Preprints of the 19th World Congress
The International Federation of Automatic Control
Cape Town, South Africa. August 24-29, 2014

A Novel Time Dependent Prandtl-Ishlinskii Model for Sensorless Hysteresis Compensation in Piezoelectric Actuators

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Abstract: Piezoelectric actuators are widely used for micromanipulation processes. High natural frequency, fine resolution and also response time are piezoelectric special properties. But the main problem is with the hysteresis nonlinear behavior which degrades its performance. Several hysteresis models have been proposed for hysteresis identification and compensation. The necessity of precise and high cost position sensors is the main drawback in hysteresis estimation. In this paper, it is analytically shown that the hysteresis compensation in mechanical domain (Voltage-Position) can be concurrently achieved by hysteresis compensation in electrical domain (Voltage-Charge). But hysteresis in electrical domain can be more complicated rather than mechanical one due to the time dependent behavior which is called “Active Hysteresis”. In fact, the output charge includes the main hysteresis compounded with an increasing behavior. As a result, conventional models cannot identify the hysteresis behavior. In this paper, a new active Prandtl-Ishlinskii model is proposed for active hysteresis estimation. This model can estimate the increasing output trend with the main hysteresis behavior, simultaneously. Experimental results confirm that the proposed model successfully estimate the time dependent hysteresis and also compensate it. It is shown that hysteresis in mechanical domain would also be compensated by this approach.

Keywords: Piezoelectric Actuators, Sensorless Hysteresis Compensation, Active Prandtl-Ishlinskii model

1 INTRODUCTION

Precise micro positioning has been an attractive research area especially in last decades. Micro positioning stages usually use piezoelectric actuators due to the properties such as high natural frequency, fine resolution and also response time. Two methods are usually utilized to derive piezoelectric actuators i.e. Charge driven and Voltage driven. In the first approach, the actuator shows a linear behavior between input charge and position output. But, electrical equipment for charge driving are so complicated and not easily applicable (Vautier et al. 2005). Voltage driven actuators are more applicable, but fine positioning suffers from the effect of hysteresis nonlinearity. This nonlinear property makes the control process a challenging problem. The major part of reported research work has been concentrated on hysteresis nonlinearity effect. Models by Preisach, Krasnosel’skii-Pokrovskii, Duhem and Bouc-Wen have been proposed for identification of hysteresis effect (Ghafarirad et al. 2011).

Prandtl-Ishlinskii (PI) model is also one of the popular hysteresis estimation approaches. The most important advantage of this model is its simplicity and that its inverse could be calculated analytically. The conventional PI model has been utilized for rate independent hysteresis estimation (Bashash et al. 2007). In addition, modified PI approaches have been proposed for non-symmetric and also rate dependent hysteresis estimation (Ang et al. 2007). By applying inverse hysteresis models as a compensator, hysteresis behavior could be eliminated.

In most previous researches, hysteresis identification has been carried out in mechanical domain i.e. Input Voltage-Position Output. Therefore, it is necessary to have a precise position sensor such as laser displacement sensor, laser vibrometer, capacitive or inductive sensors. Such sensors are too expensive especially in micro actuators as like as micro grippers and it is also difficult to incorporate into the equipment. To solve this problem, sensorless hysteresis compensation approaches such as image based methods have been proposed. One alternative approach is hysteresis identification and compensation in electrical domain. Piezoelectrics are usually modeled as a nonlinear capacitor which Voltage-Charge behavior is also hysteretic. It can be shown that hysteresis in mechanical domain can be compensated by hysteresis compensation in the electrical domain (Park et al. 2010). As a result, the necessity of high cost position sensors can be substituted by a simple charge measurement electric circuit.

