

Genetic algorithm for multi-level assembly systems under stochastic lead times

Oussama Ben Ammar*, Alexandre Dolgui*, H el ene Marian*

* * cole Nationale Sup erieure des Mines, CNRS UMR6158 LIMOS, F-42023 Saint- tienne, France*
(e-mail: {obenammar, dolgui, marian}@emse.fr)

Abstract: The aim of this paper is to propose tools to adapt and parameterize the Material Requirement Planning (MRP) method under lead time uncertainty. We study multi-level assembly systems with one type of finished products and several types of components. We consider that each component has a fixed unit inventory cost and the finished product has a backlogging cost per unit of time. The lead times of components are discrete random variables, and the customer's demand of the finished product is known. A general mathematical model for supply planning of multi-level assembly systems is presented. A Genetic Algorithm (GA) method is proposed to minimize the sum of the average inventory holding cost for components and the average backlogging and inventory holding costs for the finished product.

Keywords: Stochastic lead times, Multi-level assembly system, MRP Parameterization.

1. INTRODUCTION AND RELATED PUBLICATIONS

For assembly systems, the lead times of components may be an uncertain parameter; it is rarely deterministic and mostly has a variable value. This unpredictability may be caused by economic conditions (changes in costs increase in prices of raw materials, etc.) and to technical problems (machines breakdowns, limited capacity, delay of transport, etc.).

The literature review identified different types of supply variability. Wazed et al. (2009) identified the major factors of uncertainty in a real manufacturing environment as demand, supplier lead time, quality and capacity. Several states of the art in the field of MRP parameterization under uncertainties (Dolgui et al. (2013), Dolgui and Prodhon (2007), Damand et al. (2011), Koh et al. (2002) and Guide and Srivasta (2000)) studied and analyzed their consequences. Various techniques such as safety stocks and safety lead times are used by planners to control the supply variability in order to lead the better anticipation of uncertainties (Van Kampen Tim et al. (2010)).

For example Koh and Saad (2007) specified how the safety lead time is especially helpful in handling supply uncertainties, such as late delivery. Molinder (1997) proved that a high level of lead time variability and demand variability has a strong effect both on the level of optimal safety lead times and optimal safety stocks.

Dolgui et al. (2008) studied the MRP parameterization problem for assembly systems under uncertainties, in particular, for the control of component inventories for two-level assembly systems with random component procurement times. They explained that lead time uncertainties seem to be insufficiently studied for a long time, favoring the study of demand uncertainties.

The aim of this study is to investigate the effectiveness of lead times in the presence of supply variability. We are interested in a one period model demand for multi-level assembly systems under a fixed demand and uncertainty of components lead times.

The rest of paper is organized as follows. Firstly, we present a short review of previous work relating to the optimization of assembling systems under uncertainties for a one period model demand (section 2). The problem description is presented in section 3. The analytical model is proposed in section 4. In section 5, we present the optimization algorithm which is used to find the order release dates which minimize the expected value of the total cost. Some results are shown in sections 5 and 6. Finally, we outline the work done in the conclusion and give some perspectives of future research.

In this paper, a multi-level assembly system with stochastic lead times at each level is studied. We focus on the problem of MRP parameterization under lead time uncertainties. It continues the work of several authors (Ben Ammar et al. (2012), Hnaïen et al. (2009), Dolgui et al. 2008, Hnaïen et al. 2007 and Ould Louly et al. (2002)).

In the literature few researchers have considered lead times as discrete random variables. In papers Dolgui et al. (1995) and Dolgui (2001), authors proposed an approach based on the coupling of simulation models and an integer linear programming. A model is considered for one level assembly systems under constant demand and for stochastic lead times. The lot for lot policy was employed and several types of finished products were considered. Each finished product is assembled using various types of components. Holding cost for each item was considered. The suggested approach is applied to calculate the number of components of each type to be ordered at the beginning of each period as well as the number of products to be assembled during each period.

