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Trajectory Tracking of Nonholonomic Mobile Robots Using a Vision-based Adaptive Algorithm for Position and Velocity Estimation

Luyang Li*, Yun-Hui Liu*, Kai Wang*

**Department of Mechanical and Automation Engineering, The Chinese University of Hong Kong, Sha Tin, New Territories, Hong Kong (e-mail: lyli@mae.cuhk.edu.hk, yhliu@mae.cuhk.edu.hk, kwang@mae.cuhk.edu.hk).*

Abstract: One difficult issue for trajectory tracking control of nonholonomic mobile robots is measurements of the robot's position and linear velocity. To solve this problem, this paper presents a new controller to control a nonholonomic mobile robot to trace a desired trajectory using an omnidirectional vision system and inertial sensors without measuring the robot's position and linear velocity. Based on a new projection model of the omnidirectional vision system, a novel adaptive estimator is developed and embedded into the new controller for estimating the position and linear velocity of the robot on-line using the continuously tracked natural feature points in the omnidirectional image sequence, the robot's acceleration and orientation measured by the inertial sensors. It is proved by Lyapunov theory that the proposed controller gives rise to asymptotic tracking of a desired trajectory and convergence of the estimations of the robot's position and linear velocity to their true values. Experiments were conducted to validate the superior performance of the proposed adaptive controller.

1. INTRODUCTION

Motion control of mobile robots is a classical problem in robotics and has been extensively studied for many potential applications in various fields. In this research area, trajectory tracking control of nonholonomic mobile robots which adopt the differential drive or the steering drive mechanism is a crucial technical issue because there exists no smooth time-varying feedback controller (Brockett, 1983).

Many efforts have been made to design tracking controllers for nonholonomic mobile robots since early 1990s. Various model-based control methods have been proposed, e.g. discontinuous controllers (Yang and Kim, 1999), time-varying controllers (Kanayama *et al*., 1990; Kanayama *et al*., 1991; Jiang and Nijmeijer, 1997), and hybrid controllers (Hespanha and Morse,1999). A sliding mode control method for trajectory tracking of mobile robots was presented in Yang and Kim (1999). Besides, in Kanayama *et al*. (1990, 1991), a controller for path tracking was developed based on an important coordinate transformation of the tracking error. Using the same transformation, an asymptotic controller was presented in Jiang and Nijmeijer (1997), which proved convergence of the tracking error when the desired linear velocity is not zero. Moreover, a hybrid controller was proposed in Hespanha and Morse (1999) to logically switch several time-varying controllers to guarantee the exponential convergence to origin.

Most existing trajectory tracking controllers work under a key assumption that the pose of the robot can be precisely measured. However, despite of tremendous efforts made around the world (Bonnifait and Garcia, 1998; Hu *et al*., 2003; Nister *et al*., 2004; Davison *et al*., 2007; Klein and Murray, 2007; Xu *et al*., 2009; etc.), localization of mobile robots still remains an open research problem in robotics.

The main objective of this work is to eliminate the requirement for the pose measurement in trajectory tracking control of nonholonomic mobile robots. To achieve this objective, the idea of visual servo control is adopted. Visual servoing presents an excellent framework for controlling mobile robots without knowing their pose by the direct feedback of the information of image features (image-based visual servoing) or by employing them to estimate pose of the robot (position-based visual servoing). Existing works on visual servoing of mobile robots can be classified into regulation (Fang *et al*., 2005; Mariottini *et al*., 2007; Hu *et al*., 2009; etc.), tracking moving objects (Tsai *et al*., 2009), path following (Coulaud *et al*., 2006; Cherubini *et al*., 2011; etc.), and image-based trajectory tracking (Chen *et al*., 2006). Note that the traditional visual servoing for regulation, tracking of moving objects, and path following are different from the problem addressed in this paper. Image-based trajectory tracking proposed in Chen *et al.* (2006) has the limitation that the image sequences should be stored beforehand, which is not suitable for many applications. To the best of our knowledge, there is no position-based visual servoing controller developed for trajectory tracking of nonholonomic mobile robots.

In this paper, we propose a novel position-based visual servo controller for trajectory tracking of the mobile robots with nonholonomic constraint on the basis of the works in Kanayama *et al*. (1990, 1991) and Jiang and Nijmeijer (1997) without using the position and linear velocity feedback of the robot. An adaptive estimator, similar to that used in the model-based adaptive control (Slotine and Li, 1987), is developed and embedded into the controller to estimate the robot's position as well as linear velocity in real-time by using acceleration and orientation of the robot measured by inertial sensors, and visual feedback from an omnidirectional vision system. It should be noted that the adaptive estimator is coupled with the tracking controller, unlike many other works (Fang *et al*., 2005; etc.), where the robot's localization and tracking control are carried out in two independent loops.

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In this paper, it has been proven by Lyapunov theory that the proposed controller with the adaptive estimator leads to asymptotically tracking the desired trajectory, and convergence of the estimations of the robot's position and linear velocity to their true values, simultaneously. Experiment conducted on a differential drive wheeled mobile robot in an indoor environment and the results ascertained the superior performance of the proposed controller.

2. PRELIMINARIES

2.1 Problem Statement

Consider a differential or steering drive wheeled mobile robot moving on a plane in an unknown environment where no any global localization system are available for calculating the position and linear velocity of the robot. The robot is equipped with an omnidirectional vision system to capture images of the surrounding environment and an inertial sensor system such as Attitude Heading Reference System (AHRS) to measure its acceleration and orientation angles. In this paper, we address the problem of controlling the mobile robot to track a desired trajectory from the measurements of AHRS and the omnidirectional image sequence.

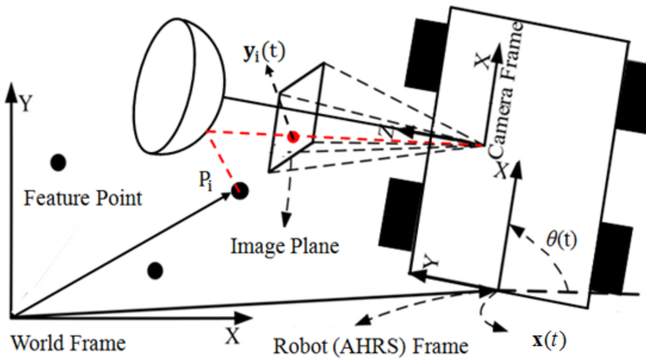


Fig. 1. The coordinate frames of the mobile robot.

In order to formulate this problem, we set up a world coordinate frame, a robot coordinate frame, a camera coordinate frame and an inertial sensor (AHRS) coordinate frame (see Fig. 1). We assume the robot frame is overlapped by the inertial sensor frame, and transformation of the camera frame with respect to the robot frame is represented to a matrix $\mathbf{T}\in\mathbb{R}^{4\times4}$. The position and orientation of the robot w.r.t. the world frame are denoted by $\mathbf{x}(t)$ and $\theta(t)$, respectively. The kinematics of the differential or steering drive mobile robot can be represented as follows:

$$\begin{pmatrix}\dot{\mathbf{x}}(t)\\ \dot{\theta}(t)\end{pmatrix}=\begin{pmatrix}\cos\theta(t) & 0\\ \sin\theta(t) & 0\\ 0 & 1\end{pmatrix}\begin{pmatrix}v(t)\\ \omega(t)\end{pmatrix} \tag{1}$$

where we denote $\mathbf{v}(t)=\dot{\mathbf{x}}(t)$, $\dot{\theta}(t)$ and $v(t)$ as the linear velocity, the angular velocity and the linear speed of the robot, respectively. The major issue in tracking control of mobile robots is designing proper $v(t)$ and $\omega(t)$ to achieve the control objective. It is well-known that (1) imposes the nonholonomic constraint on the motion of the robot, which makes the controller design difficult.

