Towards a New Generation of Relay Autotuners

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Abstract: The relay autotuner for PID control is based on the simple idea of investigating process dynamics by the oscillation obtained when the PID controller is replaced by a relay function. In this paper an asymmetric relay function is used, which provides an equation for the static gain of the process. A method to find the normalized time delay is proposed and the benefits of this is discussed. Ways to find low-order models from the experiment are described. Considerations of how to choose the relay parameters are made and some examples are given.

Keywords: Autotuning, asymmetric relay, normalized time delay, parameter estimation.

1. INTRODUCTION

The relay autotuner, introduced in Åström and Hägglund (1983), is based on the idea of finding process dynamics from a relay oscillation and determining the parameters of a PID controller using simple rules. In the original tuner, the PID controller was tuned based on just the amplitude and the period of the relay oscillation. The tuner gained strong industrial acceptance, the main reason for this was that it was simple to use. It was just to push a button - no information had to be provided by the user. It was, of course, also important that the tuning gave controllers with acceptable performance. Another important aspect was that the tuning procedure was short. Experiences of the tuner are given in Hägglund and Åström (1991).

The development of the original autotuner was limited by the computational power available at the time. The first tuner had to be squeezed into 2 kB. Understanding of PID tuning has also increased significantly during the last decades, see e.g. Åström and Hägglund (2005); Vilanova and Visioli (2012). In particular it has been shown that more process information is required to obtain efficient tuning rules. The amplitude and the period obtained from a relay feedback experiment is simply not enough.

It has been shown in Åström and Hägglund (2005) that knowledge of the normalized time delay τ (Åström and Hägglund, 2005, p 26) of the process gives important qualitative information about tuning. For example, derivative action has strong benefits for small τ but marginal effects for τ close to 1. Robust PI/PID controllers can be designed based on a first order model with time delay (FOTD), see e.g. Sell (1995); Skogestad (2003, 2006); Åström and Hägglund (2005). Significant improvements can however be obtained by using a second order model with time delay (SOTD) for processes with small τ (Åström and Hägglund, 2005, ch 7). These recent insights could be included into the design of an improved autotuner.

This paper outlines ideas for new autotuners. When applying autotuning it is desirable to use procedures that require short experimentation time and little a priori information. The conventional relay autotuner uses a symmetric relay which does not give sufficient excitation to obtain good models. In this paper, it is suggested to use an asymmetric relay. It provides the possibility to estimate an FOTD model of the process. A method to quickly determine the normalized time delay is presented. This is useful, since knowledge of τ makes it possible to assess the benefits of derivative action and better models. Parameter estimation is used to obtain better models. The methods are iterative and initial estimates are obtained from the preliminary FOTD model. Practical issues such as choice of relay amplitudes, relay hysteresis, convergence and noise are discussed in Sec. 5 and examples are presented in Sec. 6.

2. ASYMMETRIC RELAY FEEDBACK

Relay feedback with the possibility to use an asymmetric relay has been investigated earlier. In Lin et al. (2004) an analysis of the existence and stability of limit cycles for FOTD systems under relay feedback has been performed. Determination of the parameters of an FOTD model from an asymmetric relay experiment has been done in different ways in e.g. Wang et al. (1997); Kaya and Atherton (2001); Srinivasan and Chidambaram (2003); Liu and Gao (2008).

An FOTD system could be parametrized by either (K, T, L) or (a, b, L) as shown in (1)

$$P(s) = \frac{K}{1 + sT}e^{-sL} = \frac{b}{s + a}e^{-sL}. \qquad (1)$$

The normalized time delay τ is defined as

$$\tau = \frac{L}{L + T} = \frac{aL}{1 + aL}, \qquad 0 \le \tau \le 1. \qquad (2)$$

A small value of τ corresponds to a lag-dominated system and a large value to a delay-dominated system. Notice that processes with integral action cannot be represented with finite values of K and T and that pure delays cannot be represented with finite values of a and b. The parametrization (K, T, L) is thus suitable for processes with balanced or delay-dominated dynamics, while (a, b, L)
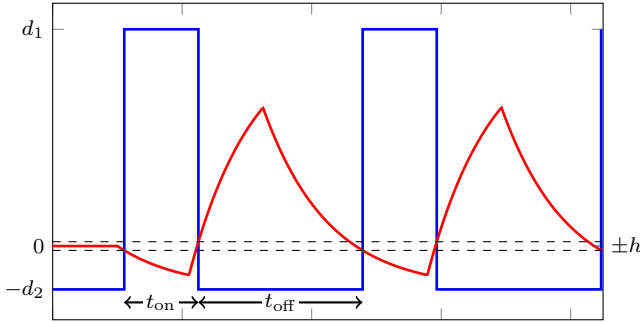
Fig. 1. Outputs from an asymmetric relay feedback experiment. The process output y is shown in red and the output from the relay, u, is shown in blue. The relay amplitudes are denoted d_1 and d_2 respectively. The hysteresis of the relay is denoted h. The time intervals t_{on} and t_{off} are also defined by the figure.

