Preprints of the 19th World Congress
The International Federation of Automatic Control
Cape Town, South Africa. August 24-29, 2014

Joint Charging and Routing Optimization for Electric Vehicle Navigation Systems ⋆

Chensheng Liu, Jing Wu, Chengnian Long

Shanghai Jiao Tong University, Shanghai, 200240 China (Corresponding author email:longcn@sjtu.edu.cn).

Abstract:
Electric Vehicle (EV) navigation system normally requires multiple charging in a long origin destination (OD) trip, which makes it different from the traditional vehicle navigation system. Fast charging is an ideal charging solution in multiple charging because of its high charging efficiency, rapid transient response and short charging time. However, EVs fast charging at on-peak hours may lead to overloading of distribution system. To solve the problem above, a real-time pricing (RTP) policy, as one of the main thrusts for load shaving application, should be introduced, which will bring new challenges in EV navigation systems due to the characteristic of time-dependence. In this paper, we address an optimization problem for EV navigation systems under the RTP policy with the consideration of both charging and routing. By using the EV arrival states and the traffic parameters, the original traffic network can be extended to a feasible state graph. An Improved Chrono-SPT (ICS) algorithm is provided to derive the optimal decision sequence, which provides an optimal routing and charging policy. Furthermore, a Simplify-Charge-Control (SCC) algorithm is also presented to reduce the computation complexity of the ICS algorithm. Simulations show the effectiveness of both ICS and SCC algorithms and the computation complexity of SCC algorithm is much simplified within acceptable deviation of optimal cost under approximation pricing (AP) than that in ICS algorithm.

Keywords: Electric Vehicle Navigation, Charging Control, Minimal cost routing.

1. INTRODUCTION

In advanced traveler information systems, recent efforts have been made in developing a new navigation concept called "eco-routing", which finds a route that requires the least amount of fuel (Boriboonsomsin et al. (2012)). However, for electric vehicles (EVs), eco-routing, mainly focusing on finding a minimum energy path, can't express the driving cost accurately because the cost is affected not only by energy consumption but also by charging price, especially when the price is time-dependent, e.g., real-time pricing (RTP). Compared with the traditional navigation systems, EV navigation systems under RTP policy consider a coupled situation to derive the lowest cost for the long origin destination (OD) trip, that is, both where to charge and how much to charge are decided simultaneously. It becomes a joint charging and routing problem rather than a common shortest path routing problem, which bring new challenges in EV navigation systems.

In literature, EV navigation has been studied in some works. For example, Sachenbacher et al. (2011) studied a energy-optimal routing problem with the consideration of recuperation, battery limitation, etc.. The same problem was considered by Artmeier et al. (2010). Both of them formulate EV routing as a constrained shortest path problem with hard and soft constraints. They focus on the navigating in only one-charge distance and take EVs as traditional vehicles with capacity limitation and recuperation. Bessler and Grønbæk (2012) discussed an routing policy towards optimal charging plan, where matching energy supply and demand is an auxiliary service providing to grid operator. Stein et al. (2013) presented a minimum delay time of an OD trip from EV drivers' point of view. By using intention-aware method, the system can accurately predict the congestion at charging stations and a path with minimum delay time is given. However, it only focuses on the multiple charging problem of the waiting/delay time.

In this paper, we address a minimum-cost path problem under RTP with multiple charging in a long distance OD trip. The optimal cost problem with a travel-time limitation is formulated as a dynamic programming, which couples the optimal path problem with a charging control problem. According to the EV arrival states and the traffic parameters, we transform the the original traffic to a feasible state graph, where an improved Chrono-SPT (ICS) algorithm is provided to derive the minimum cost path. Since the navigation systems mostly operate on embedded devices and drivers commonly expect a rapid response, the computation complexity of algorithm is strictly restricted. In order to reduce the computation complexity, a simplify-charge-control (SCC) algorithm is designed, utilizing the characteristic of charging in a constant price interval. By simulation, it's easy to see the SCC algorithm can greatly reduce the computation complexity within acceptable deviation of optimal cost under approximation pricing (AP).

The following sections are organized as follows: A detailed system modeling is presented in Section 2. In Section 3, the problem is formulated in a dynamic programming. To solve the problem, an ICS algorithm and a SCC algorithm

⋆ The research was supported in part by the Natural Science Foundation of China (NSFC) under Grant 61172064, 61104091, the Program for New Century Excellent Talents in University under Grant NCET-11-0326, the Specialized Research Fund for Doctoral Program of Higher Education under Grant 20100073120061, the SJTU Science and Technology Innovation Funding under Grant AE0300006.

are derived in Section 4. Simulations are given in Section 5. Finally, the conclusion is made in Section 6.

