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Adaptive High-Gain observer for joint state and parameter estimation: A comparison to Extended and Unscented Kalman filter

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Abstract: An adaptive High-Gain observer (AHG) as well as an Extended (EKF) and Unscented Kalman filter (UKF) are implemented for joint state and parameter estimation of a novel multi-axial electromagnetically actuated punch. These observers are compared in terms of convergence and response time to erroneous parameter and state initialization, as well as parameter modifications during operation. The AHG is further analyzed proposing an adaptive gain, which reduces the observer's high sensibility with respect to noise. Simulation results show that AHG is more suitable compared to state-of-the-art EKF and UKF.

1. INTRODUCTION

Many aspects of parameter identification have been studied during the last decades, including problems present in nonlinear systems. In practical applications unmeasured states and unknown parameters are usually necessary for control purposes. Moreover, joint state and parameter estimation algorithms are also important in fault detection and isolation (Witczak [2007]).

Basically, two approaches can be used to design an observer for joint state and parameter estimation. On the one hand, it is possible to extend the original system state vector including the unknown parameters. The observers use this extended system, and state as well as parameter are estimated simultaneously. However, the convergence of this observer is only locally ensured (Byrnes and Rantzer [2003]). The EKF and the UKF correspond to this group of observers. Both filters do not take advantages of the particularity of the parameters into account, which are normally constant. They extend the system state vector introducing the unknown parameters, transforming the structure of the original system, e.g., if the original system is linear time invariant (LTI) with unknown parameters, the transformed system is likely to become nonlinear. On the other hand, a second approach is called adaptive observer. It consists of designing an observer for the original system assuming that the parameters are known, and finding an appropriate adaptive law for estimating these parameters, ensuring the overall observer convergence. These observers are globally convergent and present numerical advantages against the above mentioned observers (Zhang [2002]).

Nonlinear systems satisfying a Lipschitz type condition (Witczak [2007]) allow for designing an AHG, which is based on the high-gain (HG) observer for system state estimation and ensures global convergence. This observer tries to 'hide' the system nonlinearities, making the system 'linearities' predominant against these.

In this paper, the authors implement and compare a AHG observer, EKF, and UKF for joint state and parameter estimation of a multi-axial electromagnetically actuated punch modeled as a single mass oscillator. Moreover, a modification of the AHG observer based on an innovation factor is presented in this paper. This helps to reduce the observer's sensibility against noise.

The paper is organized as follows. In section 2, the HG observer, AHG observer, EKF, and UKF are introduced. In section 3, the proposed adaptive gain modification for the AHG observer is presented. A simulative example and results are shown in section 4. Finally, the paper is concluded by section 5.

2. METHODS

2.1 HG observers

The HG observer presented in this section follows Gauthier et al. [1992]. It is shown in Gauthier and Bornard [1980] that, if a system has the property to be observable for any input, then there is a coordinate transformation such that a MISO system can be formulated as

$$\begin{aligned}\dot{\mathbf{x}}(t) &= \mathbf{A}_0\mathbf{x}(t) + \boldsymbol{\phi}(\mathbf{x}(t), \mathbf{u}(t)) \,, \\ y(t) &= \mathbf{c}_0\mathbf{x}(t) \,,\end{aligned} \tag{1}$$

where $\mathbf{x}(t) \in \mathbb{R}^n$, $\mathbf{u}(t) \in \mathbb{R}^m$ and $y(t) \in \mathbb{R}$ are the system states, the input vector, and the measured output, respectively. $\mathbf{A}_0$, $\mathbf{c}_0$ and $\boldsymbol{\phi}(\mathbf{x}, \mathbf{u})$ are defined as follows [1] :

$$\mathbf{A}_0 = \begin{bmatrix} 0 & 1 & & 0 \\ & & \ddots & \\ & & & 1 \\ 0 & & & 0 \end{bmatrix} \in \mathbb{R}^{nxn} \,, \tag{2}$$

$$\mathbf{c}_0 = [1,\ 0,\ \ldots,\ 0] \in \mathbb{R}^{1xn} \,, \tag{3}$$

$$\boldsymbol{\phi}(\mathbf{x}, \mathbf{u}) = \begin{bmatrix} \phi_1(x_1, \mathbf{u}) \\ \phi_2(x_1, x_2, \mathbf{u}) \\ \vdots \\ \phi_n(\mathbf{x}, \mathbf{u}) \end{bmatrix} \in \mathbb{R}^{nx1} \,. \tag{4}$$

[1] The time variable is omitted as long as no ambiguity is presented for writing convenience.

