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Weighting Logic Design of Hybrid Database Referenced Navigation Algorithm Using Multiple Geophysical Information

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Abstract: This paper presents the hybrid database referenced navigation algorithm using terrain height and gravity anomaly information together. Geophysical information database can be used for navigation with inertial sensors on the GPS denied environment. Terrain height is the most commonly used as secondary navigation information with INS. However, the terrain referenced navigation is sometimes impossible to estimate the position because of the lack of uniqueness on the flat terrain. Therefore, we propose the hybrid database referenced navigation technique which utilizes a gravity anomaly information along with terrain height. In this paper, the point mass filter is used for estimation of the vehicle's position. And weighting logic for hybrid database referenced navigation is designed based on the kurtosis of posteriori pdf (probability density function) because that of posteriori pdf has a proportional relationship to the estimation accuracy. Each estimation result is mixed together according to the kurtosis. As a result, it is shown by the numerical simulation that the proposed method has a better performance than the navigation result obtained from individual database.

1. INTRODUCTION

Inertial navigation has a number of advantages. Such as high bandwidth output can autonomously operate, it provides high resolution attitude, angular rate, and acceleration, as well as position and velocity. However, the accuracy of an inertial navigation degrades with time. Consequently, additional navigation is applied with inertial navigation (Grove, 2008). Inertial and satellite navigation systems are the most common choice as an integrated navigation systems. On the other hands, it also has drawbacks. In satellite navigation challenging environment, different methods for navigation are essential parts of integrated navigation systems. Database referenced navigation algorithm is self-sufficient and independent of external situations. Jamming is almost impossible unlike with satellite navigation systems. Hence these characteristics of DBRN help to consist more reliable navigation system. DBRN(Data-Based Referenced Navigation) can be utilized an integration algorithm which provides position information of the vehicle as correct the INS (Inertial Navigation System) position using the comparison of geophysical measurements with database. Terrain height, gravity, or geomagnetic field are the geophysical information which is used to integrated navigation. Among these geophysical information, the terrain height information is generally applied with INS correction.

TRN (Terrain Referenced Navigation) is the conventional DBRN algorithm which takes advantage of the terrain height information. Nowadays, cruise missile, fighter, transport, and helicopter are using TRN system to take the more robust navigation information (Cowie, 2008). It is divided into batch and sequential processing method depending on how it is implemented. Batch processing method uses terrain height correlation techniques directly between terrain database and terrain measurements (Metzger, 2005; Siouris, 2004; Yoo, 2013) while the sequential processing method uses the filter to integrate the INS information with terrain referenced navigation results. Extended Kalman filters, particle filters and grid-based filter are applied to the sequential processing method (Hostetler, 1977). In this paper, we use PMF (Point Mass Filter) which is one of the grid-based methods as an estimation technique. PMF is calculating the probability of a set of points on the grids and makes the position information using the probability (Anonsen, 2005). Grid update and propagation of PMF have been biased towards a grid solution well suited for navigation problem (Bergman, 1999).

TRN method is always not preferable to correct INS error. If the terrain height information does not have the uniqueness as enough as distinguish the true position from others, TRN is meaningless. In order to solve this drawback, we propose to use multiple databases together. Different type of geophysical database has a different characteristic. It means that hybrid database referenced navigation can provide a solution for the lack of uniqueness in the single database applied navigation. In this paper, the gravity anomaly is utilized along with terrain height information. Gravity anomaly is the difference between the calculated gravity from model and observed gravity. It has the different characteristics to terrain height. These characteristics help to solve the navigation on the flat terrain. For the combination of multiple databases, we propose weighting logic based on the kurtosis of posteriori pdf. Kurtosis means how the posteriori pdf of PMF is flat or rough. Degree of posteriori pdf convergence is proportional to estimated position accuracy that is acquired using PMF. And the kurtosis means the degree of posteriori pdf. Kurtosis of pdf is more useful to make a decision between the position information that come from the multiple databases. Thus, the kurtosis is proposed as a weighting logic in this paper. If the kurtosis of posteriori pdf is large, more accurate position information can be obtained.

More details on the geophysical database are given in section 2. PMF algorithm is introduced in section 3. In section 4, studies conducted to prevent false fixes when using single database are presented. Numerical simulations are performed to verify the performance of the proposed algorithm in Section 5. Finally, Section 6 gives conclusions.

