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Multimodal cyclic processes scheduling in fractal structure networks environment

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Abstract: The paper introduces the concept of a fractal-like multimodal transportation network (FMTN) in which several isomorphic subnetworks interact each other via distinguished subsets of common shared workstations as to provide a variety of demand-responsive work-piece transportation/handling services. The set of transportation modes supporting production flows within the FMTN environment is considered. In that context, a fractal-like layout of FMS equipped with AGVS where work-piece flows are treated as multimodal processes can be seen as a real-life example of this model. In opposite to the traditional approach we assume that the given network of local cyclic acting AGV services, i.e. corresponding to distinguished isomorphic subnetworks of FMS layout. The goal is to provide a declarative model enabling to state a constraint satisfaction problem aimed at multimodal transportation processes scheduling encompassing production flows.

Keywords: AGVs fleet scheduling, fractal structure, multimodal processes, cyclic scheduling

1. INTRODUCTION

Multimodal processes scheduling are found in different application domains (such as manufacturing, intercity fright transportation supply chains, multimodal passenger transport network combining several unimodal networks (bus, tram, metro, train, etc.) as well as service domains (including passenger/cargo transportation systems, e.g. ferry, ship, airline, AGV, train networks, as well as data and supply media flows, e.g., cloud computing, oil pipeline and overhead power line networks) (Abara 1989; Bielli et al. 2006; Clarke et al. 1996, Friedrich 1999). Multimodal processes executed in multimodal transportation network (MTN), i.e. a set of transport modes which provide connection from origin to destination, can be seen as passengers and/or goods flows transferred between different modes to reach their destination (Bocewicz and Banaszak 2013). The throughput of passengers and/or freight depends on geometrical and operational characteristics of MTN. In that context the solutions of the layout designs exposing the fractal like structures are frequently observed. Such a Manhattan-like regular, encompassing repeating design units of transportation structures can be seen in many irrigation and energy/data transmission systems as well as in AGVS' (Hall et al. 2001, Sharma 2012) layouts. The problems arising in these kind of networks concern multimodal routing of freight flows and supporting them multimodal transportation processes (MTP) scheduling, and are NP-hard (Levner et al. 2010). Since the transportation processes executed along unimodal networks are usually cyclic, hence the multimodal processes supported by them have also periodic character. That means, the periodicity of MTP depends on periodicity of unimodal (local) processes executed in MTN. Of course, the MTP throughput is maximized by minimization of its cycle time. Many models and methods have been considered so far (Levner et al. 2010). Among them, the mathematical programming approach (Abara 1989; Kampmeyer 2006), max-plus algebra (Polak et al. 2004), constraint logic programming (Bocewicz and Banaszak 2013), Petri nets (Song and Lee 1998) frameworks belong to the more frequently used. Most of them are oriented at finding of a minimal cycle or maximal throughput while assuming deadlock-free processes flow. The approaches trying to estimate the cycle time from cyclic processes structure and the synchronization mechanism employed (i.e. mutual exclusion instances) while taking into account deadlock phenomena are quite unique.

In that context our main contribution is to propose a new modeling framework enabling to evaluate the cyclic steady state of a given fractal system of concurrent cyclic processes (SCCP) encompassing the behavior typical for transportation services (see Fig. 1a)) in the flexible manufacturing systems. The following questions are of main interest (Bocewicz and Banaszak 2013): Can the assumed material handling system, e.g. AGVs, behavior meet the load/unload deadlines imposed by flow of scheduled work-pieces processing? Does there exist AGVS enabling to schedule the AGVs fleet as to follow lag-free service of scheduled work-pieces processing? So, the main question is: Can the MTP reach their goals subject to constraints assumed on SCCP?

In other words, the paper's objective concerns of MTN infrastructure assessment from the perspective of possible FMS oriented requirements imposed on fractal-like MTP scheduling The rest of the paper is organized as follows: Section 2 introduces a concept of multimodal network and then provides its representation in terms of systems of concurrently flowing cyclic processes and fractal structure models. Section 3 provides the problem formulation.

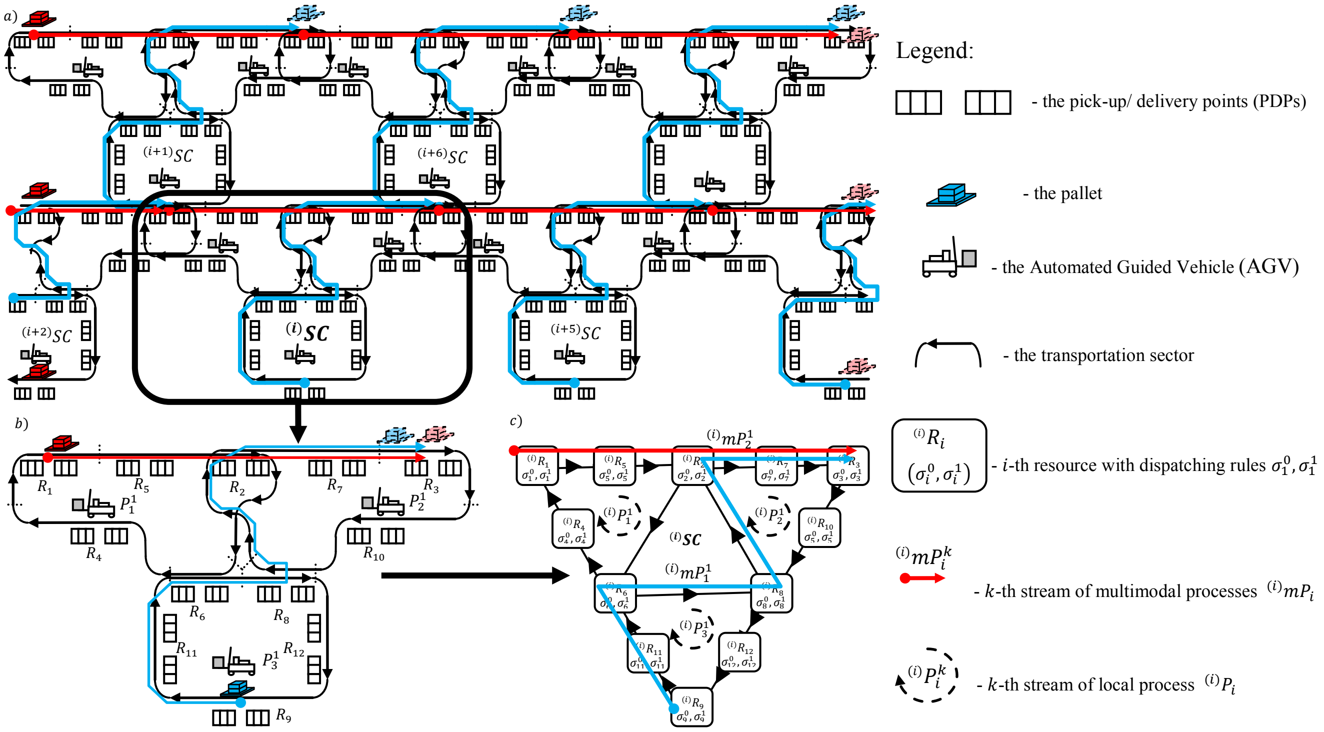

Fig. 1. FMTN structure composed of elementary substructures a), elementary isomorphic substructure ${}^{(i)}SC$ b), and its SCCP model c)

Section 4 discuses the declarative modeling driven approach to multimodal processes scheduling problems. The fractal-like material transportation structure environment is considered, and a match-up processes cyclic scheduling principle is proposed. Computational experiments and conclusions are presented in Sections 5 and 6, respectively.