If the function $\boldsymbol{\phi}(\mathbf{x}, \mathbf{u})$ is globally Lipschitz w.r.t. $\mathbf{x}$ and uniformly in $\mathbf{u}$, which should be bounded, then the system (1) admits an observer of the following form (Gauthier et al. [1992]):

$$\dot{\hat{\mathbf{x}}} = \mathbf{A}_0\hat{\mathbf{x}} + \boldsymbol{\phi}(\hat{\mathbf{x}}, \mathbf{u}) + \lambda\boldsymbol{\Lambda}(\lambda)^{-1}\mathbf{k}_0(y - \mathbf{c}_0\hat{\mathbf{x}}) , \quad (5)$$

where $\mathbf{k}_0 \in \mathbb{R}^n$ is defined such as that $\mathbf{A}_0 - \mathbf{k}_0\mathbf{c}_0$ is exponentially stable, $\boldsymbol{\Lambda}(\lambda)^{-1} = \mathrm{diag}(\left[1,\ \lambda,\ \lambda^2,\ \ldots,\ \lambda^{n-1}\right])$, and $\lambda \in \mathbb{R}^+$ large enough. $\hat{\mathbf{x}} \in \mathbb{R}^n$ corresponds to the estimated states.

This observer design is known as a HG observer which has an exponential convergence. The parameter λ determines the observer's speed of convergence.

2.2 Adaptive observers for parameter-affine systems

An adaptive observer allows to estimate both parameters and states simultaneously in dynamic systems. A collection of several adaptive observers can be found in Zhang [2005], and have been expanded to systems with parametric uncertainty in the unmeasured state dynamics in Besançon [2007]. In short, the design of the AHG with exponential state and parameter convergence can be expressed as follows.

Consider a system of the following form:

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{A}_0\mathbf{x} + \boldsymbol{\phi}(\mathbf{x}, \mathbf{u}) + \boldsymbol{\psi}(\mathbf{x}, \mathbf{u})\boldsymbol{\theta} , \\ y &= \mathbf{c}_0\mathbf{x} , \end{aligned} \quad (6)$$

where $\boldsymbol{\theta} \in \mathbb{R}^p$ is an unknown constant parameter vector and $\boldsymbol{\psi}(\mathbf{x}, \mathbf{u}) \in \mathbb{R}^{nxp}$ has the following form:

$$\boldsymbol{\psi}(\mathbf{x}, \mathbf{u}) = \begin{bmatrix} 0, & \ldots, & 0 \\ \vdots & & \vdots \\ 0, & \ldots, & 0 \\ \psi_1(\mathbf{x}, \mathbf{u}), & \ldots, & \psi_p(\mathbf{x}, \mathbf{u}) \end{bmatrix} . \quad (7)$$

Besançon [2007] designed a HG observer as in (5) but for joint estimation of $\mathbf{x}$ and $\boldsymbol{\theta}$ based on the following assumptions:

[A1]: $\boldsymbol{\phi}$ and $\boldsymbol{\psi}$ are smooth functions w.r.t. their arguments $\mathbf{x}$ and $\mathbf{u}$, and the input vector $\mathbf{u}$ is bounded generating bounded states $\mathbf{x}$.

[A2]: $\exists\, \mathbf{k}_0$ such that $\mathbf{A}_0 - \mathbf{k}_0\mathbf{c}_0$ is exponentially stable.

[A3]: $\exists\, \boldsymbol{\Upsilon}(t) \in \mathbb{R}^{nxp}$ defined as the solution of

$$\dot{\boldsymbol{\Upsilon}} = \lambda(\mathbf{A}_0 - \mathbf{k}_0\mathbf{c}_0)\boldsymbol{\Upsilon} + \lambda\boldsymbol{\psi}(\mathbf{x}, \mathbf{u}) . \quad (8)$$

Additionally, $\boldsymbol{\psi}$ should be persistently exciting so that the matrix $\boldsymbol{\Upsilon}$ satisfies, for some positive constants α, T, λ large enough and t_0, the inequality

$$\int_{t-T}^{t} \boldsymbol{\Upsilon}(\tau)^{\mathrm{T}}\mathbf{c}_0^{\mathrm{T}}\mathbf{c}_0\boldsymbol{\Upsilon}(\tau)\mathrm{d}\tau \geq \alpha\mathbf{I},\ \forall t > t_0 , \quad (9)$$

where $\mathbf{I}$ corresponds to the identity matrix.