suits lag-dominated and balanced processes. Throughout this paper the parametrization (K, T, L) will be used.

The asymmetric relay has the input-output relation

$$u(t) = \begin{cases} d_1, & \text{if } y(t) < -h \text{ or } y(t) < h \text{ and } u(t_-) = d_1 \\ -d_2, & \text{if } y(t) > h \text{ or } y(t) > -h \text{ and } u(t_-) = -d_2 \end{cases}$$

with notations explained in Fig. 1. The notation $\gamma = d_1/d_2$ will be used as a measure of the asymmetry level and it is assumed that $d_1 > d_2$.

2.1 Estimating static gain K

One of the benefits of using an asymmetric relay is that the static gain K can be derived as

$$K = \frac{\int_t^{t+t_p} y(t)dt}{\int_t^{t+t_p} u(t)dt}. \tag{3}$$

Here t_p is the period of one oscillation, so $t_p = t_{\text{on}} + t_{\text{off}}$. Equation (3) can not be used for a symmetric relay since the denominator then becomes zero. This could also happen for an asymmetric relay, but only in the specific case when $d_1 t_{\text{on}} = d_2 t_{\text{off}}$. The estimation of K for the processes in the test batch, (Åström and Hägglund, 2005, p 227) which contains 134 processes common in industry, is shown in Fig. 2 for some different asymmetry levels γ. The integrating processes have been removed in this plot since they can not be described with a finite K. All the other processes have $K = 1$ and as can be seen the estimated values stays within 10% of the true value. A close look at the dots for a certain τ shows that the accuracy of the estimation is increased as the asymmetry grows larger. It is also worth noting from the frequency plots shown in Fig. 3 that the excitation for low frequencies increases with the asymmetry level, in the symmetric case, $\gamma = 1$, there is no excitation at all for $\omega = 0$.

2.2 Equations for t_{on} and t_{off}

An advantage with the asymmetric relay is that the normalized time delay τ can be easily estimated by looking at the ratio between t_{on} and t_{off}. This is further discussed in Sec. 3, but we can conclude that it would not be possible for a symmetric relay since it always has $t_{\text{on}} = t_{\text{off}}$.

For an FOTD model under asymmetric relay feedback it is straightforward to determine how t_{on} and t_{off} depend on
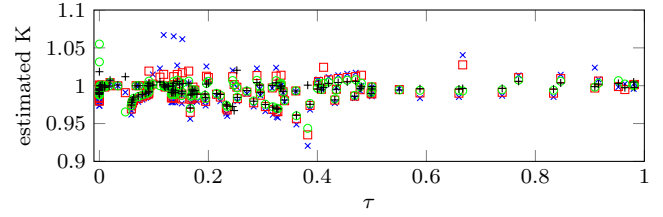

Fig. 2. Estimation of static gain K for the processes in the test batch, plotted against the normalized time delay τ. The result is shown for the asymmetries $\gamma = 1.3$ (blue), $\gamma = 1.5$ (red), $\gamma = 2$ (green) and $\gamma = 3$ (black). Other parameters used were $h = 0.1$ and $d_2 = 0.3$.
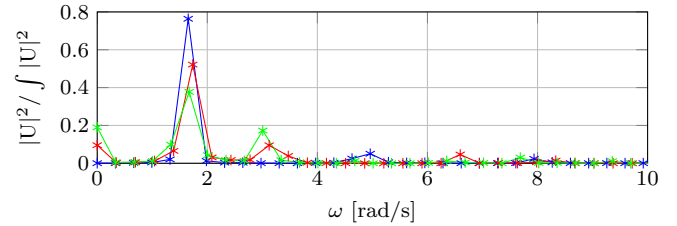