2. MULTIMODAL NETWORKS

2.1 FMTN as a system of concurrently flowing cyclic processes SCCP

The FMTN shown in Fig. 1a) can be seen as a network of AGVs circulating along cyclic routes and can be modeled in terms of SCCPs as shown in Fig. 1b) and c). In this system three **local cyclic processes** are considered, viz. P_1, P_2, P_3. The processes follow the **routes** composed of transportation sectors and workstations (distinguished in Fig. 1b) by the **set of resources** $R = \{R_1, \ldots, R_c, \ldots, R_{12}\}$, R_c – the c-th resource). The local cyclic processes P_i contain the **streams** P_i^k (the k-th stream of the i-th local process P_i is denoted as P_i^k): $P_i = \left\{P_i^1, \ldots, P_i^k, \ldots, P_i^{ls(n)}\right\}$. The streams (representing vehicles from Fig. 1b)) of the processes follow the same route while occupying different resources (sectors). In the considered case all processes P_1, P_2, P_3 contain only the unique streams: $P_1 = \{P_1^1\}$, $P_2 = \{P_2^1\}$, $P_3 = \{P_3^1\}$.

Apart from local processes, we consider two **multimodal processes** (i.e. processes executed along the routes consisting parts of the routes of local processes): mP_1, mP_2.

For example, the transportation route depicted by the blue line corresponds to the multimodal process mP_1 supported by AGVs, which in turn encompass local transportation streams P_3^1 and P_2^1. This means that the production route specifying how a multimodal process is executed can be considered as composed of parts of the routes of local cyclic processes. Similar as in the case of local processes, in the system considered each multimodal process consist of one stream: $mP_i = \{mP_i^1\}$, $i = 1{,}2$, which means along each transportation route one work-piece is processed (one pallet on the one transportation line – see Fig. 1b)). In general case the situation where multimodal processes consist many streams is possible: $mP_i = \left\{mP_i^1, \ldots, mP_i^k, \ldots, mP_i^{lsm(i)}\right\}$.

Processes can interact with each other through shared resources, i.e. the transportation sectors. The routes of the considered local processes (streams) are as follows:

$$p_1^1 = (R_1, R_5, R_2, R_6, R_4),\ p_2^1 = (R_2, R_7, R_3, R_{10}, R_8),$$
$$p_3^1 = (R_6, R_8, R_{12}, R_9, R_{11}),$$

where: R_2, R_6, R_8 are resources shared by local processes, and R_1,$R_3 - R_5$, R_7, R_9–R_{12}, are the non-shared resources.

In the general case, the route p_i^k is the sequence of resources used in order to execute the operations of the stream P_i^k.

Similarly the streams of cyclic multimodal processes: mP_1, mP_2, follow the routes (see Fig. 1c)):

$$mp_1^1 = \left((R_9, R_{11}, R_6, R_8)\frown(R_2, R_7, R_3)\right) = (R_9, R_{11}, R_6, R_8, R_2, R_7, R_3),$$

$$mp_2^2 = \left((R_1, R_5, R_2)\frown(R_7, R_3)\right) = (R_1, R_5, R_2, R_7, R_3),$$

where: (R_9, R_{11}, R_6, R_8), (R_2, R_7, R_3) – subsequences of routes p_3^1, p_2^1, defining the transportation sections of mp_1^1, (R_1, R_5, R_2), (R_7, R_3) – subsequence of routes p_1^1, p_2^1, defining the transportation sections of mp_2^1. $u\frown v$ – concatenation of sequences u and v (Bocewicz and Banaszak 2013).

A resource conflict (caused by the application of the mutual exclusion protocol) is resolved with the aid of a priority dispatching rule (Bocewicz and Banaszak 2013), which determines the order in which streams access shared resources. For instance, in the case of the resource R_2, the priority dispatching rule: $\sigma_2^0 = (P_1^1, P_2^1)$, determines the order in which streams of local processes can access the shared resource R_2, in the case considered the stream P_1^1 is allowed to access first, then the stream P_2^1 next, and then once again P_1^1, and so on. The SCCP shown in Fig. 1c) is specified by the following set of dispatching rules: $\Theta = \{\Theta^0, \Theta^1\}$, where: $\Theta^0 = \{\sigma_1^0, \dots, \sigma_c^0, \dots, \sigma_{12}^0\}$ ($\Theta^1 = \{\sigma_1^1, \dots, \sigma_c^1, \dots, \sigma_{12}^1\}$)- set of rules determining the orders of local (multimodal) processes.

In general, the following notation is used:

- a sequence $p_i^k = (p_{i,1}^k, \dots, p_{i,j}^k, \dots, p_{i,lr(i)}^k)$ specifies **the route of the stream of the local process P_i^k** (the k-th stream of the i-th local process P_i). Its components define the resources used in the execution of operations, where: $p_{i,j}^k \in R$ (the set of resources $R = \{R_1, \dots, R_c, \dots, R_m\}$) – denotes the resources used by the k-th stream of the i-th local process in the j-th operation; in the rest of the paper, **the j-th operation executed on the resource $p_{i,j}^k$ in the stream P_i^k** will be denoted by $o_{i,j}^k$.
- $x_{i,j}^k(l) \in \mathbb{N}$ - the timing of commencement of operation $o_{i,j}^k$ in the l-th cycle,
- $t_i^k = (t_{i,1}^k, t_{i,2}^k, \dots, t_{i,j}^k, \dots, t_{i,lr(i)}^k)$ specifies **the operation times of local processes**, where $t_{i,j}^k$ denotes the time of execution of operation $o_{i,j}^k$.
- $mp_i^k = \left(mpr_{i_1}^{q_1}(a_{i_1}, b_{i_1})^\frown \dots ^\frown mpr_{i_y}^{q_y}(a_{i_y}, b_{i_y})\right)$ specifies **the route of the stream mP_i^k from the multimodal process mP_i** (the k-th stream of the i-th multimodal process mP_i), where:

$$mpr_i^q(a,b) = \begin{cases} \left(p_{i,a}^q, p_{i,a+1}^q, \dots, p_{i,b}^q\right) & , a \le b \\ \left(p_{i,a}^q, p_{i,a+1}^q \dots, p_{i,lr(i)}^q, p_{i,1}^q, \dots, p_{i,b}^q\right) & , a > b \end{cases}$$