2. SYSTEM MODEL

Symbol	physical means
k_c	The constant charging rate.
t_i^s	The arrival time of node i.
t_i^u	The arrival time of the successor node decided at node i.
t_i^c	The charging times at node i.
τ_{ij}^l	The travel times of link (i, j).
p_{ij}^l	The EV's power at link (i, j).
v_{ij}^l	The average speed at link (i, j).
d_{ij}^l	The distance of link (i, j).
e_{ij}^l	The energy consumed at link (i, j).
$p(t)$	The real time electricity prices.
p_{od}	The path from origin to destination.
$c(s_k, u_k)$	The charging cost at kth stage.
$C_{p_{od}}$	The cost of path p_{od}.
$C(s_1, s_k)$	The cost between state s_1 to s_k.
V_n	Thwe minimal cost of the first n stages.
$S_{ij}^f(x)$	Feasible state set of state x link(i, j).
$FS(i)$	The link set originate from node i.
$x = (i, t_i^s, e_i^s)$	The feasible arrival state of node i.
$s_k = (v_k^s, t_{v_k^s}^s, e_{v_k^s}^s)$	The state variable at the kth stage.
$u_k = (v_k^u, t_{v_k^u}^u)$	The decision variable at the kth stage.
$\Gamma = \{\lambda_1, \lambda_2, \ldots \lambda_r\}$	The price changing time set.
$T = \{t_1^d, t_2^d, \ldots, t_q^d\}$	The discrete time instant set.
$E = \{e_1^d, e_2^d, \ldots, e_p^d\}$	The discrete energy set of interval $[e_{bas}, e_{reg}]$.

Table 1. Nomenclature Table

Consider a long origin destination trip from origin node to destination node. Due to the limit of cruise range, EVs have to drive from one charging station to another in order to arrive at the destination eventually. We assume there is an optimal path between two charging stations within one distance-per-charge, which is the basic part of a path from origin to destination.

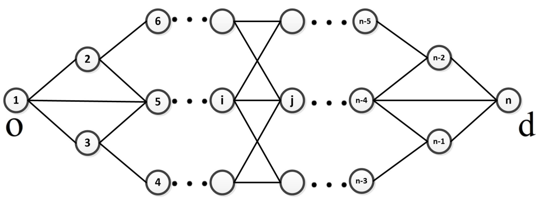


Fig. 1. An OD trip example.

The trip can be abstracted as a network topology $G = (N, A)$, as shown in Figure 1. In the network, N ($|N| = n$) is the set of nodes, which includes all en-route charging stations, origin nodes o, and destination node d. A ($|A| = m$) is the set of links, which denotes the optimal path between two stations. In the network, we denote $s_k = (i, t_i^s, e_i^s)$ as the states when EVs arrive at station i, where t_i^s is the arrival time, e_i^s [1] is the energy left. Let $u_k = (j, t_j^u)$ be the control decision, based on the state s_k, where j is the successor node of node i, t_j^u is the arrival time of node j. The nomenclature is presented in Table 1.

2.1 Fast Charging

The fast charging process can be modeled as a linear function. According to the data from Eaton whose products are based on CHAdeMO and SAE Combo standards, fast chargers can recharge the batteries to 80% capacity in as little as 30 minutes(Eaton (2013)). The charge rate is approximately constant when state of charge (SoC [2]) is less than 80% and drop steeply when SoC is greater than 80%. In order to saving time, we assume the drivers will not charge any more when SoC is greater than 80%, and define the associate energy in battery as e_{reg}, ($e_{reg} < e_{cap}$). On the other hand, batteries discharge excessively will shorten the cycle lives significantly. We define the associate energy in battery as e_{bas}. Suppose the energy in interval $[e_{bas}, e_{reg}]$ can be dispersed as $E = \{e_1^d, e_2^d, \ldots, e_p^d\}$, where $e_1^d = e_{bas}, e_p^d = e_{reg}$ respectively. The energy left when arriving at intermediate node i is e_i^s, $e_i^s \in E$.

Suppose the current state is $s = (i, t_i^s, e_i^s)$. We define $e_i'^s = e_i^s + k_c \cdot t_i^c$, where $e_i'^s$ is energy left in battery when EV leaves node i; k_c is constant charging rate; t_i^c is time charged at station i. So the energy recharged at station i is $e'^s - e^s = k_c \cdot t_i^c$. When the decision based on the current state is (j, t_j^u), the charged time can be expressed as $t_i^c = t_j^u - \tau_{ij}^l - t_i^s$, where τ_{ij}^l is the travel times of link (i, j). Above all, the energy recharge at node i can be expressed by $k_c \cdot t_i^c = k_c \cdot (t_j^u - \tau_{ij}^l - t_i^s)$.

2.2 Link Energy Consumption

Link energy consumption is a parameter used to express the energy consumed on the links, which limit the decisions. In the network shown in Figure 1, energy consumption of link (i, j) can be calculated using link distance d_{ij}^l and link travel times τ_{ij}^l. According to the power-speed curve of Tesla Roadster (TeslaMotors (2008)), the total power can be expressed as a function of speed, even though some components are not speed-dependent.

We assume that the power function of link (i, j) is $p_{ij}^l = g(v_{ij}^l)$, where v_{ij}^l is the speed of EVs at link (i, j). Then the energy consumption of link (i, j) is $e_{ij}^l = p_{ij}^l \cdot \tau_{ij}^l = g(v_{ij}^l) \cdot \tau_{ij}^l$, where $v_{ij}^l = d_{ij}/\tau_{ij}$. In all, the link energy consumption can be expressed as $e_{ij}^l = f(d_{ij}^l, \tau_{ij}^l)$, which is an important limiting condition of decision making.

