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Combined Approach of Fuzzy Decision Making and Predictive Functional Control to Minimize Variations of Manipulated Variables in Processes with Dead Time

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Abstract: Basically conventional controllers operate on a crisp set point, even if the control task does not require an exact value of the controlled variable. In fact the requirements for the controlled variable can often be described as intervals of acceptable or ideal ranges. In this paper we present an approach of a predictive functional controller combined with fuzzy decision making, which leads to a controller that operates on complete fuzzy goals. This approach is demonstrated for simple processes with dead time and the performance is analyzed by comparison with conventional controllers.

1. INTRODUCTION

The aim of many control applications is to reach a specified range of the process variable x and therefore it is not necessary to hold the controlled variable x at an exact set point w. The requirements of the control task may then be defined by intervals of ideal, acceptable and inappropriate ranges, whereby the controlled variable x should stay at least in the acceptable range, preferably in the ideal range. Such a linguistically formulated range can easily be described by formulating a fuzzy goal with the parameters of a trapezoidal membership function μ_W (figure 1):

$$\mu_W(x) := f_T(\underline{W}, x) = f_T\left([W_1 \quad W_2 \quad W_3 \quad W_4]^T, x\right) \tag{1}$$

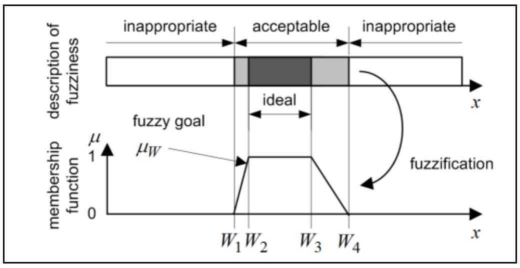

Fig. 1. Fuzzification of stationary requirements for controlled variable x .

It is obvious, that one requirement of the control task is, to manipulate the process in such a way, that the controlled variable x owns a high degree of membership to the stationary fuzzy goal μ_W represented in Fig.1. Besides of reaching the stationary goal, the reducing of variations of the manipulated variable u is desirable. By reducing the variations of the manipulated variable u ,it is possible to take care of the actuators durability. Hence, an additional fuzzy goal μ_C for the dynamic behaviour has to be formulated. Therefore, initially the maximum (forced) variation of the controlled variable $\dot{x}_{\max}$ can be formulated based on the maximum gradients of the manipulated variable $\dot{u}_{\max}$:

$$\dot{x}_{\max} = f(\dot{u}_{\max}) \approx \Delta x / \Delta T \tag{2}$$

Using equation (2) and doing some simple transformations leads to the dynamic fuzzy goal μ_C . Regarding the actuators durability it is evident, that no variations of the manipulated variable u is “ideal”, which is the case, if there is no need for an adjustment of the controlled variable x. Consequently we define the core of the membership function only for $\dot{x} = 0$. Hence we will get a resulting triangular membership function using the sample time ΔT with the optimal case of $\dot{x} = 0$ and the spread defined by the maximum accepted variation $\dot{x}_{\max}$:

$$\mu_C(\Delta x) := f_T\left(\Delta T \cdot [-\dot{x}_{\max} \quad 0 \quad 0 \quad \dot{x}_{\max}]^T, \Delta x\right) \tag{3}$$

In many technical applications that deal with SISO-Systems simple continuous controllers (like PID) or discontinuous controllers (like two- or three point controllers) are used. For that type of controllers it is basically necessary to define a crisp set point w and it is not possible to operate on a fuzzy goal like shown in figure 1. As one consequence, the variations of the manipulated variable $|\Delta u|$ can not be taken into account. In case of using a continuous controller it may be suitable to use a nonlinear transfer element at the input of the controller, which operates as a dead zone element. For discontinuous controllers we can define a hysteresis Δh to reduce the variations of the manipulated variable $|\Delta u|$. An approach for defining the parameters for hysteresis Δh or a dead zone, may be to use an alpha cut of the stationary fuzzy goal μ_W and to define the supremum and infimum of the resulting set as parameters of the set point interval $[w_{\min}, w_{\max}]$, as shown in figure 2.