$a, b \in \{1, \dots, lr(i)\}$,
is the subsequence of the route $p_i^q = (p_{i,1}^q, \dots, p_{i,j}^q, \dots, p_{i,lr(i)}^q)$ containing elements from $p_{i,a}^q$ to $p_{i,b}^q$. The transportation route mp_i^k is a sequence of parts of routes of local processes. In the rest of the paper, **the j-th operation executed on the resource $mp_{i,j}^k$ in the stream mP_i^k** will be denoted by $mo_{i,j}^k$,
- $mx_{i,j}^k(l) \in \mathbb{N}$ - the timing of commencement of operation $mo_{i,j}^k$ in the l-th cycle.
- $mt_i^k = (mt_{i,1}^k, mt_{i,2}^k, \dots, mt_{i,j}^k, \dots, mt_{i,ldm(i)}^k)$ specifies **the operation times of multimodal processes**, where $mt_{i,j}^k$ denotes the time of execution of operation $mo_{i,j}^k$,
- $\Theta = \{\Theta^0, \Theta^1\}$ is the set of **priority dispatching rules**, $\Theta^i = \{\sigma_1^i, \sigma_2^i, \dots, \sigma_c^i, \dots, \sigma_m^i\}$ is the set of priority dispatching rules for local ($i = 0$) / multimodal ($i = 1$) processes where: $\sigma_c^i = (s_{c,1}^i, \dots, s_{c,d}^i, \dots, s_{c,lp(c)}^i)$ are sequence components which determine the order in which the processes can be executed on the resource R_c, $s_{c,d}^0 \in H$ (where: H – is the set of local streams).

Using the above notation, a SCCP can be defined as a tuple (Bocewicz and Banaszak 2013):

$$SC = \left((R, SL), SM\right), \tag{1}$$

where: $R = \{R_1, \dots, R_c, \dots, R_m\}$ – the set of resources,
$SL = (U, T, \Theta^0)$ – the structure of local processes:
U – the set of routes of local process, T – the set of sequences of operation times in local processes, Θ^0 – the set of priority dispatching rules for local processes,
$SM = (M, mT, \Theta^1)$ – the structure of multimodal processes:
M – the set of routes of a multimodal process, mT – the set of sequences of operation times in multimodal processes,
Θ^1 – the set of priority dispatching rules for multimodal processes.

The behavior of the structure of SCCP (1) will be characterized by the schedule (2):

$$X' = \left((X, \alpha), (mX, m\alpha)\right) \tag{2}$$

where: $X = \left\{x_{1,1}^1, \dots, x_{i,j}^k, \dots, x_{n,lr(n)}^{ls(n)}\right\}$ – a set of the timings of commencement of local processes operations in $l = 0$ of the cycle, $x_{i,j}^k$ – determines the value $x_{i,j}^k(l)$: $x_{i,j}^k(l) = x_{i,j}^k + \alpha \cdot l$, $\alpha -$ periodicity of local processes executions, $mX = \left\{mx_{1,1}^1, \dots, mx_{i,j}^k, \dots, mx_{w,lm(w)}^{lsm(w)}\right\}$ – a set of the timings of commencement of operations of multimodal processes in $l = 0$ of cycle, $mx_{i,j}^k$ – determines the value $mx_{i,j}^k(l)$: $mx_{i,j}^k(l) = mx_{i,j}^k + m\alpha \cdot l$, $m\alpha -$ periodicity of multimodal processes executions.

2.2 Fractal-like structure

In a special case, SCCP structures may have a fractal form. An example of such a structure is shown in Fig. 1a). Structures of this kind consist of repeatable constant fragments of the system (sub-structures SC_i). The structure presented in Fig. 1a) was created as a result of multiple composition of the structure shown in Fig. 1b).

Formally, the fractal-like structure is defined as SC (1) structure, that can be decomposed into the set of isomorphic substructures : $SC^* = \{SC_1, \dots, SC_i, \dots, SC_{lc}\}$. In such case, an assumption is made that:

a) each substructure $SC_i \in SC^*$ of the structure SC is defined analogically as (1):

$$SC_i = \left((Rp_i, SLp_i), SMp_i\right), \tag{3}$$

where: Rp_i – the set of resources of sub-structure SC_i, $Rp_i \subset R$, SLp_i –level of local processes of substructure SC_i, including local processes $Pp_i \subset P$ and corresponding route sequences: $Up_i \subset U$, of the operation times $Tp_i \subset T$. The set of routes Up_i includes all the resourcesRp_i. The set of dispatching rules is characterized by $\Theta_i^0 = \{\sigma_{k,i}^0 = (s_{k,1,i}^0, \dots, s_{k,d,i}^0, \dots, s_{k,lh(k,0),i}^0) \mid k = 1 \dots lk\}$, where $\sigma_{k,i}^0$ – dispatching rule for the resource $R_k \in Rp_i$ in i^{th} substructure, $s_{k,d,i}^0$ – stream of a local process belonging to Pp_i, $lh(k, i, 0)$ – the length of rule $\sigma_{k,i}^0$. SMp_i –level of multimodal processes of substructure SC_i, the level includes fragments of $mP_j(a, b)$ of multimodal processes forming the set mPp_i, where: $mP_j(a, b)$ – means fragment of the process mP_j related with executing the operation from a, $a+1$, … ,b. In the substructure SC_i there are only such fragments of multimodal processes which are performed based on local processes Pp_i.

b) $SC^* = \{SC_1, \dots, SC_i, \dots, SC_{lc}\}$ is a set of substructures of the structure SC if:

$\bigcup_{i=1}^{lc} Rp_i = R$ – substructures include all resources of the structure SC,

$\bigcup_{i=1}^{lc} Pp_i = P$; $\prod_{i=1}^{lc} Pp_i = \emptyset$ and $\bigcup_{i=1}^{lc} Up_i = U$; $\prod_{i=1}^{lc} Up_i = \emptyset$ – substructures use all the local processes and one process occurs in exactly one structure,

$\prod_{i=1}^{lc} mPp_i = \emptyset$ – each fragment of a multimodal process occurs in exactly one substructure; moreover, within the substructures all fragments of multimodal processes are used.

c) Two substructures SC_a,$SC_b \in SC^*$ are called isomorphic if:

- each resource $R_a \in Rp_a$ of substructure SC_a is corresponding to exactly one resource $R_b \in Rp_b$ of the structure SC_b: $R_b = f(R_a)$,
- each process P_a /mP_a (local as well as multimodal) of the substructure SC_a is corresponding to exactly one process P_b /mP_b of the structure SC_b: $P_b = f(P_a)$, $mP_b = f(mP_a)$,
- routes p_b/mp_b i p_a/mp_a of the corresponding processes are sequences consisting of corresponding resources,
- each operation $o_{a,j}^h/mo_{a,j}^h$ executed within the substructure SC_a is corresponding to exactly one operation $o_{b,j}^h/mo_{b,j}^h$ executed within the substructure SC_b: $o_{a,j}^h = f(o_{b,j}^h)$ / $mo_{a,j}^h = f(mo_{b,j}^h)$; the corresponding operations are executed at the same time: $t_{a,j} = t_{b,j}$ / $mt_{a,j} = mt_{b,j}$,
- dispatching rules σ_a^l / σ_b^l of the corresponding resources are sequences consisting of elements $s_{a,d}^l$ /$s_{b,d}^l$ indicating the streams of corresponding processes.

The structure shown in Fig. 1a) consists of one type of isomorphic substructures presented in Fig. 1c). The substructures it consists of, denoted as ${}^{(i)}SC$, are corresponding to the structure illustrated in Fig 1. Each of them includes twelve resources (R_1-R_{12}), three local processes (${}^{(i)}P_1, {}^{(i)}P_2, {}^{(i)}P_3$) and two fragments of multimodal processes (it is assumed that each fragment is related with one stream of multimodal process - ${}^{(i)}mP_1^1$, ${}^{(i)}mP_2^1$).

3. PROBLEM FORMULATION

The considered problem is related to evaluating the parameters of SCCP with fractal structure. Formally, this problem is defined as follows:

A fractal structure SC (1) is given, where the values of operation times (T, mT) and dispatching rules Θ are unknown. An answer is sought to the question whether there are such values T, mT and Θ that can guarantee that the cyclic behavior represented by the schedule X' (2) will be attainable in the structure SC (1).

The fractal structure SC (1) can be decomposed into a set of isomorphic substructures $SC^* = \{SC_1, \dots, SC_i, \dots, SC_{lc}\}$. Therefore, the selection of parameters T, mT and Θ can be carried out independently for each substructure. If, for every substructure SC_i, there is a subset of parameters T, mT and Θ that guarantee its cyclic behavior, then the considered problem should provide an answer to the following question: **Is there such a way of composing the substructures SC^*, that can guarantee the cyclic work of the system SC?**

In order to answer this question the **operator of substructure composition** $\oplus$ is introduced. An assumption is made that the result of compositing two substructures SC_a, SC_b through mutually shared resources ($Rp_a \cap Rp_b \neq \emptyset$): $SC_a \oplus SC_b = SC_c$ is the structure SC_c defined as follows:

$$SC_c = \big((Rp_c, SLp_c), SMp_c\big) \tag{4}$$

where: $Rp_c = Rp_a \cup Rp_b$ - the set of resources, and

- variables characterizing SLp_c are detrmined in the following way:

  $Pp_c = Pp_a \cup Pp_b$; $Up_c = Up_a \cup Up_b$; $Tp_c = Tp_a \cup Tp_b$

  $\Theta_c^0 = \{\sigma_{k,c}^0 | k = 1 \dots lk\}$, where:

  $$\sigma_{k,c}^l = \begin{cases} \sigma_{k,a}^l & \text{for } R_k \in Rp_a \text{ and } R_k \notin Rp_b \\ \sigma_{k,b}^l & \text{for } R_k \in Rp_b \text{ and } R_k \notin Rp_a \\ \vartheta(\sigma_{k,a}^l, \sigma_{k,b}^l) & \text{for } R_k \in Rp_a \text{and } R_k \in Rp_b \end{cases} \tag{5}$$

  $\vartheta(\sigma_{k,a}^0, \sigma_{k,b}^0)$ – function determining the dispatching rules for the mutual resource R_k of the composed structures.

- variables characterizing SMp_c are determined in the following way:

  mPp_c- the set including all fragments of multimodal processes of the sets mPp_a and mPp_b except for fragments meeting the condition below.

  If in the set $mPp_a \cup mPp_b$ there are such two fragments: $mP_j(a_{j_1}, a_{j_2})$, $mP_j(b_{j_1}, b_{j_2})$, that $a_{j_2} = b_{j_1}$, then in the set mPp_c these fragments are replaced by a fragment of a multimodal process in the form of $mP_j(a_{j_1}, b_{j_2})$. The set mPp_c attained in this way determines the set of routes mUp_c, of the operation and the their execution times mTp_c, $\Theta_c^1 = \{\sigma_{k,c}^1 | k = 1 \dots lk\}$, where $\sigma_{k,c}^1$ is determined analogically as (5).

4. CYCLIC SCHEDULING OF FRACTAL-LIKE SCCP

4.1 Determining cyclic steady processes

Fig. 1c) shows in detail the substructure arrangement of the system from Fig. 1a). There is one type of elementary isomorphic substructures ${}^{(i)}SC$ which are put together by means of integrating mutual resources.

As Fig. 1c) shows, for every substructure ${}^{(i)}SC$ processes are implemented in the same manner: operations are performed along the same routes, the same dispatching rules are applied, etc. In this context the introduced operator of substructures composition ($\oplus$) SC can be shown as a multiple composition of substructures ${}^{(i)}SC$:

$$SC = \oplus_{i=1}^{lc}\big({}^{(i)}SC\big) \tag{6}$$

where: $\oplus_{i=1}^{lc}\big({}^{(i)}SC\big) = {}^{(1)}SC \oplus \dots \oplus {}^{(i)}SC \oplus \dots \oplus {}^{(lc)}SC$ – means composition according to (4), (5), (8) i.e. each substructure ${}^{(i)}SC$ is put together with the others by means of integrating the resources belonging to the same set of corresponding resources.

For example, the structure ${}^{(i)}SC$ from Fig. 2a) is put together with the others by the resources ${}^{(i)}R_1$, ${}^{(i)}R_3$, ${}^{(i)}R_9$. The resource ${}^{(i)}R_1$ plays the role of the resource ${}^{(i+2)}R_3$ of the structure ${}^{(i+2)}SC$ and the resource ${}^{(i+1)}R_9$ of the structure ${}^{(i+1)}SC$. In other words, each isomorphic structure such as ${}^{(i)}SC$ shares the following resources with the neighboring structures: ${}^{(i)}R_1$ treated also as ${}^{(i+2)}R_3$ and ${}^{(i+1)}R_9$ (contiguity with ${}^{(i+1)}SC$ and ${}^{(i+2)}SC$), ${}^{(i)}R_3$ treated as ${}^{(i+5)}R_1$ and ${}^{(i+6)}R_9$ and ${}^{(i)}R_9$ treated as ${}^{(i+3)}R_3$ and ${}^{(i+4)}R_1$.