The desired trajectory of the robot is denoted by the desired pose and velocity shown as follows:

$$\mathbf{x}_{pd}(t)=\left(\mathbf{x}_d^{\,T}(t)\;\;\theta_d(t)\right)^T\in\mathbb{R}^3 \tag{2.1}$$

$$\mathbf{v}_{pd}(t)=\dot{\mathbf{x}}_{pd}(t)=\left(\dot{\mathbf{x}}_d^{\,T}(t)\;\;\dot{\theta}_d(t)\right)^T\in\mathbb{R}^3 \tag{2.2}$$

From (2.1) and (2.2), the desired linear speed and angular velocity of the robot are denoted by:

$$v_d(t)=\left\|\dot{\mathbf{x}}_d(t)\right\|_2\in\mathbb{R} \tag{3.1}$$

$$\omega_d(t)=\dot{\theta}_d(t)\in\mathbb{R} \tag{3.2}$$

where $\|\bullet\|_2$ represents *Frobenius norm*.

To clarify this problem, we have the following assumption and remark:

Assumption 1. The robot's acceleration and orientation can be measured by inertial sensors (AHRS) with good accuracy.

Problem. Under assumption 1, design proper $v(t)$ and $\omega(t)$ to control the mobile robot subject to the nonholonomic constraint (1) to track a desired trajectory (2.1) - (2.2) using the robot's acceleration and orientation measured by inertial sensors (AHRS), and visual feedback of the omnidirectional vision system.

2.2 Projection Model of the Omnidirectional Vision System

Suppose that the omnidirectional vision system equipped on the mobile robot captures a number of fixed feature points whose 3-D positions w.r.t. the world frame are denoted by $\mathbf{p}_i\in\mathbb{R}^3$. The projection of the feature point on the image plane of the omnidirectional vision system is denoted by $\mathbf{y}_i(t)=[u_i(t),v_i(t)]^T\in\mathbb{R}^2$. Based on a new projection model of the omnidirectional vision proposed in our previous work (Li *et al.*, 2013), the projection of a 3-D point on image plane of the omnidirectional vision system can be written to:

$$\begin{pmatrix}\mathbf{y}_i(t)\\ 1\end{pmatrix}=\frac{1}{z_i(t)}\mathbf{M}(\mathbf{y}_i(t))\mathbf{R}^T(\theta(t))\left(\mathbf{p}_i-\begin{pmatrix}\mathbf{x}(t)\\ 0\end{pmatrix}\right) \tag{4}$$

where $\mathbf{M}(\mathbf{y}_i(t))\in\mathbb{R}^{3\times3}$ is the projection matrix of the omnidirectional vision system. It is clear that the projection model above is represented to a similar form to that of the perspective vision, but $\mathbf{M}(\mathbf{y}_i(t))$ is not only depending on the transformation matrix $\mathbf{T}$ and the intrinsic parameters of the omnidirectional vision system, but the projections of the feature points. $\mathbf{m}_j(\mathbf{y}_i(t))\in\mathbb{R}^3$ denotes the *j-th* row vector of $\mathbf{M}(\mathbf{y}_i(t))$. $\mathbf{R}(\theta(t))\in\mathbb{R}^{3\times3}$ is the rotation matrix of the robot frame with respect to the world frame. $z_i(t)\in\mathbb{R}$ is the depth of the feature point with respect to the camera frame given by:

$$z_i(t)=\underbrace{\left\{\mathbf{m}_3(\mathbf{y}_i(t))\mathbf{R}^T(\theta(t))\right\}}_{\mathbf{A}(\theta(t),\mathbf{y}_i(t))}\left(\mathbf{p}_i-\begin{pmatrix}\mathbf{x}(t)\\ 0\end{pmatrix}\right) \tag{5}$$

The projection equation (4) can be easily rewritten to the following equation:

$$z_i(t)\,\mathbf{y}_i(t) = \underbrace{\left\{\begin{pmatrix}\mathbf{m}_1(\mathbf{y}_i(t))\\ \mathbf{m}_2(\mathbf{y}_i(t))\end{pmatrix}\mathbf{R}^T(\theta(t))\right\}}_{\mathbf{B}(\theta(t),\mathbf{y}_i(t))}\left(\mathbf{p}_i - \begin{pmatrix}\mathbf{x}(t)\\ 0\end{pmatrix}\right) \quad (6)$$

Moreover, by differentiating (6) and noting (5), we have:

$$z_i(t)\,\dot{\mathbf{y}}_i(t) = \underbrace{\left\{\dot{\mathbf{B}}(\theta(t),\mathbf{y}_i(t)) - \mathbf{y}_i(t)\,\dot{\mathbf{A}}(\theta(t),\mathbf{y}_i(t))\right\}}_{\mathbf{C}(\theta(t),\dot{\theta}(t),\dot{\mathbf{y}}_i(t),\mathbf{y}_i(t))}\left(\mathbf{p}_i - \begin{pmatrix}\mathbf{x}(t)\\ 0\end{pmatrix}\right)$$
$$+\underbrace{\left\{\mathbf{y}_i(t)\,\mathbf{A}(\theta(t),\mathbf{y}_i(t)) - \mathbf{B}(\theta(t),\mathbf{y}_i(t))\right\}}_{\mathbf{D}(\theta(t),\mathbf{y}_i(t))}\begin{pmatrix}\mathbf{v}(t)\\ 0\end{pmatrix} \quad (7)$$

where $\dot{\mathbf{y}}_i(t)$ is the moving velocity of the image feature point, which can be calculated by *optical flow*.

3. TRACKING CONTROL OF NONHOLONOMIC MOBILE ROBOTS WITHOUT POSE MEASUREMENTS

This section presents a new position-based visual servo controller for trajectory tracking of a nonholonomic mobile robot without using the position and linear velocity feedback of the robot. This controller is developed on the basis of the work presented in Kanayama *et al*. (1990, 1991), and the controller proposed in Jiang and Nijmeijer (1997).

3.1 Review of Tracking Control with Pose Measurements

Assume that the pose of the mobile robot can be obtained, and the pose error of the robot is given by:

$$\begin{pmatrix}\Delta\mathbf{x}(t)\\ \Delta\theta(t)\end{pmatrix} = \begin{pmatrix}\mathbf{x}_d(t)\\ \theta_d(t)\end{pmatrix} - \begin{pmatrix}\mathbf{x}(t)\\ \theta(t)\end{pmatrix} \quad (8)$$

Based on (8), the following useful error transformation was introduced in Kanayama *et al*. (1990, 1991):

$$\mathbf{e}(t) = \begin{pmatrix}e_x(t)\\ e_y(t)\\ e_\theta(t)\end{pmatrix} = \begin{pmatrix}\cos\theta(t) & \sin\theta(t) & 0\\ -\sin\theta(t) & \cos\theta(t) & 0\\ 0 & 0 & 1\end{pmatrix}\begin{pmatrix}\Delta\mathbf{x}(t)\\ \Delta\theta(t)\end{pmatrix} \in \mathbb{R}^3 \quad (9)$$