The main problem is hysteresis behavior in electrical domain. Piezoelectrics do not usually behave as a pure capacitor. In fact, a resistor is also exerted in parallel with the capacitor (Ivanet al. 2009). Thus, charge output increases by time due to the resistor effect. It would generate a time dependent hysteresis behavior which is called "Active Hysteresis" in this paper. In fact, this behavior is a combination of main hysteresis caused by the nonlinear capacitor with the increasing charge due to the resistor. As a result, previous time independent hysteresis estimation approaches (Passive Models) will not be more applicable.

In this paper, it is analytically shown that hysteresis compensation in mechanical domain can be implemented by electrical domain. Then, an active time dependent PI model is proposed for active hysteresis estimation in electrical domain. This model can estimate the charge increasing trend and also identify the main hysteresis behavior simultaneously. Experimental results evaluate the proposed model performance in hysteresis identification and also compensation. It is shown that hysteresis is also compensated in mechanical domain by this approach.

2 HYSTERESIS NONLINEAR BEHAVIOR IN PIEZOELECTRICS

2.1 *Nonlinear Constitutive Equations*

Governing linear constitutive equations for a piezoelectric actuator driven under the coercive electric field (E_c) are as follows:

$$S = s^E T + d\,E \tag{1}$$

$$D = d\,T + \varepsilon^T E \tag{2}$$

S , T , E and D are Strain, Stress, Electric Field and Electric Displacement, respectively. d is the piezoelectric strain constant which defines the electromechanical coupling. s^E and ε^T are compliance in the zero electric field and permittivity in the zero stress, respectively. Equations (1) and (2) define the electromechanical behavior of piezoelectric. In fact, the first equation expresses the mechanical domain as an actuator and the second one is the electrical domain as a sensor.

Linear constitutive equations are not usually applied in piezoelectric actuators. If the driven electric field becomes greater than coercive field, the piezoelectric shows hysteretic nonlinear output e.g. Strain (S) or Electric Displacement (D).

In this condition, experimental investigation has confirmed that a linear relation between electric displacement and strain is still existed (Goldfarb et al. 1997). In addition, the linear strain-stress relation in the zero electric displacement would be remained (Goldfarb et al. 1997). Therefore the mechanical constitutive equation can be still stated in a linear form as follow

$$S = s^D T + g\,D \tag{3}$$

s^D is compliance in the zero electric displacement and g is the strain constant.

But, a hysteretic nonlinear relation is existed in electrical domain between electric field and electric displacement. In such a case, the total electric displacement can be divided to two parts (Cao et al. 2012).

$$D = D^r + D^{ir} \tag{4}$$

D^r is the reversible part containing the linear electrical behavior and D^{ir} is the irreversible part to show the hysteresis effect. Reversible part, D^r , can be defined by the equation (2). Therefore, the nonlinear constitutive relation would be as follows:

$$D = d\,T + \varepsilon^T E + D^{ir} \tag{5}$$

Electric displacement (Charge) is considered as an independent state variables in equation (3). It means that the actuator is charge driven. But, the electric field (Voltage) is the common driving input for the piezoelectric actuators. To be more compatible with conventional applications, electric displacement should be substituted by the electric field from the equation (5). As a result, new nonlinear constitutive equations with stress and electric field as independent variables can be found after some manipulation.

$$S = s^E T + d\,E + g\,D^{ir} \tag{6}$$

$$D = d\,T + \varepsilon^T E + D^{ir} \tag{7}$$

where $s^E = s^D + g\,d$ and $d = g\,\varepsilon^T$. It can be easily seen that the hysteresis relation has been entered in both equations i.e. $E-S$ and $E-D$, by different coefficients. Therefore, it can be deduced that by identification of each nonlinear behavior, another one can also be achieved.

Conventional approaches use external position sensors to measure the hysteresis relation in mechanical domain i.e. $E-S$. But precise position sensors such as laser sensors and laser vibrometers are so expensive. As an alternative approach, the nonlinear relation can be identified in electrical domain i.e. $E-D$, and then it would be scaled to the mechanical domain.

