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Some Refinements on Stability Analysis and Stabilization of Second Order T-S Models Using Line-Integral Lyapunov Functions

Kevin Guelton, Abdelmadjid Cherifi
Laurent Arcese

CReSTIC EA3804 – Université de Reims Champagne-Ardenne, Moulin de la Housse BP1039, 51687 Reims cedex 2, France (Tel: +33 3 26 91 32 61; e-mail: name.surname@univ-reims.fr).

Abstract: This paper deals with new conditions for stability analysis and controller design for Takagi-Sugeno (T-S) models in the non-quadratic framework. The aim of this study is to provide some improvements on results that do not require the knowledge of the bounds of the membership function derivatives, i.e. the ones that employ line-integral Lyapunov candidate functions. First, sight improvements on recent stability conditions are proposed by the use of the Finsler's lemma and with unconstrained slack decision variables. Then, non-PDC controller design is proposed through two new approaches. The first proposed approach is based on the use of the Finsler's lemma but requires a scalar parameter to be fixed in advance. The second proposed approach employs another gimmick to avoid any unknown parameters and introducing new slack decision variables. It is finally noticed that these results can be solved as linear matrix inequalities (LMI) for first and second order systems, bilinear matrix inequalities (BMI) for the third order and then remain more complex as the system's order increases.

Keywords: Takagi-Sugeno models, Non-quadratic stability, Line-integral Lyapunov functions, LMI.

1. INTRODUCTION

Takagi-Sugeno (T-S) models constitute a convenient way for dealing with nonlinear systems stability analysis and stabilisation (Takagi and Sugeno, 1985). Indeed, using the sector nonlinearity approach (Tanaka and Wang, 2001), a nonlinear system can be exactly rewritten as a T-S one on a compact set of the state space. Hence, a T-S model is a collection of linear dynamics blended together by convex nonlinear membership functions. Therefore, it is possible to extend some results of the linear control theory to such nonlinear systems.

Stability analysis and stabilization of T-S models have been widely investigated through the use of common quadratic Lyapunov functions; see e.g. (Tanaka and Wang, 2001). However, such approaches are conservative since they require finding common decision matrices from a set of linear matrix inequalities (LMI). Therefore, a lot of efforts are done in the framework of conservatism reduction of LMI stability and stabilization conditions for T-S models; see (Sala, 2009) for an overview on the sources of conservatism. Among the conservatism reduction approaches, the use of non-quadratic fuzzy Lyapunov functions appears as very convenient since they are based on a similar interpolation fuzzy structure as the T-S system to be analysed (Tanaka et al., 2003). However, standard results in the non-quadratic framework suffer from the occurrence of the time derivative of the membership functions in the stability/stabilization conditions. The most commonly employed technique consists in bounding these derivative terms (Tanaka et al., 2003; Mozelli et al., 2009a). Nonetheless, these bounds are very difficult (if not impossible) to be found in practical control engineering problems. Thus, numerous works has been done to circumvent this drawback. For instance, some authors have reconsidered the goal of finding global asymptotical stability conditions by proposing local approaches maximizing a domain of attraction where the time derivative bounds are guaranteed (Guerra et al., 2012). However, these local approaches lead to very complex LMI conditions which may be difficult to apply by non-specialists. Some other recent studies have considered Sum-Of-Squares (SOS) conditions as an alternative to LMIs but requiring a very restrictive modelling assumption (Guelton et al., 2013; Duong et al., 2013). Another alternative, but less investigated, are very interesting to cope with the occurrence of the time derivatives of the membership functions. It consists in considering line-integral fuzzy Lyapunov functions introduced by Rhee and Won (2006). Hence, the path independency of the line-integral is guaranteed by a special structure of the Lyapunov matrices (Rhee and Won, 2006; Guelton et al., 2010) and the obtained Lyapunov stability conditions do not depend on any time derivative terms.

In (Rhee and Won, 2006), stability analysis has been first investigated and leads to LMI condition while in stabilization the result remain BMI. An improvement has been proposed in (Mozelli et al., 2009b) with a LMI result in stabilization. More recently, a less conservatism stabilization result has been proposed by Tognetti et al. (2011) but through a two steps LMI algorithm. Moreover, the special case of a second order fuzzy model with two rules has been investigated for non-PDC controller design (Márquez et al., 2013), which complexity suggests how a generalization is challenging.

