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Optimal LED-based Illumination Control via Distributed Convex Optimization

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Abstract: Achieving illumination and energy consumption targets is essential in indoor lighting design. The provision of localized illumination to occupants and the utilization of natural light and light-emitting diode (LED) luminaires can help meet both objectives. This paper presents several distributed optimal illumination control schemes that provide localized illuminance to occupants. The lighting system consists of multiple LED-based luminaires, each governed by an individual local controller. The illuminance requirements and energy costs for the lighting system are expressed as a linear programming problem that is solved in a distributed manner, using only local communication amongst the controllers. State-of-the-art accelerated first-order methods are applied to parallelize the optimization among multiple controllers. Practical aspects such as convergence rate, computational complexity and communication requirements are investigated via simulations.

Keywords: optimal illumination control, light-emitting diodes, smart energy buildings, distributed convex programming

1. INTRODUCTION

Providing sufficient illuminance while reducing energy costs are primary objectives in the design and control of indoor lighting for commercial and office buildings. In particular, the use of occupancy information and the integration of natural and artificial light are effective strategies for serving both requirements (Roisin et al., 2008). Illumination control schemes use occupancy information in several ways. In the simplest case, luminaires are switched on if an occupant is present (Guo et al., 2010). In large rooms, such strategies may not be as energy-efficient as more advanced schemes that provide *localized* illuminance to occupants (Singhvi et al., 2005; Pandharipande and Caicedo, 2010). In such control schemes, luminaires in an occupant's vicinity provide the desired illuminance, and other luminaires are switched off or dimmed. The use of occupancy and illuminance sensors enables localized illuminance control to adequately meet both illuminance and energy requirements. Moreover, these schemes are well-suited for lighting systems that use light-emitting diode (LED) luminaires. Compared to traditional technologies such as incandescent light bulbs and fluorescent lights, LEDs have longer life times, are more energy-efficient, and offer greater design and control flexibility (Tsao et al., 2010).

In indoor environments with multiple luminaires, localized illuminance control schemes require suitable coordination mechanisms to obtain luminaire dimming levels that achieve the desired illuminance and energy levels. In *centralized* coordination, sensor measurements are gathered by, and luminaire dimming levels are issued from a central controller (Pandharipande and Caicedo, 2010). Modular LED-based luminaires with integrated micro-controllers, sensors and communication capabilities can help overcome the need for global connectivity in centralized illumination coordination. This results in a *distributed* control architecture, where luminaires determine their dimming levels locally without the intervention of a central controller. As indoor lighting solutions transition towards modular LED-based luminaires, there is a need for fast and simple distributed coordination mechanisms.

In this paper, the *centralized* coordination mechanism for LED-based office lighting systems developed in (Pandharipande and Caicedo, 2010) is extended to a distributed setting. The lighting system consists of multiple ceiling-mounted LED luminaires, equipped with micro-controllers and communication devices, and with integrated illuminance and occupancy sensors. A target illumination pattern, consisting of illuminance specifications for each region, is imposed via linear constraints on the luminaire dimming levels. Each controller is responsible for providing the desired illuminance levels in a certain part of the workspace plane and has control over the dimming levels of a subset of the LED luminaires. Since adjacent luminaires affect the illuminance in each other's areas, they use local communication and iterative algorithms to coordinate their dimming levels such that local illuminance requirements are met and overall energy consumption is minimized. Minimization of energy consumption is specified as a linear cost on luminaire dimming, thus yielding a linear program that can be solved in a distributed manner.

Several methods for distributed convex optimization are applicable to the above setting. These include subgradient methods (Bertsekas and Tsitsiklis, 1997), constrained consensus (Nedic et al., 2010), accelerated first-order methods (Necoara and Suykens, 2008; Dinh et al., 2013), interior-point methods (Necoara and Suykens, 2009). In addition, there are distributed variants of the simplex method for linear programming problems (Bürger et al., 2011). These algorithms differ greatly in terms of convergence rate, computational complexity, and communication. Recently, distributed illumination coordination schemes for LED-based office lighting systems were developed in (Wang et al., 2012; Caicedo and Pandharipande, 2013). In both works, the optimal illumination control problem is formulated as a linear program, and a feasible solution is obtained with communication restricted to neighboring controllers only. In (Wang et al., 2012), heuristic methods based on the simplex algorithm are proposed. In (Caicedo and Pandharipande, 2013), a constrained consensus approach is used. Sufficient conditions are provided under which this method converges to a feasible solution, and a bound on sub-optimality is obtained. Heuristic approaches for distributed illumination coordination via stochastic hill climbing were studied, e.g., in (Miki et al., 2007).

In this paper, recent results on accelerated first-order methods from (Necoara and Suykens, 2008; Dinh et al., 2013) are leveraged to obtain optimal distributed illumination coordination algorithms. Unlike most of the above alternatives, their application to the considered LED-based lighting system yields simple iterative schemes that are obtained by solving the underlying subproblems analytically.

2. OPTIMAL ILLUMINATION CONTROL

2.1 Preliminaries

Let $\mathbb{R}$, $\mathbb{R}_+$, and $\mathbb{Z}$ denote the field of real numbers, the set of non-negative reals, and the set of integer numbers, respectively. The notation $\mathbb{R}_{[c_1,c_2]}$ and $\mathbb{Z}_{[c_3,c_4]}$ denotes the sets $\{k \in \mathbb{R} : c_1 \le k \le c_2\}$, for $c_1, c_2 \in \mathbb{R}$, and $\{k \in \mathbb{Z} : c_3 \le k \le c_4\}$, for $c_3, c_4 \in \mathbb{Z}$, respectively. The vector space $\mathbb{R}^n$ is endowed with inner product $\langle x, y\rangle := x^\top y$ for $x, y \in \mathbb{R}^n$ and Euclidean norm $\|x\| = \sqrt{\langle x, x\rangle}$. For a matrix $A \in \mathbb{R}^{m\times n}$, the operator norm is given by $\|A\| = \max_{\|x\|=1} \|Ax\|$, and $A^\top$ denotes the transpose. For sets $\mathcal{A}$ and $\mathcal{B}$, $\mathcal{A} \setminus \mathcal{B} := \{a : a \in \mathcal{A}, a \notin \mathcal{B}\}$ denotes the set difference, and $|\mathcal{A}|$ denotes the cardinality of $\mathcal{A}$. For a set $\{M_i\}_{i\in\mathbb{Z}_{[1,N]}}$, $M_i \in \mathbb{R}^{m_i\times n_i}$, the notation $\mathrm{col}\big(\{M_i\}_{i\in\mathbb{Z}_{[1,N]}}\big)$ and $\mathrm{col}(M_1, \cdots, M_N)$, denotes the matrix $(M_1^\top, \cdots, M_N^\top)^\top$. The notation $\mathbf{0}$ and $\mathbf{1}$ is used for vectors, whose dimension will be clear from the context, with all entries equal to zero and one, respectively. A function $f : \mathbb{R}^n \to \mathbb{R}$ is *convex* if for all $x, y \in \mathbb{R}^n$ and $\alpha \in [0, 1]$,