Considering the system (6) satisfying the previous assumptions then

$$\begin{aligned} \dot{\hat{\mathbf{x}}} &= \mathbf{A}_0\hat{\mathbf{x}} + \boldsymbol{\phi}(\hat{\mathbf{x}}, \mathbf{u}) + \boldsymbol{\psi}(\hat{\mathbf{x}}, \mathbf{u})\hat{\boldsymbol{\theta}} \\ &\quad + \boldsymbol{\Lambda}(\lambda)^{-1}\lambda(\mathbf{k}_0 + \hat{\boldsymbol{\Upsilon}}\boldsymbol{\Gamma}\hat{\boldsymbol{\Upsilon}}^{\mathrm{T}}\mathbf{c}_0^{\mathrm{T}})(y - \mathbf{c}_0\hat{\mathbf{x}}) , \\ \dot{\hat{\boldsymbol{\theta}}} &= \lambda^n\boldsymbol{\Gamma}\hat{\boldsymbol{\Upsilon}}^{\mathrm{T}}\mathbf{c}_0^{\mathrm{T}}(y - \mathbf{c}_0\hat{\mathbf{x}}) , \\ \dot{\hat{\boldsymbol{\Upsilon}}} &= \lambda(\mathbf{A}_0 - \mathbf{k}_0\mathbf{c}_0)\hat{\boldsymbol{\Upsilon}} + \lambda\boldsymbol{\psi}(\hat{\mathbf{x}}, \mathbf{u}) , \end{aligned} \quad (10)$$

is a global exponential observer for the system (6), i.e., $\forall\mathbf{x}(0)$, any bounded $\hat{\mathbf{x}}(0)$ and $\hat{\boldsymbol{\theta}}(0)$, the errors $\|\hat{\mathbf{x}} - \mathbf{x}\|_2$ and $\left\|\hat{\boldsymbol{\theta}} - \boldsymbol{\theta}\right\|_2$ tend exponentially to zero when $t \to \infty$.

$\boldsymbol{\Gamma} \in \mathbb{R}^{pxp}$ is a symmetric positive definite tuning matrix used to balance the convergence speeds of state and parameter estimation.

See Besançon [2007] for the proof and further details.

2.3 EKF and UKF observers

The EKF is the nonlinear version of the well-known Kalman filter. The EKF linearizes the model system around the current estimate using partial derivatives of the system and measurement functions.

Consider a system of the following form

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{f}(\mathbf{x}, \mathbf{u}) + \mathbf{w} , \\ \mathbf{y} &= \mathbf{h}(\mathbf{x}) + \mathbf{v} , \end{aligned} \quad (11)$$

where

$$\begin{aligned} \mathbf{w} &\sim N(\mathbf{0}, \mathbf{Q}) , \\ \mathbf{v} &\sim N(\mathbf{0}, \mathbf{R}) . \end{aligned} \quad (12)$$

The EKF can be written as

$$\dot{\hat{\mathbf{x}}} = \mathbf{f}(\hat{\mathbf{x}}, \mathbf{u}) + \mathbf{K}(\mathbf{y} - \mathbf{h}(\hat{\mathbf{x}})) , \quad (13)$$

where

$$\begin{aligned} \mathbf{K} &= \mathbf{P}\mathbf{H}^{\mathrm{T}}\mathbf{R}^{-1} , \\ \dot{\mathbf{P}} &= \mathbf{F}\mathbf{P} + \mathbf{P}\mathbf{F}^{\mathrm{T}} - \mathbf{K}\mathbf{H}\mathbf{P} + \mathbf{Q} , \\ \mathbf{F} &= \left.\frac{\partial\mathbf{f}}{\partial\mathbf{x}}\right|_{\hat{\mathbf{x}}^-, \mathbf{u}} , \\ \mathbf{H} &= \left.\frac{\partial\mathbf{h}}{\partial\mathbf{x}}\right|_{\hat{\mathbf{x}}^-, \mathbf{u}} . \end{aligned} \quad (14)$$

This observer allows the estimation of only the system states. In order to estimate the unknown parameters, it is possible to extend the state vector into

$$\mathbf{X} := \begin{pmatrix} \mathbf{x} \\ \boldsymbol{\theta} \end{pmatrix} , \quad (15)$$

being the resulting extended observer for parameter and state estimation

$$\begin{aligned} \dot{\mathbf{X}} &= \mathbf{F}(\mathbf{X}, \mathbf{u}) + \mathbf{w} , \\ \mathbf{y} &= \mathbf{H}(\mathbf{X}) + \mathbf{v} , \end{aligned} \quad (16)$$

with $\dot{\boldsymbol{\theta}} = \mathbf{0}$. It should be noted that, LTI systems turn usually into a nonlinear system. See Welch and Bishop [2006] for further details on EKF.

