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Nonquadratic stabilization of switching TS systems

Zs. Lendek * **P. Raica** * **J. Lauber** ** **T. M. Guerra** **

* *Department of Automation, Technical University of Cluj-Napoca, Memorandumului 28, 400114 Cluj-Napoca, Romania(e-mail: {zsofia.lendek,paula.raica}@aut.utcluj.ro).*
** *University of Valenciennes and Hainaut-Cambresis, LAMIH UMR CNRS 8201, Le Mont Houy, 59313 Valenciennes Cedex 9, France (e-mail: {jlauber,guerra}@univ-valenciennes.fr).*

Abstract: In this paper we consider stabilization of switching nonlinear systems represented by TS models. To develop the conditions we use two different switching Lyapunov functions. For each Lyapunov function a set of conditions is developed. The conditions are formulated as LMIs and relaxed using delays in the controller and the Lyapunov function. The application of the conditions is illustrated on numerical examples.

Keywords: stabilization, TS systems, swicthing systems, non-quadratic Lyapunov function.

1. INTRODUCTION

Switching systems are a class of hybrid systems that switch between a family of modes or subsystems. In the last decades, stabilization of such systems has attracted much attention, mostly in the continuous-time case. For instance, linear switching systems where the switching laws can be arbitrarily chosen have been considered by Altafini [2002]. Stabilization and tracking conditions for continuous-time linear switching systems have been developed in [Baglietto et al., 2013, Battistelli, 2013], delay-dependent stabilization by Kim et al. [2008]. State-feedback controller design for nonlinear switching systems has been presented by Blanchini et al. [2007]. Switching models can be found in various domains [Zwart et al., 2010, Venkataramanan et al., 2002, Pasamontes et al., 2011, Widyotriatmo and Hong, 2012, Moustris and Tzafestas, 2011, Zhao and Spong, 2001], such as automotive, networked control, DC converters, mobile robots, etc.

Most of the results on switching systems concern linear subsystems, such as [Jungers et al., 2011], where stabilization in the presence of input saturation and uncertainties is considered, or [Dehghan and Ong, 2013] which considers the computation of the mode-dependent dwell-time. A linear controller with integral action has been used for the stabilization of switching systems in [Blanchini et al., 2007]. Other recent approaches have been reported in [Chen et al., 2012, Duan and Wu, 2012, Hetel et al., 2011].

We represent the switching nonlinear models by switching Takagi-Sugeno (TS) fuzzy systems [Takagi and Sugeno, 1985], which are nonlinear, convex combinations of local linear models. Stabilization conditions for TS models have recently been developed using nonquadratic Lyapunov functions [Guerra and Vermeiren, 2004, Kruszewski et al., 2008, Mozelli et al., 2009]. The design conditions are generally derived in the form of linear matrix inequalities (LMIs).

In this paper we consider switching discrete-time TS fuzzy models and develop conditions for their stabilization by a switching law. We use a graph representation of the switching system and to develop stabilization conditions we employ a nonquadratic switching Lyapunov function. Switching TS systems have been investigated in the last decades mainly in the continuous case where the stability is based on the use of a quadratic Lyapunov function [Tanaka et al., 2001, Lam et al., 2002, 2004, Ohtake et al., 2006] or a piecewise one [Feng, 2003, 2004]. Although results are available for discrete-time linear switching systems [Daafouz et al., 2002], for discrete-time TS models, few results exist [Doo et al., 2003, Dong and Yang, 2009].

We derive relaxed LMI conditions for the stabilization of switching TS systems. Two different switching nonquadratic Lyapunov functions are used and therefore two different sets of stabilizing conditions are developed. We assume that the set of the admissible switches is known, but the exact switching sequence is not known in advance. This is a worst-case assumption. However, by taking into account the admissible switches, it is possible to develop conditions for the stabilization of some systems with uncontrollable local models.

The structure of the paper is as follows. Section 2 presents the notations used in this paper. Conditions for the stabilization of switching systems are developed in Section 3. Section 4 illustrates their use on a numerical example. Section 5 presents further extensions of the developed stabilization conditions.