To measure the electric displacement, the piezoelectric electric charge Q_C can be considered instead as follows.

$$Q_C = \int_A D.dA \tag{8}$$

As a result, the hysteresis identification in electrical domain i.e. ($E-Q_C$), can be an alternative solution for sensorless hysteresis compensation in piezoelectric actuators. In addition, this hysteretic behavior i.e. $E-Q_C$, is also utilized in different researches such as physical study in electric domain behavior, self-sensing position control, external force estimation and etc.

But, the hysteresis behavior in electrical domain can be more complex rather than the mechanical one, especially in small size actuators.

2.2 *Hysteresis Behavior in Mechanical and Electrical Domains*

Hysteretic identification in electrical domain is often more complicated than mechanical domain. To investigate this effect, Fig 1 shows the hysteresis behavior in mechanical domain i.e. Voltage-Position ($V - S$).

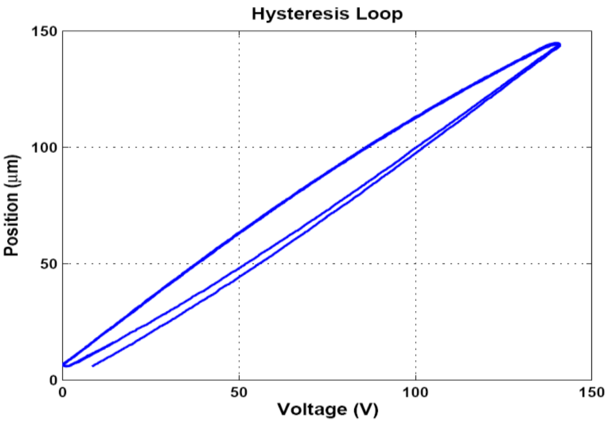


Fig 1. Hysteresis Behavior in mechanical domain

This general hysteretic behavior is almost similar in different actuators. But, the charge output behavior in electrical domain would be more complicated for some types of piezoelectric actuators such as piezoelectric cantilever due to their small size. In such cases, the input-output relation would be as Fig 2.

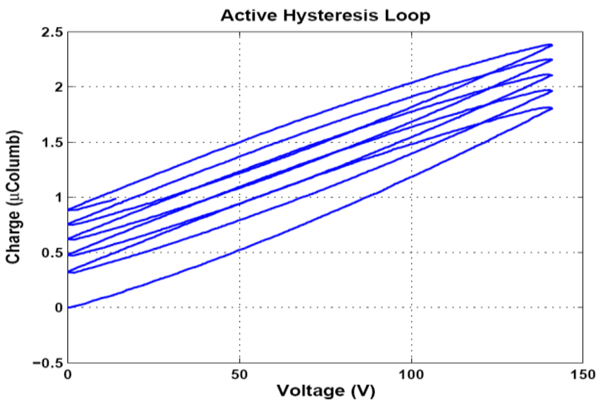


Fig 2. Input – Output hysteretic behavior

It is clear that the conventional hysteresis behavior cannot be seen in these actuators. Therefore, conventional hysteresis estimation approaches cannot be more useful.

The reason can be investigated in the more realistic electrical equivalent of piezoelectrics. In fact, piezoelectrics do not behave as a pure capacitor, but they have a resistor in parallel with the capacitor as shown in Fig 3.

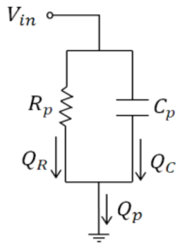


Fig 3. Electrical equivalent of piezoelectrics

In this structure, the measuring charge output Q_p is combination of nonlinear capacitance charge Q_C and resistor charge Q_R .

In many piezoelectric actuators, the resistor magnitude is high enough to not affect on the output charge. But, this impedance would be decreased due to the small size and therefore an increasing trend is appeared in the charge output. Such an increasing behavior is not existed in the mechanical domain. So it can be deduced that this increasing charge has not any effect on the actuator position. It can also be concluded that the hysteresis relation mentioned in equation (7), is expressed just for the capacitor charge output i.e. Q_C . Therefore the resistor charge Q_R should be omitted to achieve the pure capacitive charge and also main hysteresis effect.