The Unscented Kalman filter was first proposed by Julier and Uhlmann [1997]. A central and vital operation performed in a Kalman filter is the propagation of a Gaussian random variable (GRV) through the system dynamics. In the EKF, the state distribution is approximated by a GRV and then propagated through the linearization of the nonlinear system. This is sub-optimal and can introduce errors in the true mean and covariances of the transformed GRV, particularly when the nonlinearities are predominant. The UKF addresses this propagation error by using a deterministic sampling approach. In contrast to the EKF, the state distribution is approximated using a minimal set of chosen sample points, which capture the true and covariance of the GRV. These points are propagated through the true nonlinear system, allowing a better accuracy of the transformed mean and covariance.

This approach was extended for joint state and parameter estimation by Wan et al. [1999]. This filter is based on the extended state vector presented in (15) and (16). See Van der Merwe and Wan [2001] and references therein for more details on UKF and further observer designs.

3. AHG MODIFICATION

The high sensitivity of HG observers is a well-known drawback: the observers ensure a global exponential convergence, but increase noise effects. It can be proved that, with larger gains the convergence of these observers is faster, but the noise is amplified.

Motivated by Boizot et al. [2010], a solution proposed in this paper is to compensate this drawback with a varying gain. When the error in the parameter and state estimation is large, then the observer gain should be large enough providing global convergence, otherwise it should converge to a minimum value ensuring the observer convergence, but reducing the noise effects.

The proposed varying λ is defined as the solution of the following differential equation:

$$\begin{aligned} \dot{\lambda} &= K_{\text{up}} S(\text{Inn})(\lambda_{\max} - \lambda) \\ &\quad + K_{\text{down}}(\lambda_{\min} - \lambda)(1 - S(\text{Inn}))\,, \\ \lambda(0) &= \lambda_{\max}\,. \end{aligned} \tag{17}$$

$K_{\text{up}} \in \mathbb{R}^+$ and $K_{\text{down}} \in \mathbb{R}^+$ define the convergence rates in direction to the boundaries $\lambda_{\max}$ and $\lambda_{\min}$, respectively. $S(\text{Inn})$ is an innovation factor which can be defined such as

$$S(\text{Inn}) = \begin{cases} 1, & \text{Inn} \geq \gamma \\ 0, & \text{Inn} < \gamma \end{cases}\,. \tag{18}$$

However, this transition is abrupt. Hence, the following function, which has a continuous first derivative is proposed

$$\begin{aligned} S(\text{Inn}) &= 1/(1 - \mathrm{e}^{-\beta(\text{Inn}-\gamma)})\,, \\ \text{Inn} &= \int_{t-T}^{t} \|y(s) - \overline{y}_{t-T}\|_2 \mathrm{d}s\,. \end{aligned} \tag{19}$$

The tuning factors γ, $\beta \in \mathbb{R}^+$ are the bounds and the switching speed of the function S between 0 and 1, and $T \in \mathbb{R}^+$ represents the length of the window used to calculate the innovation. The value of the variable y corresponds to the measured output, and $\overline{y}_{t-T}$ is the prediction of the output trajectory of the system (6) over the time interval $[t-T, t]$, using the initial state and parameter values at the time $t - T$.

4. RESULTS

4.1 System description and mathematical model

The simulation model corresponds to the multi-axial electromagnetically actuated punch (MEAP) described in Riva et al. [2013].

The MEAP is shown in Fig. 1 and consists of four crosswise arranged electromagnetic actuators sharing one armature. Heavy duty spring packs are used to keep the armature in the mid-position and generate additional forces at the upper and lower end of the stroke. The cutting tool is located at the upper side and is connected with the armature via rods, which are guided using ball-bearing, reducing bending and vibration problems. The position of the armature is controlled and measured with four commercial eddy current sensors located at the side of each electromagnet and fixed to the base frame.

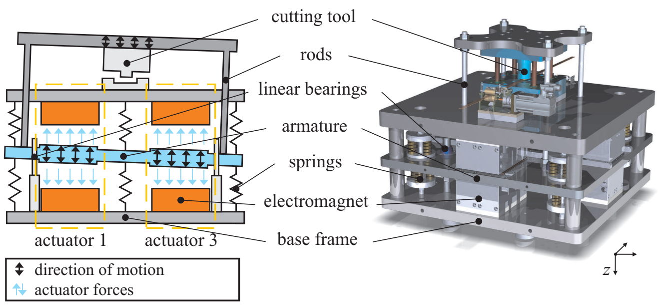

Fig. 1. Structure and functionality of the MEAP.

Table 1. Specification of the MEAP.

