Garnier and Young [2014]).

3.3 SRIVC Method for COE Models

When the measurement noise in (5) is assumed to be white, the model set to be estimated takes the form of a continuous-time output error (COE) model structure

$$\mathcal{M}_{coe} \begin{cases} x(t) = G(p, \boldsymbol{\rho})u(t) \\ y(t_k) = x(t_k) + e(t_k) \end{cases} \quad (12)$$

where $e(t_k)$ is a zero-mean white DT measurement noise while the model is formulated in continuous-time terms

$$\mathcal{G} : G(p, \boldsymbol{\rho}) = \frac{B(p, \boldsymbol{\rho})}{A(p, \boldsymbol{\rho})} = \frac{b_0 p^{n_b} + b_1 p^{n_b-1} + \cdots + b_{n_b}}{p^{n_a} + a_1 p^{n_a-1} \cdots + a_{n_a}} \quad (13)$$

and $\boldsymbol{\rho}$ is defined in (7). In this COE model setting, the optimal filter defined in (10) reduces to

$$F^{\mathrm{opt}}(p) = \frac{1}{A_o(p)} \quad (14)$$

The SRIVC parameter estimates are obtained, at iteration j, from:

$$\hat{\boldsymbol{\rho}}^j = \left[\sum_{k=1}^{N} \boldsymbol{\zeta}_f(t_k, \hat{\boldsymbol{\rho}}^{j-1}) \boldsymbol{\varphi}_f^T(t_k, \hat{\boldsymbol{\rho}}^{j-1}) \right]^{-1} \left[\sum_{k=1}^{N} \boldsymbol{\zeta}_f(t_k, \hat{\boldsymbol{\rho}}^{j-1}) y_f^{(n_a)}(t_k, \hat{\boldsymbol{\rho}}^{j-1}) \right] \quad (15)$$

with

$$\begin{cases} \boldsymbol{\zeta}_f(t_k, \hat{\boldsymbol{\rho}}^{j-1}) = F(p, \hat{\boldsymbol{\rho}}^{j-1}) \hat{\mathring{\boldsymbol{\varphi}}}(t_k, \hat{\boldsymbol{\rho}}^{j-1}) \\ \boldsymbol{\varphi}_f^T(t_k, \hat{\boldsymbol{\rho}}^{j-1}) = F(p, \hat{\boldsymbol{\rho}}^{j-1}) \boldsymbol{\varphi}^T(t_k) \\ y_f^{(n_a)}(t_k, \hat{\boldsymbol{\rho}}^{j-1}) = F(p, \hat{\boldsymbol{\rho}}^{j-1}) y^{(n_a)}(t_k) \\ F(p, \hat{\boldsymbol{\rho}}^{j-1}) = \dfrac{1}{A(p, \hat{\boldsymbol{\rho}}^{j-1})} \end{cases} \quad (16)$$

and where $\boldsymbol{\varphi}^T(t_k)$ is defined in (9) and $\hat{\mathring{\boldsymbol{\varphi}}}(t_k, \hat{\boldsymbol{\rho}}^{j-1})$ is an estimate of the noise-free regression vector defined in (11) where the unobserved noise-free output is replaced by an estimate obtained from an auxiliary model based on the parameters estimated at the previous iteration

$$\hat{x}(t_k, \hat{\boldsymbol{\rho}}^{j-1}) = G(p, \hat{\boldsymbol{\rho}}^{j-1}) u(t_k) \quad (17)$$

Note that the SRIVC method will deliver optimal parameter estimates when the additive noise is purely white in form. If the noise happens to be coloured, the inherent instrumental variable aspects of the algorithm ensure that the SRIVC parameter estimates are asymptotically unbiased. However, the SRIVC estimates are not, in this situation, statistically efficient (minimum variance) because the prefilters are not designed to account for the colour in the noise process and a more sophisticated refined IV-based method is required [Young et al., 2008].