2. PRELIMINARIES

In this paper we consider stabilization of discrete-time switching TS systems. We consider subsystems of the form

$$\begin{aligned}\boldsymbol{x}(k+1) &= \sum_{j=1}^{r_i} h_{i,j}(\boldsymbol{z}(k))(A_{i,j}\boldsymbol{x}(k) + B_{i,j}\boldsymbol{u}(k)) \\ &= A_{i,z}\boldsymbol{x}(k) + B_{i,z}\boldsymbol{u}(k)\end{aligned} \tag{1}$$

where i is the number of the subsystem, $i = 1, 2, \ldots, n_s$, n_s being the number of the subsystems, $\boldsymbol{x}$ denotes the state vector, $\boldsymbol{u}$ is the input, r_i is the number of rules in the ith subsystem, $\boldsymbol{z}$ is the scheduling vector, $h_{i,j}$, $j = 1, 2, \ldots, r_i$ are normalized membership functions, and $A_{i,j}$, and $B_{i,j}$, $j = 1, 2, \ldots, r_i$, $i = 1, 2, \ldots, n_s$, are the local models.

For the easier notation, we use a directed graph representation of the switching system (1). The graph associated to (1) is $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$, where $\mathcal{V}$ denotes the set of vertices or subsystems and $\mathcal{E}$ denotes the set of admissible switches. As such, $(v_i, v_j) \in \mathcal{E}$ if a switch from subsystem i to subsystem j is possible. Note that we assume that self-transitions are also possible: these correspond to the subsystem being active for more than one sample.

A path $\mathcal{P}(v_i, v_j)$ between two vertices v_i and v_j in the graph $\mathcal{G}$ is a sequence of vertices $\mathcal{P}(v_i, v_j) = [v_{p_1}, v_{p_2}, \ldots, v_{p_{n_p}}]$ so that $v_i = v_{p_1}$, $v_j = v_{p_{n_p}}$, and $(v_{p_k}, v_{p_{k+1}}) \in \mathcal{E}$, $p_k = 1, 2, \ldots, n_p - 1$. A path in a graph associated to a switching system corresponds to a switching law. The length of a path is given by the number of edges it contains.

Let us illustrate the notations above on an example.

Example 1. Consider a switching system composed of 4 subsystems of the form

$$\boldsymbol{x}(k+1) = A_{i,z}\boldsymbol{x}(k) + B_{i,z}\boldsymbol{u}(k)$$

for $i = 1, 2, 3, 4$, and with admissible switches $(1, 2)$, $(1, 4)$, $(2, 3)$, $(3, 2)$, $(3, 4)$, $(4, 1)$. The 1st and 3rd subsystems can be active for more than one sample. The corresponding graph representation is illustrated in Figure 1.

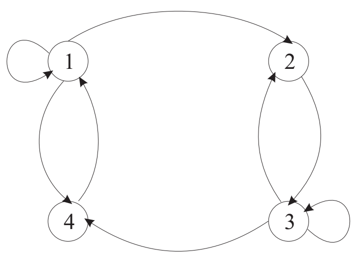

Fig. 1. Graph representation of the switching system in Example 1.

The graph is $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$, with $\mathcal{V} = \{1, 2, 3, 4\}$ and

$$\mathcal{E} = \{(1,1), (1,2), (1,4), (2,3), (3,2), (3,3), (4,1)\}$$

A path is $\mathcal{P}(1, 3) = [1, 2, 3]$. The length of the path $\mathcal{P}(1, 3) = [1, 2, 3]$ is 2.

Our goal is to develop conditions under which the switching system is stabilized, with any admissible switching law. We assume that the switching sequence cannot be influenced, i.e., all possible switches must be taken into account.

0 and I denote the zero and identity matrices of appropriate dimensions, and a $(*)$ denotes the term induced by symmetry in matrix expressions and the symmetrical of the left-hand side in inline expressions. The subscript $z + m$ (as in A_{1z+m}) stands for the scheduling vector being evaluated at the current sample plus mth instant, i.e., $\boldsymbol{z}(k + m)$.