$$f(\alpha x + (1-\alpha)y) \le \alpha f(x) + (1-\alpha)f(y),$$

and *strongly convex*, with convexity parameter $\sigma > 0$, if for all $x, y \in \mathbb{R}^n$ and $\alpha \in [0, 1]$,

$$f(\alpha x+(1-\alpha)y) \le \alpha f(x)+(1-\alpha)f(y)-\frac{1}{2}\sigma\alpha(1-\alpha)\|x - y\|^2.$$

It is concave if $-f$ is convex. A set $\mathcal{X} \subseteq \mathbb{R}^n$ is convex if each point on the line segment connecting any two points in $\mathcal{X}$ is also in $\mathcal{X}$, i.e., for all $x, y \in \mathcal{X}$ and $\alpha \in [0, 1]$, $\alpha x + (1-\alpha)y \in \mathcal{X}$. A *prox-function* $\phi_{\mathcal{X}} : \mathbb{R}^n \to \mathbb{R}_+$ of a closed convex set $\mathcal{X} \subseteq \mathbb{R}^n$ is a function which is continuous on $\mathcal{X}$ and strongly convex with convexity parameter $\sigma_{\mathcal{X}}$. The *prox-center* $x^0 := \arg\min_{x\in\mathcal{X}} \phi_{\mathcal{X}}(x)$ satisfies $\phi_{\mathcal{X}}(x^0) = 0$. A continuously differentiable function f has *L-Lipschitz continuous gradient* with respect to the norm $\|\cdot\|$ if, $\forall x, y \in \mathbb{R}^n$, $\|\nabla f(x) - \nabla f(y)\| \le L\|x - y\|$. The scalar $L \in \mathbb{R}_+$ is called the *Lipschitz constant* of ∇f.

Definition 1: Suppose $f : \mathbb{R}^n \to \mathbb{R}$ is convex and ∇f is L-Lipschitz continuous on a convex set $\mathcal{X} \subseteq \mathbb{R}^n$. The *gradient mapping* of f at $x \in \mathcal{X}$ is given by:

$$\mathcal{G}(x) := \arg\min_{\hat{x}\in\mathcal{X}} \big\{ \langle \nabla f(x), \hat{x}\rangle + \frac{L}{2}\|\hat{x} - x\|^2 \big\}.$$

2.2 LED Lighting System

The considered lighting system consists of LED luminaires located at fixed points of an office ceiling. The luminaire dimming levels are controlled individually by adjusting the duty cycles of the respective pulse-width modulated driving waveforms. The workspace plane, denoted by $\mathcal{W} \subseteq \mathbb{R}^2$, is a horizontal plane parallel to the ceiling and at a fixed perpendicular distance from it. The i-th occupant is located at $p_i \in \mathcal{W}$, for $i \in \mathcal{I}_O := \mathbb{Z}_{[1,n_O]}$, where n_O is the total number of occupants in the room. The i-th occupied region is described by a circle of radius r_O centered at p_i, i.e., $\mathcal{R}_O^i := \{w \in \mathcal{W} : \|w - p_i\| \le r_O\}, i \in \mathcal{I}_O$. The total occupied region is given by the union of the individual occupied regions, i.e., $\mathcal{R}_O := \bigcup_{i\in\mathcal{I}_O} \mathcal{R}_O^i$. The remaining part of the workspace plane constitutes the non-occupied region, i.e., $\mathcal{R}_U := \mathcal{W}\setminus\mathcal{R}_O$. The target illumination pattern consists of illuminance specifications for these regions. The realization of this pattern is facilitated by characterizing illuminance intensities at a finite number of evaluation points in $\mathcal{W}$. Let $\mathcal{P} \subset \mathcal{W}$ denote the set of evaluation points, with n_P elements, and $x \in \mathbb{R}_{[0,1]}^{n_L}$ denote the vector of luminaire dimming levels, where n_L is the number of luminaires. Define index sets $\mathcal{I}_P := \mathbb{Z}_{[1,n_P]}$ and $\mathcal{I}_L := \mathbb{Z}_{[1,n_L]}$. Furthermore, let $\Psi \in \mathbb{R}_+^{n_P}$ denote the vector whose i-th entry is the illuminance due to natural light at the i-th evaluation point. The illuminance $\mathcal{E}_i : \mathbb{R}_{[0,1]}^{n_L} \times \mathbb{R}_+^{n_P} \to \mathbb{R}_+$ at the i-th evaluation point is given by

$$\mathcal{E}_i(x, \Phi) := \langle F_i, x\rangle + \Psi_i, \quad i \in \mathcal{I}_P, \tag{1}$$

where the j-th entry of $F_i \in \mathbb{R}_+^{n_L}$ represents the contribution of the j-th luminaire to the illuminance at the i-th evaluation point, for $(i, j) \in \mathcal{I}_P \times \mathcal{I}_L$. The target illumination pattern consists of the following illuminance specifications: (i) uniform illumination of intensity $L_t \in \mathbb{R}_+$ across each occupied region $\mathcal{R}_O^i$, $i \in \mathcal{I}_O$; (ii) illumination intensity above $L_m \in \mathbb{R}_+$ in $\mathcal{R}_U$. *Illumination uniformity* is measured in terms of the *illuminance contrast* (Boyce, 2003), which describes the deviation of intensity with respect to the target illuminance L_t. Specifically, the illuminance contrast $C_i : \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}_+$ at the i-th evaluation point is defined as follows:

$$C_i(\mathcal{E}_i, L_t) := \frac{|\mathcal{E}_i - L_t|}{L_t}, \quad i \in \mathcal{I}_P. \tag{2}$$