Specification	Value
maximum stroke	4 mm
maximum cutting force	60 kN
translational inertia (m_z)	125 kg
spring stiffness (c_z)	20 kN/m
dampers (d_z)	1500 N s/m

In z direction of displacement, the system can be modeled as a single-mass oscillator and the state space representation of the MEAP can be described by

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{A}\mathbf{x} + \mathbf{b}(\mathbf{x}, u)\,, \\ y &= \mathbf{c}\,\mathbf{x}\,, \end{aligned} \tag{20}$$

in which

$$\begin{aligned} \mathbf{A} &= \begin{bmatrix} 0 & 1 \\ -\dfrac{c_z}{m_z} & -\dfrac{d_z}{m_z} \end{bmatrix}, \\ \mathbf{b} &= \begin{bmatrix} 0 \\ -\dfrac{d_{z2}}{m_z}\tanh(d_{z3}x_2) + F_z \end{bmatrix}, \\ \mathbf{c} &= [1,\ 0]\,, \end{aligned} \tag{21}$$

where m_z corresponds to the armature and the cutting tool masses, c_z represents the heavy duty springs, d_z is the velocity proportional dampers, and $d_{z2,z3}$ are nonlinear dampers, due to friction between armature and linear bearings as well as the Coulomb friction, and F_z are the electromagnetic forces. The output $y \in \mathbb{R}$ corresponds to the measured armature position and the system states $\mathbf{x} = [x_1,\ x_2]^{\mathrm{T}} = [x_z,\ \dot{x}_z]^{\mathrm{T}}$ are the armature's measured position and speed. Specifications of the MEAP can be found in Table 1.

In order to control the armature position of the MEAP a linear state feedback control is applied. See Dagen et al. [2012] for further details.

4.2 AHG design

The system (20) fits into the form (6). Hence, an AHG can be designed. The observer matrices are defined as follows:

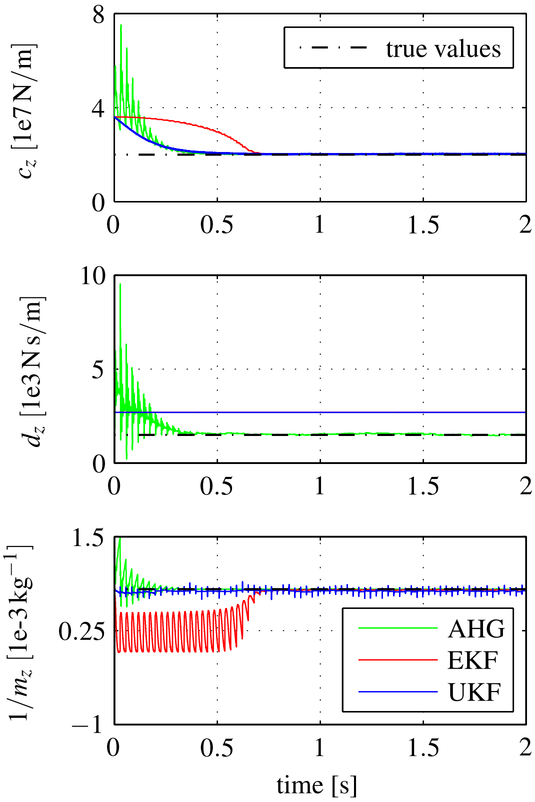

Fig. 2. Comparison between AHG, UKF, and EKF being $\hat{\mathbf{p}}(0) = 1.8 \cdot \mathbf{p}$.

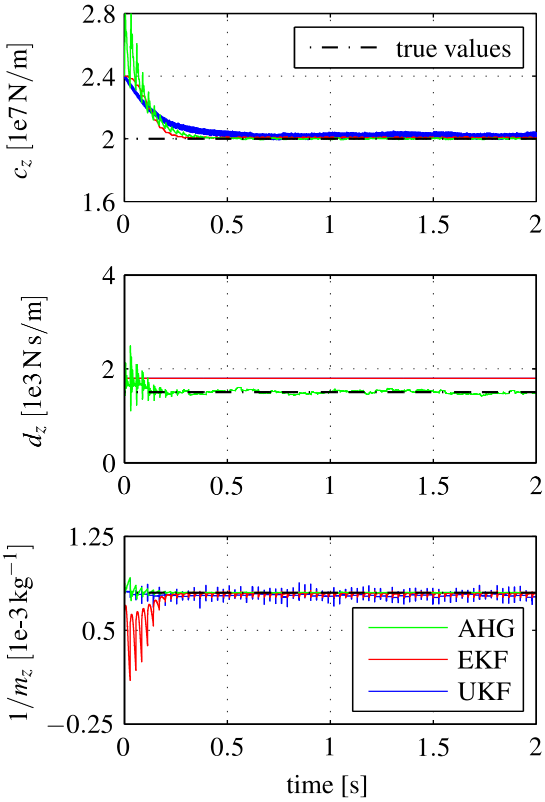

Fig. 3. Comparison between AHG, UKF, and EKF being $\hat{\mathbf{p}}(0) = 1.2 \cdot \mathbf{p}$.