The total input-output behavior is called active hysteresis in this paper. It includes the main hysteresis effect combined with an increasing trend. The purpose is to identify both the increasing trend and also real hysteresis behavior. After that, it can be used for growth rate elimination to achieve main hysteresis loop and also hysteresis compensation.

The resistance measurement is not possible. It is caused by nonlinear behavior of piezoelectric actuator impedances in different voltages. Experimental results done by the author show that different resistances are achieved for different driving voltages. This behavior is also reported in previous works (Zhang et al. 2013). In addition, the increasing behavior in output has some other sources in charge measurement circuit components such as Op-Amp current biases. Thus, the growth rate cannot be simply identified.

3 TIME DEPENDENT ACTIVE HYSTERESIS IDENTIFICATION

The increasing hysteretic behavior cannot be generally identified by conventional passive hysteresis estimation methods. Due to its time dependent behavior, identification requires some active time based estimation methods. As a result, a new active Prandtl-Ishlinskii (PI) model is proposed in this paper. In contrast to previous conventional models, the output not only depends on the input and its history, but the time has also an effect in this model. First, the conventional model is expressed and then the modification would be elaborated.

3.1 Conventional PI Hysteresis Models

Rate independent backlash operator is the primary operator in conventional PI model. Fig 4 shows its operator.

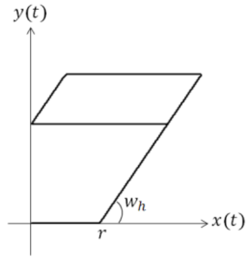


Fig 4. Conventional time independent backlash operator

This operator can be defined as:

$$y(t) = w_h\, H_r\left[x, y_0\right](t) \qquad (9)$$
$$H_r = max\left[x(t) - r, \min\left\{x(t) + r, y(t-T)\right\}\right]$$

$x(t)$ and $y(t)$ are control input and actuator output, respectively. r is the input threshold value or magnitude of backlash and T is the sampling time. The weight w_h defines the slope of operator.

The initial condition for the actuator output can be assumed as

$$y(0) = max\left[x(0) - r, \min\left\{x(0) + r, y_0\right\}\right] \qquad (10)$$

It can be usually equalled to zero.

A real complicated hysteresis loop can be identified by a linear superposition of many primary back lash operators with different thresholds and weights.

$$y(t) = \sum_{i=0}^{n} w_{hi} H_{ri}\left[x, y_{0i}\right](t) = \vec{w}_h^T \vec{H}_r\left[x, \vec{y}_0\right](t) \qquad (11)$$

where the vector parameters are as follows:

$$\vec{w}_h^T = \left[w_{h1} \;\dots\; w_{hn}\right] \qquad (12)$$
$$\vec{H}_r\left[x, \vec{y}_0\right] = \left[H_{r1}\left[x, y_{01}\right] \dots H_{rn}\left[x, y_{0n}\right]\right]$$
$$\vec{r} = \left[r_1 \;\dots\; r_n\right]$$
$$\vec{y}_0 = \left[y_{01} \;\dots\; y_{0n}\right]$$

3.2 Active Time Dependent PI Hysteresis Models

The primary backlash operator is modified to express the time dependent increasing trend of actuator output. For this purpose, the output is composed of two functions.

$$y(t) = y_1(t) + R\, y_2(t) \qquad (13)$$
$$y_1(t) = w_h\, H_r\left[x, y_{1_0}\right](t)$$
$$= w_h max\left[x(t) - r, \min\left\{x(t) + r, y_1(t-T)\right\}\right]$$
$$y_2(t) = x(t)T + y_2(t-T)$$

$y_1(t)$ is the primary backlash operator with the same definition as previous. $y_2(t)$ is an integrator function to model the increasing trend. R is the growth rate which determines the rate of increasing.