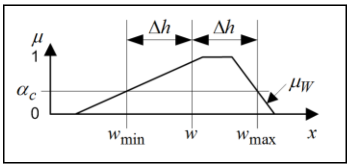

Fig. 2. Defining the limiting set points out of a stationary membership function depended on the parameter of the alpha cut.

Now it is possible to define the parameters of hysteresis Δh and dead zone as well as the desired value w of the controlled variable x for the controllers by the following equations:

$$
\begin{aligned}
w(\mu_W,\alpha_c) &= \frac{w_{\max}(\mu_W,\alpha_c)+w_{\min}(\mu_W,\alpha_c)}{2} \\
\Delta h(\mu_W,\alpha_c) &= \frac{w_{\max}(\mu_W,\alpha_c)-w_{\min}(\mu_W,\alpha_c)}{2}
\end{aligned} \tag{4}
$$

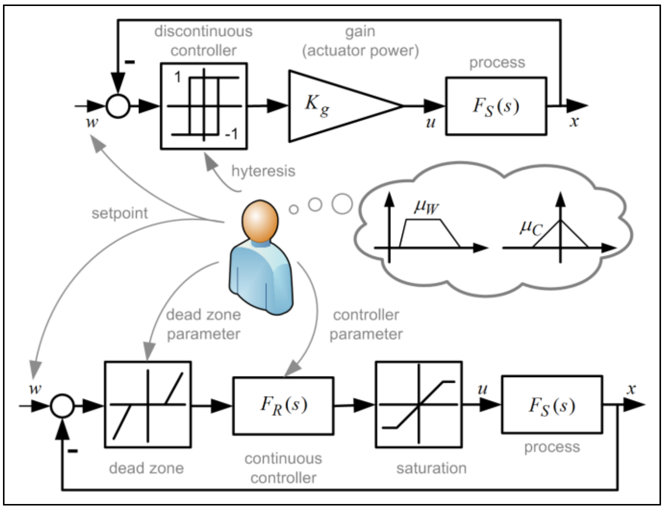

Fig. 3. Adjusting the parameters for a discontinuous (above) and a continuous controller (below). The user has to define a compromise out of stationary requirements and the variations in the manipulations.

As we can see in figure 3 there are a lot of parameters to set up by the user, which can lead to a confusing tuning task. To avoid the defining of the parameters for controllers, hysteresis, dead zone, etc., it may be suitable to use a controller which operates on the complete fuzzy goal, as shown in figure 4.

In this paper we will describe an approach of a fuzzy goal predictive functional controller, which operates on the complete fuzzy goal. Therefore, we will combine the theories of predictive functional control and fuzzy decision making. We will use fuzzy decision making to replace the calculation of the reference trajectory for future process states for the predictive functional controller. First of all we will describe some theoretical aspects in section 2 followed by the presentation of the approach of a fuzzy goal predictive functional controller in section 3. Some simulation results will be presented in section 4. Finally, some concluding remarks and a short outlook will be given in section 5.

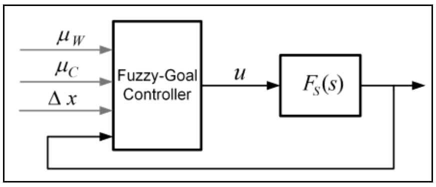

Fig. 4. Approach for a fuzzy goal predictive functional controller.

2. FUNDAMENTALS

In this chapter we will introduce some basics of predictive functional control (PFC) and the theory of fuzzy decision making, which is necessary to explain the approach of the fuzzy goal predictive functional controller (FGPFC) in the next section.

2.1 Predictive Functional Control

The theory of predictive functional control was developed out of the Model based predictive control (MPC), which is a very common method of advanced control strategies. The idea of MPC can easily be described by the following steps: initially the free process behavior without control is predicted in a prediction horizon n_P. The prediction is done by an internal process model. The process model requires some measurements or observations for the initial state. The future disturbances of the process can also be used if they are known. With the prediction of future process states, it is possible to evaluate a control sequence in such a way, that the futures control deviation (difference between set point and controlled process variable) is minimized.