2.3 Real-time Electricity Pricing

Real-time electricity pricing is one category of time-based electricity pricing, which is used to reflect dynamic cost of generation and motivate load shifting. Empirically, the real-time prices fluctuate by an order of magnitude from low-demand night-time hours to high-demand afternoons. According to the statistical analysis of the real-time prices used by Illinois Power Company from January 2007 to December 2009(Mohsenian-Rad and Leon-Garcia (2010)), the electricity prices fluctuate in a small interval during the off-peak or on-peak period, while fluctuate varied violently in hourly at boundary hours. In this paper, we assume there is a *price predictor* unit, which estimates the upcoming prices of 24 hours by applying a weighted averaging filter to past prices. And we will solve the minimum cost problem based on the prediction.

Referring to the instance of Illinois Power Company hourly varying RTP prices, we model RTP prices as a step

[1] e_i^s is the amount of energy left in battery, whose units are Joules.

[2] SoC is the equivalent of a fuel gauge for the battery pack in a EV, whose units are percentage points. It can be expressed as $SoC = e^s/e_{cap}$ in this paper.

function $p(t)$, whose definition domain is discreted as $T = \{t_1^d, t_2^d, \dots, t_q^d\}$, where t_q^d is the latest arrival time set by drivers. There is a price changing time instant set $\Gamma = \{\lambda_1, \lambda_2, \dots, \lambda_r\}$, where the price is constant in the half closed price interval, e.g., the price is constant in the interval $[\lambda_n, \lambda_{n+1})$. In the price changing time instant set Γ, $\forall \lambda_n, \lambda_{n+1} \in \Gamma$, $min\{\lambda_{n+1} - \lambda_n\} \geq 1\text{h}$, i.e., all the constant price intervals are equal or longer than one hour.

2.4 The Cost of Intermediate Stations

The cost at intermediate stations is determined by time to charge and the energy recharged. On condition that the arrival state of node i is $s_k = (i, t_i^s, e_i^s)$ and the decision is $u_k = (j, t_j^u)$, the charging cost at node i based on the arrival state and decision can be expressed as

$$c(s_k, u_k) = \int_{t_i^s}^{t_j^u - \tau_{ij}^l} p(t) \cdot k_c \cdot dt.$$

According the description in 2.3, the constant price intervals are all larger than one hours, while the fast charging time is less than half an hour. So the charging period should be in one or two adjacent price intervals. The charging cost of node i based on state $s_k = (i, t_i^s, e_i^s)$ and $u_k = (j, t_j^u)$ can be expressed as follows:

$$c(s_k, u_k) = \begin{cases} k_c \cdot p(t_i^s) \cdot (t_j^u - t_i^s - \tau_{ij}^l) \\ \qquad\qquad\qquad \lambda_n \leq t_i^s \leq t_j^u \leq \lambda_{n+1} \\ k_c \cdot [p(t_i^s) \cdot (\lambda_n - t_i^s) + p(t_j^u) \cdot (t_i^s - \lambda_n)] \\ \qquad\qquad\qquad \lambda_{n-1} \leq t_i^s \leq \lambda_n \leq t_j^u \leq \lambda_{n+1} \end{cases} \tag{1}$$

where $\lambda_{n-1}, \lambda_n, \lambda_{n+1} \in \Gamma$.

3. PROBLEM FORMULATION

Consider a minimum cost path problem under RTP pricing in a discrete time set T. Suppose EVs depart from origin node with a full charged battery at time $t_o^s, t_o^s \in T$. The traffic network $G = (N, A)$ is shown in Figure 1. The RTP prices is modeled as a step function $p(t)$ with a price changing time instant set $\Gamma = \{\lambda_1, \lambda_2, \dots, \lambda_r\}$. A dynamic programming formulation is given below.

3.1 State Variables

We define state variables at intermediate nodes $s_k = (v_k^s, t_{v_k^s}^s, e_{v_k^s}^s)$ as the starting state of the kth stage, i.e., the states when EVs arrive at charging station v_k^s. In the state variable above, $v_k^s \in N$, $t_{v_i^s}^s \in T$, $e_{basic} \leq e_{v_k^s}^s \leq e_{reg}$, Since that EVs depart from origin with full charged battery e_{cap} at time t_o^s, the starting state is $s_1 = (o, t_o^s, e_{cap})$, where $e_{cap} \geq e_{reg}$. Since less energy left when arriving at destination will lead to less cost, the state variable at destination node d at the beginning of Kth stage, can be expressed as $s_K = (d, t_d^s, e_{bas})$, where $t_d^s \leq t_q^d$,