3.4 Implementation issues

Initialisation of the iterative search. The initial selection of $A(p, \hat{\boldsymbol{\rho}}^0)$ does not have to be particularly accurate provided the prefilter $F(p, \hat{\boldsymbol{\rho}}^0)$ based on it does not seriously attenuate any signals within the passband of the system being modelled. It can be based on various approaches

(1) The selection of the single breakpoint frequency parameter λ of the filter,
$$F(p) = \frac{\lambda^{n_a}}{(p+\lambda)^{n_a}} \tag{18}$$
which is chosen so that it is equal to, or larger than, the bandwidth of the system to be identified.
(2) The incorporation of an algorithm for DT model estimation, such as the DT version of SRIVC using a coarser sampling interval if necessary, from which the CT model polynomial can be inferred.
(3) The specification of an *a priori* polynomial $A(p, \hat{\boldsymbol{\rho}}^0)$ based on prior studies.

Of these, 2) is more automatic but not so robust because of the problems that can arise in estimating a DT model parameter from rapidly sampled data and so is not appropriate here. While 1) is simple and, based on practical experience, is the best method for rapidly sampled data and therefore will be used in the considered application.

Digital implementation of the continuous-time filtering operations. It is worth noticing that the computation of the SRIVC parameter estimates at iteration j given in (15) requires the value of prefiltered signals at the time-instants $t_k, k = 1, \ldots, N$ in both regression and instrument vectors, expressed below under their developed forms
$$\boldsymbol{\varphi}_f^T(t_k, \hat{\boldsymbol{\rho}}^{j-1}) = [-y_f^{(n_a-1)}(t_k, \hat{\boldsymbol{\rho}}^{j-1}) \ \cdots \ -y_f(t_k, \hat{\boldsymbol{\rho}}^{j-1}) \\ u_f^{(n_b)}(t_k, \hat{\boldsymbol{\rho}}^{j-1}) \ \cdots \ u_f(t_k, \hat{\boldsymbol{\rho}}^{j-1})] \tag{19}$$
$$\boldsymbol{\zeta}_f^T(t_k, \hat{\boldsymbol{\rho}}^{j-1}) = [-\hat{x}_f^{(n_a-1)}(t_k, \hat{\boldsymbol{\rho}}^{j-1}) \ \cdots \ -\hat{x}_f(t_k, \hat{\boldsymbol{\rho}}^{j-1}) \\ u_f^{(n_b)}(t_k, \hat{\boldsymbol{\rho}}^{j-1}) \ \cdots \ u_f(t_k, \hat{\boldsymbol{\rho}}^{j-1})] \tag{20}$$
The digital implementation issues of the CT filtering operations are well-known in CT model identification but they should be treated in an appropriate way since errors generated by the digital implementation can have severe influence on the quality of the estimated model. When the filter input inter-sample behaviour is known (e.g. piecewise constant or piecewise linear) or takes a particular form (e.g. steady-state response to a sinusoid or sum of sinusoid), an exact solution to the filtering operation at specified time-instants can be obtained by using an appropriate digital simulation method. The digital implementation method has to be selected carefully according to the filter input intersample behaviour. However, if the latter is not known, the filtered output obtained via digital simulation will then only be an approximation of the closed form solution.

Let $\xi(t)$ be an unknown continuous-time signal whose bandwidth is below the Nyquist frequency, and $F_c(s)$ is a given continuous-time filter. The problem is, given sampled data $\xi(t_k)$ where $t_k = kT_s$, compute
$$\xi_f(t_k) = [F_c * \xi](t_k) \tag{21}$$
where the notation $[F_c * \xi]$ is used to denote the convolution.