$$\begin{aligned} \mathbf{A}_0 &= \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} , \\ \boldsymbol{\phi}(\mathbf{x}, u) &= \mathbf{0} , \\ \boldsymbol{\psi}(\mathbf{x}, u) &= \begin{bmatrix} 0 & 0 & 0 \\ -x_1 & -x_2 & F_z - d_{z2}\tanh(d_{z3}x_2) \end{bmatrix} , \\ \boldsymbol{\theta} &= \begin{bmatrix} \frac{c_z}{m_z} & \frac{d_z}{m_z} & \frac{1}{m_z} \end{bmatrix}^{\mathrm{T}} , \\ \mathbf{c}_0 &= [1,\ 0] , \end{aligned} \tag{22}$$

and $\mathbf{p} = [m_z,\ c_z,\ d_z]$ is the unknown parameter vector with positive constants.

Additionally, the following conditions hold

- F_z is bounded,
- m_z, c_z, d_z are unknown positive constants,
- $\mathbf{x}$ is bounded.

4.3 EKF and UKF design

In order to design an EKF and UKF for joint state and parameter estimation, the extended state vector of the system (20) is defined as

$$\mathbf{X}(t) := \begin{bmatrix} x_z \\ \dot{x}_z \\ c_z \\ d_z \\ 1/m_z \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} , \tag{23}$$

resulting the system's model in

$$\begin{aligned} \dot{\mathbf{X}}(t) &= \begin{bmatrix} x_2 \\ x_5(-x_3x_1 - x_4x_2 - d_{z2}\tanh(d_{z3}x_2) + F_z) \\ 0 \\ 0 \\ 0 \end{bmatrix} , \\ y(t) &= x_1 . \end{aligned} \tag{24}$$

The previous equations allow the design of both an EKF and UKF.

4.4 Parameter estimation

In order to evaluate the convergence rate between AHG, EKF, and UKF with respect to erroneous state and parameter initialization, two simulations with initial parameter values $\hat{\mathbf{p}}(0) = 1.8 \cdot \mathbf{p}$ and $\hat{\mathbf{p}}(0) = 1.2 \cdot \mathbf{p}$ were performed, respectively. The initial system state values were chosen to be $\hat{\mathbf{x}}(0) = \mathbf{0}$.

Due to the high value of the spring stiffness, it is difficult to persistently excite the system and identify the parameters. The MEAP can achieve maximal cutting frequencies of up to 50 Hz and cutting speeds of up to 200 mm/s. Thus, a s-curve profile as the desired armature position was considered taking into account these limitations. White noise was added to the simulated system output representing measurement noise.

It should be noted that, the observer tuning parameters are different. $\{\mathbf{K}_0,\ \lambda,\ \boldsymbol{\Gamma}\}$, $\{\mathbf{Q},\ \mathbf{R},\ \mathbf{P}\}$, and $\{\mathbf{Q},\ \mathbf{R},\ \mathbf{P}\}$ together with the parameters related with the spread of the sigma points and Gaussian distributions (Wan and Van der Merwe [2000]) tune the AHG, EKF, and UKF,

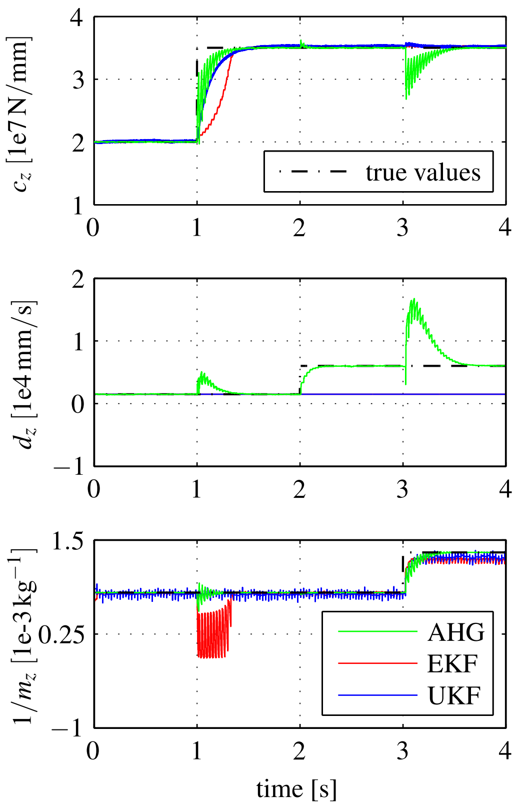

Fig. 4. Comparison between AHG, UKF, and EKF: Parameter modifications.

respectively. Hence, it is difficult to find a compatible tuning standard for comparing these observers. Therefore, the same tuning procedure was used for the three observers to get the 'best' estimation condition for the MEAP for each observer. Only the state estimation was considered first, then the parameter estimation was tuned. The goal was to maximize the convergence rate avoiding oscillations, driftings, and offsets in the state and parameter estimation.