3.2 Decision Variables

Based on current state $s_k = (v_k^s, t_{v_k^s}^s, e_{v_k^s}^s)$ and global network information, EVs make decision $u_k = (v_k^u, t_{v_k^u}^u)$, where v_k^u is the successor node, $t_{v_k^u}^u$ is the arrival time at the successor node. The relationship of state and decision variables can be expressed as follows:

$$\begin{aligned} &v_{k+1}^s = v_k^u; t_{v_{k+1}^s}^s = t_{v_k^u}^u \\ &e_{v_{k+1}^s}^s = e_{v_k^s}^s + (t_{v_k^u}^u - t_{v_k^s}^s - \tau_{v_k^s v_k^u}^l) \cdot k_c - e_{v_k^s v_k^u}^l \end{aligned} \tag{2}$$

Due to the energy limitation in state variable, $e_{bas} \leq e_{v_k^s}^s \leq e_{reg}, k > 1$, the energy in battery when departing from v_k^s should be large enough to arrive at the successor station v_i^u, i.e.,

$$e_{v_k^s v_k^u}^l + e_{bas} \leq e_{v_k^s}^s + k_c \cdot (t_{v_k^u}^u - t_{v_k^s}^s - \tau_{v_k^s v_k^u}^l) \leq e_{reg} \tag{3}$$

Some specific definitions should be given based on the state of origin and destination nodes. We define that the decision at origin node is $u_1 = (v_1^u, t_{v_1^u}^u)$, where $t_{v_1^u}^u = t_1 + \tau_{ov_1^u}^l$. The decision at the $(K-1)$th stage is $u_{K-1} = (d, t_d^u)$, where $t_d^u \leq t_q^d$.

3.3 Recursive Value Equation

During the travel from origin to destination with the starting state s_1, there is a decision sequence $U = \{u_1, \dots, u_{K-1}\}$, and a state sequence $S = \{s_1, \dots, s_K\}$ correspondingly. The path from origin to destination can be defined as follows:

***Definition** 1.* A path between the origin and destination for starting state s_1 is a sequence of ordered triplets.

$$p_{od} = ((v_1^s, t_{v_1^s}^s, e_{v_1^s}^s), (v_2^s, t_{v_2^s}^s, e_{v_2^s}^s), \dots, (v_K^s, t_{v_K^s}^s, e_{v_K^s}^s))$$

Considering the relationship of state variables and the decision variables in Equation (2), the state s_k can be calculated by the sequence $(s_1, u_1, \dots, u_{k-1})$. Then the path can be expressed by the decision variables as well:

$$p_{od} = ((v_1^s, t_{v_1^s}^s, e_{v_1^s}^s), (v_1^u, t_1^u), \dots, (v_{K-1}^u, t_{K-1}^u))$$

Since the cost of slow charge is much cheaper than fast charge, EVs depart from origin with full charged battery, and arrive at destination with the least energy left, the cost of the path can be expressed as the sum of the cost at intermediate nodes.

$$C_{p_{od}} = \sum_{k \in \{2,3,\dots K-1\}} c(s_k, u_k)$$

where $c(s_k, u_k)$ is modeled in Equation (1), the relationship of decision and state variables are modeled in Equation (2), and the decision variables are subject to Inequality (3).

In all, the minimum path problem can be translated to find the optimal decision sequences based the starting state.

$$U^* = arg \min_u \sum_{k \in \{2,3,\dots K-1\}} c(s_k, u_k)$$

Define the value function of sequence $(s_1, u_1, \dots, u_n), n \leq K-1$ as $V_n \triangleq \min \sum_{k \in \{2,3,\dots,n\}} c(s_k, u_k)$, the cost of sequences $(s_1, u_1, \dots, u_n, u_{n+1})$ can be rewritten as follows:

$$\begin{aligned} V_{n+1} &= \min \sum_{k \in \{2,3\dots n+1\}} c(s_k, u_k) \\ &= \min\{c(s_{n+1}, u_{n+1}) + \min \sum_{k \in \{2,3\dots n\}} c(s_k, u_k)\} \\ &= \min\{c(s_{n+1}, u_{n+1}) + V_n\} \end{aligned} \tag{4}$$

Note that the discussion given above is based on the state variables, e.g., V_n denotes the cost between s_1 and s_n implicitly.

4. JOINT CHARGING AND ROUTING OPTIMIZATION

In this section, we transform the original problem into a classical shortest path problem by extending the origin traffic network to a feasible state graph based on several definitions. An ICS algorithm is present based on the feasible state graph. In order to simplify the computation complexity, a simplified state graph and a SCC algorithm are present using the property of charging control in constant price intervals.

4.1 Feasible State Graph

In this paragraph, the feasible state graph is structured based on the definitions below. The process of extending the origin traffic network to feasible state graph is demonstrated in Figure 2.