Note that the coefficient matrix in (9) is of full rank, so convergence of $\mathbf{e}(t)$ to zero guarantees convergences of $\Delta\mathbf{x}(t)$ and $\Delta\theta(t)$ to zero. By differentiating (9), the following error dynamics can be obtained:

$$\dot{\mathbf{e}}(t) = \begin{pmatrix}\dot{e}_x(t)\\ \dot{e}_y(t)\\ \dot{e}_\theta(t)\end{pmatrix} = \begin{pmatrix}v_d(t)\cos e_\theta(t)\\ v_d(t)\sin e_\theta(t)\\ \omega_d(t)\end{pmatrix} + \begin{pmatrix}-1 & e_y(t)\\ 0 & -e_x(t)\\ 0 & -1\end{pmatrix}\begin{pmatrix}v(t)\\ \omega(t)\end{pmatrix} \in \mathbb{R}^3 \quad (10)$$

Based on the error dynamics (10), the following controller was developed in Jiang and Nijmeijer (1997):

$$\begin{pmatrix}v(t)\\ \omega(t)\end{pmatrix} = \begin{pmatrix}v_d(t)\cos e_\theta(t) + k_x e_x(t)\\ \omega_d(t) + k_\theta e_\theta(t) + v_d(t)e_y(t)\left(\sin e_\theta(t)/e_\theta(t)\right)\end{pmatrix} \quad (11)$$

where $k_x, k_\theta \in \mathbb{R}$ are positive gains and $\lim_{t\to\infty}\left(\sin e_\theta(t)/e_\theta(t)\right) = 1$.

It has been proven that the controller (11) leads to asymptotic convergence of the pose error to zero. However, when there is no global localization system, the pose of the robot is not available and controller (11) cannot be implemented directly.

3.2 Tracking Control without Pose Measurements

To solve this problem, this paper proposes a novel controller which has a similar form to (11) but employs the estimations of the position $\hat{\mathbf{x}}(t) \in \mathbb{R}^2$ and the orientation $\hat{\theta}(t) \in \mathbb{R}$ of the robot. From assumption 1, $\hat{\theta}(t)$ can be measured by inertial sensors with good accuracy, that is, $\hat{\theta}(t) = \theta(t)$. Along this paper, the tracking controller is designed on the basis of the accurate orientation of the robot. The corresponding estimation of the pose error of the robot is given by:

$$\begin{pmatrix}\Delta\hat{\mathbf{x}}(t)\\ \Delta\hat{\theta}(t)\end{pmatrix} = \begin{pmatrix}\mathbf{x}_d(t) - \hat{\mathbf{x}}(t)\\ \theta_d(t) - \hat{\theta}(t)\end{pmatrix} = \begin{pmatrix}\mathbf{x}_d(t) - \hat{\mathbf{x}}(t)\\ \theta_d(t) - \theta(t)\end{pmatrix} \quad (12)$$

The estimated and true pose errors of the robot are related by:

$$\begin{pmatrix}\Delta\hat{\mathbf{x}}(t)\\ \Delta\hat{\theta}(t)\end{pmatrix} = \begin{pmatrix}\Delta\mathbf{x}(t)\\ \Delta\theta(t)\end{pmatrix} + \begin{pmatrix}\tilde{\mathbf{x}}(t)\\ 0\end{pmatrix} \quad (13)$$

where $\tilde{\mathbf{x}}(t) = \mathbf{x}(t) - \hat{\mathbf{x}}(t) \in \mathbb{R}^2$ is denoted by the estimation error of the robot's position.

The estimation of the transformed error $\hat{\mathbf{e}}(t) \in \mathbb{R}^3$ is given by:

$$\hat{\mathbf{e}}(t) = \begin{pmatrix}\hat{e}_x(t)\\ \hat{e}_y(t)\\ e_\theta(t)\end{pmatrix} = \begin{pmatrix}\cos\theta(t) & \sin\theta(t) & 0\\ -\sin\theta(t) & \cos\theta(t) & 0\\ 0 & 0 & 1\end{pmatrix}\begin{pmatrix}\Delta\hat{\mathbf{x}}(t)\\ \Delta\hat{\theta}(t)\end{pmatrix} \in \mathbb{R}^3 \quad (14)$$

Replacing the true error $\mathbf{e}(t)$ in the controller (11) by the estimated one $\hat{\mathbf{e}}(t)$ leads to the following new controller:

$$\begin{pmatrix}v(t)\\ \omega(t)\end{pmatrix} = \begin{pmatrix}v_d(t)\cos e_\theta(t) + k_x\hat{e}_x(t)\\ \omega_d(t) + k_\theta e_\theta(t) + v_d(t)\hat{e}_y(t)\left(\sin e_\theta(t)/e_\theta(t)\right)\end{pmatrix} \quad (15)$$

It is clear that the controller in (15) includes the unknown estimation of the robot's position. Our idea is to design an adaptive estimator to estimate the robot's position, and embed it into (15) for tracking control of the mobile robot. To design the adaptive estimator, we substitute the controller (15) into error dynamics (10) leading to following closed-loop system:

$$\dot{\mathbf{e}}(t) = \begin{pmatrix}-k_x\hat{e}_x(t) + \hat{e}_y(t)\omega(t)\\ v_d(t)\sin e_\theta(t) - \hat{e}_x(t)\omega(t)\\ -k_\theta e_\theta(t) - v_d(t)\hat{e}_y(t)\left(\sin e_\theta(t)/e_\theta(t)\right)\end{pmatrix} + \begin{pmatrix}\tilde{e}_y(t)\\ -\tilde{e}_x(t)\\ 0\end{pmatrix}\omega(t) \quad (16)$$

where

$$\tilde{e}_x(t) = e_x(t) - \hat{e}_x(t), \quad \tilde{e}_y(t) = e_y(t) - \hat{e}_y(t) \quad (17)$$

From (9), (13) and (17), we can obtain that:

$$\begin{pmatrix}\tilde{e}_x(t)\\ \tilde{e}_y(t)\end{pmatrix} = \underbrace{\begin{pmatrix}\cos\theta(t) & \sin\theta(t)\\ -\sin\theta(t) & \cos\theta(t)\end{pmatrix}}_{\mathbf{Y}_1(\theta(t))}\tilde{\mathbf{x}}(t) \quad (18)$$

where $\mathbf{Y}_1(\theta(t)) \in \mathbb{R}^{2\times 2}$ is a coefficient matrix. From (18), we have:

$$\begin{pmatrix} \tilde{e}_y(t) \\ -\tilde{e}_x(t) \end{pmatrix} \omega(t) = \underbrace{\begin{pmatrix} -\sin\theta(t) & \cos\theta(t) \\ -\cos\theta(t) & -\sin\theta(t) \end{pmatrix}}_{\mathbf{Y}_2(\theta(t))} \omega(t)\, \tilde{\mathbf{x}}(t) \tag{19}$$

where $\mathbf{Y}_2(\theta(t)) = -\mathbf{Y}_1(\theta(t) - \frac{\pi}{2}) \in \mathbb{R}^{2\times 2}$ is not depending on the estimation error of the robot's position. Note that the right hand side of (19) is a linear form of the estimation error. From (18), (19), and the closed-loop equation (16), we have:

$$\begin{pmatrix} \tilde{e}_x(t) \\ \tilde{e}_y(t) \\ 0 \end{pmatrix}^T \dot{\mathbf{e}}(t) = \begin{pmatrix} \tilde{e}_x(t) \\ \tilde{e}_y(t) \end{pmatrix}^T \begin{pmatrix} -k_x\hat{e}_x(t) + \hat{e}_y(t)\omega(t) \\ v_d(t)\sin e_\theta(t) - \hat{e}_x(t)\omega(t) \end{pmatrix}$$

$$= \underbrace{\begin{pmatrix} -k_x\hat{e}_x(t) + \hat{e}_y(t)\omega(t) \\ v_d(t)\sin e_\theta(t) - \hat{e}_x(t)\omega(t) \end{pmatrix}^T}_{\mathbf{h}(t)} \mathbf{Y}_1(\theta(t))\, \tilde{\mathbf{x}}(t) \tag{20}$$