The parameter estimation results of the AHG, EKF, and UKF are shown in Fig. 2 and Fig. 3 for the two initial parameter values sets $\hat{\mathbf{p}}(0) = 1.8 \cdot \mathbf{p}$ and $\hat{\mathbf{p}}(0) = 1.2 \cdot \mathbf{p}$, respectively. The first subplot corresponds to the stiffness spring constant, the second one refers to the linear damper parameter, and the last one is the MEAP armature mass.

The AHG converged faster than the EKF, and UKF. The difference between the two initializations did not considerably affect the convergence time of the AHG. The EKF and UKF converged with almost the same rate w.r.t the spring constant, but the UKF reached the true mass value faster.

With the selected position profile, the EKF and UKF could not identify the damper constant d_z. The reason is that, due to the high spring stiffness c_z, and the maximal stroke of the MEAP, this parameter could not be enough exited. Although these complicate the estimation of d_z, AHG overcomes this obstacle by persistently exciting the system parameters using the adaptive law, allowing the identification of them.

4.5 Parameter variation

In the previous subsection, neither the initial parameter values, nor the initial system states were the true values. In this subsection a parameter variation is proposed in order to compare the observer response time. Once the observers reached the true values, a stiffness spring constant modification was introduced, then after converging to stable values, a damper and finally a mass changes were added, being $\mathbf{x}(t_s^-) \approx \mathbf{x}_{\text{true}}$, where t_s corresponds to the switching times.

Fig. 4 presents the parameter estimation results for the three observer. The AHG converged faster than the EKF and UKF. Although, only a single system parameter was changed at once, the others oscillated and then converged again to the true value. The EKF mass estimation was the most affected parameter. The oscillating phenomenon appeared on the AHG too, but on the damper constant estimation. In this case, no comparison with the EKF and UKF is possible, because these observers were not able to estimate this parameter.

The EKF and UKF remained with a small bias after the mass change. These observers did not follow the damper constant change. Thus, this estimation error was probably derived to the mass estimation, causing this bias.

4.6 Adaptive gain

Two AHG were used for joint state and parameter estimation of the MEAP. The first observer used a fix gain λ. On the second observer the adaptive gain λ was implemented. Thus, γ, β, K_{up}, K_{down}, $\lambda_{\max}$, and $\lambda_{\min}$ were defined. The parameters γ and β define the switching boundaries and switching speed of the innovation function, i.e., if the innovation value remains under the value γ, then no parameter modification has been occurred, otherwise, a parameter was modified. Additionally, K_{up} and K_{down} were defined to get a fast convergence rate to $\lambda_{\max}$, but a slower rate to $\lambda_{\min}$.This ensured that, the noise effects were reduced, but the overall observer convergence rate was not considerably reduced.

Fig. 5 shows on the first subplot the AHG with fix and adaptive gain λ, and the evolution of the adaptive gain on the second subplot. The λ value increased up to its maximum value immediately after a modification in the parameters. The convergence rate did not change considerably, both observers needed the same time to reach the true values, but the sensitivity of the AHG with adaptive λ to noise was lower, resulting in a smaller parameter variation. Otherwise, the response to parameter changes remained almost the same for both observers. Thus, the adaptive gain λ improve the AHG performance against noise without enlarging the response time.

5. CONCLUSION

The AHG performed best compared to the EKF and UKF in parameter identification on the simulated system. The EKF and UKF had almost the same complexity and efficiency. The UKF is easy to implement in comparison with the EKF. The later needs the calculation of the analytical

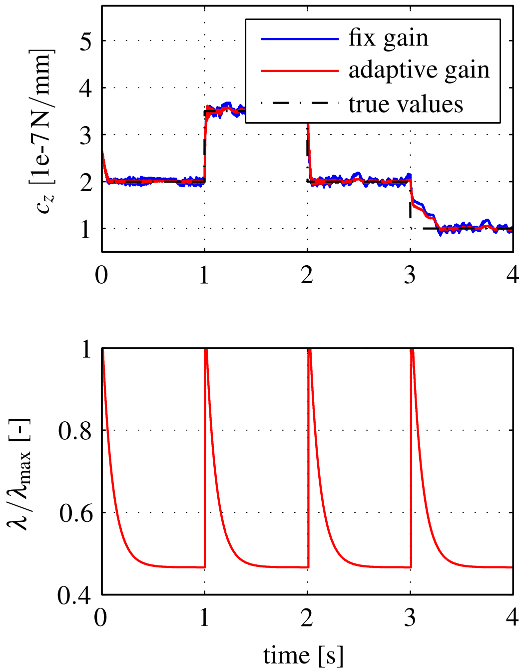

Fig. 5. Comparison between fix and adaptive gain AHG (upper): Spring constant parameter. Adaptive λ for AHG (lower).