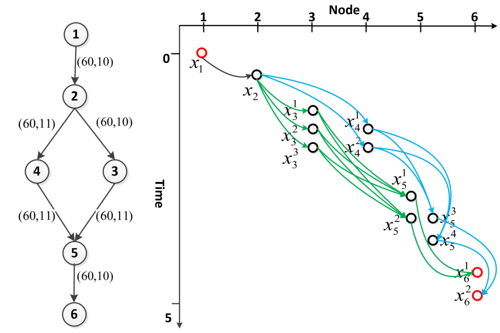
Suppose there is a traffic network $G = (N, A)$ shown in Figure 2(a). The labels pasted on the links are link travel times and link energy consumption, e.g., $(60, 10)$ on link (1,2) mean that the link travel time is 60 minutes and the link energy consumption is 10kWh. Suppose there is a battery with a capacity of 20kWh on the EV, the charging rate is 30kW (recharge the battery to 80% of capacity within 30 minutes), and EV will leave the charging station no latter than the time when the battery is recharged to 80% of capacity. Suppose the EV start the trip from node 1 to node 6 at time $t = 0{:}00$ with a full charged battery, i.e., the starting state is $x_1 = (1, 0{:}00, 20)$, several definitions based on the traffic network are given as follows:

***Definition** 2.* In network $G = (N, A)$, $\forall i \in N$, define the forward link set $FS(i) \triangleq \{(i, j)|(i, j) \in A\}$. For example in Figure 2(a), $FS(2) = \{(2, 3), (2, 4)\}$.

***Definition** 3.* Suppose the current state is $x = (i, t_i^s, e_i^s)$, the feasible states set of x under link $(i, j) \in FS(i)$ can be defined as $S_{ij}^f(x) \triangleq \{(j, t_j^s, e_j^s)|\ t_j^s = t_i^s + \tau_{ij}^l + \frac{e_j^s + e_{ij}^l - e_i^s}{k_c}$, $e_j^s \in \{e_m^d | e_{bas} \leq e_m^d \leq e_{reg} - e_{ij}^l\}\}$. Specially, when i is the origin node, $t_j^s = t_i^s + \tau_{ij}^l$, $e_j^s = e_{cap} - e_{ij}^l$. For example, in Figure 2(a) the feasible state set of $x = (2, 1{:}00, 10)$ under link $(2, 4)$ is $S_{24}^f = \{(4, 2{:}10, 4), (4, 2{:}12, 5)\}$, specially $S_{12}^f = \{(2, 1{:}00, 10)\}$.

***Definition** 4.* For the given starting feasible state $x = (o, t_o^s, e_{cap})$, we define $S_1^x = \bigcup_{oi \in FS(o)} S_{oi}^f(x)$; $S_2^x = \bigcup_{ij \in FS(i)} S_{ij}^f(x)$, $x = (i, t_i^s, e_i^s), x \in S_2^x$; $S_{k+1}^x = \bigcup_{ij \in FS(i)} S_{ij}^f(x)$, $x = (i, t_i^s, e_i^s), x \in S_k^x$. The feasible state graph $R = (V, E)$ can be defined as $V = \{x = |x \in S_k^x, k \in N^+\}$, $E = \{(x, x')|x = (i, t_i^s, e_i^s), x' \in S_{ij}^f(x), ij \in FS(i)\}$.

Based on the definitions above, the feasible state graph can be structured. Since the EV depart with a full charged battery, i.e., it is unnecessary to recharge at node 1, the state when arriving at node 2 is (2, 1:00, 10), where the arrival time is 1:00, energy left in battery is 10kWh. For link (2,3), limited by the stop discharging energy and the leaving energy assumption, the leaving energy should be in interval $[14, 18]$. Disperse the energy in every 1 kWh, the feasible arrival states at node 3 is $S_{23}^f(x_2) = \{x_3^1 = (3, 2{:}08, 4), x_3^2 = (3, 2{:}10, 5), x_3^3 = (3, 2{:}12, 6)\}$. Similarly $S_{24}^f(x_2) = \{x_4^1 = (4, 2{:}10, 4), x_4^2 = (4, 2{:}12, 4)\}$, $S_{35}^f(x_3^1) = S_{35}^f(x_3^2) = S_{35}^f(x_3^1) = \{x_5^1 = (5, 3{:}30, 4), x_5^2 = (5, 3{:}32, 5)\}$, $S_{45}^f(x_4^1) = S_{45}^f(x_4^2) = \{x_5^3 = (5, 3{:}32, 4), x_5^4 = (5, 3{:}34, 5)\}$. Select the feasible states from previous feasible state sets, and structure new feasible state sets based on the selected feasible states. The feasible state graph can be structured by iterating the steps above. The feasible state graph $R = (V, E)$ extended from $G = (N, A)$ is shown in Figure 2(b).



(a) A traffic network. (b) Feasible state graph.

Fig. 2. An example of structuring a feasible state graph.

4.2 Improved Chrono-SPT Algorithm

Similar to the process transforming origin traffic network into feasible state graph, an improved chrono-SPT (Dial (1969)) algorithm is designed. In ICS algorithm, the minimal cost charging and routing policy can be searched chronologically in a classical shortest path algorithm manner, and the feasible state graph can be formed simultaneously.

In ICS algorithm, we use a bucket-list $\boldsymbol{B} = \{B_{t_o^s}, \ldots, B_{t_i^s}, \ldots, B_{t_d^s}\}$, $t_d^s \leq t_q^d$, to efficiently perform the selection operations, where $B_{t_i^s}$ denotes the bucket containing the feasible states to be visited at the time instant t_i^s, $t_i^s \in T$. In our notation, $A(x)$ indicates the feasible state ahead of the current node x in R, while $C(x)$ associated with state x denotes the the cost between the starting state s_1 and the current state x.