In this paper, after a presentation of required preliminaries in the section 2, a slight improvement of the stability conditions proposed by Mozelli et al. (2009b) via line-integral Lyapunov functions is presented in section 3. Then, stabilization conditions are investigated, following the second order special case depicted in (Márquez et al., 2013), in section 4 through two approaches. The first one consists in a result obtained by applying the Finsler's lemma (Skelton et al., 1998) and the second one provide new conditions that are free of unknown parameters. A comparison is done with the results of Márquez et al. (2013) since it is noticed that these proposed approaches can be solved as LMI for first and second order systems, bilinear matrix inequalities (BMI) for the third order and then remain more complex as the system's order increases.

2. DEFINITIONS, NOTATIONS AND USEFUL LEMMAS

Consider the Takagi-Sugeno fuzzy model given by:

$$\dot{x}(t) = \sum_{i=1}^{r} h_i(z(t))\left(A_i x(t) + B_i u(t)\right) \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the state vector, $z(t)$ is the vector of premises, for $i \in \{1,...,r\}$, $h_i(z(t)) \in [0,1]$ are convex membership functions with $\sum_{i=1}^{r} h_i(z(t)) = 1$, $A_i \in \mathbb{R}^{n\times n}$ and $B_i \in \mathbb{R}^{n\times m}$ are real constant matrices defining the i^{th} vertex.

Assumption 1: $z(t)$ only depends on the states $x(t)$.

Notations: In the sequel, if not explicitly stated, matrices are assumed to have appropriate dimensions. Moreover, when there is any ambiguity, the time t argument will be omitted to lighten mathematical expressions. $M \succ 0$ and I denote respectively a positive definite matrix and an identity matrix. An asterisk $(*)$ denotes a transpose quantity in a matrix or for inline expressions, the transpose of its left-hand side term. Consider a set of real matrices M_i and N_{ij}, for all $(i,j) \in \{1,...,r\}^2$, one denotes $M_h = \sum_{i=1}^{r} h_i(z) M_i$, $N_{hh} = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z) h_j(z) N_{ij}$, and so on.

The goal of the next sections is to propose new LMI based conditions for the stability analysis and controller design for the class of T-S systems (1). The following lemmas will be useful to derive these new conditions.

Lemma 1 (*Finsler's lemma*, Skelton et al., 1998): Let $x \in \mathbb{R}^n$, $Q = Q^T \in \mathbb{R}^{n\times n}$ and $R \in \mathbb{R}^{m\times n}$ such that $rank(R) < n$. The following statements are equivalent.

$$x^T Q x \prec 0,\ \forall x \in \left\{x \in \mathbb{R}^n : x \neq 0, Rx = 0\right\} \tag{2}$$

$$\exists X \in \mathbb{R}^{n\times m} : Q + XR + R^T X^T \prec 0 \tag{3}$$

Lemma 2 (Tuan et al., 2001): Let Γ_{ij}, for $(i,j) \in \{1,...,r\}^2$, be matrices of appropriate dimensions. $\Gamma_{hh} \prec 0$ is satisfied if both the following conditions hold:

$$\begin{cases} \Gamma_{ii} \prec 0, & \forall i \in \{1,...,r\} \\ \dfrac{2}{r-1}\Gamma_{ii} + \Gamma_{ij} + \Gamma_{ji} \prec 0, & \forall (i,j) \in \{1,...,r\}^2 / i \neq j \end{cases} \tag{4}$$

3. STABILITY CONDITIONS

In this section new LMI based conditions are proposed as a slight extension of the ones given in Mozelli et al. (2009b), allowing to guarantee the stability of autonomous T-S fuzzy models, i.e. unforced systems (1), given by:

$$\dot{x}(t) = \sum_{i=1}^{r} h_i(z(t)) A_i x(t) \tag{5}$$

Now, consider the following line-integral Lyapunov function candidate (Rhee and Won, 2006):

$$v(x(t)) = 2\int_{\Gamma(0,x(t))} f^T(\varphi)\, d\varphi \tag{6}$$

where $\Gamma(0,x(t))$ is the path from the origin 0 to the current state $x(t)$, $\varphi \in \mathbb{R}^n$ is a dummy vector for the integral.