In what follows, we will make use of the following results:

Lemma 1. [Skelton et al., 1998] Consider a vector $\boldsymbol{x} \in \mathbb{R}^{n_x}$ and two matrices $Q = Q^T \in \mathbb{R}^{n_x \times n_x}$ and $R \in \mathbb{R}^{m \times n_x}$ such that $\text{rank}(R) < n_x$. The two following expressions are equivalent:

(1) $\boldsymbol{x}^T Q \boldsymbol{x} < 0$, $\boldsymbol{x} \in \{\boldsymbol{x} \in \mathbb{R}^{n_x}, \boldsymbol{x} \neq 0, R\boldsymbol{x} = 0\}$
(2) $\exists M \in \mathbb{R}^{n_x \times m}$ such that $Q + MR + R^T M^T < 0$

Analysis and design for TS models often lead to double-sum negativity problems of the form

$$\boldsymbol{x}^T \sum_{i=1}^{r} \sum_{j=1}^{r} h_i(\boldsymbol{z}(k)) h_j(\boldsymbol{z}(k)) \Gamma_{ij} \boldsymbol{x} < 0 \tag{2}$$

where Γ_{ij}, $i, j = 1, 2, \ldots, r$ are matrices of appropriate dimensions.

Lemma 2. [Wang et al., 1996] The double-sum (2) is negative, if

$$\begin{aligned}&\Gamma_{ii} < 0 \\ &\Gamma_{ij} + \Gamma_{ji} < 0, \quad i, j = 1, 2, \ldots, r,\ i < j\end{aligned}$$

Lemma 3. [Tuan et al., 2001] The double-sum (2) is negative, if

$$\begin{aligned}&\Gamma_{ii} < 0 \\ &\frac{2}{r-1}\Gamma_{ii} + \Gamma_{ij} + \Gamma_{ji} < 0, \quad i, j = 1, 2, \ldots, r,\ i \neq j\end{aligned}$$

Proposition 1. (Congruence) Given a matrix $P = P^T$ and a full column rank matrix Q it holds that

$$P > 0 \ \Rightarrow\ QPQ^T > 0$$

Proposition 2. Let A and B be matrices of appropriate dimensions and ranks, with $B = B^T > 0$. Then

$$(A - B)^T B^{-1} (A - B) \geq 0 \Longleftrightarrow A^T B^{-1} A \geq A + A^T - B$$

3. NONQUADRATIC STABILIZATION

Consider the switching system

$$\boldsymbol{x}(k+1) = A_{i,z}\boldsymbol{x}(k) + B_{i,z}\boldsymbol{u}(k) \tag{3}$$

and the switching control law

$$\boldsymbol{u}(k) = -F_{i,z} H_{i,z}^{-1} \boldsymbol{x}(k) \tag{4}$$

The closed-loop system is expressed as

$$\boldsymbol{x}(k+1) = (A_{i,z} - B_{i,z} F_{i,z} H_{i,z}^{-1}) \boldsymbol{x}(k) \tag{5}$$

In what follows, we use two switching Lyapunov functions:

$$V = \boldsymbol{x}(k)^T P_{i,j,z}^{-1} \boldsymbol{x}(k) \tag{6}$$

and

$$V = \boldsymbol{x}(k)^T H_{i,z}^{-T} P_{i,j,z} H_{i,z}^{-1} \boldsymbol{x}(k) \tag{7}$$

respectively, defined during the switches, i.e., on the edges of the associated graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$, with $(v_i, v_j) \in \mathcal{E}$. The subscript indices i, j denote that the corresponding Lyapunov function is active if we switch from subsystem i to subsystem j.

Let us consider first (6). The difference in the Lyapunov function is

$$\Delta V = \boldsymbol{x}(k+1)^T P_{j,l,z+1}^{-1} \boldsymbol{x}(k+1) - \boldsymbol{x}(k)^T P_{i,j,z}^{-1} \boldsymbol{x}(k) \\ = \begin{pmatrix} \boldsymbol{x}(k) \\ \boldsymbol{x}(k+1) \end{pmatrix}^T \begin{pmatrix} -P_{i,j,z}^{-1} & 0 \\ 0 & P_{j,l,z+1}^{-1} \end{pmatrix} \begin{pmatrix} \boldsymbol{x}(k) \\ \boldsymbol{x}(k+1) \end{pmatrix}$$

where $[v_i, v_j, v_l]$ is an admissible path.