Dolgui et al. (2009) and Ould Louly et al. (2008) studied multi-period one-level assembly systems under components lead times uncertainties. The demand was considered as deterministic and the capacity of the assembly system was assumed unlimited. They used a generalization of discrete Newsboy model proposed in the paper Ould Louly and Dolgui (2002) to minimize the average inventory holding cost for components while maintaining a high customer service level for the finished product. The same problem was solved by a Branch and Bound approach in Ould Louly and al (2008a).

Tang and Grubbström (2003) studied a two-level assembly system with stochastic lead times for components and fixed demand for the finished product. The due date is assumed to be known and the process time for components at level one is also stochastic. The Laplace transform procedure was proposed to minimize the total backlogging and inventory holding costs. The optimal safety lead times, which are the difference between planned and expected lead times are determined. The same problem was treated by Hnaïen et al. (2009). A GA is suggested to find the release dates for the components at level 2 and to minimize the total expected cost which equals to the sum of the inventory holding costs for components and the backlogging cost for the finished product. In the paper of Fallah-Jamshidi et al. (2011), the same problem is considered but within a multi-objective context. For minimizing both costs at the same time, authors reinforce the GA with a developed evolutionary algorithm, called the Electromagnetism-like Mechanism.

In the paper Ben Ammar et al. (2010), authors study the same problem but for multi-level assembly systems. The main aim is to find the values of planned lead times which minimize the sum of the average component holding cost and the average finished product backlogging and holding costs. They proposed a simulation model coupled with a GA which Hnaïen et al. (2009) used in their studies. To validate their model, they compared, for two-level assembly systems, their approach with a mathematical model coupled with the same GA. The last approach appears more accurate, efficient and to converge faster than the simulation model coupled with the same GA. However, the simulation model allows the study of multi-level assembly systems.

2. PROBLEM DESCRIPTION

To get closer to the industrial methods of planning, we consider a discrete temporal environment. The figure (1) shows that the finished product is produced from components themselves obtained from other components. We limit our study into a single period. We assume that the demand D for the finished product is deterministic and known as well as its due date T . A unit backlogging cost and a unit inventory holding cost for the finished products, and a unit inventory cost for each component are considered. Actual lead times are modelled as independent random discrete variables with known probability distributions.

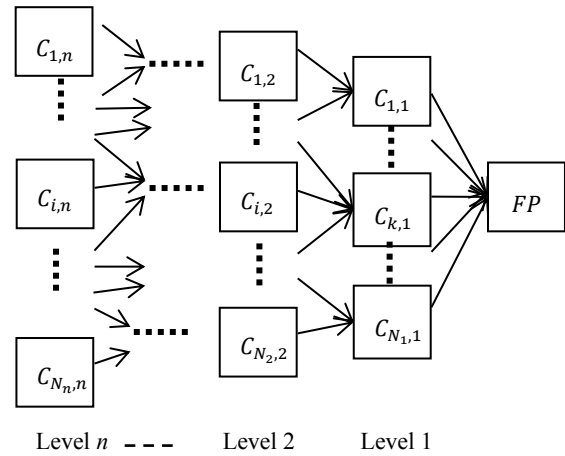


Fig. 1. A multi-level assembly system.

The following notations are used in this paper:

Table 1. Notation

Parameters	
T	Due date for the finished product
D	Demand for the finished product, without loss of generality, let $D = 1$
l	Level in a bill of material (BOM), $l = 1, \dots, m$
$c_{i,l}$	Component i of level l of BOM
N_l	Number of types of components of level l
$S_{i,l}$	Set of the “sons” of $c_{i,l}$ in a BOM tree
$L_{i,l}$	Random lead time for component $c_{i,l}$
$u_{i,l}$	Maximum value of $L_{i,l}$; each $L_{i,l}$ varies in $[1, u_{i,l}]$
$U_{i,m}$	The longest time between the release date for component $c_{i,m}$ and T . It is equal to the maximum value of $\sum_{v=1}^m L_{i,v}$; $\sum_{v=1}^m L_{i,v}$ varies in $[T - \sum_{v=1}^m u_{i,v}, T - m]$, $\forall \theta \in [1, m - 1], c_{i_{\theta+1},l} \in S_{i_{\theta},l-1}$
$h_{i,l}$	Unit holding cost for component $c_{i,l}$
b	Unit backlogging cost of the finished product
r	Unit inventory holding cost for the finished product
Variables	
$X_{i,m}$	Decision variable: release date for component $c_{i,m}$ (this type of variable is defined only for components at level m)
Functions	
$E[\cdot]$	Expected value
$F_{i,l}(\cdot)$	Cumulative distribution function of $L_{i,l}$
$Q(\cdot)$	The recursive function used to calculate $E[\cdot]$ value

