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A distributed obstacle avoidance MPC strategy for leader-follower formations

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Abstract: In this paper we address the obstacle avoidance motion planning problem for leader-follower vehicles configurations operating in static environments. By resorting to set-theoretic ideas, a receding horizon control algorithm is proposed for robots modelled by linear time-invariant (LTI) systems subject to input and state constraints. Terminal robust positively invariant regions and sequences of precomputed inner approximations of the one-step controllable sets are on-line exploited to compute the commands to be applied in a receding horizon fashion. Moreover, we prove that the design of both terminal sets and one-step ahead controllable regions is achieved in a distributed sense. An illustrative example is used to show the effectiveness of the proposed control strategy.

Keywords: Distributed Model predictive control schemes, Obstacle avoidance, Leader-follower networks, Constrained control.

1. INTRODUCTION

In recent years, control and coordination of multi-agent network systems have attracted the attention of many researchers, see e.g. Fax & Murray [2004], Jadbabaie et al. [2003], Lin et al. [2004], Liu et al. [2006]. This is partly due to broad applications of multi-agent systems in cooperative control of unmanned air vehicles, scheduling of automated highway systems, formation control of satellite clusters, and congestion control in communication networks, etc. Essentially, a cooperative multi-agent system is composed of a set of autonomous agents that interact each another in a shared environment in order to reach a common goal or optimize a global performance measure. Due to the large number of inputs and outputs of this class of systems, distributed control is often required. In this contribution the interest is devoted to the class of Distributed Model Predictive Control (DMPC)-based approaches that have been successfully proposed in the last decade, see Christofides et al. [2013] for an update and detailed literature review. Noticeable contributions on this isuue can be found in Dunbar [2007], Magni & Scattolini [2006], Maestre et al. [2011] and Liu et al. [2010].
For most real mobile robotics applications, a basic requirement is the capability to safely operate in environments where the presence of obstacles could encumber the "normal" dynamical behavior, see Huang et al. [2013]. Avoidance of collisions is a key component of the safe navigation whose typical objective is to reach a target through the obstacle-free part of the environment, see Du Toit & Burdick [2012] and references therein. Specifically, we are here interested to consider constrained Receding Horizon Control schemes which are an extremely appealing methodology for dealing with the obstacle avoidance motion planning problem in virtue of their intrinsic capability to generate at each time instant feasible trajectories that allow to safely reach a given goal, see Yoon et al. [2009].

Moving from these considerations, in this paper we will focus on a novel distributed discrete-time receding horizon strategy for solving the obstacle avoidance motion planning problem for leader-follower vehicles configurations operating in static environments. To deal with the latter, the idea is to exploit the distributed set-theoretic approach proposed in Franzè & Tedesco [2013] so that the prescribed saturation and geometric constraints are always fulfilled and large regions of attractions can be achieved. The key ingredients of the proposed strategy are below summarized:

- Stabilizing state feedback laws associated to robust positively invariant ellipsoidal sets;
- One-step controllable sets that can be steered to a target in a finite number of steps;
- On line low demanding one-step ahead prediction and receding horizon control philosophy.

Finally, the theoretical results of the proposed strategy are illustrated by means of a simulation campaign on a point mobile robot team whose navigation within a planar environment is limited by the presence of three obstacles.

PRELIMINARIES AND NOTATIONS

Given a set $S \subseteq \mathbb{R}^n$, $In[S] \subseteq S$ denotes its inner ellipsoidal approximation. Given a set $S \subseteq X \times Y \subseteq \mathbb{R}^n \times \mathbb{R}^m$, the projection of the set S onto X is defined as $\mathrm{Proj}_X(S) := \{x \in X \mid \exists y \in Y \text{ s.t. } (x, y) \in S\}$. Let us consider the following discrete-time plant description

$$x_p(t+1) = \Phi x_p(t) + Gu(t) \quad (1)$$

where $t \in \mathbb{Z}_+ := \{0, 1, ...\}$, $x_p(t) \in \mathbb{R}^n$ denotes the plant state and $u(t) \in \mathbb{R}^m$ the control input. Moreover, the system (1) is subject to the following set-membership state and input constraints:

$$u(t) \in \mathcal{U},\ \forall t \geq 0,\ \ \mathcal{U} := \{u \in \mathbb{R}^m \,|\, u^T u \leq \bar{u}\}, \quad (2)$$

$$x_p(t) \in \mathcal{X},\ \forall t \geq 0,\ \ \mathcal{X} := \{x_p \in \mathbb{R}^n |\, x_p^T x_p \leq \overline{x}\}, \quad (3)$$

with $\overline{u} > 0$, $\overline{x} > 0$, and $\mathcal{U}$, $\mathcal{X}$ compact subsets of $\mathbb{R}^m$ and $\mathbb{R}^n$, respectively.