Remark: If a subsystem i may be active for several samples, the Lyapunov function above is in fact used to prove its stability. However, if a subsystem is active for only one sample, it is not necessary for it to be stable.

On the edge $[v_i, v_j]$, the dynamics of the system are described by

$$\begin{pmatrix} A_{i,z} - B_{i,z} F_{i,z} H_{i,z}^{-1} & -I \end{pmatrix} \begin{pmatrix} \boldsymbol{x}_k \\ \boldsymbol{x}_{k+1} \end{pmatrix} = 0$$

Using Lemma 1, the difference is the Lyapunov function is negative, if there exists M such that

$$M \begin{pmatrix} A_{i,z} - B_{i,z} F_{i,z} H_{i,z}^{-1} & -I \end{pmatrix} + (*) \\ + \begin{pmatrix} -P_{i,j,z}^{-1} & 0 \\ 0 & P_{j,l,z+1}^{-1} \end{pmatrix} < 0$$

By choosing

$$M = \begin{pmatrix} 0 \\ P_{j,l,z+1}^{-1} \end{pmatrix}$$

we have

$$\begin{pmatrix} -P_{i,j,z}^{-1} & (*) \\ P_{j,l,z+1}^{-1} A_{i,z} - P_{j,l,z+1}^{-1} B_{i,z} F_{i,z} H_{i,z}^{-1} & -P_{j,l,z+1}^{-1} \end{pmatrix} < 0$$

Congruence with

$$\begin{pmatrix} H_{i,z}^T & 0 \\ 0 & P_{j,l,z+1} \end{pmatrix}$$

leads to

$$\begin{pmatrix} -H_{i,z}^T P_{i,j,z}^{-1} H_{i,z} & (*) \\ A_{i,z} H_{i,z} - B_{i,z} F_{i,z} & -P_{j,l,z+1} \end{pmatrix} < 0$$

and applying Proposition 2 we have

$$\begin{pmatrix} -H_{i,z} - H_{i,z}^T + P_{i,j,z} & (*) \\ A_{i,z} H_{i,z} - B_{i,z} F_{i,z} & -P_{j,l,z+1} \end{pmatrix} < 0$$

The condition developed above can be formulated as follows.

Theorem 1. The switching TS system (3) is asymptotically stabilized by the switching control law (4) if there exist matrices $H_{i,j}$ and symmetric positive definite matrices $P_{i,j,m} = P_{i,j,m}^T > 0$, $i, j = 1, 2, \ldots, n_{\mathrm{s}}$, $(v_i, v_j) \in \mathcal{E}$, $(v_j, v_l) \in \mathcal{E}$, $m, n = 1, 2, \ldots, r_i$, $o = 1, 2, \ldots, r_j$ so that Lemma 3 holds with

$$\Gamma_{m,n,o}^{i,j,l} = \begin{pmatrix} -H_{i,m} - H_{i,m}^T + P_{i,j,m} & (*) \\ A_{i,n} H_{i,m} - B_{i,n} F_{i,m} & -P_{j,l,o} \end{pmatrix}$$

Let us now consider (7). The difference in the Lyapunov function is

$$\Delta V = \boldsymbol{x}(k+1)^T H_{j,z+1}^{-T} P_{j,l,z+1} H_{j,z+1}^{-1} \boldsymbol{x}(k+1) \\ - \boldsymbol{x}(k)^T H_{i,z}^{-T} P_{i,j,z} H_{i,z}^{-1} \boldsymbol{x}(k) \\ = \begin{pmatrix} \boldsymbol{x}(k) \\ \boldsymbol{x}(k+1) \end{pmatrix}^T \\ \cdot \begin{pmatrix} -H_{i,z}^{-T} P_{i,j,z} H_{i,z}^{-1} & 0 \\ 0 & H_{j,z+1}^{-T} P_{j,l,z+1} H_{j,z+1}^{-1} \end{pmatrix} \begin{pmatrix} \boldsymbol{x}(k) \\ \boldsymbol{x}(k+1) \end{pmatrix}$$

where $[v_i, v_j, v_l]$ is an admissible path.