Illuminance uniformity for the i-th occupied region $\mathcal{R}_O^i$, $i \in \mathcal{I}_O$, is achieved by bounding the illuminance contrast at each evaluation point in $\mathcal{R}_O^i$ and requiring the *average* *illuminance* across evaluation points in $\mathcal{R}_O^i$ to equal L_t. Let $\mathcal{V}_i \subset \mathcal{P}$, $i \in \mathcal{I}_O$, denote the set of evaluation points, with n_{V_i} elements, for the i-th occupied region $\mathcal{R}_O^i$, and $\mathcal{U} \subset \mathcal{P}$ denote the set of evaluation points, with n_U elements, for the non-occupied region $\mathcal{R}_U$. Define index sets $\mathcal{I}_{V_i} := \mathbb{Z}_{[1,n_{V_i}]}$, $i \in \mathcal{I}_O$, $\mathcal{I}_U := \mathbb{Z}_{[1,n_U]}$, and $\mathcal{I}_V := \mathbb{Z}_{[1,n_V]}$, where n_V is the number of elements in the set $\mathcal{V} := \bigcup_{i\in\mathcal{I}_O} \mathcal{V}_i$. The target illumination pattern can then be described as follows:

$$\frac{1}{n_{V_i}} \sum_{j\in\mathcal{I}_{V_i}} \mathcal{E}_j(x,\Psi) = L_t, \quad i \in \mathcal{I}_O, \tag{3a}$$

$$\underline{L} \le \mathcal{E}_i(x,\Psi) \le \overline{L}, \quad i \in \mathcal{V}, \tag{3b}$$

$$\mathcal{E}_i(x,\Psi) \ge L_m, \quad i \in \mathcal{U}, \tag{3c}$$

where $\underline{L} := L_t(1 - C_{th})$, $\overline{L} := L_t(1 + C_{th})$ and $C_{th} \in \mathbb{R}_+$ is the upper bound on the illuminance contrast.

2.3 Problem Formulation

The objective of the optimal illumination control scheme is to realize the target illumination pattern in (3) with minimum power consumption of the LED luminaires. The *average power consumption* of the LED luminaires over an integer number of waveform cycles is given by $P(x) := \langle P_{\text{on}}, x\rangle + \langle P_{\text{off}}, \mathbf{1} - x\rangle$, where the i-th entries of $P_{\text{on}} \in \mathbb{R}_+^{n_L}$ and $P_{\text{off}} \in \mathbb{R}_+^{n_L}$ denote the power consumption of the i-th luminaire, $i \in \mathcal{I}_L$, in the *on* and *off* states, respectively. Under the assumption that $P_{\text{off}} = \mathbf{0}$ and $P_{\text{on}} := p_{\text{on}}\mathbf{1}$, the power consumption of the lighting system is given by

$$P(x) = p_{on} \langle \mathbf{1}, x\rangle . \tag{4}$$

In this case, minimizing P is equivalent to minimizing the sum of luminaire dimming levels. Note that the target illumination pattern in (3) is described by linear constraints in the variable $x \in \mathbb{R}_{[0,1]}^{n_L}$. Defining appropriate matrices $A \in \mathbb{R}^{q\times n_L}$ and $B \in \mathbb{R}^{n_O \times n_L}$ and vectors $a \in \mathbb{R}^q$ and $b \in \mathbb{R}^{n_O}$, where $q := 2n_V + n_U$, the optimal dimming vector can be described as the solution of the following linear programming problem:

$$x^* := \arg\min_x \quad \psi_0(x) := \langle \mathbf{1}, x\rangle , \tag{5a}$$

$$\text{s.t.:} \quad Ax \le \mathbf{a}, \tag{5b}$$

$$Bx = \mathbf{b}, \tag{5c}$$

$$x \in \mathcal{X} := \mathbb{R}_{[0,1]}^{n_L}. \tag{5d}$$

Next, the decision variables and constraints are divided amongst N controllers such that there is no overlap between the variables and constraints of any two controllers. The following discussion makes this assignment more precise. Divide the workspace plane $\mathcal{W}$ into N non-overlapping regions $\mathcal{W}_i$, $i \in \mathcal{I}_C := \mathbb{Z}_{[1,N]}$, such that $\mathcal{W} = \bigcup_{i\in\mathcal{I}_C} \mathcal{W}_i$. The inequality constraints corresponding to points $w \in \mathcal{W}_i$, $i \in \mathcal{I}_C$, are assigned to the i-th controller. The location of the j-th occupant is used to determine the assignment of the corresponding equality constraint. More precisely, the j-th equality constraint is assigned to the i-th controller, where $p_j \in \mathcal{W}_i$, $(i,j) \in \mathcal{I}_C \times \mathcal{I}_O$. Re-arranging the rows and columns of the matrices A and B, and the entries of the vectors a and b, the linear constraints in (5b)-(5c) can be expressed as follows:

$$\sum_j A_{ij} x_j \le a_i, \quad i \in \mathcal{I}_C, \tag{6a}$$

$$\sum_j B_{ij} x_j = b_i, \quad i \in \mathcal{I}_C, \tag{6b}$$

where $A_{ij} \in \mathbb{R}^{q_i \times n_j}$ (respectively, $B_{ij} \in \mathbb{R}^{l_i\times n_j}$) denote the contribution of the luminaires assigned to the j-th controller to the inequality (equality) constraints assigned to the i-th controller.