In this model, the MRP system is considered as a push-system. Thus, for each level, when all the necessary components are available, level m delivers the components to level $m - 1$ with a random discrete lead time. When the semi-finished product arrives at the final level (level 0), it undergoes the necessary operations and afterwards the

finished product is delivered to the customer in order to satisfy the demand D . It is assumed that each component of level m is used to assemble only one type of component at level $m - 1$.

We use the following notations to simplify several expressions:

- Assembly date for $c_{i,m-1}$:

$$M_{i,m-1} = \max_{c_{k,m} \in S_{i,m-1}} (L_{k,m} + X_{k,m})$$

- Assembly date for $c_{i,l-1}$:

$$M_{i,l-1} = \max_{c_{k,l} \in S_{i,l-1}} (M_{k,l} + L_{k,l}), l = 2, \dots, m - 1$$

- Assembly date for the finished product:

$$M_{PF} = \max_{i=1, \dots, N_1} (M_{i,1} + L_{i,1})$$

- Maximum between M_{PF} and the due date T :

$$M_{PF}^+ = \max(M_{PF}, T)$$

- Minimum between M_{PF} and the due date T :

$$M_{PF}^- = \min(M_{PF}, T)$$

- $R = r + \sum_{i=1}^{N_1} h_{i,1}$

- $K = \sum_{i=1}^{N_1} h_{i,1} + b$

- $\sum_{i=1}^{N_l} H_i = \sum_{i=1}^{N_l} \left(h_{i,l} - \sum_{c_{k,l+1} \in S_{i,l}} h_{k,l+1} \right)$

- $\langle Y \rangle^+ : \max(0, Y)$

- $\langle Y \rangle^- : \min(0, Y)$

3. MATHEMATICAL MODEL

The objective is to find the component release dates at level m in order to minimize the expected value of the total cost which equals to the sum of the inventory holding cost for components and the backlogging and inventory holding costs for the finished product (Fig. 2).

Proposition 1

An explicit form for the total cost is the following:

$$C(X, L) = b \times (M_{PF}^+ - T) - r \times (T - M_{PF}^-) + \sum_{i=1}^{N_1} h_{i,1} M_{PF} - \sum_{l=1}^{m-1} \sum_{i=1}^{N_l} H_i M_{i,l} - \sum_{l=1}^m \sum_{i=1}^{N_l} h_{i,l} L_{i,l} - \sum_{i=1}^{N_m} h_{i,m} X_{i,m} \quad (1)$$

With

$$L = (L_{1,1}, \dots, L_{i,1}, \dots, L_{N_1,1}, \dots, L_{1,m}, \dots, L_{i,m}, \dots, L_{N_m,m})$$

and $X = (X_{1,m}, \dots, X_{i,m}, \dots, X_{N_m,m})$.

The complete proof was published in Ben Ammar et al. (2013).

Proposition 2

The mathematical expectation of the total cost $EC[X]$ is given by the next expression:

$$E[C(X, L)] = b \times (E[M_{PF}^+] - T) + r \times (T - E[M_{PF}^-]) + \sum_{i=1}^{N_1} h_{i,1} E[M_{PF}] - \sum_{l=1}^{m-1} \left(\sum_{i=1}^{N_l} H_i \times E[M_{i,l}] \right) - \sum_{l=1}^m \left(\sum_{i=1}^{N_l} h_{i,l} E[L_{i,l}] \right) - \sum_{i=1}^{N_m} h_{i,m} E[X_{i,m}] \quad (2)$$

The complete proof was published in Ben Ammar et al. (2013).