2. GEOPHYSICAL INFORMATION REFERENCED NAVIGATION

2.1 Geophysical Information

Geophysical information for the navigation can be the terrain, gravity, and geomagnetic data. Terrain height information is generally used to navigation system. It is called TRN (Terrain Referenced Navigation). As TRN is used to navigation with INS, it is possible to obtain the high performance navigation result if we have the high resolution digital height elevation data. The terrain height databases which have applied in TRN system are represented by SRTM (Shuttle Radar Topography Mission), GTOPO30, GLOBE. SRTM is the digital terrain database which is acquired along 80% of global terrain using C-band, X-band radar of spacecraft in February 2000. SRTM is an international project spearheaded by the NGA (National Geospatial-Intelligence Agency) and the NASA (National Aeronautics and Space Administration). 3 arc seconds terrain database; DTED (Digital Terrain Elevation Data) level1 SRTM is provided for public use as a typical terrain database. It has more precise databases. However they have a limit purpose for military. GTOPO30 has 30 arc seconds resolution terrain database; DTED level 0 and is planned by USGS (U. S. Geological Survey) until 1996. GLOBE has a same resolution with GTOPO30. It is produced by NOAA (National Oceanic and Atmosphere Administration) and NGDC (National Geophysical Data Center) in NOAA. We use 3 arc seconds resolution DTED level 1 SRTM terrain database like Fig. 1.

In this paper, we also used gravity anomaly database with terrain. Gravity anomaly is the difference between the observed acceleration of earth's gravity and a calculated from a model. Gravity anomaly has a positive relation with terrain height information. If the accurate terrain height information

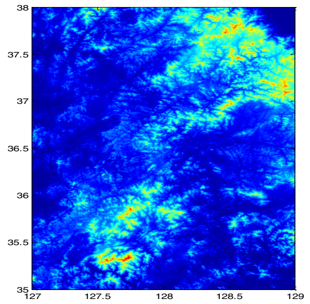

Fig. 1. Terrain Height Database

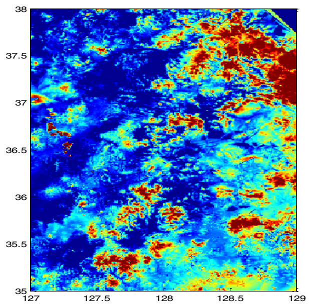

Fig. 2. Gravity anomaly Database

is available, the gravity anomaly can be calculated using it. Gravity anomaly based on terrain height information has a theoretical relationship. Airy's isostasy theory can explain about the relationship between the terrain height and gravity anomaly. Gravity anomaly calculation method which is based on ground, air, and satellite gravity measurements has been researched for a long time. In the several countries, gravity database is regularly updated and serviced on the web for a public usage. Fig. 2 is applied gravity anomaly database which has 30 arc seconds resolution.

2.2 Database Referenced Navigation Algorithm

Database referenced navigation algorithm is a method which provides position information of the vehicle using the comparison of geophysical measurement with a digital geophysical database. TRN is the conventional geophysical database referenced algorithm which takes advantage of the terrain height. Nowadays, cruise missile, fighter, transport, and helicopter are using terrain based database referenced navigation system with inertial system to take the more robust navigation information. And it is generally divided into sequential and batch processing method depending on how it is implemented. Sequential processing method uses the filter to integrate the inertial navigation system information and terrain referenced navigation results. Extended Kalman filters, particle filters and grid-based methods are applied in the sequential processing method. And PMF is also a sort of sequential processing algorithm. We will refer to the detail of PMF in the next section. SITAN (Sandia Inertial Terrain Aided Navigation) and Heli/SITAN are the representative sequential processing technique. These approaches need relatively less computational power and gives robust navigation information using the terrain height information without any infra network. On the other hand, it can be impossible to estimate the position because of terrain gradient calculation error and similar height false location. Batch processing method uses terrain height correlation techniques directly between terrain database and terrain measurements. By using a series of terrain data, it is possible to estimate more robust vehicle position than sequential processing method (Lee, 2012). However this method needs greater computational burden compared to sequential processing method. Database referenced navigation systems currently available on the market include TERCOM (TERrain COntour Matching). In this paper, we are focusing on the hybrid database referenced navigation algorithm based on PMF. And it is consisted using multi-geophysical information.