One solution is to use Shannon reconstruction to compute $\xi(t)$ from $\xi(t_k)$. The signal $[F_c * \xi](t_k)$ can then be obtained via filtering and sampling. Practically, the Shannon reconstruction can be approximated by some higher order hold circuits. Assume that the CT filter $F_c(s)$ is given under state space form:
$$\begin{cases} \dot{x}(t) = A_c x(t) + B_c u(t) \\ y(t) = C_c x(t) \end{cases} \tag{22}$$
The simplest case is when the filter input is piecewise constant corresponding to a zero-order hold (ZOH). The equivalent ZOH DT state-space model takes the form
$$\begin{cases} x(t_{k+1}) = A_d x(t_k) + B_d u(t_k) \\ y(t_k) = C_d x(t_k) \end{cases} \tag{23}$$
where
$$A_d = e^{A_c T_s}, \quad B_d = A_c^{-1}(A_d - I)B_c, \quad C_d = C_c \tag{24}$$
When the input $u(t)$ is linear during each sampling interval, as it is when driven by a first-order hold, that is
$$u(t) = u(t_k) + (t - t_k)\frac{u(t_{k+1}) - u(t_k)}{t_{k+1} - t_k} \quad \text{for} \quad t_k \le t < t_{k+1} \tag{25}$$
we can also obtain an exact relationship. The simplest way to achieve this link is to reason as follows: if $u(t)$ is piecewise linear, then its time-derivative $\dot{u}(t)$ is piecewise constant. Therefore, it is easy to form a state representation for the integrated filter
$$\frac{1}{s}F_c(s) \tag{26}$$
and run it with $\dot{u}(t)$. The equivalent DT state-space model of the CT filter $F_c(s)$ when the input is piecewise linear is then
$$\begin{cases} x(t_{k+1}) = A_d x(t_k) + B_d \dfrac{u(t_{k+1}) - u(t_k)}{t_{k+1} - t_k} \\ y(t_k) = C_d x(t_k) \end{cases} \tag{27}$$
If the true filter input is neither piecewise constant nor piecewise linear, the expressions are not exactly valid. The discretization techniques described above are implemented in the Matlab *lsim* routine. The latter is utilized to implement the CT filtering operations in the SRIVC algorithm, available in the CONTSID [1] toolbox (Garnier et al. [2008]).

4. APPLICATION TO THE LED STEP RESPONSE

The following CT model structure is assumed
$$\begin{cases} x(t) = \displaystyle\sum_{i=1}^{n} \frac{K_i}{1+\tau_i p} u(t) = \frac{\sum_{i=0}^{n-1} b_{n-i} p^i}{\sum_{i=0}^{n} a_{n-i} p^i}, \quad a_0 = 1 \\ y(t_k) = x(t_k) + e(t_k) \end{cases}$$
Different model orders for n between 4 and 7 were tested. Each model structure was estimated and the coefficient of determination R_T^2 was computed as
$$R_T^2 = 1 - \frac{\sigma_{\hat{e}}^2}{\sigma_y^2} \tag{28}$$
$\sigma_{\hat{e}}^2$ is the variance of the estimated noise $\hat{e}(t_k)$ and σ_y^2 is the variance of the measured output $y(t_k)$. In other words, R_T^2 measures how much the noisy output variance is explained by the estimated model. However, it is well known that this measure, on its own, is not sufficient to avoid over-parametrisation and identify a parsimonious model, so that other model order identification statistics are required. In this regard, the Young Information Coefficient (YIC) (see e.g. [Young, 2011]) is useful because it exploits the covariance matrix estimates provided by

[1] see `www.cran.uhp-nancy.fr/contsid/`

the SRIVC method. A large negative value for YIC provides an indication of the best parsimonious models. The R_T^2 and YIC statistics for n between 4 and 7 are given in Table 1. From this table, it is difficult to make a clear cut choice since the fit is extremely good for the 4 models ($R_T^2 > 0.99$, i.e., 99% of the output variance is explained by the models) with relatively large negative YIC values. However, the selection becomes easier when we compare the model responses plotted in a logarithmic (base 10) scale (see Figure 2). From this figure, it is clear that both 4th and 5th order models do not identify so well the smallest time-constants of the system. Both 6th and 7th order model responses are almost identical and so we select for parsimonious reason the 6th order model in the following.

model order n	YIC	R_T^2
4	–16.00	0.9984
5	–14.13	0.9997
6	–12.53	0.9997
7	–10.35	0.9997

Table 1. SRIVC model order determination.