We describe the typical algorithm iteration about state $x = (i, t_i^s, e_i^s)$ and $x' = (j, t_j^s, e_j^s)$ in Algorithm 1, where B_h denotes the current non-empty bucket. In the initial steps, we initialize $B_1 = \{(o, t_o^d, e_{cap})\}$ according to departing state variable $s_1 = (o, t_o^d, e_{cap})$, while the other bucket are all empty. The initial cost label of starting state, $C(s_1)$, is zero. The stop condition is verified when all the buckets are empty (when this happens, the minimum of the labels associated with each feasible states gives the optimum path cost from origin to destination).

In the algorithm above, lines $4 \sim 8$ and lines $11 \sim 13$ are the feasible states graph initialization, the *refresh steps* are steps of classical shortest path searching algorithm in order to mark the minimum cost path from starting state to current state. In *refresh steps*, the original cost label will be refreshed if the current cost is smaller. When the algorithm stops, the feasible states graph is formed, where the feasible states are marked with the minimal cost labels between starting state and current state. Search the minimal cost label of the possible arrival states when arriving destination node, the optimal charging and routing policy can be rebuilt in a backward recursive manner.

Algorithm 1: Improved Chrono-STP Algorithm

$*$ main iteration $*$
1: **select** $x = (i, t_i^s, e_i^s)$ from $B_{t_i^s}$;
2: $B_{t_i^s} \leftarrow B_{t_i^s} \backslash \{(i, t_i^s, e_i^s)\}$;
3: **for each** $(i, j) \in FS(i)$
4: $e_{ij}^l = f(d_{ij}^l, \tau_{ij}^l)$;
5: **if** $e_{reg} - e_{ij}^l \geq e_{bas}$ **then**
6 **if** $i = 1$ **then**
7: $t_j^s = t_i^s + \tau_{ij}^l$;
8: $c(x, x') = 0$;
9: *refresh steps*;
10: **else**
11: **for each** $e_j^s \in \{e_m^d | e_m^d \in E, e_{bas} \leq e_m^d \leq e_{reg} - e_{ij}^l\}$
12: $t_j^s = t_i^s + \tau_{ij}^l + (e_j^s + e_{ij}^l - e_i^s)/k_c$;
13: $c(x, x') = \int_{t_i^s}^{t_j^s - \tau_{ij}^l} p(t) \cdot k_c \cdot dt$;
14: *refresh steps*;
15: **end for**
16: **end if**
17: **end if**
18: **end for**

$*$ *refresh steps* $*$
if $C(x) + c(x, x') < C(x')$ **then**
$C(x') = C(x) + c(x, x')$;
$A(x') = x$;
if $x' \in B_{t_j^s}$ **then**
$B_{t_j^s} = B_{t_j^s} \cup x'$;
end if
end if

It is easy to prove that the ICS algorithm is correct and it runs in $O(|E|)$ time, where E denotes the links in feasible state graph R implicitly generated by the ICS algorithm. Suppose $E_j(ij) = \{e_m^d | e_m^d \in E, e_1^d \leq e_m^d \leq e_p^d - e_{ij}^l\}$ in line 11, $\forall j \in N, ij \in FS(i)$, such that $max \, \|E_j(ij)\| \leq p$, i.e., $E \leq mp^2$. In the worst case the algorithm time complexity is $O(mp^2)$.

Since navigation systems mainly operate on embedded devices with a low computation power and drivers commonly expect a rapid response, the computation complexity of algorithm is strictly restricted. In order to reduce the computation complexity, a simplify-charge-control algorithm is design in the following section.

4.3 Simplify-charge-control Algorithm

In this section, a simplify-charging-control algorithm are designed using the characteristic of charging in a constant price interval.

Theorem 1. In traffic network $G = (N, A)$ and feasible state graph $R = (V, E)$, suppose link $(i, j) \in A$, $j \neq d$, i.e., j is not the destination node. Feasible states at node i and j are $x = (i, t_i^s, e_i^s)$, $y = (j, t_j^s, e_j^s)$, $y' = (j, t'^s_j, e'^s_j)$, where $x, y, y' \in V$. Charging control strategies are denoted by links $(x, y), (x, y') \in E$. Suppose $\lambda_n \leq t_i^s < t_j^s < t'^s_j < \lambda_{n+1}$, if $e'^s_j \leq \min_{jk \in FS(j)} \{e_{jk}^l + e_{bas}\}$, the charging control strategies at state x can be dominated by the minimal charge-control strategy.

Proof. Since $e_j^s = (t_j^s - t_i^s - \tau_{ij}^l) \cdot k_c + e_i^s - e_{ij}^l$, $e'^s_j = (t'^s_j - t_i^s - \tau_{ij}^l) \cdot k_c + e_i^s - e_{ij}^l$, and $t'^s_j > t_j^s$, then $e'^s_j > e_j^s$. Since $e'^s_j \leq \min_{jk \in FS(j)} \{e_{jk}^l + e_{bas}\}$, then $e_j^s < \min_{jk \in FS(j)} \{e_{jk}^l + e_{bas}\}$, i.e., charging at state y is necessary if $j \neq d$. The energy left in battery is $e = e_j^s + k_c \cdot (t - t_j^s)$, where e is the energy left in battery, t is the time. When time $t = t'^s_j$, $e = e_i^s + k_c \cdot (t'^s_j - t_i^s - \tau_{ij}^l) - e_{ij}^l = e'^s_j$, state y turns into y' at time t'^s_j after charging, i.e., the charging strategies at state x can be dominated by the minimal charging control strategy.