3. POINT MASS FILTER

This paper is focused on a combination of database referenced navigation. And it is based on PMF. PMF is the nonlinear estimation method which is based on Bayes' rule. General Bayes' rule to estimate the state variable is difficult to obtain the analytical solution and requires a lot of computational power. Therefore PMF uses grid approximation. Grid approximation for the numerical integration is utilized in the PMF instead of Bayes' rule integration. As it uses grid based numerical integration, the computation burden for pdf calculation is reduced. The number of grids and spacing are adjusted following the distribution of posteriori pdf. The less precise and detailed PMF means the generality and explicitness of estimation result. The reason for this trade-off between those values is that we should need the efficient algorithm to estimate every detail in the several situations.

3.1 Bayes's Rule in Database Referenced Navigation

We assume the system and measurement model to introduce general Bayes' rule which is used in database referenced navigation like below.

$$\begin{aligned} x_{t+1} &= x_t + u_t + w_t \\ y_t &= h(x_t) + v_t \end{aligned} \tag{1}$$

where, x_t is a state variable that indicates the position. u_t is a moving distance from time t to $t+1$. And w_t is a process noise which has normal distribution, y_t is the height measurement ($y_t = y_{baro} - y_{radar}$), $h(x_t)$ is the function for height gradient calculation , v_t is a measurement noise.

The purpose of database referenced navigation based on Bayes' rule is the estimation of x_t using set of measurements, $Y_t = \{y_i\}_{i=0}^{t}$. The point is a precise calculation of conditional pdf, $p(x_t \mid Y_t)$ at the time t for recursive estimation algorithm. As we follow Bayes' rule, conditional pdf of random variable, z can be represented like (2).

$$p(z \mid w) = \frac{p(z,w)}{p(w)} = \frac{p(w \mid z)\,p(z)}{p(w)} \tag{2}$$

The pdf, $p(x_t \mid Y_t)$ related to vehicle's position can be obtain recursively using known pdf, $p(x_t \mid Y_{t-1})$ and all of measurements.

$$p(x_t \mid Y_t) = \frac{p(y_t \mid x_t, Y_{t-1})\,p(x_t \mid Y_{t-1})}{p(y_t \mid Y_{t-1})} \tag{3}$$

where, $p(x_t \mid x_t, Y_{t-1})$ can be calculated from likelihood pdf, $p(x_t \mid Y_{t-1})$ is a priori pdf and $p(y_t \mid Y_{t-1})$ is normalization factor.

As system and measurement model are assigned in (3), equation (4) is summarized.

$$p(x_t \mid Y_t) = \frac{p_{v_t}\left(y_t - h(x)\right) p(x_t \mid Y_{t-1})}{\int p_{v_t}\left(y_t - h(x)\right) p(x_t \mid Y_{t-1}) dx_t} \tag{4}$$

$$p(x_{t+1}, x_t) = p(x_{t+1} \mid x_t)\,p(x_t) \tag{5}$$

Joint probability distribution of state variable is expressed like (5). And equation (6) is priori pdf update equation.

$$p(x_{t+1}, Y_t) = \int p_{w_t}(x_{t+1} - x_t - u_t)\,p(x_t \mid Y_t)\,dx_t \tag{6}$$

Finally, MMSE (Minimum Mean Square Error) result for position estimation can be calculated like (7).

$$\hat{x}_t^{MMSE} = \int x_t\,p(x_t \mid Y_t)\,dx_t \tag{6}$$

3.2 Database Referenced Navigation Using PMF

As we mentioned above, Bayes' rule has integral term, and it makes a difficulty for database referenced navigation algorithm because the analytical solutions are not possible and the calculation burden is heavy. Therefore, N. Bergman introduced the uniform grid based PMF for database referenced navigation. Fig. 3 shows discretized PMF scheme. Integral of Bayesian estimation can be approximated using a finite number of additions. Equation (7) means the approximation.

$$\int_{R^2} f(x)\,dx_t \approx \sum_{k=1}^{N} f\left(x_t(k)\right)\delta^2 \tag{6}$$

where, Grid on the terrain is defined with distance, δ , and the number of all grid point is N .