Fig. 3. Frequency spectrum of the relay oscillations for $P(s) = \frac{1}{s+1}e^{-s}$. The simulation was run for 5 periods with $h = 0.1$ and $d_2 = 0.3$. The curves correspond to the three different relay asymmetries $\gamma = 1$ (blue), $\gamma = 3$ (red) and $\gamma = 5$ (green).

the parameters of the relay and the process, see e.g. Lin et al. (2004). We have

$$t_{\text{off}} = T \ln \frac{\varphi - \gamma + \eta}{1 - \eta}, \qquad t_{\text{on}} = T \ln \frac{\varphi - 1 + \eta}{\gamma - \eta} \tag{4}$$

where $\varphi = (1+\gamma)e^{L/T}$ and $\eta = \frac{h}{Kd_2} < 1$.

The upper bound on η is due to conditions for the existence of an oscillation. If there is no hysteresis, then $\eta = 0$ and simple expressions for the limiting cases when L/T goes to zero or infinity can be derived. Taking limits of (4) and using that $e^x \approx 1 + x$ and $\ln(1+x) \approx x$ for small x gives

$$t_{\text{off}} = \begin{cases} L(\gamma + 1) & \text{if } L/T \to 0 \\ L & \text{if } L/T \to \infty \end{cases}$$
$$t_{\text{on}} = \begin{cases} L(1 + 1/\gamma) & \text{if } L/T \to 0 \\ L & \text{if } L/T \to \infty \end{cases} \tag{5}$$

Notice that $t_{\text{on}} = t_{\text{off}}$ for delay dominated processes even if the relay is asymmetric, and that $t_{\text{off}}/t_{\text{on}} = \gamma$ for processes with lag dominated dynamics.

With K known from (3), equation (4) can be solved for the remaining parameters T and L. However this can not be done analytically and in order to solve them numerically appropriate initial guesses are needed.

3. ESTIMATING THE NORMALIZED TIME DELAY

The normalized time delay τ has been proven to be an important parameter in the tuning of PID controllers (Åström and Hägglund, 2005, ch 7). A way to rapidly determine the normalized time delay is therefore of significant value, since it provides information on how to continue the autotuning procedure.

It turns out that asymmetric relay feedback offers an effective way of estimating τ. As shown in (5) a lag-
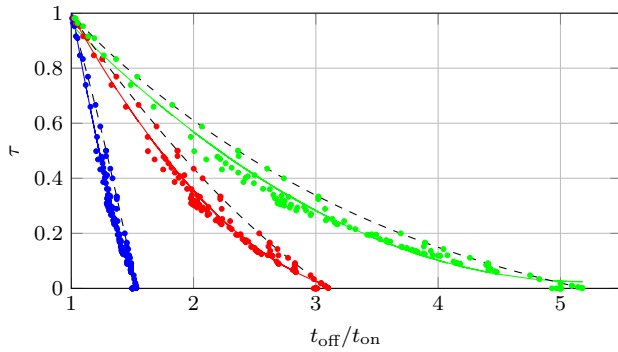
Fig. 4. τ as a function of $t_{\text{off}}/t_{\text{on}}$ for $\eta = 1/3$ and the asymmetry levels $\gamma = 5$ (green), $\gamma = 3$ (red) and $\gamma = 1.5$ (blue). The dots are the different processes in the test batch, the solid lines are second order polynomials fitted to the data and the dashed lines are the theoretical curves for FOTD processes.

dominated FOTD system will get $t_{\text{off}}/t_{\text{on}} \approx \gamma$ while a delay-dominated FOTD system will get $t_{\text{off}}/t_{\text{on}} \approx 1$ if $\eta = 0$. The dependence between τ and the ratio $t_{\text{off}}/t_{\text{on}}$ for an FOTD system is shown in the dashed lines in Fig. 4 for some different asymmetry levels. Here $\eta \neq 0$, but as long as η is not too close to its upper bound, the limits are only slightly modified compared to (5).

Somewhat surprisingly other process types also seem to more or less follow this relation. In addition to the theoretical FOTD curves Fig. 4 also shows data for the test batch together with second order polynomials fitted to the data. As can be seen in the figure the other process types also follow the limits in (5). For delay-dominated processes ($\tau \approx 1$) the times become symmetrical, $t_{\text{off}}/t_{\text{on}} = 1$, while for lag-dominated processes the times instead reflect the asymmetry of the relay amplitudes $t_{\text{off}}/t_{\text{on}} = \gamma$, no matter the structure of the process.