Moreover, from (17), it is important to note that:

$$\mathbf{e}^T(t)\, \dot{\mathbf{e}}(t) = \begin{pmatrix} \tilde{e}_x(t) & \tilde{e}_y(t) & 0 \end{pmatrix} \dot{\mathbf{e}}(t) + \hat{\mathbf{e}}^T(t)\, \dot{\mathbf{e}}(t) \tag{21}$$

By substituting (20) into (21), we have:

$$\mathbf{e}^T(t)\, \dot{\mathbf{e}}(t) = \hat{\mathbf{e}}^T(t)\, \dot{\mathbf{e}}(t) + \mathbf{h}^T(t)\, \mathbf{Y}_1(\theta(t))\, \tilde{\mathbf{x}}(t) \tag{22}$$

The equation (22) plays an important role in design of the adaptive estimator of the mobile robot.

4. ADAPTIVE ALGORITHM FOR ESTIMATING POSITION AND LINEAR VELOCITY OF THE ROBOT

This section presents an adaptive algorithm for estimating the position as well as the linear velocity of the robot in real time, and embeds it into the tracking controller (15) for controlling a mobile robot to track a desired trajectory without directly using its position and linear velocity feedbacks. The stability of the tracking controller embedded with the adaptive estimator is proved by Lyapunov theory.

4.1 Nominal Estimation Errors

As shown in subsection 3.2, $\hat{\mathbf{x}}(t)$ is the estimation of the robot's position and $\tilde{\mathbf{x}}(t)$ is that of estimation error. Besides, we denote $\hat{\mathbf{v}}(t) \in \mathbb{R}^2$ and $\hat{\mathbf{p}}_i \in \mathbb{R}^3$ as the estimations of $\mathbf{v}(t)$ and $\mathbf{p}_i$, respectively, whose estimation errors are given by:

$$\tilde{\mathbf{v}}(t) = \mathbf{v}(t) - \hat{\mathbf{v}}(t) \in \mathbb{R}^2 \tag{23}$$

$$\tilde{\mathbf{p}}_i(t) = \mathbf{p}_i - \hat{\mathbf{p}}_i(t) \in \mathbb{R}^3 \tag{24}$$

However, the estimation errors $\tilde{\mathbf{x}}(t)$, $\tilde{\mathbf{v}}(t)$, $\tilde{\mathbf{p}}_i(t)$ cannot be obtained without knowing the true values of $\mathbf{x}(t)$, $\mathbf{v}(t)$ and $\mathbf{p}_i$. To solve this problem, we design new nominal estimation errors $\mathbf{n}_i(t) = \begin{pmatrix} \mathbf{n}_{1i}(t) & \mathbf{n}_{2i}(t) \end{pmatrix}^T \in \mathbb{R}^4$ for each feature point, which satisfies the following three conditions:

(1) $\mathbf{n}_i(t) = \mathbf{0}$ for the true values of $\mathbf{x}(t)$, $\mathbf{v}(t)$ and $\mathbf{p}_i$.

(2) $\mathbf{n}_i(t)$ can be represented as a linear function of $\tilde{\mathbf{x}}(t)$, $\tilde{\mathbf{v}}(t)$ and $\tilde{\mathbf{p}}_i(t)$.

(3) $\mathbf{n}_i(t)$ can be calculated without knowing the true values of $\mathbf{x}(t)$, $\mathbf{v}(t)$ and $\mathbf{p}_i$.

Proposition 1. Based on (6) and (7), under the following definition of two components $\mathbf{n}_{1i}(t), \mathbf{n}_{2i}(t)$, nominal estimation errors $\mathbf{n}_i(t)$ satisfy the three conditions mentioned above.

$$\mathbf{n}_{1i}(t) = \hat{z}_i(t)\mathbf{y}_i(t) - \mathbf{B}(\theta(t), \mathbf{y}_i(t)) \left(\hat{\mathbf{p}}_i(t) - \begin{pmatrix} \hat{\mathbf{x}}(t) \\ 0 \end{pmatrix} \right) \tag{25}$$

$$\mathbf{n}_{2i}(t) = \hat{z}_i(t)\dot{\mathbf{y}}_i(t) - \mathbf{C}(\theta(t), \dot{\theta}(t), \dot{\mathbf{y}}_i(t), \mathbf{y}_i(t)) \left(\hat{\mathbf{p}}_i(t) - \begin{pmatrix} \hat{\mathbf{x}}(t) \\ 0 \end{pmatrix} \right)$$

$$-\mathbf{D}(\theta(t), \mathbf{y}_i(t)) \begin{pmatrix} \hat{\mathbf{v}}(t) \\ 0 \end{pmatrix} \tag{26}$$

Proof. Based on (6) and (7), if we replace the estimations $\hat{\mathbf{x}}(t)$, $\hat{\mathbf{v}}(t)$ and $\hat{\mathbf{p}}_i(t)$ in (25) and (26) by their true values $\mathbf{x}(t)$, $\mathbf{v}(t)$ and $\mathbf{p}_i$, $\mathbf{n}_{1i}(t), \mathbf{n}_{2i}(t)$ are equal to zero. The first condition can be satisfied. Moreover, from (5) - (7), we can transform (25) and (26) as follows:

$$\mathbf{n}_{1i}(t) = \left(\hat{z}_i(t) - z_i(t)\right)\mathbf{y}_i(t) + z_i(t)\mathbf{y}_i(t) - \mathbf{B}(\theta(t), \mathbf{y}_i(t)) \left(\hat{\mathbf{p}}_i(t) - \begin{pmatrix} \hat{\mathbf{x}}(t) \\ 0 \end{pmatrix} \right)$$

$$= (\hat{z}_i(t) - z_i(t))\mathbf{y}_i(t) + \mathbf{B}(\theta(t), \mathbf{y}_i(t)) \left(\tilde{\mathbf{p}}_i(t) - \begin{pmatrix} \tilde{\mathbf{x}}(t) \\ 0 \end{pmatrix} \right)$$

$$= \underbrace{\left\{ \mathbf{B}(\theta(t), \mathbf{y}_i(t)) - \mathbf{y}_i(t)\, \mathbf{A}(\theta(t), \mathbf{y}_i(t)) \right\}}_{\mathbf{E}(\theta(t), \mathbf{y}_i(t))} \left(\tilde{\mathbf{p}}_i(t) - \begin{pmatrix} \tilde{\mathbf{x}}(t) \\ 0 \end{pmatrix} \right) \tag{27}$$

$$\mathbf{n}_{2i}(t) = \left(\hat{z}_i(t) - z_i(t)\right)\dot{\mathbf{y}}_i(t) + z_i(t)\dot{\mathbf{y}}_i(t) - \mathbf{D}(\theta(t), \mathbf{y}_i(t)) \begin{pmatrix} \hat{\mathbf{v}}(t) \\ 0 \end{pmatrix}$$