On the edge $[v_i, v_j]$, similarly to the previous case, the dynamics of the system are described by

$$\begin{pmatrix} A_{i,z} - B_{i,z} F_{i,z} H_{i,z}^{-1} & -I \end{pmatrix} \begin{pmatrix} \boldsymbol{x}_k \\ \boldsymbol{x}_{k+1} \end{pmatrix} = 0$$

Using Lemma 1, the difference is the Lyapunov function is negative, if there exists M such that

$$M \begin{pmatrix} A_{i,z} - B_{i,z} F_{i,z} H_{i,z}^{-1} & -I \end{pmatrix} + (*) \\ + \begin{pmatrix} -H_{i,z}^{-T} P_{i,j,z} H_{i,z}^{-1} & 0 \\ 0 & H_{j,z+1}^{-T} P_{j,l,z+1} H_{j,z+1}^{-1} \end{pmatrix} < 0$$

By choosing

$$M = \begin{pmatrix} 0 \\ H_{j,z+1}^{-1} \end{pmatrix}$$

we have

$$\begin{pmatrix} -H_{i,z}^{-T} P_{i,j,z} H_{i,z}^{-1} & (*) \\ \begin{pmatrix} H_{j,z+1}^{-1} A_{i,z} \\ -H_{j,z+1}^{-1} B_{i,z} F_{i,z} H_{i,z}^{-1} \end{pmatrix} & \begin{pmatrix} -H_{j,z+1}^{-T} + (*) + \\ H_{j,z+1}^{-T} P_{j,l,z+1} H_{j,z+1}^{-1} \end{pmatrix} \end{pmatrix} < 0$$

Congruence with

$$\begin{pmatrix} H_{i,z}^T & 0 \\ 0 & H_{j,z+1}^T \end{pmatrix}$$

leads to

$$\begin{pmatrix} -P_{i,j,z} & (*) \\ A_{i,z} H_{i,z} - B_{i,z} F_{i,z} & -H_{j,z+1} - H_{j,z+1}^T + P_{j,l,z+1} \end{pmatrix} < 0$$

The condition developed above can be formulated as follows.

Theorem 2. The switching TS system (3) is asymptotically stabilized by the switching control law (4) if there exist matrices $H_{i,j}$ and symmetric positive definite matrices $P_{i,j,m} = P_{i,j,m}^T > 0$, $i, j = 1, 2, \ldots, n_{\mathrm{s}}$, $(v_i, v_j) \in \mathcal{E}$, $(v_j, v_l) \in \mathcal{E}$, $m, n = 1, 2, \ldots, r_i$, $o = 1, 2, \ldots, r_j$, so that Lemma 3 holds with

$$\Gamma_{m,n,o}^{i,j,l} = \begin{pmatrix} -P_{i,j,m} & (*) \\ A_{i,m} H_{i,n} - B_{i,m} F_{i,n} & -H_{j,o} - H_{j,o}^T + P_{j,l,o} \end{pmatrix}$$

4. EXAMPLE AND DISCUSSION

Let us discuss the developed conditions on an example.

Example 2. To illustrate the application of the conditions, consider a switching TS system composed of three subsystems, each having two local models. The switching graph is defined as $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$, with $\mathcal{V} = \{1, 2, 3\}$ and

$$\mathcal{E} = \{(1,1), (1,2), (2,3), (3,1), (3,2)\}$$

The edge $(1,1)$ is introduced in order to take into account that subsystem 1 can be active for several samples. The graph is illustrated in Figure 2.

The Lyapunov functions are defined for all the possible switches, i.e., we have $P_{1,1,z}$, $P_{1,2,z}$, $P_{2,3,z}$, etc. Consider the following local models of the switching TS system above:

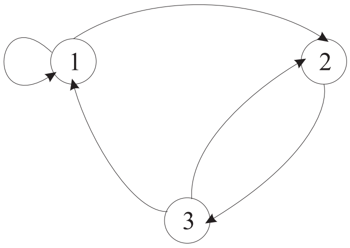

Fig. 2. Graph representation of the switching system in Example 2.