The optimal illumination control problem (5) differs from the ones formulated in (Pandharipande and Caicedo, 2010; Caicedo and Pandharipande, 2013) in two main aspects. Firstly, in (5), an equality constraint is defined for evaluation points in each of the sets $\mathcal{U}^i$, for $i \in \mathcal{I}_\mathcal{O}$. These constraints are, in general, coupled across a subset of the decision variables. In (Pandharipande and Caicedo, 2010), a single equality constraint is defined for all evaluation points in the set $\mathcal{U} = \cup_{i\in\mathcal{I}_\mathcal{O}}\mathcal{U}^i$. This constraint can, in some situations, be coupled across the dimming vectors of all controllers. Secondly, (5) defines illuminance requirements for a grid of points in the workspace plane. A finer resolution leads to better illumination rendering but increases the number of illuminance constraints. In (Caicedo and Pandharipande, 2013), illuminance requirements are instead specified at luminaire-integrated illuminance sensors (one sensor per luminaire) which measure average illuminance values in their respective fields of view. After careful calibration of the illuminance sensors, this approach is used to formulate an optimal illumination control problem where the number of illuminance constraints is equal to the number of illuminance sensors.

3. DISTRIBUTED OPTIMAL ILLUMINATION CONTROL

This section presents several algorithms for solving (5) in a *distributed* manner over a network of inter-connected controllers. The algorithms are obtained by applying the distributed optimization methods proposed in (Necoara and Suykens, 2008; Dinh et al., 2013) for separable convex optimization. Sections 3.1 and 3.2 present the algorithms, and Section 3.3 describes their distributed implementation with local communication between controllers. In what follows, (5) is referred to as the *primal problem*, and x as the *primal variable*. Furthermore, it is assumed that (5) is feasible and its optimal value is given by $f^* := \psi_0(x^*)$. The constraints (5b)-(5c) are relaxed by introducing the *dual variable* $y := \text{col}(\lambda, \mu) \in \mathbb{R}^m$, where λ and μ are Lagrange multipliers associated with the inequality and equality constraints in (5b)-(5c), respectively. The Lagrange function $\mathcal{L}_0 : \mathbb{R}^{n_L} \times \mathbb{R}^m \to \mathbb{R}$ is defined as $\mathcal{L}_0(x, y) := \psi_0(x) + \langle y, Cx - c\rangle$, where $C := \text{col}(A, B)$ and $c := \text{col}(a, b)$. The corresponding dual function is defined as:

$$d_0(y) := \min_{x\in\mathcal{X}} \mathcal{L}_0(x, y). \tag{7}$$

The *dual problem* associated with (5),

$$d^* := \max_{y\in\mathcal{Y}} d_0(y), \tag{8}$$

where $\mathcal{Y} := \{y = \text{col}(\lambda, \mu) \in \mathbb{R}^m : \lambda \ge \mathbf{0}\}$, satisfies $d^* = f^*$ (Bertsekas, 1999, Prop. 5.2.2). The following functions play an important role in the methods used in this section:

$$d_{\beta_1}(y) := \min_{x\in\mathcal{X}}\{\mathcal{L}_0(x,y) + \beta_1\phi_{\mathcal{X}}(x)\}, \tag{9a}$$
$$f_{\beta_2}(x) := \max_{y\in\mathcal{Y}}\{\mathcal{L}_0(x,y) - \beta_2\phi_{\mathcal{Y}}(y)\}, \tag{9b}$$

where $\beta_1 > 0$, $\beta_2 > 0$, and $\phi_{\mathcal{X}}$ and $\phi_{\mathcal{Y}}$ are prox-functions, with convexity parameters $\sigma_{\mathcal{X}}$ and $\sigma_{\mathcal{Y}}$, for the sets $\mathcal{X}$ and $\mathcal{Y}$, respectively. Let $x(y;\beta_1)$ and $y(x;\beta_2)$ denote the optimal solutions for the problems in (9), i.e.,

$$x(y;\beta_1) := \arg\min_{x\in\mathcal{X}}\{\mathcal{L}_0(x,y) + \beta_1\phi_{\mathcal{X}}(x)\}, \tag{10a}$$
$$y(x;\beta_2) := \arg\max_{y\in\mathcal{Y}}\{\mathcal{L}_0(x,y) - \beta_2\phi_{\mathcal{Y}}(y)\}. \tag{10b}$$

In accordance with (Nesterov, 2005, Thm. 1), $\nabla d_{\beta_1}(y) = Cx(y;\beta_1) - c$ is L_{β_1}-Lipschitz continuous with Lipschitz constant $L_{\beta_1} = \frac{\|C\|^2}{\beta_1}$. The *gradient mapping* of d_{β_1} is defined as:

$$\mathcal{G}(y;\beta_1) := \arg\max_{\hat{y}\in\mathcal{Y}}\left\{\langle\nabla d_{\beta_1}(y),\hat{y}\rangle - \frac{L_{\beta_1}}{2}\|\hat{y}-y\|^2\right\}. \tag{11}$$

Likewise, $\nabla f_{\beta_2}(x) = \mathbf{1} + C^\top y(x;\beta_2)$ is L_{β_2}-Lipschitz continuous with Lipschitz constant $L_{\beta_2} = \frac{\|C\|^2}{\beta_2}$. The gradient mapping of f_{β_2} is defined as:

$$\mathcal{H}(x;\beta_2) := \arg\min_{\hat{x}\in\mathcal{X}}\left\{\langle\nabla f_{\beta_2}(x),\hat{x}\rangle + \frac{L_{\beta_2}}{2}\|\hat{x}-x\|^2\right\}. \tag{12}$$

The following prox-functions are used:

$$\phi_{\mathcal{X}} := \frac{1}{2}\|x-x^0\|^2, \quad \phi_{\mathcal{Y}}(y) := \frac{1}{2}\|y-y^0\|^2, \tag{13}$$

where $x^0 := \frac{1}{2}\mathbf{1}$, $y^0 := \mathbf{0}$, $\sigma_{\mathcal{X}} = \sigma_{\mathcal{Y}} = 1$, and

$$\Phi_{\mathcal{X}} := \max_{x\in\mathcal{X}}\phi_{\mathcal{X}}(x) = \frac{n_L}{8}. \tag{14}$$