When we have to deal with complex processes, like nonlinear processes or processes with contraints, it is often necessary to use a numerical solver with an (for example iterative) optimization method to calculate the future control sequence ((Adamy, 2009), (Dittmar *et al*., 2004)). Otherwise, for processes that can be described as linear state space systems without constraints, there is an analytic way to calculate the control sequence. In that case, we are talking about predictive functional control (PFC), which was presented first by Richalet in (Richalet *et al*., 1978).

As mentioned above, the the PFC has the advantage that a numerical optimization for calculating a control sequence is not necessary. The existence of an analytic solution leads to an easy implementation in practice. There are many examples which demonstrate, that the PFC is getting more and more successful applied in industrial applications (see (Richalet *et al*., 1978), (Richalet *et al*., 2009), (Luft, 2009)). Richalet described PFC as "easy to understand, to implement, to tune".

Some important modifications by Richalet can simplify the optimization problem significant (Valencia-Palomo *et.al.*, 2012). On the one hand, there is the choice of coincidence points, which means that the control error has to be minimized only at a random point and not in the complete prediction horizon (for example by mean square error). This leads to an easier objective function of the PFC. On the other hand, only basic functions are used to describe the behavior of the manipulated variable. Consequently, the solution space for possible control actions gets reduced.

The application of PFC will be demonstrated in this paper by the control of a first-order lag element in combination with a dead time. The results of using the proposed fuzzy goal PFC to control a first order system without dead time are presented in (Arnold *et.al.*, 2013). The transfer function of the system regarded in this paper, which is a suitable example of a transport application, can be represented as

$$F_S(s) = \frac{K_S}{1+\tau_S \cdot s} \cdot e^{-s\tau_t} \tag{5}$$

The aim is now to calculate the control equation by using this transfer function as an internal model. Therefore the dead time τ_t is initially neglected and will be considered later. A time-discrete formulation of the resulting first order system using the sample time ΔT and a first order hold is then defined by using the parameter $\alpha = \exp(-\Delta T/\tau_s)$:

$$\begin{aligned} x(k+1) &= e^{-\frac{\Delta T}{\tau_S}} \cdot x(k) + K_S \cdot \left(1 - e^{-\frac{\Delta T}{\tau_S}}\right) \cdot u(k) \\ &= \alpha_S \cdot x(k) + K_S \cdot (1-\alpha_S) \cdot u(k) \end{aligned} \tag{6}$$

Usually, the dynamic behavior of the PFC is defined by a reference trajectory $x_R(k+1)$. It is obvious that the requirement of the control task is, that the internal model should follow the reference trajectory. Hence, we have the premise $\hat{x}(k+1) = x_R(k+1)$. The future process state can easily be calculated by the first order system (equation (6)) and the futures disturbance:

$$\hat{x}(k+1) = \alpha_S \cdot x(k) + K_S \cdot (1-\alpha_S) \cdot u(k) + z(k+1) \tag{7}$$

Assuming that the disturbance of the next sample $z(k+1)$ will be the same as in the current one (in a simplified approach), we can calculate the disturbance by the current process states in the following way:

$$z(k+1) \approx z(k) = x(k) - \hat{x}(k) \tag{8}$$

Some mathematical transformations of equations (6), (7) and (8) lead to the control sequence equation:

$$u(k) = \frac{x_R(k+1) - x(k) \cdot (1+\alpha_S) + \hat{x}(k)}{K_S \cdot (1-\alpha_S)} \tag{9}$$

The influence of the dead time on the process is obviously given by the discrete formulation of the dead time $\tau_t = d \cdot \Delta T$:

$$\hat{x}(k-d) = x(k) \qquad x(k+d) = \hat{x}(k) \tag{10}$$

Both equations can be combined to:

$$x(k+d) = \hat{x}(k) + \hat{x}(k-d) - x(k) \tag{11}$$

To control the dead time system we have to replace $x(k)$ in equation (9) by $x(k+d)$ of equation (11). The structure of the PFC controller is shown in figure 5. It has to be mentioned that constraints of the manipulation variable $u'(k)$ may be taken into account by using the constrained value of the manipulation variable $u(k)$ as input of the process and the inner model. Besides that, it is still not possible to operate on the complete fuzzy goal μ_W, so that a crisp set point is still necessary (figure 5). Therefore, we replace the calculation reference trajectory $x_R(k+1)$ by a fuzzy decision making approach.