Also, the proposed modified operator can be equally represented as follow.

$$y(t) = y_1(t) + y_2(t) \qquad (14)$$
$$y_1(t) = w_h\, H_r\left[x, y_{1_0}\right](t)$$
$$= w_h\, max\left[x(t) - r, \min\left\{x(t) + r, y_1(t-T)\right\}\right]$$
$$y_2(t) = R\, x(t)T + y_2(t-T)$$

The modified back lash operator can be seen in Fig 5.

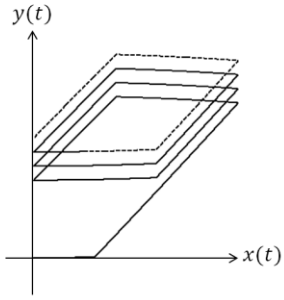


Fig 5. Active time dependent backlash operator

It can be seen that the modified backlash operator has three quantities to be identified i.e. threshold, weight and also growth rate.

An active hysteresis loop can be generated by a linear superposition of several primary backlash operators $y_1(t)$ with different thresholds and weights and just one integrating function $y_2(t)$ with one growth rate.

$$y(t) = \sum_{i=0}^{n} y_{1i}(t) + R\, y_2(t) \qquad (15)$$
$$= \sum_{i=0}^{n}\left\{w_{hi} H_{ri}\left[x, y_{1_{0i}}\right](t)\right\} + R\left\{x(t)T + y_2(t-T)\right\}$$
$$= \vec{w}_h^T \vec{H}_r\left[x, \vec{y}_{1_0}\right](t) + R\left\{x(t)T + y_2(t-T)\right\}$$

Weights, thresholds and also growth rate can be identified through the identification process.

Therefore by ignoring the effect of growth rate i.e. $x(t) - y_2(t)$, the main hysteresis behavior i.e. $x(t) - y_1(t)$ can be achieved. Therefore the hysteresis compensation can be implemented.

3.3 Identification Process and Inverse PI Model

The final stage is how to determine suitable operator parameters by the real input and output. For this purpose, the total output should be considered in a full vector form. Therefore the output is considered as follows:

$$y(t) = \vec{w}_h^T \vec{H}_r \left[x, \vec{y}_{1_0} \right](t) + R\{x(t)T + y_2(t-T)\} \quad (16)$$
$$= W^T X$$
$$W^T = \left[w_{h1}, \dots, w_{hn}, R \right]$$
$$X = \begin{bmatrix} H_{r1}\left[x, y_{1_{01}}\right], \dots, H_{rn}\left[x, y_{1_{0n}}\right], \\ \{x(t)T + y_2(t-T)\} \end{bmatrix}$$

For a set of input and output, the error function is defined.

$$E(w_{h1}, r, R, t) = y(t) - W^T X \quad (17)$$

Therefore, an optimization method can effectively estimate proper gains.

As a result, the pure hysteresis behavior $y_1(t)$ can be found by ignoring the increasing trend effect $y_2(t)$. If the main hysteresis is assumed as operator $y_1(t) = HYS(x)$, Then the inverse model of HYS operator is achieved as follows:

$$x = HYS^{-1}(y_1(t)) = \vec{w}_h'^T \vec{H}_r' \left[y_1, \vec{x}_0 \right](t) \quad (18)$$
$$w_{h0}' = \frac{1}{w_{h0}}$$
$$w_{hi}' = \frac{-w_{hi}}{\left[\sum_{j=0}^{i} w_{hj}\right]\left[\sum_{j=0}^{i-1} w_{hj}\right]} \qquad i = 1 \dots n$$
$$r_i' = \sum_{j=0}^{i} w_{hj}(r_i - r_j)$$
$$x_{0i} = \sum_{j=0}^{i} w_{hj}\, y_{1_{0i}} + \sum_{j=i+1}^{n} w_{hj}\, y_{1_{0j}} \qquad i = 1 \dots n$$

By applying the inverse model, the feedforward hysteresis compensation can be done in electrical domain as well as mechanical domain.