For the primal-dual pair $(x,y)\in\mathcal{X}\times\mathcal{Y}$, the *primal gap* $\psi_0(x) - f^*$ is bounded as follows (Necoara and Suykens, 2008, Lemma 3.3):

$$-\|y^*\|\|[Cx-c]^+\| \le \psi_0(x) - f^* \le \psi_0(x) - d_0(y),$$

where y^* denotes an optimal solution of the dual problem (8), and the vector $[Cx-c]^+ := \mathrm{col}(\max\{\mathbf{0}, Ax-a\}, Bx-b)$ is the *residual*, with the maximum being taken componentwise. The terms $\psi_0(x) - d_0(x)$ and $\|[Cx-c]^+\|$ are referred to as the *primal-dual gap* and the *feasibility gap*, respectively. The parameters $\beta_1 > 0$ and $\beta_2 > 0$ bound the primal-dual and feasibility gaps for the primal-dual pair $(x,y)\in\mathcal{X}\times\mathcal{Y}$. In particular, suppose the following condition holds for some $(x,y)\in\mathcal{X}\times\mathcal{Y}$ and $\beta_1,\beta_2 > 0$:

$$f_{\beta_2}(x) \le d_{\beta_1}(y). \tag{15}$$

Then, from (Dinh et al., 2013, Lemma 3), it follows that:

$$\psi_0(x) - d_0(y) \le \beta_1\Phi_{\mathcal{X}}, \tag{16a}$$
$$\|[Cx-c]^+\| \le \beta_2\left(\|y^*\| + \sqrt{\|y^*\|^2 + \frac{2\beta_1}{\beta_2}\Phi_{\mathcal{X}}}\right). \tag{16b}$$

The condition (15) is referred to as the *excessive gap* condition.

3.1 Proximal Center Algorithm

The function d_{β_1} forms a smooth approximation of the dual function d_0 (Necoara and Suykens, 2008, Thm. 3.1). In particular,

$$d_{\beta_1}(y) - \beta_1\Phi_{\mathcal{X}} \le d_0(y) \le d_{\beta_1}(y), \quad y\in\mathcal{Y}.$$

The proximal center algorithm (Necoara and Suykens, 2008) solves the following optimization problem:

$$\max_{y\in\mathcal{Y}} d_{\beta_1}(y), \tag{17}$$

to a desired accuracy $\epsilon > 0$ where $\beta_1 = \frac{\epsilon}{\Phi_{\mathcal{X}}}$.

Algorithm 1: Choose $\epsilon > 0$ and set $\beta_1 := \frac{\epsilon}{\Phi_{\mathcal{X}}}$. For $k\in\mathbb{Z}_+$, perform the following steps:

(1) Compute $\hat{x}^k$ from (10) as $\hat{x}^k := x(y^k;\beta_1)$.
(2) Compute $\hat{y}^k$ as follows:

$$\hat{y}^k := \arg\max_{y\in\mathcal{Y}}\left\{\sum_{l=0}^{k}\frac{l+1}{2}\langle\nabla d_{\beta_1}(y^l), y\rangle - \frac{L_{\beta_1}}{\sigma_{\mathcal{Y}}}\phi_{\mathcal{Y}}(y)\right\}.$$

(3) Compute $\bar{x}^k$ as follows:

$$\bar{x}^k := \frac{2}{(k+1)(k+2)}\sum_{l=0}^{k}(l+1)\hat{x}^l.$$

(4) Compute $\bar{y}^k$ from (11) as $\bar{y}^k := \mathcal{G}(y^k;\beta_1)$.
(5) Update $y^{k+1} := \frac{2}{k+3}\hat{y}^k + \frac{k+1}{k+3}\bar{y}^k$. ■

For the primal-dual pair $(\bar{x}^k,\bar{y}^k)\in\mathcal{X}\times\mathcal{Y}$ generated after k iterations of Algorithm 1, the primal-dual and feasibility gaps are bounded as in (16) with $\beta_1 = \frac{\epsilon}{\Phi_{\mathcal{X}}}$ and $\beta_2 = \frac{4\|C\|^2}{\beta_1(k+1)^2}$ (Necoara and Suykens, 2008, Thm. 3.7).

3.2 Excessive Gap Algorithms

Excessive gap algorithms (Dinh et al., 2013) generate primal-dual pairs $(\bar{x}^k,\bar{y}^k)\in\mathcal{X}\times\mathcal{Y}$, for $k\in\mathbb{Z}_+$, which satisfy the excessive gap condition (15). This section presents three variants.

Algorithm 2: (*Primal Excessive Gap Algorithm*) Set $\tau_0 := 0.5$ and $\beta_1^0 = \beta_2^0 := \|C\|$. Compute $\bar{x}^0$ and $\bar{y}^0$ from (10) and (12) as follows:

$$\bar{y}^0 := y(x^0;\beta_2^0), \quad \bar{x}^0 := \mathcal{H}(x^0;\beta_2^0).$$

For $k\in\mathbb{Z}_+$, perform the following steps:

(1) Update $\beta_1^{k+1} := (1-\tau_k)\beta_1^k$, $\beta_2^{k+1} := (1-\tau_k)\beta_2^k$, and $\tau_{k+1} := \frac{1}{k+3}$.
(2) Compute $\tilde{x}$ from (10) as $\tilde{x} := x(\bar{y}^k;\beta_1^k)$ and update $\hat{x} := \tau_k\tilde{x} + (1-\tau_k)\bar{x}^k$.
(3) Compute $\tilde{y}$ from (10) as $\tilde{y} := y(\hat{x};\beta_2^{k+1})$ and update $\bar{y}^{k+1} := \tau_k\tilde{y} + (1-\tau_k)\bar{y}^k$.
(4) Compute $\bar{x}^{k+1}$ from (12) as $\bar{x}^{k+1} := \mathcal{H}(\hat{x};\beta_2^{k+1})$. ■

For the primal-dual pair $(\bar{x}^k,\bar{y}^k)\in\mathcal{X}\times\mathcal{Y}$ generated after k iterations of Algorithm 2, the primal-dual and feasibility gaps are bounded as in (16) with $\beta_1 = \beta_2 = \frac{2\|C\|}{k+2}$ (Dinh et al., 2013, Thm. 2).