By measuring the time intervals t_{on} and t_{off} and using the fitted polynomial, for the relay amplitudes that are used in the experiment, an easy way to estimate τ is achieved. The errors in τ when using the fitted polynomial are shown in Fig. 5. It can be seen that almost all processes in the batch stays within 0.1 of the correct value, and that the median errors are about 0.02, by using this estimation method with these choices of η and γ. The achieved results are accurate enough to use τ for classifying the process and decide on what, if any, additional steps that need to be taken by the autotuner algorithm.

4. MODELING

After a few relay switches K and τ can be estimated. With this information different choices on how to continue the autotuning procedure can be made. There is a possibility to calculate the parameters of an FOTD model directly from the relay feedback equations, this is described in Sec. 4.1. Another option is to use the parameter estimation method in Sec. 4.2 which uses all the experiment data. This approach is less sensitive to noise and could also be used to estimate higher order models. One alternative is also to use the information about τ to change the experiment setup, this is discussed in Sec. 4.3.
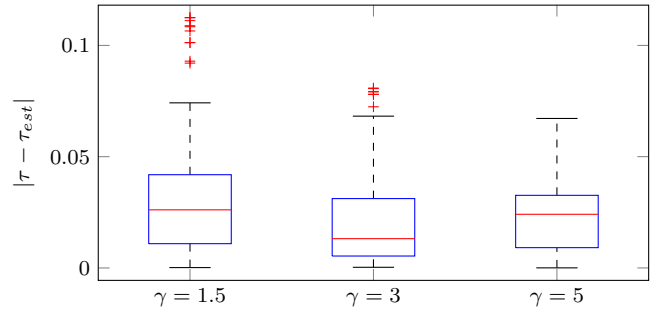

Fig. 5. Boxplots of the absolute errors when using the fitted polynomial instead of the real τ-values on the process batch. The three different asymmetry levels have the median errors 0.026, 0.013 and 0.024 respectively.

4.1 Calculate FOTD directly

The FOTD model has three parameters that needs to be found, K, T and L. With asymmetric relay feedback K can be calculated from (3). The parameters T and L could be found in a number of different ways. From the estimated τ the definition (2) is used to get the ratio L/T,

$$L/T = \frac{\tau}{1 - \tau}. \qquad (6)$$

This gives φ and from (4) T can then be calculated as

$$T = \frac{t_{\text{off}}}{\ln\left(\dfrac{\varphi - \gamma + \eta}{1 - \eta}\right)} = \frac{t_{\text{on}}}{\ln\left(\dfrac{\varphi - 1 + \eta}{\gamma - \eta}\right)}. \qquad (7)$$

When T is found L is known from (6). The equality in (7) is only true if there are no disturbances and t_{on} and t_{off} are measured correctly. In real experiments T can be estimated by taking the mean value of these two expressions.

4.2 Parameter estimation methods

With the increased computing power available today we can also consider estimating the parameters directly. Let (u_m, y_m) be the input output data obtained from a relay experiment of length t_m, and let $P(s)$ be the transfer function of the process model with parameters p. The output generated by $P(s)$ with the input u_m is y. The parameters p can then be obtained by minimizing the quadratic loss function

$$J(p) = \frac{1}{2} \int_0^{t_m} (y(t) - y_m(t)))^2 dt = \frac{1}{2} \int_0^{t_m} e(t)^2 dt. \qquad (8)$$

Optimization can be executed by computing the gradient J_p and the Hessian J_{pp} given by

$$J_p = \int_0^{t_m} y_p(t)e(t)dt, \qquad (9)$$

$$J_{pp} = \int_0^{t_m} y_p(t)y_p^T(t)dt + \int_0^{t_m} y_{pp}(t)e(t)dt. \qquad (10)$$

The sensitivity derivatives y_p and y_{pp} can be computed efficiently, see Åström and Bohlin (1965). A good approximation of the Hessian is obtained by dropping the second term in (10). Newton's method can then be used to obtain the parameters that minimize $J(p)$. The initial parameters required by Newton's method can be obtained from the approximate fitting described in Sec. 4.1.