4 EXPERIMENTAL CASE STUDY

The proposed active PI model is experimentally utilized for hysteresis estimation and compensation.

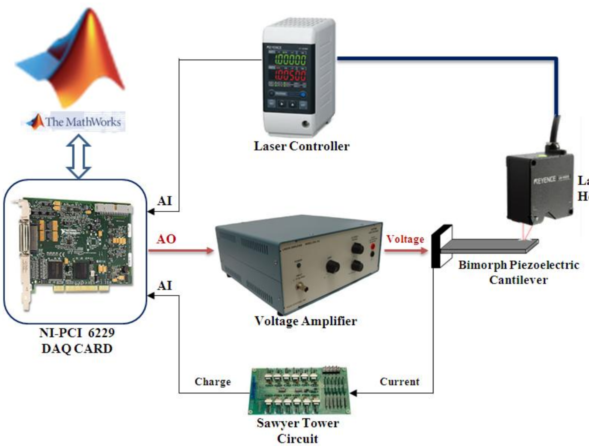


Fig 6. Experimental Setup

A bimorph piezoelectric actuator T226-H4-203X from the Piezo System Company is used for this purpose. NI data acquisition card is utilized for data capturing with the frequency of 1 KHz. A Keyence laser displacement sensor LK-H020 is used to measure the position of piezoelectric cantilever endtip by the resolution of 10 nm. In addition, an active Sawyer Tower circuit is utilized for piezoelectric charge measurement. Fig 6 shows the experimental setup.

4.1 *Hysteresis Identification and Growth Rate Compensation*

First, the actuator is driven by a quasi-static input voltage with the fixed amplitude of 140 Volt. The output charge is measured by the active Sawyer Tower circuit presented in the last section. By achieving the active hysteresis behavior, the proposed PI model is utilized for hysteresis estimation. Fig 7 shows the hysteresis curve and its estimation for fixed and multi amplitudes input voltages.

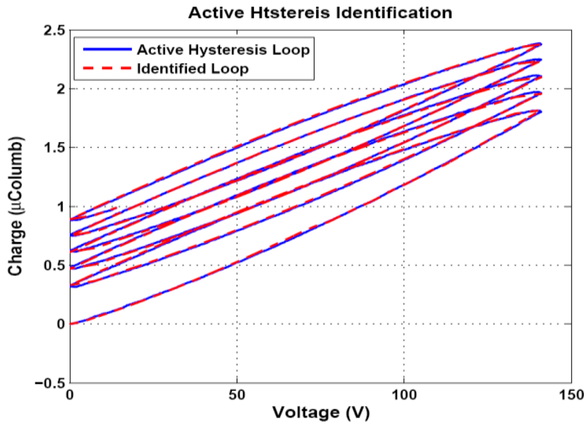


Fig 7. Active hysteresis estimation for fixed amplitude input

As mentioned before, the proposed model can simultaneously estimate the main hysteresis ($V - Q_p$) and also growth rate, R. To evaluate the validity of growth rate estimation, the identified increasing trend has been numerically eliminated from the output charge to achieve pure capacitance charge.

$$y_1(t) = y(t) - y_2(t) \quad (19)$$
$$y_2(t) = R\,x(t)T + y_2(t-T)$$
$$y_2(0) = 0$$

Fig 8 is the result of numerical growth rate cancelation. It is obvious that the proposed method could effectively estimate the increasing rate.

4.2 *Hysteresis Compensation and System Linearization*

The proposed method not only can estimate the increasing rate, but can also estimate the main hysteresis and also compensate it. By using the inverse model of identified hysteresis, the actuator output can be linearized as seen in Fig 9 .