One assumes:

$$f(x(t)) = P(x)x(t) \tag{7}$$

where $P(x)$ has to satisfy path independency conditions (Rhee and Won, 2006; Guelton et al. 2010), so that:

$$P(x) = P_0 + \sum_{i=1}^{r} h_i(x) P_i \tag{8}$$

with $P_0 = \begin{bmatrix} 0 & p_{1,2} & \cdots & p_{1,n} \\ p_{1,2} & 0 & \ddots & \vdots \\ \vdots & \ddots & \ddots & p_{n-1,n} \\ p_{1,n} & \cdots & p_{n-1,n} & 0 \end{bmatrix}$,

$P_i = diag\begin{bmatrix} p_{1,1}^{\alpha_{i1}} & \dots & p_{n,n}^{\alpha_{in}} \end{bmatrix}$ and α_{ij}, for $\{i,j\} \in \{1,...,r\}^2$, as defined in Rhee and Won (2006).

Therefore, (6) is a Lyapunov function and the T-S fuzzy system (1) is stable to the origin if conditions (9), (10) and (11) hold.

$$P_0 + P_h \succ 0 \tag{9}$$

$$for\ x(0) = 0,\ v(x(0)) = 0 \tag{10}$$

$$\forall x(0) \neq 0,\ \dot{v}(x(t)) < 0 \tag{11}$$

Note that (9) is satisfied if, for all $i \in \{1,...,r\}$, $P_0 + P_i \succ 0$. Moreover, (10) is obviously verified considering (6). Hence, stability conditions, satisfying (9), (10) and (11) are summarized in the following theorem.

Theorem 1: The T-S model (1) is globally asymptotically stable if there exist the matrices P_0 and P_i defined in (8) such that, for all $i \in \{1,...,r\}$, $P_0 + P_i \succ 0$ and conditions (4) are verified with:

$$\Gamma_{ij} = \begin{bmatrix} U_j A_i + A_i^T U_j^T & (*) \\ P_0 + P_i + V_j A_i - U_j^T & -V_j - V_j^T \end{bmatrix} \tag{12}$$

Proof: Considering the path-independency conditions, one can rewrite (11) from (6) as:

$$\begin{aligned} \dot{v}(x) &= \dot{x}^T \left(P_0 + P_h\right) x + x^T \left(P_0 + P_h\right) \dot{x} \\ &= \begin{bmatrix} x \\ \dot{x} \end{bmatrix}^T \begin{bmatrix} 0 & P_0 + P_h \\ P_0 + P_h & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \end{bmatrix} < 0 \end{aligned} \tag{13}$$

Moreover, from (1) one can write:

$$A_h x(t) - \dot{x}(t) = \begin{bmatrix} A_h & -I \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \end{bmatrix} = 0 \tag{14}$$

Now, consider U_h and V_h, two slack decision fuzzy matrices with appropriate dimensions, one may apply lemma 1, therefore (13) holds if the next inequality is satisfied:

$$\begin{bmatrix} 0 & (*) \\ P_0 + P_h & 0 \end{bmatrix} + \left(\begin{bmatrix} U_h \\ V_h \end{bmatrix} \begin{bmatrix} A_h & -I \end{bmatrix} + (*) \right) \prec 0 \tag{15}$$

That is to say:

$$\begin{bmatrix} U_h A_h + A_h^T U_h^T & (*) \\ P_0 + P_h + V_h A_h - U_h^T & -V_h - V_h^T \end{bmatrix} \prec 0 \tag{16}$$

So now, applying lemma 2, one obtains the conditions expressed in theorem 1. ■

Remark 1: The conditions given in theorem 1 generalize the ones proposed in Mozelli et al. (2009b) since here, the slack decision variables hold a fuzzy structure when they remain common in Mozelli's work. Therefore, theorem 4 in Mozelli et al. (2009b) is included in theorem 1 as a special case where $U_h = U$ and $V_h = V$ are common decision variables and without applying Tuan's relaxation (Lemma 2).

Example 3.1: To compare the result proposed in theorem 1 regarding to previous works (Rhee and Won, 2006; Mozelli et al., 2009b), let us consider a T-S fuzzy model (5) with 4 rules ($r = 4$) defined by the vertices:

$$A_1 = \begin{bmatrix} -5 & -4 \\ -1 & a \end{bmatrix},\ A_2 = \begin{bmatrix} -4 & -4 \\ \dfrac{3b-2}{5} & \dfrac{3a-4}{5} \end{bmatrix},$$

$$A_3 = \begin{bmatrix} -3 & -4 \\ \dfrac{2b-3}{5} & \dfrac{2a-6}{5} \end{bmatrix} \text{ and } A_4 = \begin{bmatrix} -2 & -4 \\ b & -2 \end{bmatrix},$$

where a and b are two parameters dedicated to evaluate the conservatism of the considered LMI conditions.