$$-\mathbf{C}(\theta(t), \dot{\theta}(t), \dot{\mathbf{y}}_i(t), \mathbf{y}_i(t)) \left(\hat{\mathbf{p}}_i(t) - \begin{pmatrix} \hat{\mathbf{x}}(t) \\ 0 \end{pmatrix} \right)$$

$$= \underbrace{\left\{ \mathbf{C}(\theta(t), \dot{\theta}(t), \dot{\mathbf{y}}_i(t), \mathbf{y}_i(t)) - \dot{\mathbf{y}}_i(t)\, \mathbf{A}(\theta(t), \mathbf{y}_i(t)) \right\}}_{\mathbf{F}(\theta(t), \dot{\theta}(t), \dot{\mathbf{y}}_i(t), \mathbf{y}_i(t))} \left(\tilde{\mathbf{p}}_i(t) - \begin{pmatrix} \tilde{\mathbf{x}}(t) \\ 0 \end{pmatrix} \right)$$

$$+\mathbf{D}(\theta(t), \mathbf{y}_i(t)) \begin{pmatrix} \tilde{\mathbf{v}}(t) \\ 0 \end{pmatrix} \tag{28}$$

where $\hat{z}_i(t) \in \mathbb{R}$ is the estimation of $z_i(t)$.

Combining (27) with (28), we can easily obtain that:

$$\mathbf{n}_i(t) = \begin{pmatrix} \mathbf{n}_{1i}(t) \\ \mathbf{n}_{2i}(t) \end{pmatrix} = \underbrace{\begin{pmatrix} -\mathbf{E}(t) & \mathbf{0}_{2\times 3} & \mathbf{E}(t) \\ -\mathbf{F}(t) & -\mathbf{D}(t) & \mathbf{F}(t) \end{pmatrix}}_{\mathbf{W}_i(\theta(t), \dot{\theta}(t), \mathbf{y}_i(t), \dot{\mathbf{y}}_i(t))} \begin{pmatrix} \tilde{\mathbf{x}}(t) \\ \tilde{\mathbf{v}}(t) \\ \tilde{\mathbf{p}}_i(t) \end{pmatrix} \tag{29}$$

where the error transformation matrix $\mathbf{W}_i(t) = \mathbf{W}_i(\theta(t), \dot{\theta}(t), \mathbf{y}_i(t), \dot{\mathbf{y}}_i(t)) \in \mathbb{R}^{4\times 7}$ is independent of the estimation errors.

From (29), the second condition can be fulfilled. Based on the first two conditions, it is clear that:

$$\mathbf{n}_i(t) = -\mathbf{W}_i(t) \begin{pmatrix} \hat{\mathbf{x}}(t) \\ \hat{\mathbf{v}}(t) \\ \hat{\mathbf{p}}_i(t) \end{pmatrix} \tag{30}$$

Therefore, the nominal estimation errors can be calculated without using the true values of $\mathbf{x}(t)$, $\mathbf{v}(t)$ and $\mathbf{p}_i(t)$. The third condition can be satisfied. ■

In order to improve robustness of the algorithm, we employ a set of feature points $\mathbf{p} = \left(\mathbf{p}_1^T \cdots \mathbf{p}_N^T\right)^T$ to estimate the position and linear velocity of the robot. Nominal estimation errors $\mathbf{n}(t) \in \mathbb{R}^{4N}$ with N feature points can be represented to:

$$\mathbf{n}(t) = \begin{pmatrix} \mathbf{n}_1(t) \\ \vdots \\ \mathbf{n}_N(t) \end{pmatrix} = \underbrace{\left\{ \begin{pmatrix} \mathbf{W}_1(t) & & & \\ & \mathbf{W}_2(t) & & \\ & & \ddots & \\ & & & \mathbf{W}_N(t) \end{pmatrix} \mathbf{G} \right\}}_{\mathbf{W}(\theta(t),\dot{\theta}(t),\mathbf{y}(t),\dot{\mathbf{y}}(t))} \begin{pmatrix} \tilde{\mathbf{x}}_{RP}(t) \\ \tilde{\mathbf{v}}_{RP}(t) \\ \tilde{\mathbf{p}}(t) \end{pmatrix} \quad (31)$$

where $\mathbf{W}(t) = \mathbf{W}(\theta(t),\dot{\theta}(t),\mathbf{y}(t),\dot{\mathbf{y}}(t)) \in \mathbb{R}^{4N\times(3N+4)}$ is the transformation matrix with N feature points. $\mathbf{G} \in \mathbb{R}^{7N\times(3N+4)}$ is a constant coefficient matrix.

4.2 Adaptive Estimator

Based on the equation (22) and the nominal estimation error (31), we introduce the following adaptive estimator to estimate the unknown $\hat{\mathbf{x}}(t)$, $\hat{\mathbf{v}}(t)$ as well as $\hat{\mathbf{p}}(t)$:

$$\begin{pmatrix} \dot{\hat{\mathbf{x}}}(t) \\ \dot{\hat{\mathbf{v}}}(t) \\ \dot{\hat{\mathbf{p}}}(t) \end{pmatrix} = \begin{pmatrix} \hat{\mathbf{v}}(t) \\ \dot{\mathbf{v}}(t) \\ \mathbf{0}_{3N\times1} \end{pmatrix} - \mathbf{J}\left\{\mathbf{q}(t) - \mathbf{W}^T(t)\,\mathbf{K}\,\mathbf{n}(t)\right\} \quad (32)$$

where

$$\mathbf{q}(t) = \begin{pmatrix} -\mathbf{Y}_1^T(\theta(t))\mathbf{h}(t) - \mathbf{Y}_2^T(\theta(t))\,\omega(t)\left(\hat{e}_x(t)\ \hat{e}_y(t)\right)^T \\ \mathbf{0}_{(3N+2)\times1} \end{pmatrix} \in \mathbb{R}^{3N+4} \quad (33)$$

and $\dot{\mathbf{v}}(t) \in \mathbb{R}^2$ is acceleration of the robot measured by accelerometers in AHRS. $\mathbf{J} \in \mathbb{R}^{(3N+4)\times(3N+4)}$ and $\mathbf{K} \in \mathbb{R}^{4N\times4N}$ are positive definite and diagonal gain matrices.

4.3 Stability Analysis

Theorem 1. The proposed controller (15) with the adaptive estimator (32) results in:

$$\lim_{t\to\infty} \hat{e}_x(t) = 0,\ \lim_{t\to\infty} e_\theta(t) = 0,\ \lim_{t\to\infty} \mathbf{n}(t) = \mathbf{0}_{4N\times1} \quad (34)$$

Moreover, if the designed speed of the robot $v_d(t) \neq 0$,

$$\lim_{t\to\infty} \hat{e}_y(t) = 0 \quad (35)$$

$$\lim_{t\to\infty} \Delta\hat{\mathbf{x}}(t) = \mathbf{0}_{2\times1},\ \lim_{t\to\infty} \Delta\hat{\theta}(t) = 0 \quad (36)$$

$$\lim_{t\to\infty} \hat{\mathbf{x}}(t) = \mathbf{x}(t),\ \lim_{t\to\infty} \hat{\mathbf{v}}(t) = \mathbf{v}(t),\ \lim_{t\to\infty} \hat{\mathbf{p}}(t) = \mathbf{p} \quad (37)$$