$$A_{1,1} = \begin{pmatrix} 0.60 & -1.02 \\ 0.94 & -0.07 \end{pmatrix} \qquad B_{1,1} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$
$$A_{1,2} = \begin{pmatrix} 0.08 & -1.78 \\ -1.77 & -0.66 \end{pmatrix} \qquad B_{1,2} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$
$$A_{2,1} = \begin{pmatrix} 1.35 & 0.16 \\ 2.13 & -1.70 \end{pmatrix} \qquad B_{2,1} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$
$$A_{2,2} = \begin{pmatrix} 0.27 & -0.09 \\ 0.39 & 0.17 \end{pmatrix} \qquad B_{2,2} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$
$$A_{3,1} = \begin{pmatrix} -1.83 & 0.81 \\ -1.50 & -0.23 \end{pmatrix} \qquad B_{3,1} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$
$$A_{3,2} = \begin{pmatrix} -1.63 & -0.79 \\ -0.31 & 0.69 \end{pmatrix} \qquad B_{3,2} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

For this particular example, both Theorems 1 and 2 provide stabilizing control laws. For instance, using Theorem 1 we obtain the gain matrices [1]

$$H_{1,1} = \begin{pmatrix} 0.21 & -0.01 \\ 0.00 & 0.32 \end{pmatrix} \qquad F_{1,1} = (0.09\ \ -0.40)$$
$$H_{1,2} = \begin{pmatrix} 0.11 & -0.08 \\ -0.03 & 0.34 \end{pmatrix} \qquad F_{1,2} = (0.05\ \ -0.56)$$
$$H_{2,1} = \begin{pmatrix} 0.05 & 0.04 \\ 0.04 & 0.33 \end{pmatrix} \qquad F_{2,1} = (0.17\ \ -0.15)$$
$$H_{2,2} = \begin{pmatrix} 0.07 & 0.07 \\ 0.04 & 0.34 \end{pmatrix} \qquad F_{2,2} = (0.14\ \ 0.04)$$
$$H_{3,1} = \begin{pmatrix} 0.14 & -0.11 \\ -0.16 & 0.44 \end{pmatrix} \qquad F_{3,1} = (-0.40\ \ 0.51)$$
$$H_{3,2} = \begin{pmatrix} 0.11 & -0.21 \\ -0.22 & 0.51 \end{pmatrix} \qquad F_{3,2} = (-0.30\ \ 0.67)$$

A trajectory of the closed-loop system, with initial states $\boldsymbol{x}(0) = (1\ 1)^T$ is presented in Figure 3 while the corresponding control input is presented in Figure 4. The switching sequence was 1 1 2 3 2 3 1 2 3 1 1 2 3 2 3. The membership functions used are the following ones:

$$h_{1,1}(\boldsymbol{x}) = \frac{1}{2}(1 - \sin(x_1)) \qquad h_{1,2}(\boldsymbol{x}) = 1 - h_{1,1}(\boldsymbol{x})$$
$$h_{2,1}(\boldsymbol{x}) = \frac{1}{2}(1 - \cos(x_1)) \qquad h_{2,2}(\boldsymbol{x}) = 1 - h_{2,1}(\boldsymbol{x})$$
$$h_{3,1}(\boldsymbol{x}) = \frac{1}{2}(1 - e^{-x_1^2}) \qquad h_{3,2}(\boldsymbol{x}) = 1 - h_{3,1}(\boldsymbol{x})$$

Although for the example above the conditions of both Theorem 1 and 2 are feasible, it has to be noted that in general, the two sets of conditions are not equivalent.

[1] Values are truncated to two decimal places.

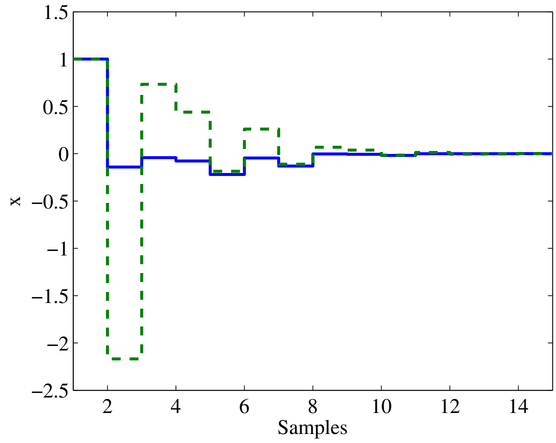

Fig. 3. Closed-loop trajectory for Example 2.