Due to the same manner of process execution, as well as the same manner of substructures composition, the cyclic schedule representing the behavior of the whole structure can be perceived as a composition of corresponding (isomorphic) schedules (Fig. 2b)):

$$X' = \uplus_{i=1}^{lc}\left({}^{(i)}X'\right) \tag{7}$$

where: ${}^{(i)}X'$ – the cyclic schedule of the substructure ${}^{(i)}SC$:

$$ {}^{(i)}X' = \left(\left({}^{(i)}X,\ {}^{(i)}\alpha\right),\left({}^{(i)}mX,\ {}^{(i)}m\alpha\right)\right) \tag{8}$$

${}^{(i)}X$/ ${}^{(i)}mX$ – set of the initiation moments of local / multimodal process operations of the substructure ${}^{(i)}SC$,

${}^{(i)}\alpha$ / ${}^{(i)}m\alpha$ – periodicity of local/multimodal processes executions, $\uplus_{i=1}^{lc}\left({}^{(i)}X'\right) = {}^{(1)}X' \uplus \ldots \uplus {}^{(i)}X' \uplus \ldots \uplus {}^{(lc)}X'$ – composition of schedules ${}^{(i)}X'$, ${}^{(a)}X' \uplus {}^{(b)}X'$ – the operation of integrating the schedule composition ${}^{(a)}X'$, ${}^{(b)}X'$:

$$ {}^{(a)}X' \uplus {}^{(b)}X' = \Big(\left({}^{(a)}X \cup {}^{(b)}X, lcm\left({}^{(a)}\alpha, {}^{(b)}\alpha\right)\right), \left({}^{(a)}mX \cup {}^{(b)}mX, lcm\left({}^{(a)}m\alpha, {}^{(b)}m\alpha\right)\right)\Big). \tag{9}$$

In order to determine the schedule X' it is enough to know the schedule ${}^{(i)}X'$ of a single substructure ${}^{(i)}SC$. However, to make the composition (7) possible, it is necessary to make sure that the operations executed according to ${}^{(i)}X'$, do not lead to deadlocks. And in the mutual resources (${}^{(i)}R_1$, ${}^{(i)}R_3$ and ${}^{(i)}R_9$) the streams belonging to various substructures must not collide, i.e. they must be implemented alternately.

In order to determine such parameters as dispatching rules ${}^{(i)}\Theta$ and operation times ${}^{(i)}T, {}^{(i)}mT$ of the substructure ${}^{(i)}SC$ (Fig. 2a) that guarantee the attainability of the cyclic schedule ${}^{(i)}X'$ within the structure, it is possible to apply the constraint satisfaction problem (10):

$$PS'_{REX_i} = \left(\left(\{{}^{(i)}T',\ {}^{(i)}X',\ {}^{(i)}\Theta,\ {}^{(i)}\alpha'\}, \{D_T, D_X, D_\Theta, D_\alpha\}\right), \{C_L, C_M, C_D\}\right) \tag{10}$$

where: ${}^{(i)}T'$, ${}^{(i)}X'$, ${}^{(i)}\Theta$, ${}^{(i)}\alpha'$ – decision variables,

${}^{(i)}T' = \left({}^{(i)}T,\ {}^{(i)}mT\right)$ – sequence of operation times of substructure ${}^{(i)}SC$, ${}^{(i)}X'$– cyclic schedule (8) of substructure ${}^{(i)}SC$, ${}^{(i)}\Theta = \left\{{}^{(i)}\Theta^0,\ {}^{(i)}\Theta^1\right\}$ – the set of priority dispatching rules for substructure ${}^{(i)}SC$, ${}^{(i)}\alpha' = \left({}^{(i)}\alpha,\ {}^{(i)}m\alpha\right)$ – periodicity of local/multimodal processes executions for substructure ${}^{(i)}SC$,

$D_T, D_X, D_\Theta, D_\alpha$ – domains determining admissible value of decision variables: D_T: ${}^{(i)}mt_{i,j}^k, {}^{(i)}t_{i,j}^k \in \mathbb{N}$; D_X: ${}^{(i)}mx_{i,j}^k, {}^{(i)}x_{i,j}^k \in \mathbb{Z}$; D_α: ${}^{(i)}m\alpha, {}^{(i)}\alpha \in \mathbb{N}$;

$\{C_L, C_M, C_D\}$ – the set of constraints C_L and C_M describing SCCP behavior, C_L , C_M– constraints determining cyclic steady state of local /multimodal processes, i.e. their cyclic schedule, C_D – constraints that guarantee the smooth implementation of the stream operation executed on mutual resources, (in case of ${}^{(i)}SC$ from Fig. 2a) of the resources ${}^{(i)}R_1$, ${}^{(i)}R_3$ i ${}^{(i)}R_9$).

The solution of the problem (10) is, among other things, the schedule ${}^{(i)}X'$ that meets all the constraints from the given set $\{C_L, C_M, C_D\}$. It means that, if such schedule exists within the substructure ${}^{(i)}SC$, it is possible to smoothly execute the operations of processes occurring in ${}^{(i)}SC$ as well as in neighboring substructures (${}^{(i+1)}SC$, ${}^{(i+2)}SC$,…, ${}^{(i+6)}SC$).

4.2 The conditions for cyclic implementation of processes

The constraints C_L, C_M occurring in the problem (10) are meant to guarantee deadlock-free and smooth execution of the operations of substructure ${}^{(i)}SC$.

They are typical of the relationship between the structure parameters ${}^{(i)}\Theta$, ${}^{(i)}T'$, ${}^{(i)}U$, ${}^{(i)}M$ and its behavior ${}^{(i)}X'$, ${}^{(i)}\alpha'$ (meeting the accepted conditions: mutual exclusion protocol, etc.) and the mutual relationships between local and multimodal processes.

In case of the two levels structure model, i.e. including levels SL and SM as shown in Fig. 2, the constraints C_L and C_M determining ${}^{(i)}x_{a,b}^k$ / ${}^{(i)}mx_{a,b}^k$ were described in (Bocewicz and Banaszak 2013) .

4.3 Principle of match-up structures coupling

The constraints C_L, C_M guarantee that in the substructure ${}^{(i)}SC$ from Fig. 2a) the processes will be executed in a cyclic and deadlock-free manner. These constraints, however, cannot ensure the lack of interferences between the operations of neighboring substructure streams (${}^{(i+1)}SC$, ${}^{(i+2)}SC$,…, ${}^{(i+6)}SC$) with the substructure ${}^{(i)}SC$. In order to avoid interferences of this kind, additional constraints C_D, are introduced, which describe the relationships between the process operations of the constituted structures. For that purpose the principle of match-up structures coupling is applied.