As an example we consider modeling of the system $P(s) = (s + 1)^{-4}$. The upper plot in Fig. 6 shows the results
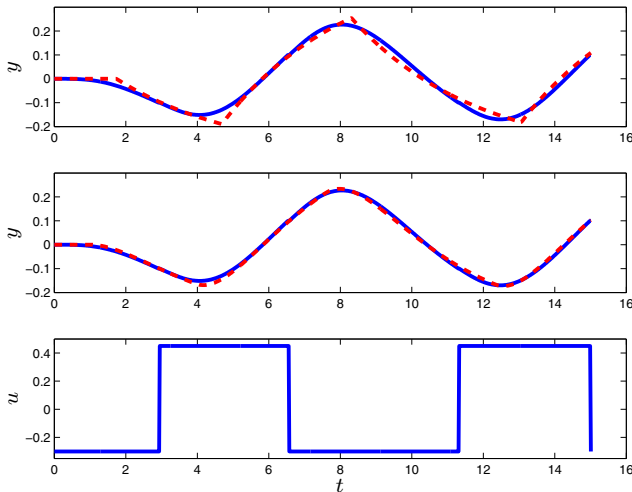
Fig. 6. Parameter estimation for $P(s) = (s+1)^{-4}$. In the upper plot an FOTD model has been fitted to the data, in the middle plot an SOTD model has been fitted. The lower plot shows the input signal from the relay, with parameters given in Sec. 6.

when fitting an FOTD model to the data from a relay experiment. The minimal loss function is $J = 4.9 \cdot 10^{-3}$ and the estimated parameters are $K = 1.113, L = 1.736, T = 3.511$. The figure shows that the FOTD model fits the data reasonably well. An SOTD model on the form

$$P(s) = \frac{K}{(1+sT)^2} e^{-sL}$$

which only has three parameters was also fitted to the data. The initial parameters for this model were the same as for the FOTD model except that T_{start} was halved. The SOTD model gives a better fit, see the middle plot in Fig. 6, with $J = 7.4 \cdot 10^{-4}$. The SOTD parameters are $K = 1.020, L = 1.008, T = 1.648$. Notice that L is smaller than for the FOTD model and T is approximately half the size as for the FOTD model. The condition number of J_{pp} is 80 which indicates that the excitation is reasonable. Attempting to fit models with more parameters gives Hessians with much larger condition numbers. To fit models with more parameters it is thus necessary to make additional experiments with more excitation.

4.3 Use knowledge of τ to modify the experiment

Controller tuning could be improved if the information given from τ is considered. Depending on τ the choice of whether to use a PI controller or a PID controller can be made. This makes it possible to adjust the experiment to get a model better suited for the controller type chosen. A model that a PID controller should be tuned for needs to be accurate in a higher frequency interval than a model that should be used for PI tuning. There are different ways the relay experiment could be modified to change the excitation to the wanted frequencies. Fig. 3 shows that the excitation changes a little by changing the asymmetry level of the relay. Other methods to move the excitation is e.g., to change the hysteresis, add an integrator in the feedback loop, use a Two-Channel Relay like Friman and Waller (1997), use the method described in Soltesz and Hägglund (2011) or to use a chirp signal.

For large τ an FOTD model is a good approximation, while a $\tau < 0.5$ indicates that a higher order model might be needed for tuning purposes (Åström and Hägglund, 2005, ch 7). How to get this model is something that needs to be investigated. One possibility is to use the parameter estimation methods mentioned earlier with appropriate initial guesses and experiment data with improved excitation.

5. PRACTICAL CONSIDERATIONS

There are two different situations worth separating when it comes to the use of the autotuner. Apart from the use of autotuning for industrial controllers, a new application has also appeared recently. When large systems are explored by simulation there is often the need to keep certain variables at prescribed levels. This can be accomplished by PID control and autotuning can be useful for finding suitable controller parameters. This application is different from the typical process control problem since there are no difficulties with measurement noise and much greater freedom in experimentation. Different aspects of how to choose the autotuner parameters, depending on situation, will be described shortly in this section.

5.1 Noise and choice of hysteresis

The first step of an autotuning procedure is to measure the noise level of the signal. The hysteresis can then be chosen to be about 2-3 times the noise level. If the noise level is too large the signals need to be filtered before starting the relay experiment, otherwise the output amplitudes required for the experiment will be too large. In the noise-free simulation environment the hysteresis could be chosen arbitrarily. In the simulations in this paper the hysteresis $h = 0.1$ has been used.