Figure 1 shows the feasibility fields computed, using the MATLAB LMI Toolbox, from theorem 1 given above, theorem 4 in Mozelli et al. (2009b) and theorem 3 in Rhee and Won (2006). As expected, the stability conditions proposed in this paper provide the less conservative results.

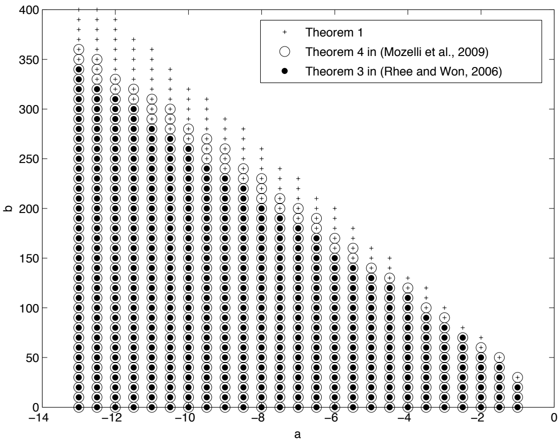

Fig. 1. Comparison of the feasibility fields obtained through theorem 1, theorem 4 in Mozelli et al. (2009b) and theorem 3 in Rhee and Won (2006).

4. NON-PDC CONTROLLERS DESIGN

In this section, we are interested in stabilizing T-S fuzzy models depicted in (1). Thus, let us consider the following non-PDC control law (Jaadari et al., 2012):

$$u(t) = F_h H_h^{-1} x(t) \tag{17}$$

where $F_i \in \mathbb{R}^{m\times n}$ and $H_j \in \mathbb{R}^{n\times n}$ are constant gain matrices to synthesize. The closed-loop dynamics may be expressed as:

$$\dot{x}(t) = \left(A_h + B_h F_h H_h^{-1}\right) x(t) \tag{18}$$

Therefore, the goal is now to propose new LMI based conditions allowing to design $F_i \in \mathbb{R}^{m\times n}$ and $H_i \in \mathbb{R}^{n\times n}$ such that the closed-loop dynamics (18) is stable. Let us consider the following line-integral Lyapunov function candidate:

$$v(x(t)) = 2\int_{\Gamma(0,x(t))} g^T(\varphi)\, d\varphi \tag{19}$$

where $\Gamma(0, x(t))$ is the path from the origin 0 to the current state $x(t)$, $\varphi \in \mathbb{R}^n$ is a dummy vector for the integral, and:

$$g(x(t)) = X^{-1}(x) x(t) \tag{20}$$

where $X(x)$ is has to satisfy path independency conditions (Rhee and Won, 2006; Guelton et al. 2010), i.e.:

$$X^{-1}(x) = P(x) = P_0 + \sum_{i=1}^{r} h_i(x) P_i \tag{21}$$

with P_0 and P_i defined as in (8) and:

$$X(x)=P^{-1}(x)=\frac{Z_h}{\det(P(x))} \tag{22}$$

where Z_h is the adjugate matrix of $P(x)$ and (Strang, 2005):

$$\det(P(x))=\frac{1}{\det(X(x))}=\det(Z_h)^{n-1}>0 \tag{23}$$

From now, (19) is a Lyapunov function and the T-S fuzzy system (1) is stabilized by the non-PDC control law (17), i.e. the closed-loop dynamics (18) is stable if conditions (24), (25) and (26) hold.

$$X_h^{-1}\succ 0 \tag{24}$$

$$for\ x(0)=0,\ v(x(0))=0 \tag{25}$$

$$\forall x(0)\neq 0,\ \dot{v}(x(t))<0 \tag{26}$$

From (22) and (23), (24) holds if, for all $i\in\{1,\dots,r\}$, $Z_i\succ 0$. Moreover, (25) is obviously verified by considering (19). The goal is now to derive LMI conditions satisfying (26) in order to design the non-PDC controller (17). In the sequel, two approaches are investigated.

4.1 First attempt:

In Jaadari et al. (2012), new stability conditions has been proposed in a local point of view through the use of the Finsler's lemma. In the same mood, the following theorem summarizes global LMI conditions through the use of the line-integral Lyapunov candidate function (19).