Definition 1.1. A set $\mathcal{T} \subseteq \mathbb{R}^n$ is robustly positively invariant for (1) if once the state $x_p(t)$ enters that set at any given time t_0, it remains inside for all future instants, i.e. $x_p(t_0) \in \mathcal{T} \rightarrow x_p(t) \in \mathcal{T}, \forall t \geq t_0$. □

In principle, it is possible to compute the sets of states i-step controllable to $\mathcal{T}$ via the following recursion (see Blanchini & Miani [2008]):

$$\begin{array}{l} \mathcal{T}_0 := \mathcal{T} \\ \mathcal{T}_i := \{x_p \in \mathcal{X} : \exists\, u \in \mathcal{U} : \Phi x_p + Gu \in \mathcal{T}_{i-1}\}, \end{array} \quad (4)$$

where $\mathcal{T}_i$ is the set of states that can be steered into $\mathcal{T}_{i-1}$ using a single move with a causal control. By induction we have that $\mathcal{T}_i$ is the set of states that can be steered into $\mathcal{T}$ in at most i control moves.

2. PROBLEM FORMULATION

In the sequel, we refer to leader-follower LTI subsystems, for which the dynamics of the first subsystem is:

$$x^1(t+1) = A^1 x^1(t) + B^1 u^1(t) \quad (5)$$

while the remaining subsystems are described by the following dynamical equations:

$$x^i(t+1) = A^i x^i(t) + B^i u^i(t) + A^{i,i-1} x^{i-1}(t) + B^{i,i-1} u^{i-1}(t), \quad i = 2, \ldots, l, \quad (6)$$

where $x^i \in \mathbb{R}^{n_i}$ and $u^i \in \mathbb{R}^{m_i}$ are the state and input vectors of the $i-th$ subsystem, $A^i \in \mathbb{R}^{n_i \times n_i}$ and $B^i \in \mathbb{R}^{n_i \times m_i}$ are the state and input matrices for the $i-th$ subsystem, while $A^{i,i-1} \in \mathbb{R}^{n_i \times n_{i-1}}$ and $B^{i,i-1} \in \mathbb{R}^{n_i \times m_{i-1}}$ are the matrices for the coupling dynamics which define the influence of the $(i-1)-th$ subsystem upon the $i-th$ one. and that the following input and state constraints are prescribed for each $i-th$ subsystem:

$$u(t) \in \mathcal{U}^i,\ \forall t \geq 0,\ \ \mathcal{U}^i := \{u^i \in \mathbb{R}^{m_i} \,|\, {u^i}^T u^i \leq \bar{u}^i\}, \quad (7)$$

$$x(t) \in \mathcal{X}^i,\ \forall t \geq 0,\ \ \mathcal{X}^i := \{x^i \in \mathbb{R}^{n_i} |\, {x^i}^T x^i \leq \overline{x}^i\}, \quad (8)$$

with $\overline{u}^i > 0$, $\overline{x}^i > 0$, and $\mathcal{U}^i$, $\mathcal{X}^i$ compact subsets of $\mathbb{R}^m$ and $\mathbb{R}^n$, respectively.

Assumption 2.1. Without loss of generality we shall consider leader-follower networks whose agents have the same state space dimension n_p, i.e. $n_1 = n_2 = \cdots = n_l = n_p$. □

In order to formally characterize the obstacle avoidance problem, the following definitions will be considered:

Definition 2.2. Let Ob_k be an object. Then an obstacle scenario $\mathcal{O}$ is defined as

$$\mathcal{O} := \{Ob_1, \ldots, Ob_{n_o}\} \quad (9)$$

where n_o denotes the number of the involved objects. □

Definition 2.3. Let $\mathcal{O}$ be an obstacle scenario. Then the non-convex free region external to the obstacle Ob^k is identified as

$$\mathcal{O}^k_{free} := \{x \in \mathbb{R}^{n_p} :\, h_k(x) > 0\} \quad (10)$$

where $h_k : \mathbb{R}^{n_p} \rightarrow \mathbb{R}^{n_f}$ represents the support function characterizing the admissibility state space region. □

Definition 2.4. The non-convex free region is identified as

$$\mathcal{O}_{free} = \bigcap_{k=1}^{n_o} \mathcal{O}^k_{free} := \{x \in \mathbb{R}^{n_p} :\, h(x) > 0\} \quad (11)$$

Hereafter, we shall assume without loss of generality that each object Ob_k has a polyhedral convex structure (see e.g. Varaiya [1998] for details on the computation of outer polyhedral approximations) described as the intersection of l_k half-spaces:

$$Ob_k :\ \left[\, (H_k)_1^T \ \ldots\ (H_k)_{l_k}^T \,\right]^T p \leq \left[\, (g_k)_1 \ \ldots\ (g_k)_{l_k} \,\right]^T \quad (12)$$

where $p := B\,x \in \mathbb{R}^2$ are the environment components of the state space $x \in \mathbb{R}^{n_p}$, with $B \in \mathbb{R}^{2 \times n_p}$ a projection matrix.