Algorithm 3: (*Primal-Dual Excessive Gap Algorithm*) Set $\tau_0 := 0.5(\sqrt{5}-1)$ and $\beta_1^0 = \beta_2^0 := \|C\|$. Compute $\bar{x}^0$ and $\bar{y}^0$ from (10) and (11) as follows:

$$\bar{x}^0 := x(y^0;\beta_1^0), \quad \bar{y}^0 := \mathcal{G}(y^0;\beta_1^0).$$

For $k\in\mathbb{Z}_+$, perform the following steps:

(1) If k is even
  (a) Compute $\tilde{x}$ from (10) as $\tilde{x} := x(\bar{y}^k;\beta_1^k)$ and update $\hat{x} := \tau_k\tilde{x} + (1-\tau_k)\bar{x}^k$.

(b) Compute $\tilde{y}$ from (10) as $\tilde{y} := y(\hat{x}; \beta_2^k)$ and update $\bar{y}^{k+1} := \tau_k \tilde{y} + (1-\tau_k)\bar{y}^k$.
(c) Compute $\bar{x}^{k+1}$ from (12) as $\bar{x}^{k+1} := \mathcal{H}(\hat{x}; \beta_2^k)$.
(d) Update β_1^k as $\beta_1^{k+1} := (1-\tau_k)\beta_1^k$.

(2) If k is odd
(a) Compute $\tilde{y}$ from (10) as $\tilde{y} := y(\bar{x}^k; \beta_2^k)$ and update $\hat{y} := \tau_k \tilde{y} + (1-\tau_k)\bar{y}^k$.
(b) Compute $\tilde{x}$ from (10) as $\tilde{x} := x(\hat{y}; \beta_1^k)$ and update $\bar{x}^{k+1} := \tau_k \tilde{x} + (1-\tau_k)\bar{x}^k$.
(c) Compute $\bar{y}^{k+1}$ from (11) as $\bar{y}^{k+1} := \mathcal{G}(\hat{y}; \beta_1^k)$.
(d) Update β_2^k as $\beta_2^{k+1} := (1-\tau_k)\beta_2^k$.

(3) Update τ_k as $\tau_{k+1} := \frac{\tau_k}{2}\left[\sqrt{\tau_k^2+4} - \tau_k\right]$. ■

For the primal-dual pair $(\bar{x}^k, \bar{y}^k) \in \mathcal{X} \times \mathcal{Y}$ generated after k iterations of Algorithm 3, the primal-dual and feasibility gaps are bounded as in (16) with $\beta_1 = \frac{2\|C\|}{k+1}$ and $\beta_2 = \frac{(\sqrt{5}+1)\|C\|}{k+1}$ (Dinh et al., 2013, Thm. 3).

Algorithm 4: (*Dual Excessive Gap Algorithm*) Choose $\epsilon > 0$ and set $\beta_1 := \frac{\epsilon}{\Phi_{\mathcal{X}}}$, $\beta_2^0 := \frac{\|C\|^2}{\beta_1}$, and $\tau_0 := 0.99$. Compute $\bar{x}^0$ and $\bar{y}^0$ from (10) and (11) as follows:

$$\bar{x}^0 := x(y^0; \beta_1), \quad \bar{y}^0 := \mathcal{G}(y^0; \beta_1).$$

For $k \in \mathbb{Z}_+$, perform the following steps:

(1) Compute $\tilde{y}$ from (10) as $\tilde{y} := y(\bar{x}^k; \beta_2^k)$ and update $\hat{y} := \tau_k \tilde{y} + (1-\tau_k)\bar{y}^k$.
(2) Compute $\tilde{x}$ from (10) as $\tilde{x} := x(\hat{y}; \beta_1)$ and update $\bar{x}^{k+1} := \tau_k \tilde{x} + (1-\tau_k)\bar{x}^k$.
(3) Compute $\bar{y}^{k+1}$ from (11) as $\bar{y}^{k+1} := \mathcal{G}(\hat{y}; \beta_1)$.
(4) Update β_2^k as $\beta_2^{k+1} := (1-\tau_k)\beta_2^k$.
(5) Update τ_k as $\tau_{k+1} := \frac{\tau_k}{2}\left[\sqrt{\tau_k^2+4} - \tau_k\right]$. ■

For the primal-dual pair $(\bar{x}^k, \bar{y}^k) \in \mathcal{X} \times \mathcal{Y}$ generated after k iterations of Algorithm 4, the primal-dual and feasibility gaps are bounded as in (16) with $\beta_1 = \frac{\epsilon}{\Phi_{\mathcal{X}}}$ and $\beta_2 = \frac{2\|C\|^2}{25(\tau_0 k+2)^2}\|y^*_{\beta_1}\|$, where $\|y^*_{\beta_1}\| \in \mathcal{Y}$ is an optimal solution of (17) (Dinh et al., 2011, Thm. 4).

3.3 Distributed implementation

This section describes the distributed implementation of Algorithms 1-4 for the networked LED-based lighting system described in Section 2. This implementation exploits the structure of the constraint matrix $C := \mathrm{col}(A, B)$ in (5b)-(5c). In accordance with the structure outlined in (6), the dual variable y is partitioned as $y = \mathrm{col}(y_1, \cdots, y_N)$, where $y_i \in \mathbb{R}^{m_i}$ is associated with the constraints assigned to the i-th controller, $i \in \mathcal{I}_C$. It is assumed that the i-th controller stores the matrices $C_{ii} \in \mathbb{R}^{m_i \times n_i}$ and $C_{ji} \in \mathbb{R}^{m_j \times n_i}$, $j \in \mathcal{N}_i := \{j \in \mathcal{I}_C : j \neq i, C_{ji} \neq 0\}$. In each iteration, the considered algorithms need to solve different combinations of the subproblems described in (10), (11) and (12). If the prox-functions $\phi_{\mathcal{X}}$ and $\phi_{\mathcal{Y}}$ are chosen as in (13), these sub-problems can be parallelized and solved analytically. Note that the subproblem in Step 2 of Algorithm 1 is similar to that in (10b), and thus, will not be considered separately.