According to Theorem 1, the feasible state graph in Figure 2(b) can be simplified at state x_2, when $\lambda_{n-1} = $ 0:00, $\lambda_n = $ 3:00, $\lambda_{n+1} = $ 6:00. The simplified graph shows in Figure 3.

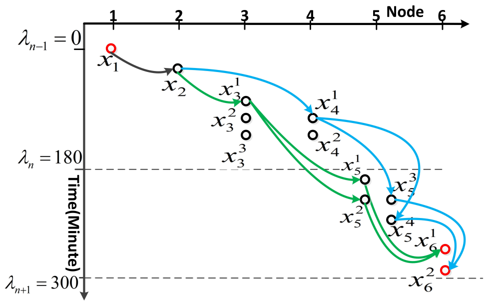


Fig. 3. An example of Simplified graph.

Empirically, the larger the constant price interval is, the more links in feasible state graph can be simplified. According to the data from Illinois Power Company, even though the RTP prices varies hourly, the fluctuation is quit small during off-peak and on-peak period. In this way, we approximate the real-time prices using their average price when the fluctuation in adjacent hours is small. Associated with the approximation pricing, a new price changing time instant set $\Gamma' = \{\lambda_1, \lambda_2, \ldots, \lambda_{r'}\}$ is obtained.

According to the process of structuring the simplified graph above, a simplify-charge-control algorithm is designed. In SCC algorithm, only the changes are present corresponding lines 11 $\sim$ 14 in ICS algorithm.

Algorithm 2: Simplify-Charge-Control Algorithm

$*$ Corresponding to lines 11$\sim$ 14 in Algorithm 1 $*$
if $\lambda_n \leq t_i^s \leq \lambda_{n+1}$ **then**
$e'^s_j = e_i^s + k_c \cdot (\lambda_{n+1} - t_i^s - \tau_{ij}^l) - e_{ij}^l$;
if $e'^s_j > e_{reg} - e_{ij}^l$ **then**
$e_j^s = e_{bas}$;
$t_j^s = t_i^s + \tau_{ij}^l + (e_{bas} + e_{ij}^l - e_i^s)/k_c$;
$c(x, x') = p(t) \cdot (e_{bas} + e_{ij}^l - e_i^s)$;
refresh steps;
elseif $e_{bas} \leq e'^s_j \leq e_{reg} - e_{ij}^l$ **then**
$e_j^s = e_{bas}$;
$t_j^s = t_i^s + \tau_{ij}^l + (e_{bas} + e_{ij}^l - e_i^s)/k_c$;
$c(x, x') = p(t_i^s) \cdot (e_{bas} + e_{ij}^l - e_i^s)$;
refresh steps;
for each $e_j^s \in \{e_m^d | e_m^d \in E, e'^s_j \leq e_m^d \leq e_{reg} - e_{ij}^l\}$
$t_j^s = t_i^s + \tau_{ij}^l + (e_m^d + e_{ij}^l - e_i^s)/k_c$;
$c(x, x') = \int_{t_i^s}^{t_j^s - \tau_{ij}^l} p(t) \cdot k_c \cdot dt$;
refresh steps;
end for
else
for each$e_j^s \in \{e_m^d | e_m^d \in E, e_{bas} \leq e_m^d \leq e_{reg} - e_{ij}^l\}$
$t_j^s = t_i^s + \tau_{ij}^l + (e_j^s + e_{ij}^l - e_i^s)/k_c$;
$c(x, x') = \int_{t_i^s}^{t_j^s - \tau_{ij}^l} p(t) \cdot k_c \cdot dt$;
refresh steps;
end for
end if
end if

In SCC algorithm, e'^s_j is a level used to determine the feasible state within the same constant price interval $[\lambda_n, \lambda_{n+1})$. For the possible charge-control strategies of state x in constant interval, they can be dominated by the minimal charg-control strategy.

Suppose the simplified graph is $R' = (v', E')$, the feasible state graph is $R = (V, E)$. Similar to ICS algorithm, the computation complexity of SCC algorithm is $O(|E'|)$. However $|E'| \ll |E|$ especially under approximated real-time prices.

5. SIMULATION

In this section, we simulate the ICS and SCC algorithms in a traffic network with 19 nodes and 42 links. According to the technical parameters of current EVs, we assume the battery has a capacity of 20kW·h, the stop-discharge energy is 4kWh, and the cruise range is 150km. The link distances are randomly selected between 60km and 120km, the link travel times are selected between 40min and 80min subject to the limitation that link energy consumption should be less than 12kWh so that the EVs departure from fast charging station with 80% SoC can reach the next charging station. The RTP and AP are given based on the data of Illinois Power Company on 15 December 2009, shown as in Figure 4.