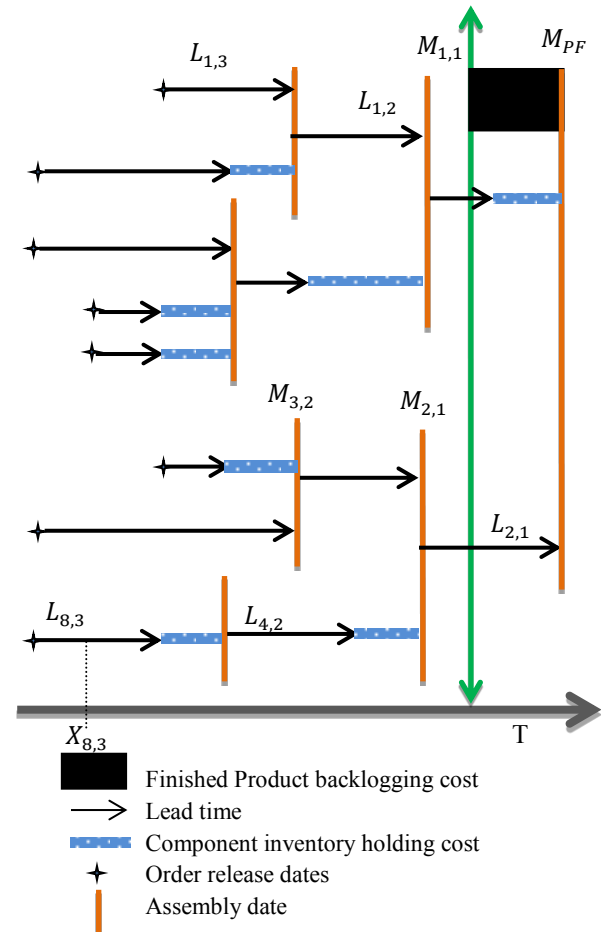


Fig. 2. A three-level assembly system.

Proposition 3

In order to decrease the complexity of the forthcoming algorithms, the research space of possible solutions $[T - U_{i,m}, T - m]$ is reduced to $[T - U_{i,m}, T - \sum_{v=1}^m E[L_{i,v,v}]]$, $\forall \theta \in [1, m - 1], c_{i_{\theta+1},l} \in S_{i_{\theta},l-1}$.

Proof 3

By contradiction, suppose that there exists an optimal solution $X^* = (X_{1,m}^*, \dots, X_{k,m}^*, \dots, X_{N_m,m}^*)$, with, $\forall c_{k,m} \in$

$$S_{i,m-1}, X_{k,m}^* \in [T - U_{i,m}, T - m] \text{ and } \forall c_{j,m} \in S_{i,m-1}, \forall \theta \in [1, m - 1], \forall c_{i_{\theta+1},l} \in S_{i_{\theta,l-1}}, X_{j,m}^* \in [T - \sum_{v=1}^m E[L_{i_v,v}], T - m].$$

We demonstrate that there is a dominant solution $A^* = (A_{1,m}^*, \dots, A_{k,m}^*, \dots, A_{N_m,m}^*)$ such that $\forall c_{k,m} \in S_{k,m-1}, A_{k,m}^* \in [T - U_{i,m}, T - E[L_{i,1}]] - E[L_{j,m}]$ [and $\exists! \delta \in IN, A_{k,m}^* = X_{k,m}^* - \delta$].

Let $\varepsilon(A^*, X^*) = E[C(A^*, L)] - E[C(X^*, L)]$ and $S_{sup} = T + m \times (u - 1)$.

After simplifications, we have:

$$\varepsilon(A^*, X^*) = b \sum_{T \leq s < T + \delta} (1 - Pr[M_{FP\{X^*\}} \leq s]) - r \sum_{T \leq s < T + \delta} (Pr[M_{FP\{X^*\}} \leq s])$$

It can easily be proven that for $\delta \in IN^*$ and for $b > \sum_{i=1}^{N_1} (h_{i,1})$:

$$\varepsilon(A^*, X^*) = 2b \times \left(\sum_{s=T}^{T+\delta+1} (Pr[M_{FP\{X^*\}} > s] - \frac{1}{2}) \right) > 0$$

4. GENETIC ALGORITHM

To solve the model of the problem, the Genetic Algorithm proposed in Ben Ammar, O. et al. (2010) and Hnaïen, F. et al. (2009) is applied. An Elitist strategy is employed. The initial population is formed by individuals built by randomized algorithm. Crossover, mutation, selection procedures and local search (LS) are used to create better individuals (chromosomes). A fitness function is available to evaluate each solution.