PMF applied to the navigation can be expressed as follows.

$$p(x_t(k) \mid Y_t) = \alpha_t^{-1} p_{e_t}(y_t - h(x_t(k)))\,p(x_t(k) \mid Y_{t-1}) \tag{7}$$

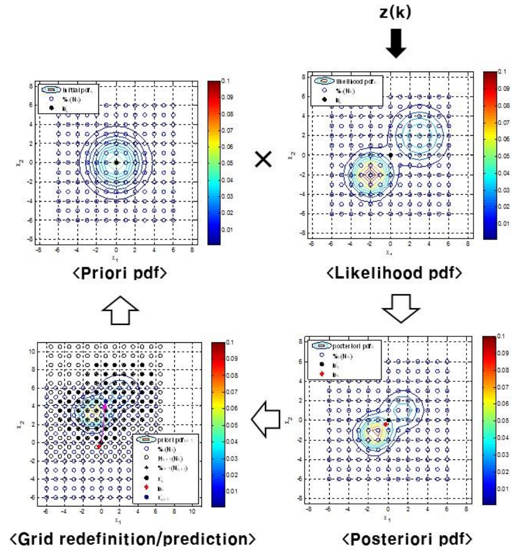

Fig. 3. Point Mass Filter Scheme

$$x_{t+1}(k) = x_t(k) + u_t, \quad k = 1, 2, \cdots, N \tag{8}$$

$$\alpha_t = \sum_{k=1}^{N} p_{e_t}(y_t - h(x_t(k)))p(x_t(k) \mid Y_{t-1})\delta^2 \tag{9}$$

$$p(x_{t+1}(k) \mid Y_t) = \sum_{n=1}^{N} p_{v_t}(x_{t+1}(k) - x_t(n))p(x_t(n) \mid Y_t)\delta^2 \tag{10}$$

$$\hat{x}_t = \sum_{k=1}^{N} x_t(k)p(x_t(k) \mid Y_t)\delta^2 \tag{11}$$

where, $x_t(k)$ is the position of an arbitrary grid point at time k, $\hat{x}_t$ is the estimation position by MMSE. A probability of grids is $p\left(x_t(k) \mid Y_t\right)$, α means the normalization factor.

Using (7), (8), (9) and (10), the position can be updated like (11) based on PMF. At every time step, this process is working recursively. Grid points are also updated and regenerated every time step according to the vehicle's dynamics.

4. DATA FUSION TECHNIQUE USING KURTOSIS

TRN is commonly used to compensate the navigation error of INS. However, it is also impossible to help the navigation on the flat terrain. The database referenced navigation which has multiple databases overcomes the drawback of the conventional TRN. We proposed the weighting logic for multiple databases referenced navigation using the kurtosis of pdf. Depending on the degree of kurtosis, the weighted navigation results are used for global navigation information. In this section we will explain the details about how to determine the weightings of each database referenced navigation result. PMF uses Bayesian estimation and calculates the position information from updated pdf. The position accuracy can be guaranteed if posteriori pdf is sharp enough. Therefore, kurtosis is effective to determine what database referenced navigation result is better than another.

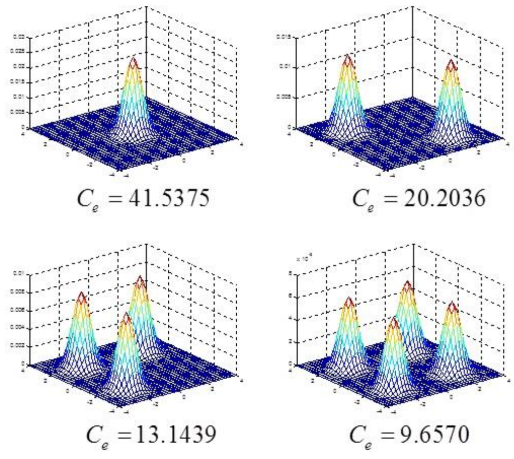

Fig. 4. Kurtosis index value according to characteristics of database

Kurtosis is a measure of whether the data are peaked or flat relative to a normal distribution. Data sets with high kurtosis tend to have a distinct peak near the mean and decline rather rapidly. Data sets with low kurtosis tend to have a flat top near the mean rather than a sharp peak. A uniform distribution would be the extreme case. Kurtosis index applied in PMF can be expressed like (12).

$$C_e = \frac{1}{l \times m} \sum_{i=0}^{l-1} \sum_{j=0}^{m-1} \frac{\left(S_{i,j} - \bar{S}\right)^4}{\sigma_T^4} \tag{11}$$

$$\sigma_T = \sqrt{\frac{1}{l \times m - 1} \sum_{i=0}^{l-1} \sum_{j=0}^{m-1} \left(S_{i,j} - \bar{S}\right)^2} \tag{12}$$

where, $S_{i,j}$ is the geophysical information at $i-th$ row, $j-th$ column. l, m mean the number of row and column. σ_T is the roughness index used in database referenced navigation.