Assumption 2.5. **Communication facilities**: At each time instant t, each $i-th$ follower subsystem knows the current state and input values of the $(i-1)-th$ subsystem and the current state of the $i+1-th$ subsystem. □

Distributed Obstacle Avoidance for leader-follower robot formations (DOA-LFF) - *Given the leader-follower formation (5)-(6), with the robot positions entries of the state vectors x^i, and the obstacle scenario (9) determine a distributed state-feedback control policy*

$$\begin{array}{ll} u^1(t) \,=\, g(x^1(t)), & i = 1 \\ u^i(t) \,=\, g(x^i(t), x^{i-1}(t), u^{i-1}(t)), & i = 2, \ldots, l \end{array} \quad (13)$$

compatible with (7), (8) and (11), such that starting from an admissible initial condition $x(0) = [x^1(0)^T, x^2(0)^T, \ldots x^l(0)^T]^T$ the state trajectory of each $i-th$ agent is driven to a target position x_f. □

In what follows, we will consider a receding horizon control that makes use of the results obtained in Angeli et al. [2008] and Franzè et al. [2013]. Strictly speaking, the basic control strategy prescribes the following computations:

- a stabilizing state-feedback control law and the corresponding robust positively invariant (terminal) region compatible with the prescribed constraints;
- the set sequence of states that can be steered into the terminal set in a finite number of steps.

In order to exploit such ideas for dealing with the **DOA-LFF** problem, the following key questions must be addressed:

- How can one define terminal ellipsoidal regions and sequences of one-step controllable sets for each subsystem within a distributed framework such that there exist feasible commands compatible with the dynamics (5) and (6)?
- How can one define a sequence of one-step controllable sets such that there exists at least a feasible path ($x(0) \rightarrow x_f$) complying with obstacle avoidance purposes?

The next two sections will provide a solution to these relevant issues.

3. ROBUSTLY POSITIVELY INVARIANT REGIONS AND ONE-STEP CONTROLLABLE SET FAMILIES: OFF-LINE PHASE

The aim of this section is to characterize terminal pairs, i.e. stabilizing controllers (K^i) and ellipsoidal regions ($\mathcal{T}_0^i$), and one-step controllable sets which comply with the prescriptions of the **DOA-LFF** problem. The main idea is, starting from the goal location x_f , derive off-line a family of one-step controllable sets able to reach the initial location $x(0)$. To this end, we have to formally build within a distributed (leader-follower) framework the following items: terminal sets, one-step controllable sets and a feasible path tube from $x(0)$ to x_f.

3.1 Terminal sets

Here, we determine a terminal constraint set which preserves the following Cartesian structure:

$$\mathcal{T}_0 := \prod_{i=1}^{l} \mathcal{T}_0^i \tag{14}$$

where $\mathcal{T}_0 \subset \mathbb{R}^n$, $n = n_1 + n_2 + \ldots + n_l$, is in principle the terminal set of the centralized system achievable by using (5) and (6).

The simplest way to design terminal pairs complying with (14) is to initially compute the pair (K^1, $\mathcal{T}_0^1$) pertaining to the leader because its dynamics (5) does not depend on the other subsystems behaviours, and then to compute the remaining (K^i, $\mathcal{T}_0^i$), $i = 2, \ldots, l$, by resorting to a "worst-case" approach. Specifically, the leader terminal pair (K^1, $\mathcal{T}_0^1$) is obtained by solving a standard semi-definite programming (SDP) problem as outlined e.g. in Kothare et al. [1996]. While the followers pairs (K^i, $\mathcal{T}_0^i$), $i = 2, \ldots, l$, require different arguments because of the coupled terms $A^{i,i-1}x^{i-1}$. Therefore, for each follower subsystem $A^{i,i-1}x^{i-1}$ can be considered as an unknown bounded disturbance in view of the fact that $x^{i-1} \in \mathcal{T}_0^{i-1}$ and, as a consequence, the robust positively invariant ellipsoid $\mathcal{T}_0^i$ can be obtained by resorting to SDP procedures based on P-difference arguments, see e.g. Kurzhanski & Valyi [1997].