5.2 Convergence of limit cycles

If an FOTD system has a limit cycle it will converge to it after the first switch of the relay, see Lin et al. (2004). However, for other processes or if noise is added it is not certain that the limit cycle will be reached that fast. One parameter to consider in the relay experiment is therefore to decide when convergence to the limit cycle has been achieved. One method is to compare the time one period take, t_p, with the time the previous period took, t_p^*. If the difference between the period times is lower than a certain threshold ϵ, i.e.,

$$\left| \frac{t_p - t_p^*}{t_p^*} \right| \leq \epsilon \qquad (11)$$

the system is considered to have reached the limit cycle.

The convergence times of the simulations for the batch was investigated for $\epsilon = [0.005, 0.01, 0.05]$. The results showed that a few processes needed 3 periods to converge for $\epsilon = 0.005$ while most had converged in 2.5 or 2 periods. If ϵ was increased to 0.01 all processes had converged within 2.5 periods while with $\epsilon = 0.05$ all processes had converged within 2 periods. The resulting values of t_{off}/t_{on} achieved with $\epsilon = 0.01$ lied within 0.25% from the ones achieved with $\epsilon = 0.005$. With $\epsilon = 0.05$ the results deviated at most 1.15% from the values achieved with $\epsilon = 0.005$.
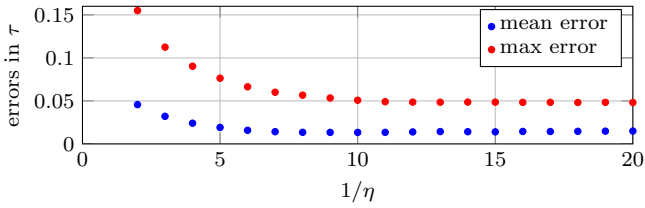
Fig. 7. Mean and maximum errors of the τ-estimations for ratios of $1/\eta$ between 2 and 20.

5.3 Relay amplitudes

The question of how to choose the relay amplitudes is subject to some different aspects. It is necessary that $1/\eta = Kd_2/h > 1$ for the output to reach above the hysteresis and create oscillations. In Fig. 7 the accuracy of the estimated τ is shown for some different values of $1/\eta$. It can be seen that you actually need some margin in this ratio to get a good estimation of τ. When the hysteresis is small enough in comparison to the relay amplitude, $1/\eta \geq 10$, the accuracy stays more or less constant.

The asymmetry level γ, i.e. the ratio between d_1 and d_2 is also something to consider. The results in Sec. 2.1 indicates that as high asymmetry as possible should be chosen to get good low-frequency excitation and good estimates of K.

By using a large ratio for $1/\eta$ and a high γ the results shown are more accurate. If the autotuner is to be used in simulation environments you could therefore use high values. However, if you are to use the autotuner on real processes in an industrial setup it is important not to deviate too much from the setpoint and you are therefore forced to keep these values low even if the performance is somewhat worse. In this paper the values have been set to $1/\eta = 3$ and $\gamma = 1.5$, these choices were made with the industrial case in mind.

6. EXAMPLES

To investigate the accuracy of the achieved models we look at the following processes that have also been explored by other methods in (Åström and Hägglund, 2005, p247-251)

$$P_1(s) = \frac{1}{(s+1)(0.1s+1)(0.01s+1)(0.001s+1)}$$

$$P_2(s) = \frac{1}{(s+1)^4} \qquad\qquad . \quad (12)$$

$$P_3(s) = \frac{1}{(0.05s+1)^2}\mathrm{e}^{-s}$$

P_1 has lag-dominated dynamics, P_2 is balanced and P_3 is delay-dominated.

By using the relay method, τ and the FOTD parameters K, T, L are achieved as described in Sec. 4.1. PI controllers are calculated according to the AMIGO rules in Åström and Hägglund (2005). The model and controller parameters are listed in Table 1. The parameters used in the relay experiments are $\epsilon = 0.01$, $h = 0.1$, $\eta = 1/3$ and $\gamma = 1.5$.