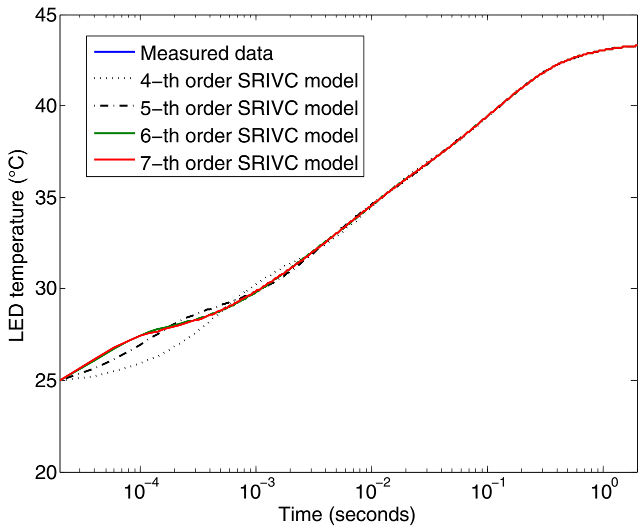

Fig. 2. Simulated SRIVC model step responses together with the measured thermal step response of the LED.

The 6th-order transfer function model takes the form

$$\begin{cases} x(t) = \dfrac{b_1p^5 + b_2p^4 + b_3p^3 + b_4p^2 + b_5p + b_6}{p^6 + a_1p^5 + a_2p^4 + a_3p^3 + a_4p^2 + a_5p + a_6} u(t) \\ y(t_k) = x(t_k) + e(t_k) \end{cases}$$

The measured step response can also be written

$$y(t_k) = \sum_{i=1}^{6} R_i \left(1 - e^{-\frac{t_k}{\tau_i}}\right) + e(t_k) \qquad (29)$$

where τ_i are the time-constants of the response, and R_i the contribution of each-time constant to the total response. The pairs $(\tau_i; R_i)$ form the so-called time-constant spectrum.

The rapidly sampled step response data from 0 to 1.76 seconds (98000 data points) (see Figure 1) was used to compute directly a CT model by using the srivc routine from the CONTSID toolbox. The latter was initiated by choosing the single breakpoint frequency parameter as $\lambda = 30000$ rad/s (see (18)). The sampled step response data was also used to estimate a CT COE model by using the coe routine in the CONTSID toolbox (which implements a Levenberg-Marquardt algorithm via sensitivity functions). The recent tfest routine (see [Ljung and Singh, 2012]) for direct CT model identification available in the latest release (version 8.3 coming out with Matlab 2013b) of the System Identification toolbox was also tested to model the LED. Both coe and tfest routines were initialized in the same way than the srivc routine (the initialization method was specified as ’svf’ with the same the cut-off frequency, see the help of tfestoptions). Finally, the prediction error estimate of a DT output error (DT OE) model was also computed by using the oe routine from the latest version 8.3 of the SID toolbox. In contrast to the IV-based SRIVC method, these algorithms rely on a numerical search procedure with a risk of getting stuck in local minima (see e.g. Ljung [2003]).

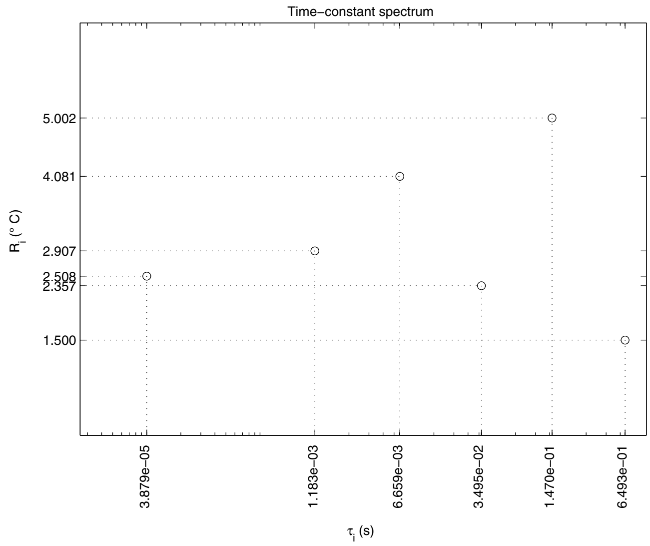

Fig. 3. Time-constant spectrum - SRIVC model.