3.2 One-step controllable sets

The basic construction of one-step controllable set families for the leader-follower architecture (5)-(6) is strictly connected to the chance to be able to reach the target x_f starting from $x(0)$. In fact because there exists an unavoidable saturation effect on the growth of the one-step controllable set sequence, it must be guaranteed that an admissible path ($x(0) \rightarrow x_f$) can be obtained by ensuring the feasibility retention of the overall distributed scheme. This reasoning leads to the need to deal with the following issues:

- Define recursions for the computation of one-step controllable regions;
- Develop a procedure to overcome the drawback due to the the saturation effect occurring on the one-step controllable sets growth so that an admissible path ($x(0) \rightarrow x_f$) could be determined.

Leader-Follower set recursions We characterize all the states one-step controllable to given target sets $\mathcal{T}_0^i$, $i = 1, \ldots, l$ by carefully taking care that the one-step state predictions are evaluated along interacting subsystem models. Then, in order to derive admissible controllable regions, it is necessary to proceed with the set construction in "backwards": the l set sequences are achieved level-by-level and starting from the last element of the leader-follower architecture, i.e. the $l - th$ follower subsystem (6). In this way, it can be ensured that each one-step controllable region $\mathcal{T}_j^i$ is compatible with the set $\mathcal{T}_j^{i-1}$ pertaining to the $(i - 1) - th$ subsystem whose the dynamics is shared with the $i - th$ agent. The following result summarizes this reasoning.

Proposition *1.* Let $\mathcal{T}_0^i \neq \emptyset$, $i = 1, \ldots, l$ be given robustly invariant ellipsoidal regions complying with the input, state and obstacle constraints (7), (8) and (11) respectively. Let $x_{aug}^1 = \left[x^{1^T}, u^{1^T}, x^{1^T}, u^{1^T}\right]^T \in \mathbb{R}^{2\times n+2\times m}$ and $x_{aug}^i = \left[x^{i-1^T}, u^{i-1^T}, x^{i^T}, u^{i^T}\right]^T \in \mathbb{R}^{2\times n+2\times m}$, $i = 2, \ldots, l$, the augmented states describing the dynamics of the subsystems (5) and (6) respectively. Then, the one-step controllable sets sequence $\{\mathcal{T}_j^i\}$, $i = 1, \ldots, l$, are derived by means of the following recursions:

$$\begin{aligned}\mathcal{T}_j^l = \{&x_{aug}^l = [x^l, u^l, x^{l-1}, u^{l-1}] \in \\ &(\mathcal{X}^{l-1} \cap \mathcal{O}_{free}) \times \mathcal{U}^{l-1} \times (\mathcal{X}^l \cap \mathcal{O}_{free}) \times \mathcal{U}^l : \\ &A^l x^l + B^l u^l + A^{l,l-1}x^{l-1} + B^{l,l-1}u^l \in \mathcal{E}_{j-1}^{x^l}\}\end{aligned} \tag{15}$$

$$\begin{aligned}\mathcal{T}_j^i = \{&x_{aug}^i = [x^{i-1}, u^{i-1}, x^i, u^i] \in \\ &\mathcal{T}_j^{i+1} \cap (\mathcal{X}^{i-1} \cap \mathcal{O}_{free}) \times \mathcal{U}^{i-1} \times (\mathcal{X}^i \cap \mathcal{O}_{free}) \times \mathcal{U}^i : \\ &A^i x^i + B^i u^i + A^{i,i-1}x^{i-1} + B^{i,i-1}u^{i-1} \in \mathcal{E}_{j-1}^{x^i}\}, \\ &i = l-1 \ldots 2\end{aligned} \tag{16}$$

$$\begin{aligned}\mathcal{T}_j^1 = \{&x_{aug}^1 = [x^1, u^1, x^1, u^1] \in \\ &\mathcal{T}_j^2 \cap (\mathcal{X}^1 \cap \mathcal{O}_{free}) \times \mathcal{U}^i \times (\mathcal{X}^1 \cap \mathcal{O}_{free}) \times \mathcal{U}^1) : \\ &A^1 x^1 + B^1 u^1 \in \mathcal{E}_{j-1}^{x^1}\}\end{aligned} \tag{17}$$

$$\mathcal{E}_{j-1}^{x^i} \times \mathcal{E}_{j-1}^{u^i} \times \mathcal{E}_{j-1}^{x^{i-1}} \times \mathcal{E}_{j-1}^{u^{i-1}} \subseteq \mathcal{T}_j^i \quad \forall i = 1 \ldots l \tag{18}$$

where

$$\mathcal{E}_j^{x^i} = \text{Proj}_{x^i}\{\mathcal{T}_j^i\} \quad \text{and} \quad \mathcal{E}_j^{u^i} = \text{Proj}_{u^i}\{\mathcal{T}_j^i\} \tag{19}$$