The idea of the principle of match-up structures coupling is to attain the cyclic schedule X'_c (that does not lead to any collisions between operations) in the substructure SC_c, gained as a result of the composition $SC_a \oplus SC_b$. The cyclic schedule is a composition of the schedules X'_a, X'_b: $X'_c = X'_a \uplus X'_b$ (7) if the following conditions hold:

- the value of the periodicity of schedule X'_a is the total multiple of the periodicity of schedule X'_b:
- $m\alpha_a\ MOD\ m\alpha_b = 0$; and $\alpha_a\ MOD\ \alpha_b = 0$
- the operations of mutual resources $Rk = Rp_a \cap Rp_b = \left\{R_{k_1}, \ldots, R_{k_i}, \ldots, R_{k_q}\right\}$ are executed without mutual interferences.

Formally, the constraints that guarantee the lack of interferences while executing the process operations on mutual resources are defined in the following way:

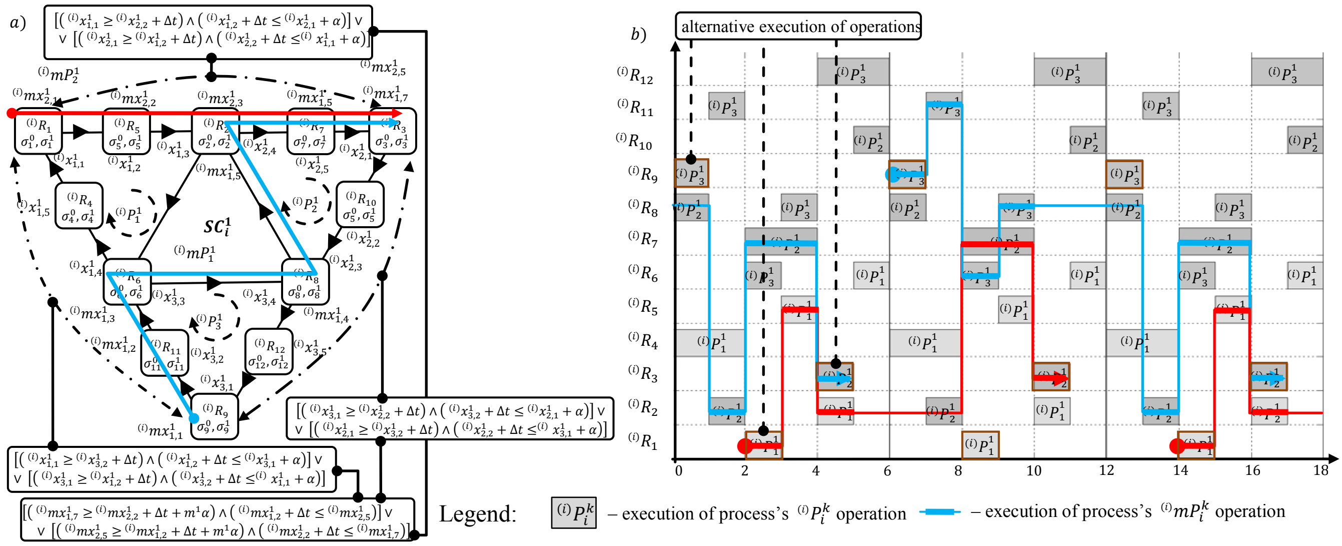

Fig. 2 Substructure $^{(i)}SC$ with constraints that guarantee the alternate execution of $^{(i)}P_1^1$, $^{(i)}P_2^1$, $^{(i)}P_3^1$ a), cyclic schedule $^{(i)}X'$ of the structure $^{(i)}SC$ b)

Constraints for local process operations. In order to guarantee the smooth process implementation on the resource $R_{k_i} \in Rk$ the extension of the conventional constraints of non-superimposition of time intervals is used (Bach et al. 2010). The two operations $o_{i,j}^h$, $o_{q,r}^s$do not interfere (on the mutually shared resource R_{k_i}) if the operation $o_{i,j}^h$ begins (moment $x_{i,j}^h$) after the release (with the delay Δt) of the resource by the operation $o_{q,r}^s$ (moment x_{q,r^*}^s of the subsequent operation initiation) and releases the resource (moment x_{i,j^*}^h of the subsequent operation initiation) before the beginning of the next execution of the operation $o_{q,r}^s$ (moment $x_{q,r}^s + \alpha$). The collision-free execution of the local process operations is possible if the constraint below is satisfied:

$$[(x_{i,j}^h \geq x_{q,r^*}^s + k'' \cdot \alpha_b + \Delta t) \wedge (x_{i,j^*}^h + k' \cdot \alpha_a + \Delta t \leq x_{q,r}^s + \alpha_b)]$$
$$\vee [(x_{q,r}^s \geq x_{i,j^*}^h + k' \cdot \alpha_a + \Delta t) \wedge (x_{q,r^*}^s + k'' \cdot \alpha_b + \Delta t \leq x_{i,j}^h + \alpha_a)] \quad (11)$$

where: $j^* = (j+1)\ MOD\ lr(i)$, $r^* = (r+1)\ MODlr(q)$,

$$k' = \begin{cases} 0 \text{ when } j+1 \leq lr(i) \\ 1 \text{ when } j+1 < lr(i) \end{cases}, k'' = \begin{cases} 0 \text{ when } r+1 \leq lr(q) \\ 1 \text{ when } r+1 < lr(q) \end{cases},$$

α_a / α_b – periodicity of schedule X_a / X_b; $lr(i)$ / $lr(q)$ – length of process route P_i / P_q; $x_{i,j}^h$ / $x_{q,r}^s$ – initiation moments of the operation $o_{i,j}^h$ / $o_{q,r}^s$ of the structure SC_a / SC_b; x_{i,j^*}^h/ x_{q,r^*}^s –moments of operation executed after $o_{i,j}^h$ / $o_{q,r}^s$.

Satisfying the constraint (11) means that on every mutually shared resource of the composed substructures SC_a, SC_b the local processes are executed alternately.

Constraints for multimodal processes. In order to guarantee an interference-free implementation of the multimodal processes (when the condition of mutual exclusion is applied) the applied conditions are similar to those used for local processes. The collision-free execution of the multimodal process operations $mo_{i,j}^h$, $mo_{q,r}^s$ is possible if the following constraint is satisfied:

$$[(mx_{i,j}^h \geq mx_{q,r^*}^s + k'' \cdot m\alpha_b + \Delta t)$$
$$\wedge (mx_{i,j^*}^h + k' \cdot m\alpha_a + \Delta t \leq mx_{q,r}^s + m\alpha_b)]$$
$$\vee [(mx_{q,r}^s \geq mx_{i,j^*}^h + k' \cdot m\alpha_a + \Delta t)$$
$$\wedge (mx_{q,r^*}^s + k'' \cdot m\alpha_b + \Delta t \leq mx_{i,j}^h + m\alpha_a)] \quad (12)$$

where: j^*, r^*, k' and k'' defined as in (11)

$mx_{i,j}^h, mx_{q,r}^s$ – initiation moments of the operations $mo_{i,j}^h$, $mo_{q,r}^s$ of substructures SC_a, SC_b, respectively; mx_{i,j^*}^h, mx_{q,r^*}^s – moments of operations executed after $mo_{i,j}^h, mo_{q,r}^s$.