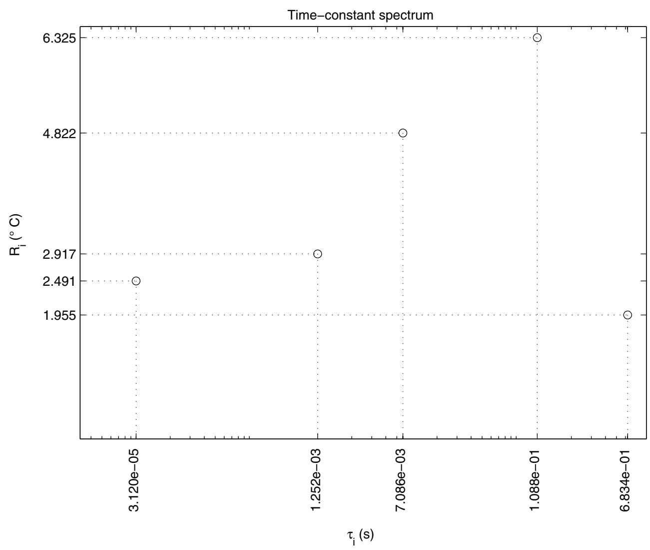

Fig. 4. Time-constant spectrum obtained by Miller *et al.* (2013).

The measured thermal step response of the LED together with the simulated CT and DT model responses in a logarithmic time scale are plotted in Figure 5.

It can be noticed that the SRIVC model output is the only one that can reproduce the thermal effects of the LED with high accuracy while the nonlinear optimization-based methods (for both CT and DT models) all fail to capture the LED dynamics. Indeed, the CT TFEST and COE methods converge clearly to a local minimum, while the DT OE model fails to capture the fast time-constant in this rapid sampling situation.

The time-constant spectrum obtained from the estimated SRIVC model is plotted in Figure 3. For comparison purpose, the spectrum obtained by the realization-based procedure presented in Miller et al. [2013] is displayed in Figure 4. The model in that latter case is a 5-th order model only but it is interesting to note that the range of the time-constants is similar and that both approaches can identify the very small time-constant.

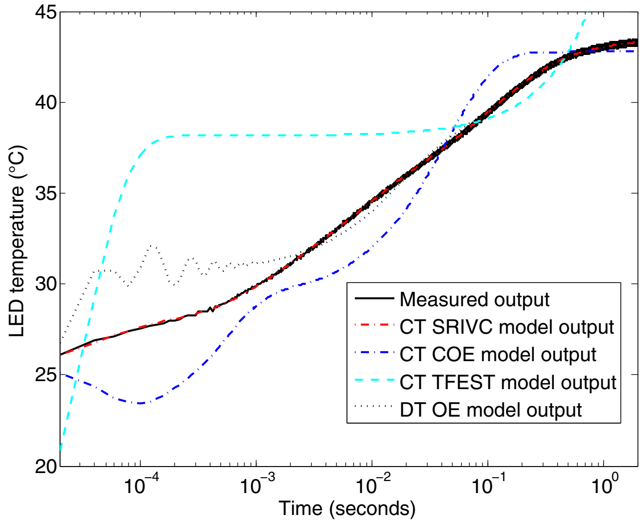

Fig. 5. Simulated SRIVC and OE model step responses together with the measured thermal step response of the LED.

Comments

- The authors acknowledge that other researchers have been able to obtain excellent fits by using other dedicated but rather more sophisticated estimation approaches (see Miller et al. [2013]). We are only able to state that the SRIVC method appears to give directly a very good fit with this rapidly sampled data.
- The poles of the model $G(p, \boldsymbol{\rho})$ in (13) were not restricted to be real-valued as done in [Miller et al., 2013]. However, the SRIVC method does give real-valued poles due to the excellent fitting of the step response data.

5. CONCLUSIONS

The iterative instrumental variable-based SRIVC method for direct continuous-time model identification was applied to the thermal stiff response of a high-powered LED. It is shown that models with six time constants can reproduce the thermal effects of the LED with high accuracy while the usual discrete-time and continuous-time model estimation schemes based on nonlinear optimization fail to deliver good models for this rapidly sampling situation imposed by the stiff behavior of the system. These results highlight the advantage of the iterative IV-based method which constitutes a quick and reliable approach to continuous-time model identification for day-to-day use.

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