Proof - It is omitted for the sake of space. □

Feasible path tube According to recursions (15)-(19), the construction of a *single* set of the one-step controllable families $\{\mathcal{E}_j^{x^i}\}$ (one family per each involved agent) does not in principle guarantee that a pre-assigned initial condition $x(0)$ lies into $\bigcup_j \{\mathcal{E}_j^{x^1} \times \mathcal{E}_j^{x^2} \times \cdots \times \mathcal{E}_j^{x^l}\}$, because a saturation effect may occur on the one-step controllable sets growth. As a consequence, the whole sequence $\bigcup_j \{\mathcal{T}_j^1 \times \mathcal{T}_j^2 \times \cdots \times \mathcal{T}_j^l\}$, could be useless for any control purposes. This drawback can be worked around by resorting to the ideas developed in Franzè et al. [2013] that are here adequately modified to comply with the leader-follower topology:

*One-step Controllable Set Procedure (***OSCSP***) -*

1. *Given the goal x_f, according to (14) and by following the lines indicated in Section 3.1 design the pairs $(K_0^i, \mathcal{T}_0^i), i = 1, \ldots, l$, with $\mathcal{T}_0^i$ robust positively invariant regions centered in x_f. Store the index* 0 *into the indices vectors hereafter named IR^i*;

2. *Derive the sequence* $\{\mathcal{T}_j^i\}_{j=1}^{N_i^1}$, $i = 1,\ldots,l$, *by using recursions (15)-(19). Integers* N_i^1, $i = 1,\ldots,l$, *are the saturation levels for each* $i-th$ *region growth;*
4. *Initialize the counter* $s := 1$;
5. *If* $x(0) \notin \mathcal{E}_{N_1^s}^{x^1} \times \cdots \times \mathcal{E}_{N_1^s}^{x^l}$ *then*
   - *Choose an equilibrium* $x_{eq}^s \in \mathcal{E}_{N_1^s}^{x^1}$ *(leader agent sequence);*
   - *Design a new terminal pair* $(K_{N_1^s+1}^1, \mathcal{T}_{N_1^s+1}^1)$ *with* $\mathcal{T}_{N_1^s+1}^1$ *centered in* x_{eq}^s. *Compute new follower terminal pairs* $(K_{N_1^s+1}^i, \mathcal{T}_{N_1^s+1}^i)$ *with* $\mathcal{T}_{N_1^s+1}^i$ *centered in* x_{eq}^1 *and such that*
   $$\mathcal{T}_{N_1^s+1}^i \subseteq \mathcal{T}_{N_1^s+1}^1,\ i = 2,\ldots,l. \tag{20}$$
   Store the integers N_i^s+1 *into the corresponding indices vector* IR^i;
6. *Derive the new leader sequence* $\{\mathcal{T}_j^1\}_{j=N_1^s+1}^{N_1^{s+1}}$ *under the following additional constraint:*
   $$\{\mathcal{T}_j^1\}_{j=N_1^s+1}^{N_1^{s+1}} \subseteq \bigcup_{i=2}^{l} \{\mathcal{T}_j^i\}_{j=1}^{N_i^s} \tag{21}$$
   while the new follower sequences $\{\mathcal{T}_j^i\}_{j=N_1^s+1}^{N_i^{s+1}}, i = 2,\ldots,l$, *are exactly obtained as prescribed in (15)-(19).*
7. *Add each new sequence* $\{\mathcal{T}_j^i\}_{j=N_1^s+1}^{N_i^{s+1}}, i = 1,\ldots,l$, *to the corresponding previously computed sequences;*
8. $s \leftarrow s+1$, *and goto Step 5.*

Remark 1. Notice that the indices vector IR^i has the aim to keep trace of all the robust positively invariant regions obtained by the **OSCSP** procedure.
The additional conditions (20)-(21) are introduced to make admissible within a receding horizon control framework the switching amongst sets of one-step controllable sequences, i.e. $\{\mathcal{T}_j^i\}_{j=N_1^s+1}^{N_i^{s+1}} \rightarrow \{\mathcal{T}_j^i\}_{j=N_1^r+1}^{N_i^{r+1}}, s \neq r$.
The non-convex constraints arising from (11) can be convexified by exploting the procedure proposed in Franzè et al. [2013]. □

4. THE DISTRIBUTED MODEL PREDICTIVE CONTROL ALGORITHM

The proposed scheme will rely on the properties of the leader-follower hierarchy in the sense that each i-th agent makes a decision just before the successive $(i+1)$-th agent. In fact, the i-th subsystem selects its local command u^i by resorting to the current state measurement $x^i(t)$ and to the information ($x^{i-1}(t)$ and $u^{i-1}(t)$) received by the $(i-1)$-th predecessor agent.