First, consider subproblem (10a). Given $y \in \mathcal{Y}$, the parameter $\beta_1 > 0$ and the prox-function $\phi_{\mathcal{X}}$ as in (13), the solution of this optimization problem is:

$$x(y; \beta_1) = \min\left\{\max\left\{\mathbf{0}, x^0 - \frac{1}{\beta_1}(\mathbf{1} + C^\top y)\right\}, \mathbf{1}\right\}, \quad (18)$$

where the minimum and maximum are taken componentwise. Let the optimizer in (18) be denoted by $\bar{x}$ and partitioned according to $\bar{x} = \mathrm{col}(\bar{x}_1, \cdots, \bar{x}_N)$. Then, the vectors $\bar{x}_i$, $i \in \mathcal{I}_N$, can be computed in parallel by exploiting the structure of the matrix C, while relying on inter-controller communication. To determine $\bar{x}_i$, the i-th controller needs to form the following vector:

$$\sum_{j \in i \cup \mathcal{N}_i} y_j^\top C_{ji}.$$

Hence, the i-th controller needs to obtain the dual variables y_j from the j-th controller, $j \in \mathcal{N}_i$. Likewise, the i-th controller needs to transmit its dual variables y_i to the j-th controller, $j \in \mathcal{M}_i := \{j \in \mathcal{I}_C : j \neq i, C_{ij} \neq 0\}$. Thus, the i-th controller needs to perform the following steps:

(1) Transmit $y_i \in \mathbb{R}^{m_i}$ to $j \in \mathcal{M}_i$.
(2) Receive $y_j \in \mathbb{R}^{m_j}$ from $j \in \mathcal{N}_i$.
(3) Compute $s_i := \sum_{j \in i \cup \mathcal{N}_i} y_j^\top C_{ji}$.

Next, consider the subproblem (10b). Given $x \in \mathcal{X}$, the parameter $\beta_2 > 0$ and the prox-function $\phi_{\mathcal{Y}}$ as in (13), the solution of this optimization problem is:

$$y(x; \beta_2) = \frac{1}{\beta_2}[Cx - c]^+. \quad (19)$$

Let the optimizer in (19) be denoted by $\bar{y}$ and partitioned according to $\bar{y} = \mathrm{col}(\bar{y}_1, \cdots, \bar{y}_N)$. To determine $\bar{y}_i$, the i-th controller needs to form the following vector:

$$\sum_{j \in \mathcal{M}_i} C_{ij} x_j.$$

Hence, the i-th controller needs to obtain the vector $C_{ij}x_j$ from the j-th controller, $j \in \mathcal{M}_i$. Likewise, the i-th controller needs to transmit the vector $C_{ji}x_i$ to the j-th controller, $j \in \mathcal{N}_i$. Thus, the i-th controller needs to perform the following steps:

(1) Compute $r_{ji} := C_{ji}x_i$ and transmit $r_{ji} \in \mathbb{R}^{m_j}$ to $j \in \mathcal{N}_i$.
(2) Receive $r_{ij} \in \mathbb{R}^{m_i}$ from $j \in \mathcal{M}_i$.
(3) Compute $r_i := C_{ii}x_i + \sum_{j \in \mathcal{M}_i} r_{ij}$.

Lastly, consider the subproblems (11) and (12). Given $(x, y) \in \mathcal{X} \times \mathcal{Y}$ and the parameters $\beta_1, \beta_2 > 0$, the solutions of these subproblems are:

$$\mathcal{G}(y; \beta_1) = \left[y + \frac{1}{L_{\beta_2}}(Cx(y; \beta_1) - c)\right]^+,$$

$$\mathcal{H}(x; \beta_2) = \min\left\{\max\left\{\mathbf{0}, x - \frac{1}{L_{\beta_1}}(\mathbf{1} + C^\top y(x; \beta_2))\right\}, \mathbf{1}\right\}.$$

The parallelization of these sub-problems and the corresponding controller steps are similar to those for the subproblems (10). Tables 1 and 2 summarize the coordination requirements and number of communication rounds, per iteration, for Algorithms 1-4.

4. SIMULATION RESULTS

The distributed optimal illumination control algorithms are tested on the office, shown in Fig. 1, which is designed

Table 1. Coordination requirements, per iteration, for the i-th controller

Compute	Data in	Data out
$x_i(y;\beta_1)$	$\sum_{j\in\mathcal{N}_i} m_j$	$m_i\lvert\mathcal{M}_i\rvert$
$y_i(x;\beta_2)$	$m_i\lvert\mathcal{M}_i\rvert$	$\sum_{j\in\mathcal{N}_i} m_j$

Table 2. Communication rounds per iteration

Compute	Alg. 1	Alg. 2	Alg. 3	Alg. 4
$x(y;\beta_1)$	1	1	1	1
$y(x;\beta_2)$	0	1	1	1
$\mathcal{G}(y;\beta_1)$	2	0	0 or 1	1
$\mathcal{H}(x;\beta_2)$	0	1	1 or 0	0

Fig. 1. Office modeled in DIALux

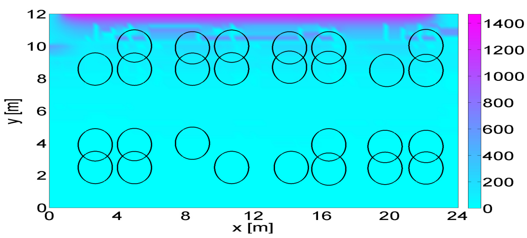

Fig. 2. Natural light distribution and occupancy pattern

in the simulation environment DIALux (v. 4.9.0.2), a simulation tool that is commonly used by lighting professionals and architects. [1] The room is 24 m long, 12 m wide, 3 m high, and contains 32 workstations. The workspace plane is located 0.75 m above the floor. A window (length 22 m, height 1.2 m, and located 0.75 m above the floor) is placed on one side of the office. LED-based luminaires of type BPS560 [2] are placed on the ceiling and arranged in a grid with 12 rows, 24 columns and separation 1 m. The workspace plane is discretized and a grid of evaluation points is obtained. The grid contains 47 points along the width of the room and 95 points along its length. The 4465 evaluation points are placed 0.25 m apart.