4.1 Perturbation

The perturbation (Fig. 3) consists of replacing 90% of the solutions who have the same cost and replacing them with solutions undergoing a special mutation (using block mutation), see figure (4). Each solution $X = (X_{1,m}, \dots, X_{i,m}, \dots, X_{N_m,m})$ undergoes a modification. The mutation concerns several genes $X_{i,m}, \dots, X_{j,m}$ which are the order release dates for components $c_{i,m}, \dots, c_{j,m}$ that needed to assemble a component $c_{k,m-1}$.

5. RESULTS

5.1 Data generation and setting

The two proposed methods described in Section 4 have been coded in C++. The experiments are carried on computer with 2.93 GHz CPU and 4 GB of RAM.

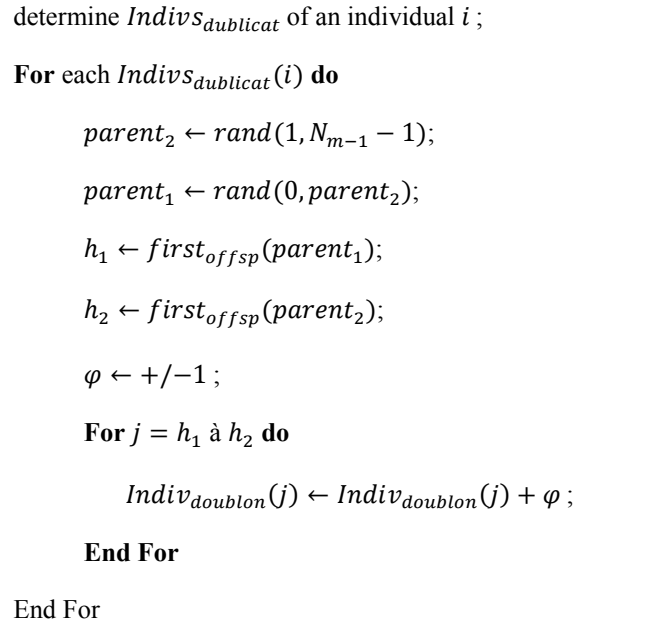


Fig. 3. Perturbation approach.

The solution approach is tested on a randomly generated instance set. We created 10 instance families for two-assembly systems. The number of components at level 2 is equal to [10,20, ...,100] in each family, 100 test instances are generated.

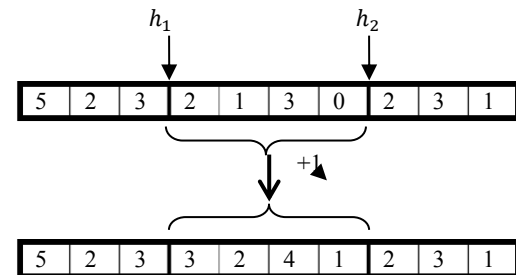


Fig. 4. Block mutation.

For parameters of the GA, the following values have been empirically chosen after preliminary tests: The population size is equal to 60 chromosomes, the crossing over probability is equal to 0.95 and the mutation probability is equal to 0.05. The number of generations (stop condition given by the maximum number of iterations) is fixed to 1000.

5.2 Experimental results

Tables (2-4) show the influence of the LS, the reducing of the search space and the perturbation on the families of instances.

The first column gives the number of components for each instance family. Second column is the average number of iterations where the best solution is found.

On the third column, the average gap between the best solution in the initial population and the best solution found by the algorithm is given:

$(gap = \frac{best_{sol_0} - best_{sol_{1000}}}{best_{sol_{1000}}} \times 100$ where $best_{sol_{1000}}$ the best solution in the population of generation 1000).