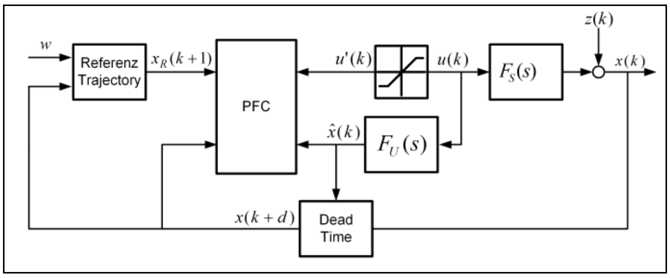

Fig. 5. Structure of the PFC controller.

2.2 *Fuzzy Goals and Fuzzy Decision Making*

As already mentioned, the requirements for the set points of some control applications are often formulated inexact or as intervals. Thus, we often have to deal with linguistically formulated requirements. An opportunity to handle those requirements is to apply the fuzzy theory by using fuzzy numbers or fuzzy intervals. In that case we can transform the knowledge of the user into a stationary fuzzy goal $\mu_W(x)$. By using a fuzzy formulation for the requirements of the control task we will be able to take care of forbidden areas of the possible states. For reasons of simplification we will only use trapezoidal membership functions as described in section 1 (see also figure 1).

In (Bellmann *et.al.*, 1970) the theory of fuzzy decision making was introduced. The idea is to find the optimal solution x^* out of a set of alternatives x, which represents the solution space. Thereby all requirements and constraints have to be formulated as fuzzy sets. Hence we formulate an amount of fuzzy goals $\mu_{Gi}(x)$ (with i=1,…n) and fuzzy contraints $\mu_{Cj}(x)$ (with j=1,…,m). The best solution x^* has to be calculated by an aggregation of all fuzzy goals $\mu_{Gi}(x)$ and constraints $\mu_{Cj}(x)$. There are several ways to find the best solution. In this paper we focus on the approach presented in (Bellmann *et.al.*, 1970), where the fuzzy decision set (possible solutions for x) $\mu_D(x)$ is defined by the intersection of all fuzzy goals and constraints. One possibility to determine the optimal solution x^* out of the fuzzy decision set $\mu_D(x)$ is given by the maximum value (Bandemer *et.al.*, 1970):

$$\mu_D(x^*) = \max\underbrace{\left\{\left(\bigcap_{i=1}^{n}\mu_{Gi}(x)\right)\cap\left(\bigcap_{j=1}^{m}\mu_{Cj}(x)\right)\right\}}_{\mu_D(x)} \quad (12)$$

It has to be mentioned, that all constraints can also be transformed into goals by a simple reversing of the set. That's the reason why we will focus on fuzzy goals from now on (Bernard, 2000).

3. CONCEPT OF A FUZZY GOAL PFC CONTROLLER

In this chapter we will present our proposed approach of a fuzzy goal PFC. The advantage of the proposed controller is, that there is no need for a crisp set point, like in a conventional PFC. The fuzzy formulation of the requirements can be used directly at the input of the controller. The approach is to replace the reference trajectory with a fuzzy decision making approach, explained shortly in section 2.