Proof. Consider the following positive-definite quadratic function $V(t) \in \mathbb{R}$:

$$V(t) = \frac{1}{2}\left(\tilde{\mathbf{x}}^T(t)\ \tilde{\mathbf{v}}^T(t)\ \tilde{\mathbf{p}}^T(t)\right)\mathbf{J}^{-1} \begin{pmatrix} \tilde{\mathbf{x}}(t) \\ \tilde{\mathbf{v}}(t) \\ \tilde{\mathbf{p}}(t) \end{pmatrix} + \frac{1}{2}\mathbf{e}^T(t)\,\mathbf{e}(t) \quad (38)$$

By differentiating (38) , we have:

$$\dot{V}(t) = \begin{pmatrix} \tilde{\mathbf{x}}(t) \\ \tilde{\mathbf{v}}(t) \\ \tilde{\mathbf{p}}(t) \end{pmatrix}^T \mathbf{J}^{-1} \left\{ \begin{pmatrix} \dot{\mathbf{x}}(t) \\ \dot{\mathbf{v}}(t) \\ \dot{\mathbf{p}} \end{pmatrix} - \begin{pmatrix} \dot{\hat{\mathbf{x}}}(t) \\ \dot{\hat{\mathbf{v}}}(t) \\ \dot{\hat{\mathbf{p}}}(t) \end{pmatrix} \right\} + \mathbf{e}^T(t)\,\dot{\mathbf{e}}(t) \quad (39)$$

Since the feature points are fixed, $\dot{\mathbf{p}}$ is equal to zero. By substituting (32) into (39) and noting (31), we have:

$$\begin{aligned} \dot{V}(t) &= \left(\tilde{\mathbf{x}}^T(t)\ \tilde{\mathbf{v}}^T(t)\ \tilde{\mathbf{p}}^T(t)\right)\mathbf{J}^{-1}\begin{pmatrix} \tilde{\mathbf{v}}(t) \\ \mathbf{0} \end{pmatrix} - \mathbf{n}^T(t)\,\mathbf{K}\,\mathbf{n}(t) \\ &\quad + \left(\tilde{\mathbf{x}}^T(t)\ \tilde{\mathbf{v}}^T(t)\ \tilde{\mathbf{p}}^T(t)\right)\mathbf{q}(t) + \mathbf{e}^T(t)\,\dot{\mathbf{e}}(t) \\ &= \tilde{\mathbf{x}}^T(t)\,\mathbf{J}_1\,\tilde{\mathbf{v}}(t) - \mathbf{n}^T(t)\,\mathbf{K}\,\mathbf{n}(t) + \left(\tilde{\mathbf{x}}^T(t)\ \tilde{\mathbf{v}}^T(t)\ \tilde{\mathbf{p}}^T(t)\right)\mathbf{q}(t) \\ &\quad + \mathbf{e}^T(t)\,\dot{\mathbf{e}}(t) \end{aligned} \quad (40)$$

where $\mathbf{J}_1 \in \mathbb{R}^{2\times2}$ is the upper left submatrix of $\mathbf{J}^{-1}$.
Based on (31), it is clear that if:

$$4N \geq 3N + 4 \Rightarrow N \geq 4 \quad (41)$$

$\mathbf{W}(t)$ has full column rank except the special cases discussed later, and $\left(\tilde{\mathbf{x}}^T(t)\ \tilde{\mathbf{v}}^T(t)\ \tilde{\mathbf{p}}^T(t)\right)^T$ can be uniquely obtained by:

$$\begin{pmatrix} \tilde{\mathbf{x}}(t) \\ \tilde{\mathbf{v}}(t) \\ \tilde{\mathbf{p}}(t) \end{pmatrix} = \underbrace{\begin{pmatrix} \mathbf{W}_{inv\_\tilde{x}}(t) \\ \mathbf{W}_{inv\_\tilde{v}}(t) \\ \mathbf{W}_{inv\_\tilde{p}}(t) \end{pmatrix}}_{\mathbf{W}^+(t)} \mathbf{n}(t) \quad (42)$$

where $\mathbf{W}^+(t) \in \mathbb{R}^{(3N+4)\times4N}$ is pseudo-inverse matrix of $\mathbf{W}(t)$. $\mathbf{W}_{inv\_\tilde{x}}(t) \in \mathbb{R}^{2\times4N}, \mathbf{W}_{inv\_\tilde{v}}(t) \in \mathbb{R}^{2\times4N}$ and $\mathbf{W}_{inv\_\tilde{p}}(t) \in \mathbb{R}^{3N\times4N}$ are the submatrices of $\mathbf{W}^+(t)$. From (46), the following equation can be satisfied:

$$\tilde{\mathbf{x}}^T(t)\,\mathbf{J}_1\,\tilde{\mathbf{v}}(t) = \left(\mathbf{W}_{inv\_\tilde{x}}(t)\,\mathbf{n}(t)\right)^T \mathbf{J}_1 \left(\mathbf{W}_{inv\_\tilde{v}}(t)\,\mathbf{n}(t)\right) \quad (43)$$

Moreover, by multiplying $\hat{\mathbf{e}}^T(t)$ to both sides of the closed-loop equation (16) , we have:

$$\hat{\mathbf{e}}^T(t)\,\dot{\mathbf{e}}(t) = -k_x\hat{e}_x^2(t) - k_\theta e_\theta^2(t) + \hat{\mathbf{e}}^T(t)\begin{pmatrix} \tilde{e}_y(t) \\ -\tilde{e}_x(t) \\ 0 \end{pmatrix}\omega(t) \quad (44)$$

By substituting (44) into (22) and noting (19), we can obtain:

$$\begin{aligned} \mathbf{e}^T(t)\,\dot{\mathbf{e}}(t) &= -k_x\hat{e}_x^2(t) - k_\theta e_\theta^2(t) + \begin{pmatrix} \hat{e}_x(t) \\ \hat{e}_y(t) \end{pmatrix}^T \mathbf{Y}_2(\theta(t))\,\omega(t)\,\tilde{\mathbf{x}}(t) \\ &\quad + \mathbf{h}^T(t)\,\mathbf{Y}_1(\theta(t))\,\tilde{\mathbf{x}}(t) \end{aligned} \quad (45)$$

By substituting (43), (45) into (40) and noting (31), we have:

$$\begin{aligned} \dot{V}(t) &= \left(\mathbf{W}_{inv\_\tilde{x}}(t)\,\mathbf{n}(t)\right)^T \mathbf{J}_1 \left(\mathbf{W}_{inv\_\tilde{v}}(t)\,\mathbf{n}(t)\right) - \mathbf{n}^T(t)\,\mathbf{K}\,\mathbf{n}(t) \\ &\quad - k_x\hat{e}_x^2(t) - k_\theta e_\theta^2(t) \\ &= -\mathbf{n}^T(t)\underbrace{\left\{\mathbf{K} - \mathbf{W}_{inv\_\tilde{x}}{}^T(t)\,\mathbf{J}_1\,\mathbf{W}_{inv\_\tilde{v}}(t)\right\}}_{\mathbf{L}(t)}\mathbf{n}(t) \\ &\quad - k_x\hat{e}_x^2(t) - k_\theta e_\theta^2(t) \end{aligned} \quad (46)$$