Theorem 2: The T-S model (1) is stabilized by the non-PDC control law (17), i.e. the closed-loop dynamics (18) is globally asymptotically stable, if there exist a scalar $\varepsilon>0$, the matrices $Z_i\succ 0$ which are the adjugate matrices of P_0+P_i defined in (8), the gain matrices F_i and H_i, for all $i\in\{1,\dots,r\}$, such that conditions (4) are verified with:

$$\Gamma_{ij}=\begin{bmatrix} A_iH_j+H_j^TA_i^T+B_iF_j+F_j^TB_i^T & (*) \\ H_j-Z_i+\varepsilon(A_iH_j+B_iF_j) & -2\varepsilon Z_i \end{bmatrix}\prec 0 \tag{27}$$

Proof: One can rewrite (26) from (19) as:

$$\begin{aligned}\dot{v}(x)&=\dot{x}^TX_h^{-1}x+x^TX_h^{-1}\dot{x}\\ &=\begin{bmatrix} x\\ \dot{x}\end{bmatrix}^T\begin{bmatrix} 0 & X_h^{-1}\\ X_h^{-1} & 0\end{bmatrix}\begin{bmatrix} x\\ \dot{x}\end{bmatrix}<0\end{aligned} \tag{28}$$

From (22), one has:

$$X^{-1}(x)=\det(P(x))Z_h^{-1} \tag{29}$$

Therefore, since $X(x)\succ 0$, $Z_h\succ 0$ and $\det(P(x))>0$, (28) is always verified if:

$$\begin{bmatrix} x\\ \dot{x}\end{bmatrix}^T\begin{bmatrix} 0 & Z_h^{-1}\\ Z_h^{-1} & 0\end{bmatrix}\begin{bmatrix} x\\ \dot{x}\end{bmatrix}<0 \tag{30}$$

Moreover, from (18) one can write:

$$\left(A_h+B_hF_hH_h^{-1}\right)x-\dot{x}=\begin{bmatrix} A_h+B_hF_hH_h^{-1} & -I\end{bmatrix}\begin{bmatrix} x\\ \dot{x}\end{bmatrix}=0 \tag{31}$$

Now, consider two free invertible decision matrices U and V with appropriate dimensions, one may apply lemma 1 (Finsler). Therefore (30) holds if:

$$\begin{bmatrix} 0 & Z_h^{-1}\\ Z_h^{-1} & 0\end{bmatrix}+\left(\begin{bmatrix} U\\ V\end{bmatrix}\left[\begin{bmatrix} A_h+B_hF_hH_h^{-1} & -I\end{bmatrix}+(*)\right]\right)\prec 0 \tag{32}$$

That is to say:

$$\begin{bmatrix} UA_h+A_h^TU^T+UB_hF_hH_h^{-1}+H_h^{-T}F_h^TB_h^TU^T & (*)\\ Z_h^{-1}+VA_h+VB_hF_hH_h^{-1}-U^T & -V-V^T\end{bmatrix}\prec 0 \tag{33}$$

Now, choosing $U=H_h^{-T}$ and $V=\varepsilon Z_h^{-1}$ with a scalar $\varepsilon>0$, and then multiplying left by $\begin{bmatrix} H_h^T & 0\\ 0 & Z_h\end{bmatrix}$ and right by its transpose, (33) becomes:

$$\begin{bmatrix} A_hH_h+H_h^TA_h^T+B_hF_h+F_h^TB_h^T & (*)\\ H_h-Z_h+\varepsilon(A_hH_h+B_hF_h) & -2\varepsilon Z_h\end{bmatrix}\prec 0 \tag{34}$$

So now, applying Tuan's relaxation (lemma 2), one obtains the conditions expressed in theorem 2. ■