3.1 Approach to a Solution

It is obvious that the optimal value of the controlled variable depends on the stationary and the dynamic requirements. Hence, it is necessary to find a compromise of both requirements. The stationary fuzzy goal μ_W is already formulated as a function of the controlled variable x (see equation (1)). Consequently the dynamic requirements have to be transformed as a function of x as well. We can do this by adding the current process state to the accepted variations mentioned in equation (3):

$$\begin{aligned}\mu_C(x) &= f_T\left(x(k) + \Delta T\cdot[-\dot{x}_{\max}\quad 0\quad 0\quad \dot{x}_{\max}]^T, x\right)\\ &= f_T\left(x(k) + [-\Delta x_{\max}\quad 0\quad 0\quad \Delta x_{\max}]^T, x\right)\end{aligned} \quad (13)$$

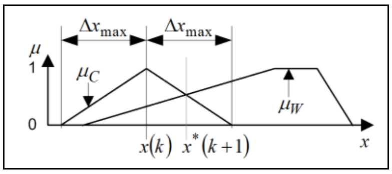

Fig. 6. Compromise of stationary and dynamical goal.

According to equation (12) the optimal solution is given by the maximum membership of the intersection set of the stationary fuzzy goal μ_W and the dynamic fuzzy goal μ_C (also shown in figure 6):

$$\mu_D(x^*) = \max\{\mu_W(\underline{W}, x)\cap\mu_C(\Delta x_{\max}, x(k), x)\} \quad (14)$$

Another benefit of using trapezoidal membership functions is that the optimal decision can easily be calculated analytically by using a case differentiation and the relations of table 1.

Table 1. Case differentiation for the optimal solution

$x^*(k+1)$	for
$x(k)+\Delta x_{\max}$	$x(k)+\Delta x_{\max} < W_1$
$\frac{(x(k)+\Delta x_{\max})\cdot W_2 - x(k)\cdot W_1}{\Delta x_{\max} + \cdot W_2 - W_1}$	$(W_2 \geq x(k))\wedge (x(k)+\Delta x_{\max} \geq W_1)$
$x(k)$	$W_2 \leq x(k) \leq W_3$
$\frac{x(k)\cdot W_4 - x(k) - \Delta x_{\max}\cdot W_3}{W_4 - W_3 + -\Delta x_{\max}}$	$(W_3 \leq x(k))\wedge (x(k)-\Delta x_{\max} \leq W_4)$
$x(k)-\Delta x_{\max}$	$x(k)-\Delta x_{\max} > W_4$

3.2 Closed loop dynamic behaviour in accepted area

For convenience we focus on the lower part of the accepted area (second case of table 1). It is evident, that the considerations are similar for the upper part of the accepted area (4th case of table 1). For the lower part we receive the following solution for the next sample:

$$x(k+1) = \frac{(x(k)+\Delta x_{\max})\cdot W_2 - x(k)\cdot W_1}{\Delta x_{\max} + W_2 - W_1} \quad (15)$$

Using the z-Transformation and some mathematical operations leads to the following equation:

$$\frac{x(z)}{W_2} = \frac{\dfrac{\Delta x_{\max}}{\Delta x_{\max} + W_2 - W_1}}{z - \dfrac{W_2 - W_1}{\Delta x_{\max} + W_2 - W_1}} = \frac{1 - e^{-(T_\Delta/\tau_{WL})}}{z - e^{-(T_\Delta/\tau_{WL})}} \quad (16)$$

The resulting system represents a first order time delay system with a static gain of one. The time constants can be determined for the lower and the upper part of the accepted area:

$$\begin{aligned}\tau_{WL} &= \frac{-T_\Delta}{\ln\left(\dfrac{W_2 - W_1}{\Delta x_{\max} + W_2 - W_1}\right)}\\ \tau_{WR} &= \frac{-T_\Delta}{\ln\left(\dfrac{W_4 - W_3}{\Delta x_{\max} + W_4 - W_3}\right)}\end{aligned} \quad (17)$$

Hence, to adjust the dynamic of the proposed fuzzy goal PFC it is only necessary to choose the accepted variations $\Delta x_{\max}$.

4. SIMULATIONS

Several simulations have been done to test and verify the approach of the fuzzy goal PFC. In this paper we will present the results of the control of a transport system with a significant dead time.

4.1 Model of the dead time system

One of the classic benchmark examples for control tasks with dead times are transport systems. The target of the control task of transport systems is to control the amount of material per time (Q_{out}) delivered to a specified place by adjusting a valve at the inflow. Hence, the valve is regulating the supply of the material (Q_{in}). It may also be possible that the process is disturbed by an unknown variable Q_z. The principle function is shown in figure 7.