From (46), with the condition in (41), it is clear that $\mathbf{L}(t) \in \mathbb{R}^{4N\times 4N}$ is a full rank matrix. Hence, by proper selecting $\mathbf{K}$, $\mathbf{L}(t)$ should be positive definite and we have:

$$\dot{V}(t) \le 0 \qquad (47)$$

Therefore, $V(t)$ is upper bounded, which directly represents boundedness of $\tilde{\mathbf{x}}(t)$, $\tilde{\mathbf{v}}(t)$, $\tilde{\mathbf{p}}(t)$, and $\mathbf{e}(t)$. From the error transformation (9) and the definition of the nominal estimation errors (31), $\Delta\mathbf{x}(t), \Delta\theta(t)$ and $\mathbf{n}(t)$ are bounded. Boundedness of $\tilde{\mathbf{x}}(t)$, $\Delta\mathbf{x}(t)$ and $\Delta\theta(t)$ imply boundedness of $\Delta\hat{\mathbf{x}}(t)$, $\Delta\hat{\theta}(t)$ and $\hat{\mathbf{e}}(t)$ according to (13) and (14). Based on this and considering the controller (15), the linear speed $v(t)$ and angular velocity $\omega(t)$ of the robot are bounded due to the bounded k_x, k_θ and $v_d(t)$. From the closed-loop equation (16), boundedness of $\dot{\mathbf{e}}(t)$ can be satisfied. Moreover, based on the adaptive estimator (32), $\dot{\hat{\mathbf{x}}}(t), \dot{\hat{\mathbf{v}}}(t)$ and $\dot{\hat{\mathbf{p}}}(t)$ are bounded, and the boundedness of $\dot{\tilde{\mathbf{x}}}(t), \dot{\tilde{\mathbf{v}}}(t)$ and $\dot{\tilde{\mathbf{p}}}(t)$ can be guaranteed because of boundedness of the true values of $\dot{\mathbf{v}}(t)$, $\theta(t)$ and $\mathbf{v}(t)$. From (14) and (31), $\dot{\mathbf{n}}(t)$ and $\dot{\hat{\mathbf{e}}}(t) = \begin{pmatrix} \dot{\hat{e}}_x(t) & \dot{\hat{e}}_y(t) & \dot{e}_\theta(t) \end{pmatrix}^T$ are bounded as well. Furthermore, grounded on (42) and (46), $\mathbf{L}(t)$ and $\dot{\mathbf{L}}(t)$ are also bounded.

By differentiating (46), we have:

$$\ddot{V}(t) = -2\,\mathbf{n}^T(t)\,\mathbf{L}(t)\,\dot{\mathbf{n}}(t) - \mathbf{n}^T(t)\,\dot{\mathbf{L}}(t)\,\mathbf{n}(t) - 2k_x\hat{e}_x(t)\dot{\hat{e}}_x(t) - 2k_\theta e_\theta(t)\dot{e}_\theta(t) \qquad (48)$$

As we have boundedness of k_x, k_θ, $\hat{\mathbf{e}}(t)$, $\dot{\hat{\mathbf{e}}}(t)$, $\mathbf{n}(t)$, $\dot{\mathbf{n}}(t)$, $\mathbf{L}(t)$, and $\dot{\mathbf{L}}(t)$, $\ddot{V}(t)$ is also bounded. Therefore, from Barbalat Lemma (Slotine and Li, 1987), we can obtain that:

$$\lim_{t\to\infty} \dot{V}(t) = 0 \qquad (49)$$

From (46) and (49), the convergence results in (34) can be proved. Moreover, by differentiating (16), $\ddot{e}_\theta(t)$ is bounded due to boundedness of $k_x, k_\theta, \hat{\mathbf{e}}(t)$, $\dot{\hat{\mathbf{e}}}(t)$. Hence, $\dot{e}_\theta(t)$ is uniformly continuous. From (49), $e_\theta(t)$ has a finite limit as t tends to infinite. From Barbalat Lemma, the following equation can be satisfied:

$$\lim_{t\to\infty} \dot{e}_\theta(t) = 0 \qquad (50)$$

Combining (49) and (50) with (16), we have:

$$\lim_{t\to\infty} \dot{e}_\theta(t) = 0 \Rightarrow \lim_{t\to\infty}\left(-k_\theta e_\theta(t) - v_d(t)\hat{e}_y(t)\frac{\sin e_\theta(t)}{e_\theta(t)}\right) = 0$$
$$\Rightarrow \lim_{t\to\infty} v_d(t)\hat{e}_y(t) = 0 \qquad (51)$$

as long as $v_d(t) \ne 0$,

$$\lim_{t\to\infty} \hat{e}_y(t) = 0 \qquad (52)$$

From (49), (51) and definition of $\hat{\mathbf{e}}(t)$ in (14), we can obtain:

$$\lim_{t\to\infty} \hat{\mathbf{e}}(t) = \mathbf{0}_{3\times 1} \Rightarrow \lim_{t\to\infty} \Delta\hat{\mathbf{x}}(t) = \mathbf{0}_{2\times 1} \text{ and } \lim_{t\to\infty} \Delta\hat{\theta}(t) = 0 \qquad (53)$$

Based on the definition of $\mathbf{n}(t)$ in (31) and the condition (41), $\mathbf{W}(t)$ is a full-rank matrix. It is clear that:

$$\lim_{t\to\infty} \mathbf{n}(t) = \mathbf{0}_{4N\times 1} \Rightarrow \lim_{t\to\infty} \begin{pmatrix} \tilde{\mathbf{x}}(t) \\ \tilde{\mathbf{v}}(t) \\ \tilde{\mathbf{p}}(t) \end{pmatrix} = \mathbf{0}_{(3N+4)\times 1} \qquad (54)$$

Hence, the equations (34) - (37) have been proven. ■

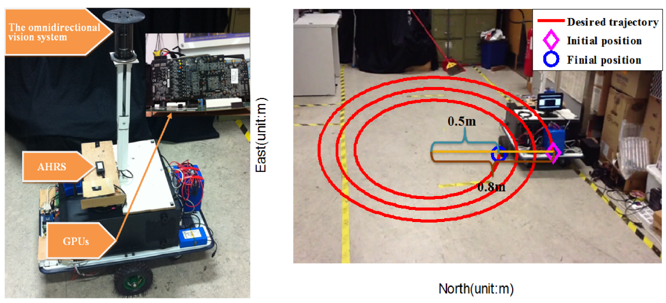
Remark 1. According to Vidal *et al.* (2008), under the condition (41), $\mathbf{W}(t)$ has full column rank except the following special cases:

- Some of tracked 3-D feature points lie in a line or a plane in space. In these cases, 3-D coordinate of one feature point can be linearly represented by that of other feature points.
- The robot performs pure rotation or pure translation with respect to some tracked 3-D feature points.

Based on the novel controller embedded with the adaptive estimator, the nonholonomic mobile robot can asymptotically track a desired trajectory without knowing its position and linear velocity. The performance of the proposed controller will be further validated by experiment.

5. EXPERIMENT

We had implemented the proposed controller on a differentially driven wheeled robot (see Fig. 2(a)) in an indoor environment. The robot was equipped with an Innalabs AHRS to measure its acceleration and orientation at 120Hz. An omnidirectional vision system was mounted on the robot to capture images with a resolution of 640×480 at a speed of 30fps. The processors included Intel Core i7-2620 CPU and NVIDIA GTX580 GPUs for extracting and tracking SURF points (Bay *et al.*, 2008) and calculating optical flow from the image sequence in real time. The frequency of the control system is 30Hz.