The distribution of natural light and the occupancy pattern, shown in Fig. 2, are assumed to be constant. Illuminance intensities are relatively high close to the window, which is located along the upper edge of the figure. Circles of radii 1 m, centered at the occupant locations, represent individual occupied regions. Each occupied region contains approximately 50 evaluation points, and up to 77 LED luminaires contribute to the average illuminance in an occupied region. The target illumination pattern parameters are $L_t = 600$ lux, $L_m = 300$ lux and $C_{th} = 0.3$. Sparsity is induced in the optimal illumination control problem (5) by ignoring luminaire illuminance contributions below a threshold of 4 lux. The so-obtained optimization problem

[1] For details, see www.dial.de/DIAL/en/dialux/.
[2] For details, see www.ecat.lighting.philips.com/.

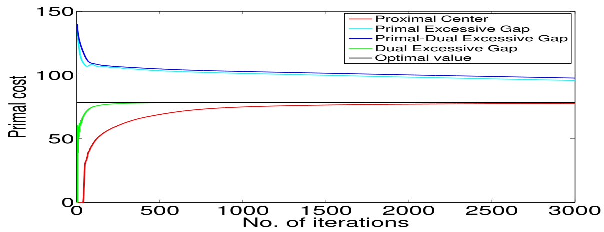

Fig. 3. Evolution of primal cost function $\psi_0(x)$

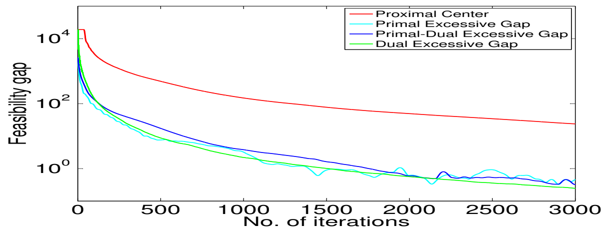

Fig. 4. Magnitude of constraint violations

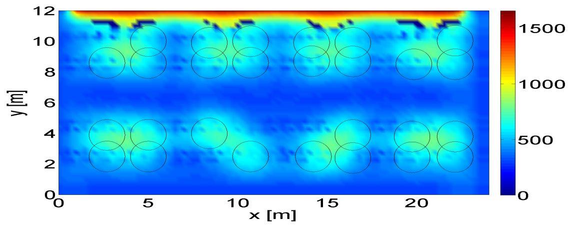

Fig. 5. Illuminance after 1000 iterations of Algorithm 4

contains 5657 inequality constraints and 27 equality constraints. The matrix norm $\|C\| = 6.82 \times 10^3$.

Algs. 1-4 are used to solve the optimal illumination control problem (5). The prox-functions $\phi_{\mathcal{X}}$ and $\phi_{\mathcal{Y}}$ are chosen as in (13), and the accuracy parameter $\epsilon = 1$ in Algs. 1 and 4. The 288 luminaires are divided equally amongst 18 controllers. This assignment of luminaires leads to a communication graph in which a controller needs to coordinate only with its immediate neighbors. Figs. 3 and 4 show the primal cost function and the feasibility gap for each algorithm, respectively. The dual excessive gap method (Alg. 4) is the quickest, in terms of number of iterations, to approach optimality. The proximal center algorithm (Alg. 1) requires less iterations than the primal and primal-dual excessive gap algorithms (Algs. 2 and 3) to approach optimality, but requires more iterations to approach feasibility. For this problem instance, the dual excessive gap method obtains an approximate solution in 1000 iterations. Fig. 5 shows the lux distribution after 1000 iterations of Alg. 4. Although not included in Figs. 3 and 4, the results for Algorithm 4 with initial step-size $\tau_0 = 0.5(\sqrt{5}-1)$ yielded curves that were almost identical to those obtained for Algorithm 1. All calculations are performed on a 2.53 GHz workstation with 3.0 GB RAM. The total computational times for 5000 iterations of Algorithms 1-4 are 19.4, 27.0, 27.5 and 36.6 seconds, respectively. These are determined using the tic-toc function in MATLAB.

In each algorithm, most of the computational effort is needed for matrix-vector multiplications. In Section 3.3, sparsity and communication were exploited to distribute the computations amongst the controllers. The structure

Table 3. Different luminaire assignments and their communication requirements.

LEDs per controller	Total links	Max. links	Max. mult.
1	7227	77	1673
4	623	25	5583
9	206	18	10518
16	50	9	14716
36	18	6	31239
144	1	1	115536
288	-	-	228109

of the matrices A and B in (5b)-(5c) defines the communication requirements, such as the number of communication links for each controller, the maximum number of links across the controllers, and the total number of links in the network. These aspects are considered for luminaires arranged in square blocks containing 1, 2, 3, 4, 6 and 12 luminaires in each row/column. Table 3 summarizes the communication requirements for different arrangements. The last column lists the maximum number of multiplications required for a single matrix-vector multiplication. Table 3 shows that as more LED luminaires are assigned to a controller, the computational effort per controller increases but the communication requirements ease. For instance, the total communication links quadruple when the number of LED luminaires per controller is decreased from 16 to 9. In this case, the maximum number of links required by a controller double. With 16 LED luminaires assigned per controller, the resulting communication graph requires neighbor-neighbor communication only. For each assignment of LED luminaires, the performance of Algs. 1-4 in terms of number of iterations is the same as that shown in Figs. 3-4.

5. CONCLUSION

This paper investigated distributed illumination coordination schemes for LED-based lighting systems that provide localized illuminance to occupants. The centralized optimization-based approach developed in (Pandharipande and Caicedo, 2010) was extended to a distributed setting. Several state-of-the-art non-centralized optimization algorithms were applied to the resulting optimization problem. For the considered lighting problem, the dual excessive gap method (Alg. 4) outperformed the proximal center, primal excessive gap and primal-dual excessive gap algorithms (Algs. 1–3). The problem formulation assumes knowledge of the occupancy pattern in the workspace plane and the illuminance contributions of natural and artificial light. Algorithm performance was evaluated using numerical simulations.

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