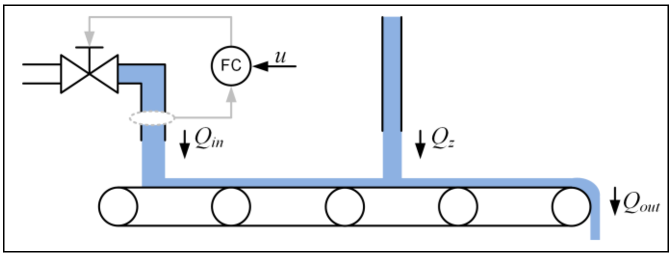

Fig. 7. Schematic representation of the process.

The dead time depends on the ratio of speed and length of the conveyor. The valve at the inflow can be described as a first order system. The resulting transfer function is given by:

$$F_s(s) = \frac{K_s}{1 + s \cdot \tau_s} \cdot e^{-s\tau_t} \tag{18}$$

The system investigated in this paper represents a first order lag element ($K_s = 0.2, \tau_s = 1s$) with dead time ($\tau_t = 10s$), which will be used as inner model of the controller.

4.2 Simulation results

All simulations were done with the simulation environment of (MATLAB/Simulink). A unified stochastic disturbance sequence Q_z was used for all executed simulations. The stationary fuzzy goal was defined as:

$$\mu_W(h) = f_T\left(Q_{out,\max} \cdot [0{,}2 \quad 0{,}4 \quad 0{,}6 \quad 0{,}9]^T, h\right) \tag{19}$$

To verify the proposed approach we compared the results of the fuzzy goal PFC with a simple PI controller and a smith predictor (Lunze, 2004), which is common practice for systems with dead time delay. The adjustment was done by zero-pole compensation. The performance of the mentioned controllers is shown in figure 8 in terms of a time plot of the controlled variable $x = Q_{in}$ and the manipulated variable $u = Q_{out}$.

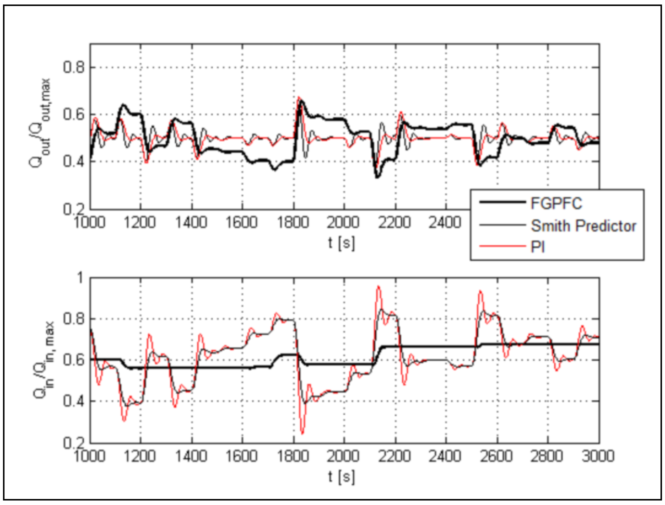

Fig. 8. Controlled (above) and manipulated signal (below) during the simulation scenario.

It is obvious, that the continuous controllers are focussed on the minimization of the control error all the time, which leads to higher variations of the manipulated variable, without achieving much better results regarding the stationary fuzzy goal. Another aspect to evaluate the performance of the controller is given by the mean fulfilment of the stationary fuzzy goal μ_W:

$$\bar{\mu}_W = \frac{1}{N} \cdot \sum_{i=1}^{N} \mu_W(h(i)) \tag{20}$$

and the mean variation of the manipulated variable (both shown in figure 9):

$$\Delta\bar{u} = \frac{1}{N} \cdot \sum_{i=1}^{N} |h(i) - h(i-1)| \tag{21}$$

It is evident, that an adaption of the controller's gain leads to an improvement (higher fulfilment of the stationary goal) to a certain degree. A higher gain for the PI controller leads to an oscillating and unstable system and therefore high variations of the manipulated variable and less fulfilment of the stationary goal. For the fuzzy goal PFC $\Delta x_{\max}$ is the parameter to adjust. The results of higher values for $\Delta x_{\max}$ are similar to the results of the PI controller or smith predictor. Because of the use of uncertain set points it is to be seen that the fuzzy goal PFC solves the control task by less variations of the manipulated variable.