Next column provides the average gap^* between the result of the algorithm and the best solution among all versions of the GA ($gap^* = \frac{best_{sol_{1000}} - best_{sol^*}}{best_{sol^*}} \times 100$ where $best_{sol^*}$ is the best known solution (BKS) found among all versions of the GA). Finally the average execution time (when the best solution is found) of the algorithm is reported on the last column.

Table 2. GA without a LS or perturbation or a RSR

Instances	Iteration	Mean gap (%)	Mean gap^* from BKS (%)	CPU Time (sec)
10	383.377	60.99	0.19	0.077
20	790.803	109.34	10.19	0.517
30	826.203	86.90	40.18	1.068
40	830.541	51.99	62.85	0.806
50	844.415	59.34	34.21	1.066
60	861.310	58.36	95.89	1.610
70	868.987	75.55	118.86	2.139
80	851.819	46.08	57.46	2.686
90	884.392	42.67	72.63	3.373
100	837.873	38.78	77.71	4.763

The results of the GA combined without LS or perturbation or a RSR are presented on the table 2. We can observe that even if there is a considerable improvement of the initial population, the average gap from the Best Known Solution is very large. The average gap on all the instances is 63%. When the space of research is reduced (table 3), the improvement is very important; the total average gap is no more than 10% and the average number of iterations where the best solution is found is significantly reduced by 49.11%.

Table 3. GA combined with a RSR

Instances	Iteration	Mean gap (%)	Mean gap^* from BKS (%)	CPU Time (sec)
10	98.096	1.89	0.00	0.008
20	537.632	15.74	1.87	0.141
30	462.932	15.78	4.44	0.238
40	441.831	8.35	10.15	0.987
50	352.059	5.30	8.40	1.057
60	351.624	6.97	52.31	0.593
70	340.999	8.03	63.54	0.831
80	466.987	10.45	10.60	1.415
90	535.24	10.25	6.54	2.269
100	473.633	8.84	8.60	2.184

We can state by the table 4 that, the inclusion of perturbation improves considerably the solution quality: the total average gap from the best known solutions on all

instances is 0.08%. But the average number of iterations where the best solution is found is more than 395 generations.

Finally, almost all the best known solutions are obtained when the use of the RSR is combined with the perturbation and with le LS (table 5). The total average gap is also less than of 0.01%. The average number of iterations where the best solution is found is significantly reduced to 113.80.

Table 4. GA combined with perturbation and LS but without a RSR

Instances	Iteration	Mean gap (%)	Mean gap^* from BKS (%)	CPU Time (sec)
10	36.016	59.67	0.00	0.010
20	91.007	133.49	0.00	0.083
30	144.038	161.19	0.00	0.243
40	225.257	147.25	0.01	0.264
50	530.231	112.87	0.23	0.776
60	363.227	170.19	0.01	0.861
70	417.974	196.10	0.01	1.307
80	675.254	141.42	0.16	2.399
90	709.28	140.65	0.10	3.293
100	754.048	144.13	0.28	4.501

We can also see on the tables 2-5 that even on the largest instances, the mean execution time of the GA is less than 5 seconds.

Table 5. GA combined with perturbation, RSR and LS

Instances	Iteration	Mean gap (%)	Mean gap^* from BKS (%)	CPU Time (sec)
10	15.436	2.20	0.00	0.002
20	45.05	8.59	0.00	0.015
30	51.567	8.89	0.00	0.042
40	85.584	0.09	0.00	0.255
50	156.958	3.31	0.08	0.050
60	68.979	8.05	0.00	0.157
70	72.504	9.61	0.00	0.231
80	119.782	1.58	0.08	0.500
90	226.911	2.25	0.01	1.060
100	295.269	1.50	0.01	1.321

6. CONCLUSIONS

The paper deals with the modeling and optimization of multi-level assembly systems under uncertainty of components lead times and a fixed demand, more precisely to determinate planned lead times when the component procurement times are independent and identically distributed discrete random variables.

Our future work will focus on the analysis of the correlation between the number of levels and the number of components in the level m of the nomenclature.

The main objective will be to use this general mathematical model and different proposed techniques to parameterize MRP system, in particular, planned lead times, when such a company deals with uncertainties of production and supply lead times.

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