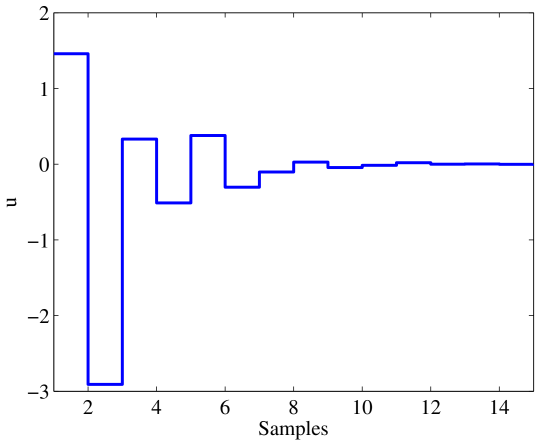

Fig. 4. Control input for Example 2.

By considering the switching possibilities in the Lyapunov matrix $P_{i,j,z}$, the developed conditions extend the case of using switching nonquadratic Lyapunov functions, i.e., simply Lyapunov matrices of the form $P_{i,z}$, for the switching system. This can be easily seen by inspecting the LMI conditions. Indeed, if there exists a solution using a switching Lyapunov function, then the conditions of Theorem 1 or 2 are satisfied. However, the reverse is not true. For instance, for Example 2, the LMI conditions corresponding to the use of a simple switching Lyapunov function in Theorem 2, i.e., the conditions

$$\frac{2}{r-1}\Gamma^{i,i}_{m,n,o} + \Gamma^{i,j}_{m,n,o} + \Gamma^{j,i}_{m,n,o} < 0$$

$i, j = 1, 2, \ldots, n_s$, $(v_i, v_j) \in \mathcal{E}$, $m, n = 1, 2, \ldots, r_i$, $o = 1, 2, \ldots, r_j$ with

$$\Gamma^{i,j}_{m,n,o} = \begin{pmatrix} -P_{i,m} & (*) \\ A_{i,m}H_{i,n} - B_{i,m}F_{i,n} & -H_{j,o} - H^T_{j,o} + P_{j,o} \end{pmatrix}$$

are not feasible.

Recall that we assume that the switching sequence is not known in advance and it is not assumed to be directly influenced. Naturally, this is the worst-case, i.e., all possible combinations on switches between the subsystems are taken into account. If the switching sequence can be chosen, or the goal is to find a stabilizing switching sequence, the conditions can be relaxed. The LMI conditions can also be relaxed by double sums in the Lyapunov function, e.g., using $P_{i,j,z,z}$ instead of $P_{i,j,z}$, or even several sums. Such relaxations are presented in the following section.

5. EXTENSIONS

In what follows, we extend the results presented in Section 3. By looking at the Lyapunov matrices, a straightforward extension of the results above is by using the control law

$$\boldsymbol{u}(k) = -F_{i,j,z}H_{i,j,z}^{-1}\boldsymbol{x}(k) \tag{8}$$

i.e., instead of choosing a control law to be applied for each subsystem, the control is applied based on the switching that takes place. Using the Lyapunov function (6) and (7), respectively, the following results can be formulated. For the Lyapunov function (6) we have

Corollary 3. The switching TS system (3) is asymptotically stabilized by the switching control law (8) if there exist matrices $H_{i,j,m}$ and symmetric positive definite matrices $P_{i,j,m} = P_{i,j,m}^T > 0$, $i, j = 1, 2, \ldots, n_\mathrm{s}$, $(v_i, v_j) \in \mathcal{E}$, $(v_j, v_l) \in \mathcal{E}$, $m, n = 1, 2, \ldots, r_i$, so that

$$\begin{pmatrix} -H_{i,j,z} - H_{i,j,z}^T + P_{i,j,z} & (*) \\ A_{i,z}H_{i,j,z} - B_{i,z}F_{i,j,z} & -P_{j,l,z+1} \end{pmatrix} < 0$$

while when using the Lyapunov function (7) we obtain

Corollary 4. The switching TS system (3) is asymptotically stabilized by the switching control law (8) if there exist matrices $H_{i,j,m}$ and symmetric positive definite matrices $P_{i,j,m} = P_{i,j,m}^T > 0$, $i, j = 1, 2, \ldots, n_\mathrm{s}$, $(v_i, v_j) \in \mathcal{E}$, $(v_j, v_l) \in \mathcal{E}$, $m, n = 1, 2, \ldots, r_i$, so that

$$\begin{pmatrix} -P_{i,j,z} & (*) \\ A_{i,z}H_{i,j,z} - B_{i,z}F_{i,j,z} & -H_{j,l,z+1} + (*) + P_{j,l,z+1} \end{pmatrix} < 0$$