derivatives of the system matrices. Otherwise, tuning the UKF was more difficult than the other observers.

Persistency of excitation should be available in order to guarantee convergence of the observers. As mentioned, the parameter d was not considerably excited for the EKF and UKF with the proposed position profile. The high spring constant hinder the damper identification. Thus, this parameter did not converge to the true value. The convergence problem did not occur with the AHG. The adaptive law helped to persistency excite the parameter in order to estimate them.

Additionally, the nonlinear part of the presented system was not predominant. If the nonlinearities were more predominant, the differences between the three observers could be greater. The AHG observer has a global exponential convergence, while the EKF and UKF ensure only local convergence. Thus, greater parameter modifications, or failures may not be tracked by the EKF and UKF, especially if the nonlinear part is predominant.

The presented observer designs were developed in continuous time. For real-time and online applications, a discrete or continuous-discrete implementation of the above observers is planned by the authors.

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REFERENCES

Gildas Besançon. *Nonlinear Observers and Applications.* Lecture Notes in Control and Information Sciences. Springer-Verlag Berlin Heidelberg, 363 edition, 2007.

N. Boizot, E. Busvelle, and J.-P. Gauthier. An adaptive high-gain observer for nonlinear systems. *Automatica*, 46(9):1483–1488, September 2010.

C. Byrnes and A. Rantzer. *Directions in Mathematical Systems Theory and Optimization.* Springer-Verlag Berlin Heidelberg, 2003.

M. Dagen, M. Riva, B. Heimann, T. Ortmaier, C. Wager, R. Krimm, and B.-A. Behrens. A multi-axial electromagnetically actuated punch for cutting microcomponents. In *American Control Conference (ACC)*, pages 4570–4575, 2012.

J.-P. Gauthier and G. Bornard. Observability for any u(t) of a class of nonlinear systems. In *19th IEEE Conference on Decision and Control including the Symposium on Adaptive Processes*, volume 19, pages 910–915, 1980.

J.-P. Gauthier, H. Hammouri, and S. Othman. A simple observer for nonlinear systems applications to bioreactors. *IEEE Transactions on Automatic Control*, 37(6): 875–880, 1992.

S. J. Julier and J. K. Uhlmann. A new extension of the kalman filter to nonlinear systems. In *Proc. of AeroSense: The 11th Int. Symp. on Aerospace/Defence Sensing, Simulation and Controls*, pages 182–193, 1997.

R. Van der Merwe and E. A. Wan. The square-root unscented Kalman filter for state and parameter-estimation. In *International Conference on Acoustics, Speech, and Signal Processing*, pages 3461–3464, 2001.

M. H. Riva, J. Díaz Díaz, M. Dagen, and T. Ortmaier. Estimation of covariances for Kalman filter tuning using autocovariance method with landweber iteration. In *14th IASTED International Symposium on Intelligent Systems and Control (ISC 2013)*, Marina del Rey, USA, 2013.

E. A. Wan and R. Van der Merwe. The unscented Kalman filter for nonlinear estimation. In *IEEE 2000 Adaptive Systems for Signal Processing, Communications, and Control Symposium*, pages 153–158, 2000.

E. A. Wan, R. Van der Merwe, and A. T. Nelson. Dual estimation and the unscented transformation. In *Neural Information Processing Systems*, pages 666–672. The MIT Press, 1999.

G. Welch and G. Bishop. An introduction to the Kalman filter. Technical report, Chapel Hill, NC, USA, 2006.

M. Witczak. *Modelling and Estimation Strategies for Fault Diagnosis of Non-Linear Systems: From Analytical to Soft Computing Approaches.* Springer-Verlag Berlin Heidelberg, 2007.

Q. Zhang. Adaptive observer for multiple-input-multiple-output (MIMO) linear time-varying systems. *IEEE Transactions on Automatic Control*, 47(3):525–529, 2002.

Q. Zhang. Revisiting different adaptive observers through a unified formulation. In *44th IEEE Decision and Control (CDC) and European Control Conference (ECC)*, pages 3067–3072, 2005.