Then, the local input $u^i(t)$ is computed according to the following optimization problem:

$$u^i(t) = \arg\min_{u^i} J_{j(t)}(x^i(t), u^i, x^{i-1}(t), u^{i-1}(t)) \quad \text{s. t.} \tag{22}$$

$$\begin{cases} A^i x^i(t) + B^i u^i + A^{i,i-1}x^{i-1}(t) + B^{i,i-1}u^{i-1}(t) \in \mathcal{E}_{j(t)-1}^{x^i} \\ u^i \in \mathcal{U}^i \end{cases} \tag{23}$$

Here, the running cost $J_{j(t)}(x^i(t), u^i, x^{i-1}(t), u^{i-1}(t))$ is chosen without loss of generality as follows:

$$\begin{aligned} &J_{j(t)}(x^i(t), u^i, x^{i-1}(t), u^{i-1}(t)) = \\ &\|A^i x^i + B^i u^i + A^{i,i-1}x^{i-1} + B^{i,i-1}u^{i-1}(t)\|^2_{P^i_{j(t)-1}} \end{aligned} \tag{24}$$

where P_{j-1}^i is the shaping matrix of the ellipsoidal region $\mathcal{E}_{j-1}^{x^i}$.

$$(x^i)^T P_{j-1}^i x^i \leq 1$$

Once the local command $u^i(t)$ has been computed, the i-th subsystem transmits to the $(i+1)$-th follower both $x^i(t)$ and $u^i(t)$. This procedure applies to all the agents in a sequential fashion. It is important to remark that the whole sequential procedure needs to be completed within the given sampling interval, i.e. all the agents have been applied their decisions.

Note that the above actions are slight different when the leader subsystem (5) is considered. In fact, the leader does not make use of any information related to the other agents and determines the command $u^1(t)$ by only using the state measurements $x^1(t)$. As a consequence, the following optimization problem results:

$$u^1(t) = \arg\min_{u^1} J_{j(t)}(x^1(t), u^1) \quad \text{s. t.} \tag{25}$$

$$\begin{cases} A^1 x^1(t) + B^1 u^1 \in \mathcal{E}_{j(t)-1}^{x^1} \\ u^1 \in \mathcal{U}^1 \end{cases} \tag{26}$$

and the running cost $J_{j(t)}(x^1(t), u^1)$ is

$$J_{j(t)}(x^1(t), u^1, u^2) = \|Ax^1(t) + B^1 u^1\|^2_{P^1_{j(t)-1}} \tag{27}$$

Finally, the leader transmits $x^1(t)$ and $u^1(t)$ to the agent 2.

All the above developments allows one to write down a computable distributed MPC scheme, hereafter denoted as **DMPC-LF**, which consists of the following algorithm.

Distributed Leader-Follower Obstacle Avoidance Receding Horizon Control Algorithm (DLFOA-RHC Algorithm) - Agent i-th

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Initialization

1.1 *Given the obstacle scenario* $\mathcal{O}$, *the initial state condition* $x(0)$ *and the goal* x_f, *compute the robust invariant ellipsoids* $\mathcal{T}_0^i \subset \mathbb{R}^{n_i}$, $i = 1,\ldots,l$, *and the corresponding stabilizing state feedback gains* K^i, $i = 1,\ldots,l$, *complying with the constraints (2), (3), (11);*

1.2 *Apply the* **OSCSP** *procedure in order to generate the sequences* $\{\mathcal{E}_j^{x^i}\}$, $i = 1,\ldots,l$, *such that*

$$x(0) \in (\mathcal{E}_{x^p}^{x^1} \times \ldots \times \mathcal{E}_{x^p}^{x^l}) \subset \left[\bigcup_j \left\{ \mathcal{E}_j^{x^1} \times \ldots \times \mathcal{E}_j^{x^1} \right\} \right] \tag{28}$$

1.3 *Store the ellipsoids* $\{\mathcal{E}_j^{x^i}\}$

On-line phase (Leader)

1. *repeat at each time t*
   - 1.1 *find* $j(t) = \min\{j : x^1(t) \in \mathcal{E}_j^{x^1}\}$
   - 1.2 *If* $j(t) \in IR^1$ *then* $u^1(t) = K^1 x^1(t)$
   - 1.3 *else solve (22)-(23)*
   - 1.4 *apply* $u^1(t)$ *and transmit* $x^1(t)$ *and* $u^1(t)$ *to the agent 2*