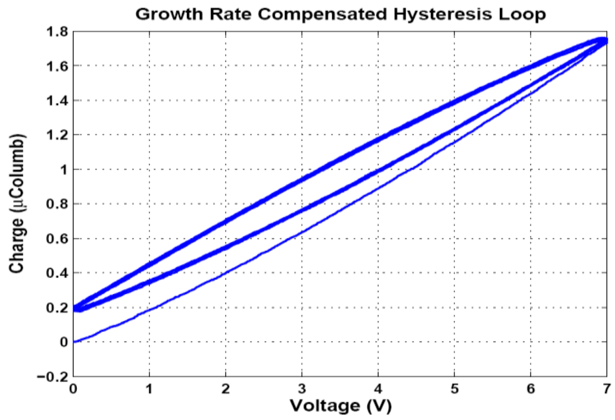


Fig 8. Numerically compensated growth rate

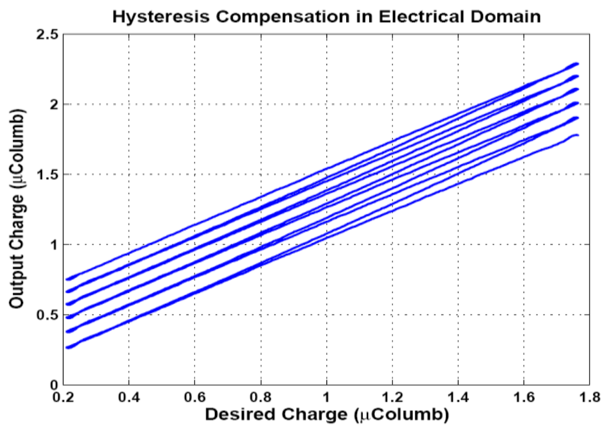
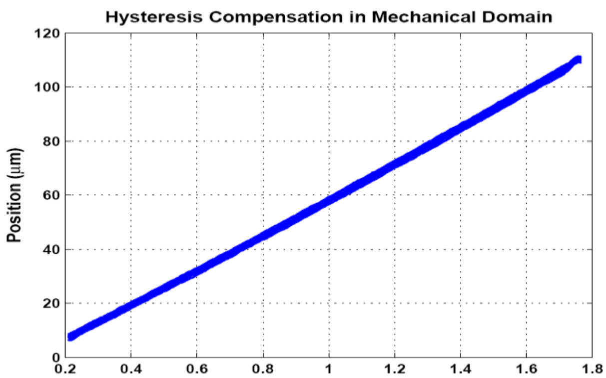


Fig 9. Hysteresis compensation in electrical domain



Desired Position Signal

Fig 10. Compensated hysteresis in mechanical domain

Although the increasing trend is seen in the charge output due to the resistance effect, but the behavior is linear. It means that the hysteresis has been compensated. It is expected that the mechanical hysteresis has been also compensated. The relation of desired signal and the actuator position has been linearized without any position sensor as seen in Fig 10.

5 CONCLUSION

Hysteresis identification and compensation in piezoelectric actuators requires high cost and precise position sensors. But, the hysteresis identification can be done in electrical domain with a simple electric circuit instead. It is analytically demonstrated that it would lead to the hysteresis compensation in mechanical domain as well. But the hysteresis in electrical domain can be time dependent. Therefore conventional passive models cannot be applicable. In this paper, a new active time dependent Prandtl-Ishlinskii model was proposed for active hysteresis identification in electrical domain for piezoelectric actuators. Experimental results confirmed that the proposed model can efficiently estimate the increasing trend of output charge. It can also identify the hysteresis behavior simultaneously. By utilizing the inverse of identified model, hysteresis can be compensated in both electrical and mechanical domain.

ACKNOWLEDGMENT

The authors would like to acknowledge the University of Malaya for providing the necessary facilities and resources for this research. This research was funded by the University of Malaya Research Grant (UMRG) Program No. RP001A-13AET.

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