Satisfying the constraint (12) means that on every mutual resource of the composed substructures SC_a, SC_b, the multimodal processes are executed alternately.

The constraints (11) and (12) must be satisfied so that the composition of two substructures $SC_c = SC_a \oplus SC_b$ of the known cyclic behaviors, is also characterized by the cyclic behavior X_c'. If these constraints are satisfied, the manner of executing operations on mutual resources Rk determines the form of dispatching rules $\sigma_{k,c}^0$ (5), and, to be more exact, the form of functions $\vartheta(\sigma_{k,a}^0, \sigma_{k,b}^0)$ and $\vartheta(\sigma_{k,a}^1, \sigma_{k,b}^1)$. The function $\vartheta(\sigma_{k,a}^l, \sigma_{k,b}^l)$ is determined based on the values of moments of operations executed on the resource R_k:

$$\vartheta(\sigma_{k,a}^l, \sigma_{k,b}^l) = (s_{k,1,c}^l, \ldots, s_{k,j,c}^l, \ldots, s_{k,lh_c,c}^l) \text{ when}$$
$$x_{k,1,c}^l < \cdots < x_{k,j,c}^l < \cdots < x_{k,lh_c,c}^l,\ l \in \{0,1\}, \quad (13)$$

where: $s_{k,j,c}^l$ – j^{th} element of the rule $\sigma_{k,c}^l$ determining the stream of the process initiating its operation on the resource R_k in the moment: $x_{k,j,c}^l$; $s_{k,j,c}^l$ is one of the elements of the rules $\sigma_{k,a}^l, \sigma_{k,b}^l$; $x_{k,j,c}^0 \in X_a \cup X_b$; $x_{k,j,c}^1 \in mX_a \cup mX_b$.

In other words, there are such dispatching rules on mutual R_k as the sequence of operations resulting from the schedules X_a', X_b' satisfying the constraints (11) and (12).

5. COMPUTATIONAL EXPERIMENTS

The evaluation of the cyclic behavior (the existence of the schedule X') of the fractal structure SC from Fig. 1a) can be obtained as a result of evaluating the parameters of isomorphic structure ${}^{(i)}SC$ from Fig. 2a). Therefore, the problem $PS'_{REX\,i}(10)$ was formulated in which the constraints C_L, C_M determining the relationships between the behavior and the structure are formulated according to (Bocewicz and Banaszak 2013). In order to formulate the constraints C_D the principle of match-up structures coupling was applied. In case of constraints C_D it is necessary that they guarantee a collision-free execution of stream operations ${}^{(i)}P_1^1$, ${}^{(i+2)}P_2^1$, ${}^{(i+1)}P_3^1$ (on the resource ${}^{(i)}R_1$), ${}^{(i)}P_2^1$, ${}^{(i+2)}P_1^1$, ${}^{(i+6)}P_3^1$ (on the resource ${}^{(i)}R_3$), ${}^{(i)}P_3^1$, ${}^{(i+3)}P_2^1$, ${}^{(i+4)}P_1^1$. In order to formulate these constraints, the isomorphic properties of substructures ${}^{(i)}SC$ are used. Owing to the fact that streams ${}^{(i)}P_1^1$, ${}^{(i+2)}P_1^1$, ${}^{(i+4)}P_1^1$ (as well as ${}^{(i)}P_2^1$, ${}^{(i+2)}P_2^1$, ${}^{(i+3)}P_2^1$ and ${}^{(i)}P_3^1$, ${}^{(i+1)}P_3^1$, ${}^{(i+6)}P_3^1$) of substructures ${}^{(i)}SC$, ${}^{(i+1)}SC$,..., ${}^{(i+6)}SC$ are executed in this manner, the collision-free operation of the streams is equal to the non-simultaneous execution of the operations of streams ${}^{(i)}P_1^1$, ${}^{(i)}P_2^1$, ${}^{(i)}P_3^1$.

The constraints C_D that guarantee this kind of process execution were shown in Fig. 2a) (distinguished by dot dashed lines). The problem $PS'_{REX\,i}$, formulated in this manner, was implemented and solved in the constraint programming environment OzMozart (CPU Intel Core 2 Duo 3GHz RAM 4 GB). The first acceptable solution was obtained in less than one second. The result of the problem solution for the substructure from Fig. 2a) are the operation times ${}^{(i)}T$ and their initiation moments ${}^{(i)}X'$ and the dispatching rules ${}^{(i)}\Theta$ shown in the Tab. 1.

To sum up, in the substructure ${}^{(i)}SC$ cyclic behavior is attainable if the operation times have such values and the dispatching rules as those in Tab. 1. The cyclic schedule attainable in this substructure was illustrated in Fig. 2b). It shows that the operations executed on the mutual resources do not superimpose on each other. According to (7) the attained schedule is a component of the schedule X'that characterizes the behavior of the whole structure SC.

The schedule $X'(7)$ being a multiple composition of the schedules ${}^{(i)}X'$ is presented in Fig. 3. It is evident that the composition of schedules ${}^{(i)}X'$of all the substructures of the structure SCdoes not lead to interferences in the execution of the operation – the schedules ${}^{(i)}X'$ on the resources ${}^{(i)}R_1$, ${}^{(i)}R_3$ i ${}^{(i)}R_9$. On the basis of the obtained schedules it is also possible to determine (according to (13) the dispatching rules for all the resources of the structure SC; the rules are presented in Tab. 1.

To sum up, the cyclic behavior in the structure SC is attainable if the operation times and the dispatching rules are such as those in Tab. 1.

Referring back to the AGVS layout presented in Fig. 1a), the obtained schedule should be treated as an illustration of AGVs movement (local processes) and the method of executing transportation routes (multimodal processes) in a network consisting of numerous fragments of the same type (Fig. 1b)). It should be emphasized that the periodicity of local processes in the network of this kind amounts to $\alpha = 6$ u.t. (units time), and the times of transporting elements of a single structure amount to 10 u.t (process ${}^{(i)}mP_1^1$) and 9 u.t. (process ${}^{(i)}mP_2^1$).