The proofs of the above corollaries follow the same lines as Theorems 1 and 2 in Section 3 and are therefore not repeated here. Similarly to the previous results, LMI conditions can be formulated using Lemmas 2 or 3. Unfortunately, the main drawback of this extension is that the switching sequence must be known in advance or directly dependent on the input.

A different possibility to extend the results presented in Section 3 is the use of delayed controller and delayed Lyapunov function [Lendek et al., 2012], as follows. Consider instead of the control law (4) the following

$$\boldsymbol{u}(k) = -F_{i,z-1,z}H_{i,z-1,z}^{-1}\boldsymbol{x}(k) \tag{9}$$

which depends not only on the current, but also on the past states through the evaluation of the scheduling variable at time $k-1$. To develop and relax the conditions, also instead of the Lyapunov functions (6) and (7), the delayed Lyapunov functions

$$V = \boldsymbol{x}(k)^T P_{i,j,z-1}^{-1}\boldsymbol{x}(k) \tag{10}$$

and

$$V = \boldsymbol{x}(k)^T H_{i,z,z-1}^{-T} P_{i,j,z,z-1} H_{i,z,z-1}^{-1}\boldsymbol{x}(k) \tag{11}$$

respectively, can be used, again defined during the switches. Then, based on the same steps as described in Section 3, the following results can be stated. Using the Lyapunov function (10), we have

Corollary 5. The switching TS system (3) is asymptotically stabilized by the switching control law (9) if there exist matrices $H_{i,j,m,n}$ and symmetric positive definite matrices $P_{i,j,m,n} = P_{i,j,m,n}^T > 0$, $i, j = 1, 2, \ldots, n_\mathrm{s}$, $(v_i, v_j) \in \mathcal{E}$, $(v_j, v_l) \in \mathcal{E}$, $m, n = 1, 2, \ldots, r_i$, so that

$$\begin{pmatrix} -H_{i,j,z,z-1} - H_{i,j,z,z-1}^T + P_{i,j,z-1} & (*) \\ A_{i,z}H_{i,j,z,z-1} - B_{i,z}F_{i,j,z,z-1} & -P_{j,l,z} \end{pmatrix} < 0$$

while when using the Lyapunov function (11) we obtain

Corollary 6. The switching TS system (3) is asymptotically stabilized by the switching control law (8) if there exist matrices $H_{i,j,m,n}$ and symmetric positive definite matrices $P_{i,j,m,n} = P_{i,j,m,n}^T > 0$, $i, j = 1, 2, \ldots, n_\mathrm{s}$, $(v_i, v_j) \in \mathcal{E}$, $(v_j, v_l) \in \mathcal{E}$, $m, n = 1, 2, \ldots, r_i$, so that

$$\begin{pmatrix} -P_{i,j,z,z-1} & (*) \\ A_{i,z}H_{i,j,z,z-1} - B_{i,z}F_{i,j,z,z-1} & \begin{pmatrix} -H_{j,l,z,z+1} + (*) \\ +P_{j,l,z,z+1} \end{pmatrix} \end{pmatrix} < 0$$

Note that similarly to the results in Section 3 and Corollaries 3 and 4, the two results above are not equivalent. In order to further reduce the conservativeness of the results, the two extensions above can also be combined, i.e., one may use the control law defined on switches with delayed Lyapunov functions.

6. CONCLUSIONS

In this paper we have considered stabilization of switching TS systems. To develop the conditions, two switching Lyapunov functions have been used, leading to two sets of conditions. Their application has been illustrated on numerical examples. The conditions have been extended using delays in the Lyapunov functions and the controller gains, in order to reduce their conservativeness.

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