The models are compared with the true processes that have PI controllers with parameters k_p and k_i based on the MIGO design method (Åström and Hägglund, 2005, p247-250). These parameters are listed for the nominal models in Table 1. As an additional comparison the FOTD
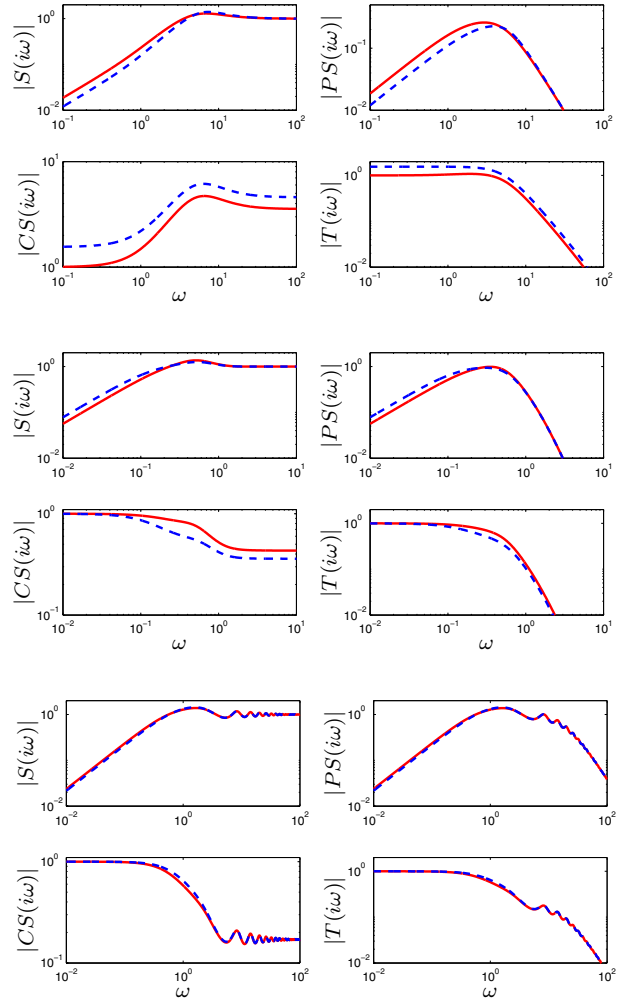

Fig. 8. Gain curves of the "gang of four" for the system obtained with a controller designed for the true process (full red lines) and the controller obtained by the experiment (dashed blue lines). The upper plots are for P_1, the middle ones for P_2 and the lower for P_3.

Table 1. Model and controller parameters

		K	T	L	τ	k_p	k_i
	Relay	0.999	1.115	0.073	0.062	4.59	8.36
P_1	Step	1	1.04	0.075			
	Nominal				0.067	3.56	5.39
	Relay	0.966	2.993	1.77	0.372	0.36	0.13
P_2	Step	1	2.90	1.42			
	Nominal				0.33	0.43	0.18
	Relay	1.006	0.078	1.03	0.929	0.17	0.46
P_3	Step	1	0.093	1.01			
	Nominal				0.92	0.17	0.42

parameters obtained with the step response method are also listed in the table. Fig. 8 illustrates the gain curves of the "gang of four", consisting of $S = 1/(1 + PC)$, $T = PCS$, PS and CS, where P and C are the transfer functions of the process and the controller. Fig. 9 shows the responses to a unit step load disturbance applied at the process input. Measures of robustness and performance given by the maximum sensitivity M_S and the Integrated Absolute Error IAE are listed in Table 2.

The results show that the FOTD parameters obtained with the relay experiment are close to the ones obtained
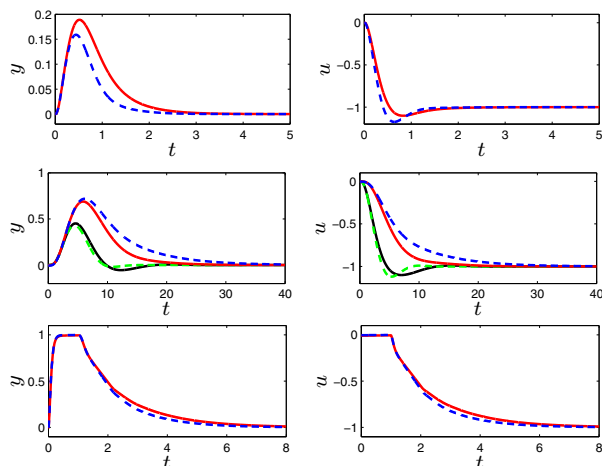
Fig. 9. Responses to a unit step load disturbance for the system obtained with a controller designed for the nominal process (full red line) and the one obtained from the relay experiment (dashed blue line).The upper plots are for P_1, the middle ones for P_2 and the lower for P_3. For P_2 the results for PID controllers derived for the true model (black) and an estimated SOTD model (dashed green) are also shown.