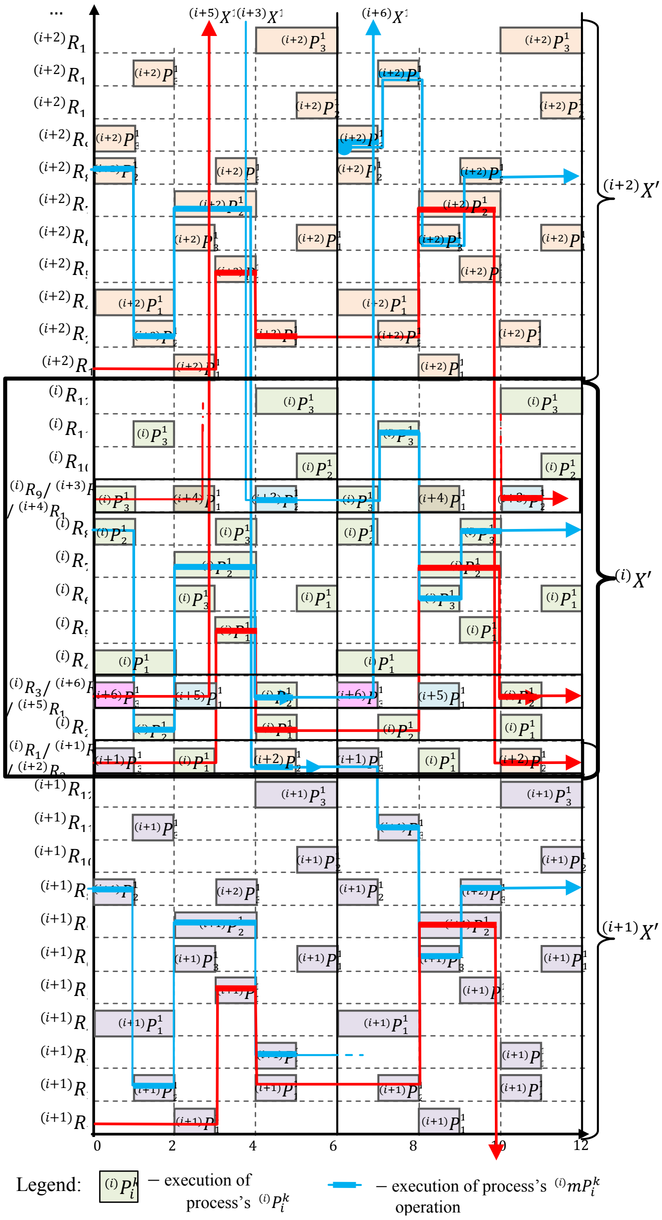

Fig. 3. Cyclic schedule for structure SC from Fig. 1

6. CONCLUSIONS

A declarative modeling approach to AGVs fleet scheduling in fractal-like AGVS multimodal networks environment is considered. Opposite to traditional approach a given network of local cyclic acting AGV services is assumed. In such a regular network, i.e. composed of elementary and structurally isomorphic subnetworks, the work-pieces pass their origin-destination routes among workstations using local AGVs, i.e. AGVs assigned to subnetworks. Since an AGVs fleet scheduling problem can be seen as a blocking job-shop one where the jobs might block either the workstations or an AGVs, and this is a NP-hard problem, hence the considered case of AGVs fleet scheduling in fractal environments also belongs to NP-hard problems. The solution proposed assumes that schedules of locally acting AGVs will match-up the given, i.e. already planned, schedules of work-pieces machining. The relevant sufficient conditions guaranteeing such a match-up exists were provided.

Table 1. The timing of commencement, operation times and the dispatching rules of ${}^{(i)}SC$ z from Fig. 1b)

	j	${}^{(i)}x^1_{j,1}$	${}^{(i)}x^1_{j,2}$	${}^{(i)}x^1_{j,3}$	${}^{(i)}x^1_{j,4}$	${}^{(i)}x^1_{j,5}$			j	${}^{(i)}t^1_{j,1}$	${}^{(i)}t^1_{j,2}$	${}^{(i)}t^1_{j,3}$	${}^{(i)}t^1_{j,4}$	${}^{(i)}t^1_{j,5}$
${}^{(i)}P^1_1$	1	2	3	4	5	6		${}^{(i)}P^1_1$	1	1	1	1	1	2
${}^{(i)}P^1_2$	2	4	5	6	7	8		${}^{(i)}P^1_2$	2	1	1	1	1	2
${}^{(i)}P^1_3$	3	6	7	8	9	10		${}^{(i)}P^1_3$	3	1	1	1	1	2

	j	${}^{(i)}mx^1_{j,1}$	${}^{(i)}mx^1_{j,2}$	${}^{(i)}mx^1_{j,3}$	${}^{(i)}mx^1_{j,4}$	${}^{(i)}mx^1_{j,5}$	${}^{(i)}mx^1_{j,6}$	${}^{(i)}mx^1_{j,7}$
${}^{(i)}m^1P^1_1$	1	6	7	8	9	13	14	16
${}^{(i)}m^1P^1_2$	2	2	3	4	8	10	-	-
	j	${}^{(i)}mt^1_{j,1}$	${}^{(i)}mt^1_{j,2}$	${}^{(i)}m^1t^1_{j,3}$	${}^{(i)}mt^1_{j,4}$	${}^{(i)}mt^1_{j,5}$	${}^{(i)}mt^1_{j,6}$	${}^{(i)}mt^1_{j,7}$
${}^{(i)}m^1P^1_1$	1	1	1	1	1	1	2	1
${}^{(i)}m^1P^1_2$	2	1	1	1	2	1	-	-

dispatching rule for local processes			
${}^{(i)}\sigma^0_1$	$({}^{(i)}P^1_1)$	${}^{(i)}\sigma^0_6$	$({}^{(i)}P^1_1, {}^{(i)}P^1_3)$
${}^{(i)}\sigma^0_2$	$({}^{(i)}P^1_1, {}^{(i)}P^1_2)$	${}^{(i)}\sigma^0_8$	$({}^{(i)}P^1_2, {}^{(i)}P^1_3)$
${}^{(i)}\sigma^0_3$	$({}^{(i)}P^1_2)$	${}^{(i)}\sigma^0_9$	$({}^{(i)}P^1_3)$
dispatching rule for multimodal processes			
${}^{(i)}\sigma^1_1$	$({}^{(i)}m^1P^1_2)$	${}^{(i)}\sigma^1_7$	$({}^{(i)}m^1P^1_2, {}^{(i)}m^1P^1_1)$
${}^{(i)}\sigma^1_2$	$({}^{(i)}m^1P^1_2, {}^{(i)}m^1P^1_1)$	${}^{(i)}\sigma^1_9$	$({}^{(i)}m^1P^1_1)$
${}^{(i)}\sigma^0_3$	$({}^{(i)}m^1P^1_2, {}^{(i)}m^1P^1_1)$		

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