On-line phase (Follower $i-th$)

1. *repeat at each time t*
   1.1 *receive $x^{i-1}(t), u^{i-1}(t)$ from the $(i-1)$-th predecessor*
   1.2 *find $j(t) = \min\{j : x^i(t) \in \mathcal{E}_j^{x^i}\}$*
   1.3 *If $j(t) \in IR^i$ then $u^i(t) = K^i x^i(t)$*
   1.4 *else solve (25)-(26)*
   1.5 *apply $u^i(t)$ and transmit $x^i(t)$ and $u^i(t)$ to the $(i+1)$-th follower*

The next proposition shows that the proposed **DLFOA-RHC** Algorithm enjoys the feasibility retention and closed-loop stability.

Proposition *2.* Let the sequences of sets $\mathcal{E}_j^{x^i}$ be non-empty such that

$$x(0) \in (\mathcal{E}_{x^p}^{x^1} \times \ldots \times \mathcal{E}_{x^p}^{x^l}) \subset \left[\bigcup_j \left\{ \mathcal{E}_j^{x^1} \times \ldots \times \mathcal{E}_j^{x^1} \right\} \right] \quad (29)$$

Then, the **DLFOA-RHC** Algorithm always satisfies the constraints and ensures asymptotic stability despite of the obstacle scenario $\mathcal{O}$ occurrence.

Proof - The existence of solutions at each time instant t implies existence of solutions at time $t+1$ because the optimization problems in Steps 1.3.1 and 1.4.1 of the on-line phase for both leader and followers are always feasible. In fact, by construction there exists an input vector u satisfying the constraints (7), (8) and (11) such that the set-membership requirements in (26) and (23) respectively hold true. Then thanks to the **OSCSP** procedure and under the additional constraint constraints (20)- (21), at the next time instant t + 1 the existence of solutions $u(t+1)$ for the Steps 1.4.1 is ensured. □

5. ILLUSTRATIVE EXAMPLE

In this section we shall consider a formation of three robots (R_1, R_2, R_3) where the robot R_1 (leader) moves independently from R_2 and R_3 (followers). For each robot we will use the point mobile robot model discussed in Kuwata et al. [2005], whose the state consists of position and velocity components $x = [p_x \; p_y \; v_x \; v_y]^T$ and motions are governed by the following discrete-time LTI model:

$$x(t+1) = \Phi x(t) + Gu(t)$$

where $u \in \mathbb{R}^2$ is the acceleration vector (m/s^2),

$$\Phi = \begin{bmatrix} I_2 & \Delta t\, I_2 \\ 0_2 & I_2 \end{bmatrix}, \; G = \begin{bmatrix} \dfrac{(\Delta t)^2 I_2}{2} \\ \Delta t I_2 \end{bmatrix},$$

with $\Delta t = 1\,sec.$ and subject to the saturation constraint

$$\|u(t)\|_2^2 \le 0.028, \;\; \forall t \ge 0. \quad (30)$$

By defining $x^{e_1} := x^1$, $x^{e_2} := x^2 - x^1$, $x^{e_2} := x^3 - x^2$, the following error state space description complying with (5)-(6) is derived:

$$\begin{aligned} x^{e_1}(t+1) &= A^1 x^1(t) + B^1 u^1(t) \\ x^{e_2}(t+1) &= A^2 x^{e_2}(t) + B^2 u^2(t) + A^{2,1} x^{e_1}(t) + B^{2,1} u^1(t) \\ x^{e_3}(t+1) &= A^3 x^{e_3}(t) + B^3 u^3(t) + A^{3,2} x^{e_2}(t) + B^{3,2} u^2(t) \end{aligned} \quad (31)$$

where

$$A^1 = A^2 = A^3 := \Phi, A^{2,1} = A^{3,2} := \Phi - \Phi = 0_{n \times n},$$

and

$$B^1 = B^2 = B^3 = G, B^{2,1} = B^{3,2} = -G$$

Moreover, we assume that the autonomous vehicles navigate in a planar environment characterized by an obstacle scenario $\mathcal{O}$ consisting of three rectangular objects, see Fig. 3. Specifically, the working space is defined as follows:

$$\begin{bmatrix} -1 & 0 \\ 1 & 0 \\ 0 & -1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} p_x \\ p_y \end{bmatrix} \le \begin{bmatrix} 0 \\ 18 \\ 0 \\ 18 \end{bmatrix} \quad (32)$$

while the obstacles are localized as below reported:

Obstacle	width	heigth	center of gravity
Ob_1	2	2	[5; 9]
Ob_2	2	2	[10; 6]
Ob_3	2	2	[10; 11]

The aim of this simulation can be stated as:

Starting from the following initial positions: $x^1(0) = [5, 5, 0, 0]^T$, $x^2(0) = [2, 10, 0, 0]^T$, $x^3(0) = [11, 2, 0, 0]^T$, *drive the leader and follower vehicles to the target* $x_f = [14, 13, 0, 0]^T$ *within the operating region (32) subject to the obstacle configuration* $\mathcal{O}$.