Table 2. Performance and robustness measures

	P_1		P_2		P_3	
	Relay	Nom.	Relay	Nom.	Relay	Nom.
IAE	0.120	0.186	7.714	5.615	2.159	2.341
M_S	1.398	1.293	1.256	1.367	1.436	1.388

with the step response method. In Fig. 8 it is seen that controllers based on the approximated models give similar gain curves as the MIGO design for the true models, especially for the delay-dominated process. Looking at the figures and Table 2 it is concluded that the controllers designed for the approximate models have a slightly worse robustness but better performance than the ones designed for the true processes for P_1 and P_3, while the opposite is true for P_2.

For P_2 we have also used the parameter estimation from Sec. 4.2 to get an SOTD model for which we have computed PID controller parameters from the formulas in (Åström and Hägglund, 2005, pp 247-251). This gives $k_p = 1.129, k_i = 0.517$ and $k_d = 1.232$ which compare favourable with those obtained by MIGO for the true process model $k_p = 1.440, k_i = 0.536$ and $k_d = 1.19$. The green curve in the middle plot in Fig. 9 shows the time responses for the PID controller calculated for the SOTD model and can be compared to the black curve that shows the PID controller based on the true process. The figure shows that the performance obtained by fitting an SOTD model to the relay experiment gives practically the same result as when applying the MIGO design method to the true process. The figure also shows the benefits of using PID control and a more complex model for this process.

7. CONCLUSION

In this paper we have outlined ideas for new relay autotuners. A method to find the normalized time delay τ from an asymmetric relay feedback experiment has been proposed. With early knowledge about τ an autotuner could modify its experiment in order to find a process model and controller tuning better suited for its purposes. The examples shows that an FOTD model derived directly from the relay experiment works good, but by using the information from τ the belief is that the performance could be improved. Some practical considerations on how to choose the experiment parameters were also discussed. However, the effects noise and disturbances have on the estimations need to be further investigated.

REFERENCES

Åström, K.J. and Bohlin, T. (1965). Numerical identification of linear dynamic systems from normal operating records. In *Proc. IFAC Conference on Self-Adaptive Control Systems*. Teddington, UK.

Åström, K.J. and Hägglund, T. (1983). Automatic tuning of simple regulators for phase and amplitude margins specifications. In *Proceedings of the IFAC Workshop on Adaptive Systems in Control and Signal Processing*. San Francisco, California.

Åström, K.J. and Hägglund, T. (2005). *Advanced PID Control*. ISA - The Instrumentation, Systems, and Automation Society, Research Triangle Park, NC 27709.

Friman, M. and Waller, K.V. (1997). A two-channel relay for autotuning. *Ind. Eng. Chem. Res.*, 36, 2662–2671.

Hägglund, T. and Åström, K.J. (1991). Industrial adaptive controllers based on frequency response techniques. *Automatica*, 27, 599–609.

Kaya, I. and Atherton, D. (2001). Parameter estimation from relay autotuning with asymmetric limit cycle data. *Journal of Process Control*, 11, 429–439.

Lin, C., Wang, Q.G., and Lee, T.H. (2004). Relay feedback: A complete analysis for first-order systems. *Ind. Eng. Chem. Res.*, 43, 8400–8402.

Liu, T. and Gao, F. (2008). Alternative identification algorithms for obtaining a first-order stable/unstable process model from a single relay feedback test. *Ind. Eng. Chem. Res.*, 47, 1140–1149.

Sell, N.J. (ed.) (1995). *Process Control Fundamentals for the Pulp & Paper Industry*. Tappi Press, Technology Park, Atlanta, GA.

Skogestad, S. (2003). Simple analytic rules for model reduction and PID controller tuning. *Journal of Process Control*, 13(4), 291–309.

Skogestad, S. (2006). Tuning for smooth PID control with acceptable disturbance rejection. *Industrial and Engineering Chemistry Research*, 45, 7817–7822.

Soltesz, K. and Hägglund, T. (2011). Extending the relay feedback experiment. In *18th IFAC World Congress*. Milano, Italy.

Srinivasan, K. and Chidambaram, M. (2003). Modified relay feedback method for improved system identification. *Computers and Chemical Engineering*, 27, 727–732.

Vilanova, R. and Visioli, A. (2012). *PID Controllers in the Third Millennium*. Springer, London, UK.

Wang, Q.G., Hang, C.C., and Zou, B. (1997). Low-order modeling from relay feedback. *Ind. Eng. Chem. Res.*, 36, 375–381.