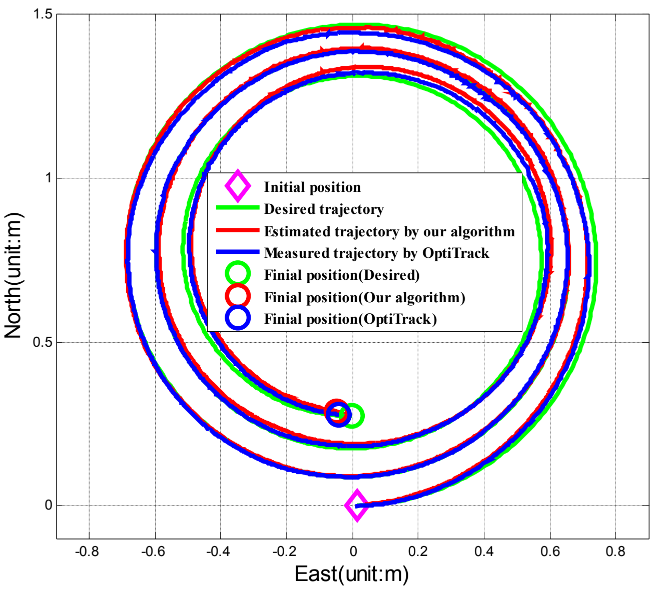
(a) (b)

Fig. 2. The experiment setup: (a) The mobile robot. (b) The desired trajectory in an indoor environment.

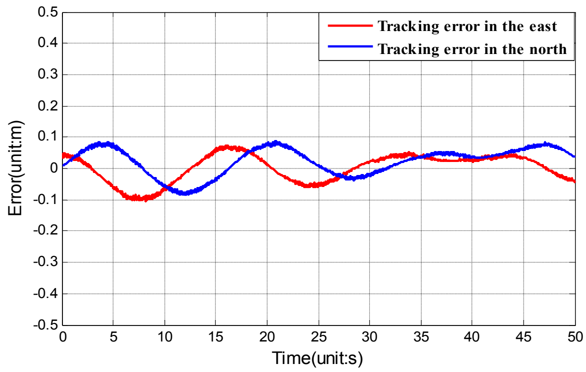
In this experiment, the robot was controlled to track a desired planar helix (see Fig. 2(b)) shown in (55). The total running time was 50s. The radius of the helix uniformly increased from 0.5m to 0.8m with time. To investigate the accuracy of the proposed algorithm, an OptiTrack vision system was installed on the ceiling to trace the motion of the robot.

$$
\begin{aligned}
&T_{total} = 50s, 0s \le t \le T_{total}, \Delta T = \frac{1}{30}s \\
&R_{\max} = 0.8m, R_{\min} = 0.5m, R = R_{\min} + \frac{(R_{\max} - R_{\min})}{T_{total}} \cdot t \\
&\omega_d(t) = \frac{6\pi}{T_{total}}, v_d(t) = \omega_d(t) \cdot R \\
&\theta_d(t) = \theta_d(t-1) + \omega_d(t) \cdot \Delta T \\
&x_d(t) = R \cdot \sin(\theta_d(t)), y_d(t) = R \cdot \cos(\theta_d(t))
\end{aligned} \tag{55}
$$

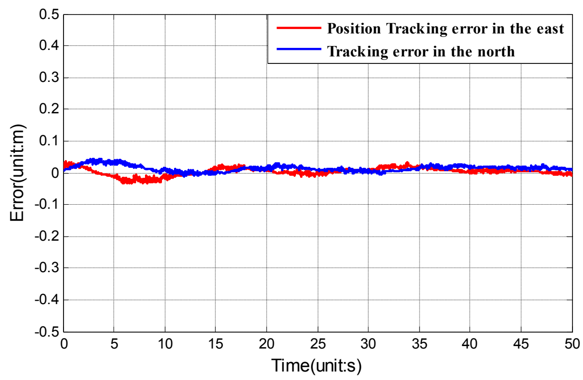
The desired trajectory, estimated trajectory by the proposed controller, and the measured trajectory by OptiTrack are shown in Fig. 3(a). Moreover, the corresponding position error, orientation error, linear velocity error, and angular velocity error of the estimated trajectory by our controller with respect to the desired trajectory and the measured trajectory by OptiTrack are shown in Fig. 3(b-f), respectively. From the experimental results of the continuous trajectory tracking by the proposed controller, the desired planar helix can be asymptotically tracked and the estimations of the robot pose, linear and angular velocities are rapidly converge to their real values measured by OptiTrack system. Moreover, the estimation errors of robot's position and orientation are not severely accumulated. The convergence and robustness of the proposed controller can be guaranteed.



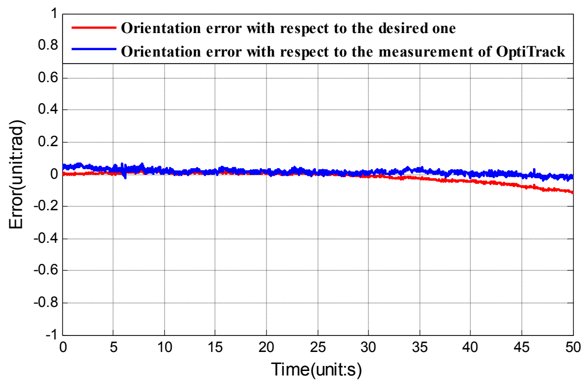
(a)



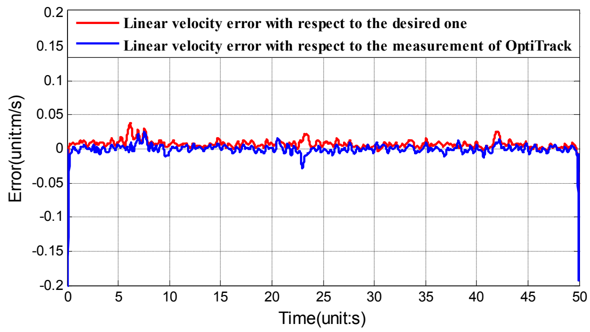
(b)



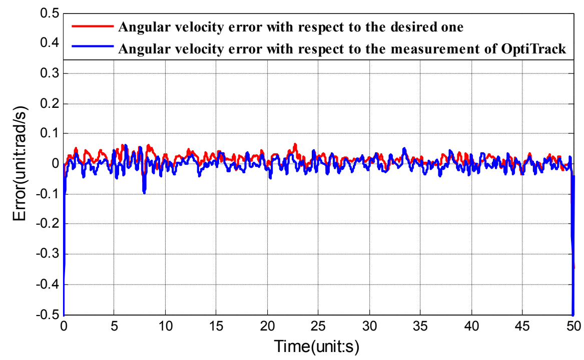
(c)



(d)



(e)



(f)

Fig. 3. Experimental results: (a) Trajectories. (b) Position error between the estimated trajectory and the desired trajectory. (c) Position error between the estimated trajectory and the measured trajectory. (d) Orientation errors of the estimated trajectory w.r.t. the measured and the desired trajectories. (e) Linear velocity errors of the estimated trajectory w.r.t. the measured and the desired trajectories. (f)

Angular velocity errors of the estimated trajectory w.r.t. the measured and the desired trajectories.

6. CONCLUSIONS

In this paper, based on a vision-based position and linear velocity estimator, we proposed a novel position-based visual servo controller for trajectory tracking of nonholonomic mobile robots by fusing the measurements of inertial sensor (AHRS) and visual feedback of an omnidirectional vision system without using directly measurements of the robot's position and linear velocity. It is proven by Lyapunov theory that the proposed controller, with the embedded position and linear velocity estimator guarantees asymptotically tracking the desired trajectory, and the convergence of the position and linear velocity of the mobile robot to their true values, simultaneously. The experimental results validate the superior performance of the proposed controller.

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