Remark 3: The conditions of theorem 2 are generic and they guarantee the path-independency of (19) if and only if one can express the matrices Z_i such that they are the adjugate matrices of P_0+P_i as defined in (8). However, as state by Márquez et al. (2013), it is not always trivial to obtain such matrices from LMI processing. Indeed, for first order dynamical systems, it is easy since $Z_i=1$. For second order dynamical systems, one has to check the existence of:

$$Z_i=\begin{bmatrix} p_{2,2}^{\alpha_{i2}} & -p_{1,2}\\ -p_{1,2} & p_{1,1}^{\alpha_{i1}}\end{bmatrix} \tag{35}$$

where $p_{1,1}^{\alpha_{i1}}$, $p_{2,2}^{\alpha_{i2}}$ and $p_{1,2}$ are scalar decision variables and the result is still LMI. For third order dynamical systems, it becomes:

$$Z_i=\begin{bmatrix} p_{2,2}^{\alpha_{i2}}p_{3,3}^{\alpha_{i3}}-p_{2,3}^2 & (*) & (*)\\ p_{1,3}p_{2,3}-p_{1,2}p_{3,3}^{\alpha_{i3}} & p_{1,1}^{\alpha_{i1}}p_{3,3}^{\alpha_{i3}}-p_{1,3}^2 & (*)\\ p_{1,2}p_{2,3}-p_{1,3}p_{2,2}^{\alpha_{i2}} & p_{1,2}p_{1,3}-p_{1,1}^{\alpha_{i1}}p_{2,3} & p_{1,1}^{\alpha_{i1}}p_{2,2}^{\alpha_{i2}}-p_{1,2}^2\end{bmatrix} \tag{36}$$

and the result is now BMI and may be solved by a two step algorithm. However, it becomes drastically more and more complex and difficult to solve as the systems order increases.

Remark 4: Even for dynamical systems with order $n \le 2$, the conditions summarized in theorem 2 are not strictly LMI since they involve a parameter ε, which has to be fixed in advance. They are usually prefixed values belonging to a logarithmically spaced family of values such as $\varepsilon \in \{10^{-6}, 10^{-5}, ..., 10^{6}\}$, which avoid performing an exhaustive linear search (Jaadari et al., 2012). Nevertheless, this non-optimal way may be understood as a drawback of such approaches. The following section provides new conditions, which overcome this drawback.

4.2 Second attempt:

In order to improve the result proposed above, the following theorem summarizes new conditions that do not depend on any prefixed parameters (like ε, see remark 4 above).

Theorem 3: The T-S model (1) is stabilized by the non-PDC control law (17) with $H_i = Z_i$, i.e. the closed-loop dynamics (18) is globally asymptotically stable, if there exist the matrices $Z_i \succ 0$ which are the adjugate matrices of $P_0 + P_i$ defined in (8), the gain matrices F_i and the slack matrices Q_{1ij}, Q_{2ij}, for all $i \in \{1, ..., r\}$, such that the conditions (4) are verified with:

$$\Gamma_{ij} = \begin{bmatrix} -Q_{1ij} - Q_{1ij}^T & (*) \\ A_i Z_j + B_i F_j + Q_{1ij}^T + Q_{2ij} & -Q_{2ij} - Q_{2ij}^T \end{bmatrix} < 0 \quad (37)$$

Proof: Consider the line-integral fuzzy Lyapunov function candidate (19), (26) may be rewritten as:

$$\dot{v}(x) = \dot{x}^T X_h^{-1} x + x^T X_h^{-1} \dot{x} < 0 \quad (38)$$

From (29) and since $X(x) \succ 0$, $Z_h \succ 0$ and $\det(P(x)) > 0$, (38) is always verified if:

$$\dot{x}^T Z_h^{-1} x + x^T Z_h^{-1} \dot{x} < 0 \quad (39)$$

Following the way open in (Liu et al., 2013), let us consider two invertible free decision matrices $S_1 \in \mathbb{R}^{n \times n}$ and $S_2 \in \mathbb{R}^{n \times n}$, (39) can trivially be rewritten as:

$$\begin{bmatrix} x \\ \dot{x} \end{bmatrix}^T \begin{bmatrix} Z_h^{-1} & S_1 \\ 0 & S_2 \end{bmatrix} \begin{bmatrix} \dot{x} \\ 0 \end{bmatrix} + (*) < 0 \quad (40)$$

Now from (18), one has $-\dot{x} + \left(A_h + B_h F_h H_h^{-1}\right) x = 0$. Therefore, (26) can be rewritten as:

$$\begin{bmatrix} x \\ \dot{x} \end{bmatrix}^T \begin{bmatrix} Z_h^{-1} & S_1 \\ 0 & S_2 \end{bmatrix} \begin{bmatrix} 0 & I \\ A_h + B_h F_h H_h^{-1} & -I \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \end{bmatrix} + (*) < 0 \quad (41)$$

which is obviously verified, $\forall x$ and $\forall \dot{x}$, if:

$$\begin{bmatrix} Z_h^{-1} & S_1 \\ 0 & S_2 \end{bmatrix} \begin{bmatrix} 0 & I \\ A_h + B_h F_h H_h^{-1} & -I \end{bmatrix} + (*) < 0 \quad (42)$$