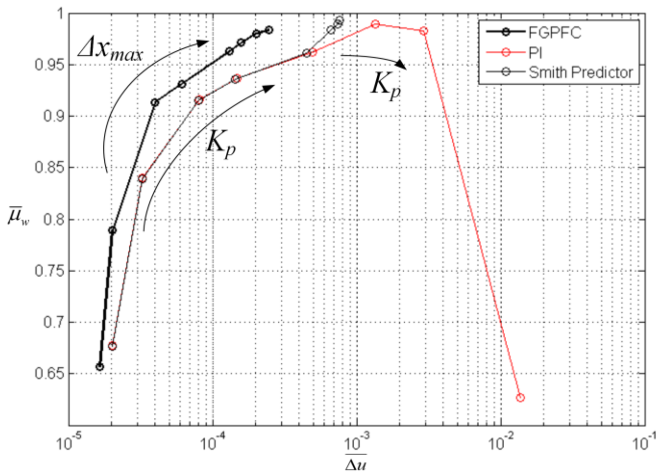

Fig. 9. Various control strategies regarding the fulfillment of the stationary fuzzy goal and the mean variation of the manipulated variable depending on K_p or $\Delta x_{\max}$.

For a last test the influence of the dead time to the robustness of the controllers were investigated. Therefore we analyzed what happens to the fulfilment of the stationary fuzzy goal for several dead times; the controllers parameters are not changed. The results are shown in figure 10. Apparently the PI-Controller is not suitable to handle larger dead-times. The Smith Predictor achieves good results, although the performance of the fuzzy goal PFC becomes relatively better for larger dead times by less variations of the manipulated variable.

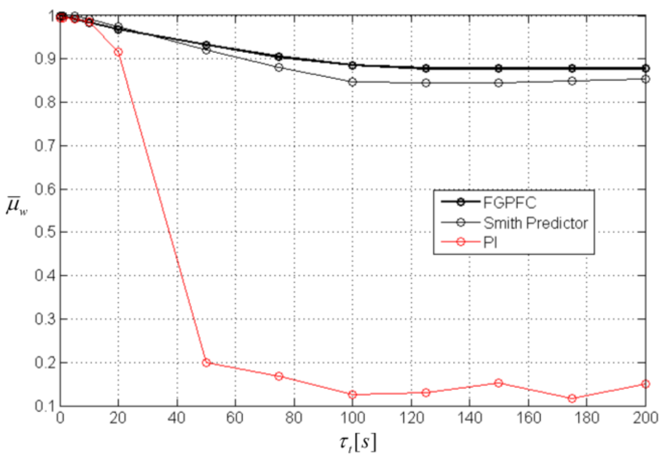

Fig. 10. Various control strategies regarding the fulfilment of the stationary dependent of the time delay τ_t

5. CONCLUSION AND OUTLOOK

In this paper we presented a fuzzy goal PFC, which operates on fuzzy formulated requirements. The way of doing this is to combine the conventional PFC with the theory of fuzzy decision making. Thereby, the calculation of a reference trajectory is replaced by a fuzzy approach. Hence, the requirements of the control task can easily be formulated linguistically. The adjustment of the proposed controller will be much easier to handle than finding several parameters of a conventional controller. We could show that the proposed approach provides good results in simulative experiments. Because of the uncertain set point the variations of the manipulated variable can be reduced without loosing stationary accuracy. In the simulations we also found out, that the proposed controller is robust against changing dead times.

In future works we will try to transmit the proposed approach to MIMO-Systems and processes with more than one inner state and we will verify the fuzzy goal PFC in practical experiments.

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