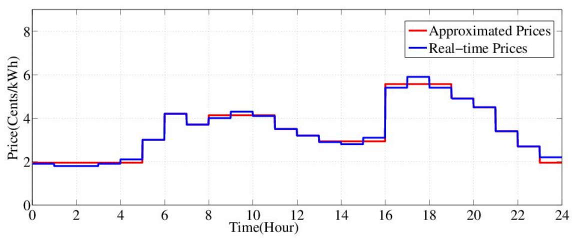


Fig. 4. The real-time prices and approximated prices in simulation.

In order to evaluate the accuracy and the computation complexity of algorithms, we calculate the minimum cost and the links of feasible state graph and simplified graph under RTP and AP. We simulate with different departure time, which locate at different part of the RTP. For the same departure time, we repeat the simulation in five different traffic conditions. The accurate optimal cost in ICS algorithm under RTP (the blue solid), the cost in SCC algorithm under RTP (the red solid), and the cost in SCC algorithm under AP (the green solid) are expressed in Figure 5. In the lines above, the cost in SCC algorithm under RTP coincide with the accurate optimal cost, and the cost in SCC algorithm under AP deviates the optimal cost slightly.

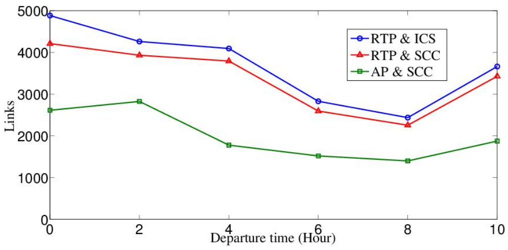


Fig. 5. Minimum cost and the average deviation rate.

Since the computation complexity is $O(|E|)$, we compare the link quantity of feasible state graph and simplified graph in Figure 6. The link amount in SCC algorithm under RTP is about 90% of that in ICS algorithm under RTP, while link amount in SCC algorithm under AP is about 50% of that in ICS algorithm under RTP. According to the simulation results, the SCC algorithm under AP can simplify the computation complexity greatly within an acceptable deviation rate of optimal cost.

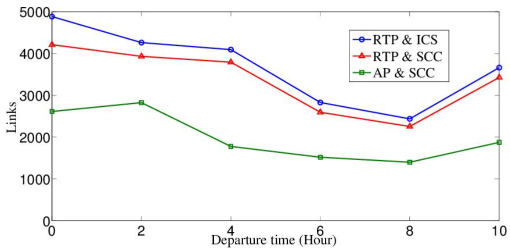


Fig. 6. Link quantity and the average simplification rate.

6. CONCLUSION

In this work, we study an EV navigation system, which aims to find the optimal charging and routing policy during a long OD trip under RTP. The ICS and SCC algorithm are designed. Simulations show that SCC algorithm under AP can simplify the computation complexity greatly within a acceptable deviation rate of optimal cost.

REFERENCES

Artmeier, A., Haselmayr, J., Leucker, M., and Sachenbacher, M. (2010). The optimal routing problem in the context of battery-powered electric vehicles. In *Workshop: CROCS at CPAIOR-10, Second International Workshop on Constraint Reasoning and Optimization for Computational Sustainability*. Bologna, Italy.

Bessler, S. and Grønbæk, J. (2012). Routing ev users towards an optimal charging plan. In *International Battery, Hybrid and Fuel Cell Electric Vehicle Symposium*. Los Angeles, California.

Boriboonsomsin, K., Barth, M.J., Zhu, W., and Vu, A. (2012). Eco-routing navigation system based on multisource historical and real-time traffic information. *IEEE Transactions on Intelligent Transportation Systems*, 13(4).

Dial, R.B. (1969). Algorithm 360: Shortest-path forest with topological ordering [h]. *Communications of the ACM*, 12(11), 632–633.

Eaton (2013). Dc quick charger. [Online]. `http://www.eaton.com/Eaton/ProductsServices/Electrical/ProductsandServices/ElectricalDistribution/ElectricVehicleChargingSolutions/DCQuickCharger/index.htm`, [Accessed 11-November-2013].

Mohsenian-Rad, A.H. and Leon-Garcia, A. (2010). Optimal residential load control with price prediction in real-time electricity pricing environments. *IEEE Transactions on Smart Grid*, 1(2), 120–133.

Sachenbacher, M., Leucker, M., Artmeier, A., and Haselmayr, J. (2011). Efficient energy-optimal routing for electric vehicles. In *Twenty-Fifth AAAI conference on Artificial Intelligence*, 1402–1407. San Francisco, America.

Stein, S., Gerding, E., Robu, V., de Weerdt, M., and Jennings, N.R. (2013). Intention-aware routing to minimise delays at electric vehicle charging stations. [Online]. `http://www.orchid.ac.uk/eprints/137/1/deweerdt.pdf`, [Accessed 11-November-2013].

TeslaMotors (2008). Roadster efficiency and range. [Online]. `http://www.teslamotors.com/blog/roadster-efficiency-and-range`, [Accessed 11-November-2013].