Let us choose S_1 and S_2 be such that one has two slack matrices $Q_{1hh} = Z^T S_1 S_2^{-T}$ and $Q_{2hh} = S_2^{-1}$. Hence one has

$\begin{bmatrix} Z_h^{-1} & 0 \\ S_1^T & S_2^T \end{bmatrix}^{-1} = \begin{bmatrix} Z_h & 0 \\ -Q_{1hh}^T & Q_{2hh}^T \end{bmatrix}$. So, multiplying (42) left by $\begin{bmatrix} Z_h & -Q_{1hh} \\ 0 & Q_{2hh} \end{bmatrix}$ and right by its transpose yields:

$$\begin{bmatrix} -Q_{1hh} - Q_{1hh}^T & (*) \\ A_h Z_h + B_h F_h H_h^{-1} Z_h + Q_{1hh}^T + Q_{2hh} & -Q_{2hh} - Q_{2hh}^T \end{bmatrix} < 0 \quad (43)$$

Then, choosing $H_h = Z_h$ and applying lemma 2, one obtains the conditions expressed in theorem 3. ■

Remark 5: The same remark as for theorem 2 may be done for theorem 3 regarding to the particular decision structure required for Z_i, i.e. the applicability of the result regarding to the systems order (see remark 3). Nevertheless, theorem 3 is still easier to compute than theorem 2 since it does not require any parameters to be fixed prior to LMI computation.

Example 4.1: To compare the result proposed in theorem 2 and 3 regarding to previous recent work (Márquez et al., 2013), let us consider a T-S fuzzy model (5) with 2 rules defined by the vertices:

$$A_1 = \begin{bmatrix} 2 & -10 \\ 2 & 0 \end{bmatrix},\ A_2 = \begin{bmatrix} a & -5 \\ 1 & 2 \end{bmatrix},\ B_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix},\ B_2 = \begin{bmatrix} b \\ 2 \end{bmatrix},$$

where a and b are two parameters dedicated to evaluate the controller design conditions conservatism.

Figure 2 shows the feasibility fields computed, using the MATLAB LMI Toolbox, from theorem 2 and 3 given above and theorem 2 in Márquez et al. (2013).

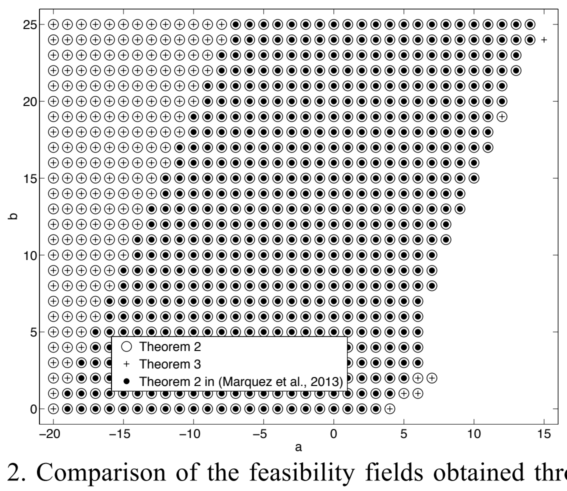

Fig. 2. Comparison of the feasibility fields obtained through theorem 2 in Márquez et al. (2013), theorem 2 and theorem 3.

As one can see, the result proposed in Márquez et al. (2013) are included in the ones of theorem 2 and theorem 3. Moreover, theorem 2 and 3 provide almost the same feasibility fields. Nevertheless, the computation of theorem 3 is strictly LMI since no parameters are required when the value of ε has to be prefixed using theorem 2.

5. CONCLUSION

In this paper, new LMI based conditions have been proposed as slight improvements of previous works dealing with stability analysis and non-PDC controller design using line-integral Lyapunov functions. The main drawback of these approaches remains on their applicability to a restricted class of T-S fuzzy systems. Indeed, it has been highlighted that these conditions are LMI for T-S models of orders 1 or 2, then BMI for 3rd order ones. However, it becomes drastically more and more complex and difficult to solve as the systems order increases. This point has motivated our further studies (Cherifi et al., 2014) where a generalization to n-th order systems has been proposed.

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