In order to implement the **DLFOA-RHC** Algorithm the following choices on the running cost have been made: $R_u = I_2$ and $R_{u^2} = I_2$. All the relevant numerical results are reported in Figs. 1-3. First, Figs. 1 and 3 show that the prescribed saturation and geometric constraints are always fulfilled. Then, to better appreciate the *modus operandi* of the proposed scheme, it is important to analyze the set-membership signal shown in Fig. 2, because it depicts the level of contraction provided by the **DLFOA-RHC** algorithm during the system evolution and represent at each time instant the smaller ellipsoids $\mathcal{E}_j^{x^i}$ of the pre-computed families containing the subsystem states $x^i(t)$.

Finally, Fig. 3 accounts for the state dynamical behaviours of the robot formation, by putting in evidence the following interesting phenomenon: though the followers (R_2 and R_3) start from initial conditions far from the leader (R_1) initial point and their *ideal* trajectories are obstructed by the obstacle occurrence, the proposed strategy is capable to drive all the follower vehicles to the target x_f by tracking *as better as possible* the leader path.

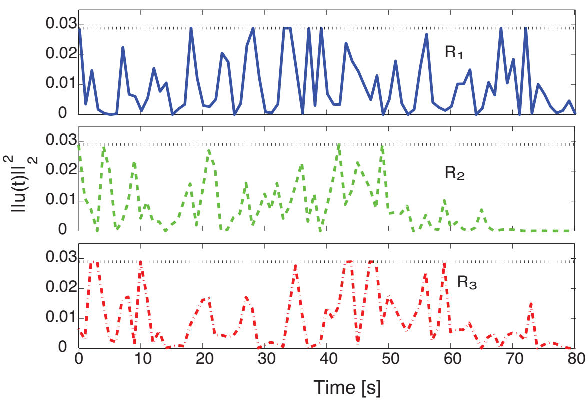

Fig. 1. Applied command inputs. The dotted lines represent the prescribed constraints

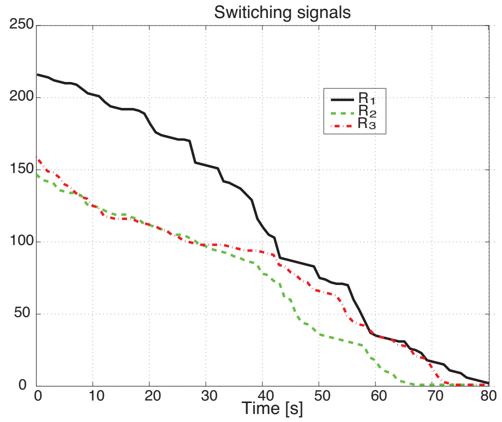

Fig. 2. Set-membership signals

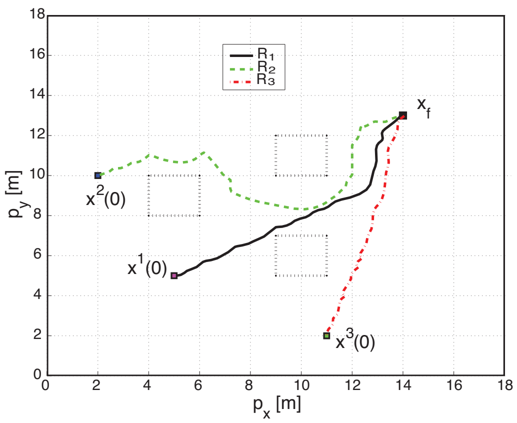

Fig. 3. Robots trajectories

6. CONCLUSIONS

In this paper we have presented a novel distributed model predictive control strategy for solving the the obstacle avoidance motion planning problem for autonomous vehicle leader-follower networks described by linear time-invariant systems within static environments. Set-theoretic ideas have been exploited to take care of all the dynamical connections amongst the involved agents. As one of its main merits, the proposed strategy carries out off-line most of the computations so that the overall framework becomes appealing in practical applications. Moreover, feasibility retention, viz. constraints fulfilment, and asymptotic stability of the closed-loop system have been formally proved. Finally, the benefits of the **DLFOA-RH** Algorithm are illustrated by means of a point mobile robots team.

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