In the Fig. 4, the kurtosis index values are represented according to the characteristics of the geophysical information. If posteriori pdf has a unique candidate position, the probability of false fix is low and kurtosis value would be high. Therefore we use kurtosis value as the accuracy of the position information.

Single database referenced navigation algorithm, generally TRN, is difficult to update the position on the flat terrain. Therefore, we applied additional database, gravity anomaly with terrain height. For the combination of multiple databases, there are many kinds of possibility to deal with. In this paper, we especially propose the weighting logic for hybrid database referenced navigation technique. It helps to overcome the drawbacks of conventional navigation methods. Weighting factor is based on kurtosis because it is proportional to estimated position accuracy. If the kurtosis of posteriori pdf is large, more accurate position information can be obtained. Contrary, the case kurtosis is low means the posteriori pdf is flat. It is not adequate to estimate the precise position

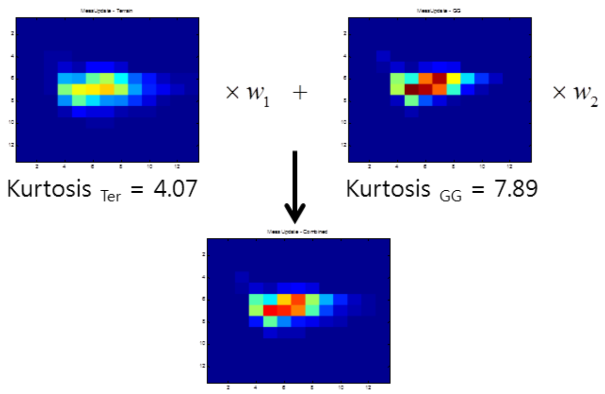

Fig. 5. Concept of weighting logic using kurtosis

information. To design the weighting logic using kurtosis, we adjust these facts. Equation (13) and Fig. 5 show calculation for weighted posteriori pdf. And weight factor can be expressed using (14).

$$pdf^{measUp} = pdf_1^{measUp} w_1 + pdf_2^{measUp} w_2 + \cdots + pdf_N^{measUp} w_N \quad (13)$$

$$w_i = \frac{C^*_{e,i}}{\sum_{k=1}^{N} C^*_{e,k}} \quad (14)$$

where, w_i is normalized weighting, $C^*_{e,k}$ is kurtosis of $k-th$ database and N is the number of referenced database.

As check the size of kurtosis, the combined algorithm decides which position information is more reliable. The estimation results are reflected in the global solution calculation by kurtosis. According to the size of kurtosis, the weighting factor is decided. By applying weighting technique, more robust estimated results can be achieved.

5. SIMULATION

To verify the theoretical improvement of hybrid database referenced method, simulation results are given. Flights were simulated on two different positions in South Korea. The first case is a rough terrain around a mountain. However in the case of gravity anomaly has a medium roughness. The second case is a flat terrain and gravity anomaly with a large rate of change. All flights were done at an average height of 3km over the ground with constant velocity. The terrain database used has a horizontal resolution of 3 arcsec and relative accuracy of 10m standard deviation. Gravity anomaly database has 30 arcsec resolution and 0.01mGal of noise standard deviation. Navigation grade inertial sensor is considered. IMU sensor specification is listed in Table 1. Initial position error along latitude and longitude is 200m each. We assume that the measurements are acquired by sampling the altitude of the vehicle and the interval of each measurement is 1 second. Position compensation using PMF based on multiple databases is performed every second.

Table 1. IMU sensor specification

Sensor	Error	Specification
Accelerometer	Bias	$25\,\mu g$
	*VRW	$5\,\mu g / \sqrt{hr}$
Gyroscope	Bias	$0.003\,\deg/hr$
	*ARW	$0.0015\,\deg/\sqrt{hr}$

*VRW : Velocity Random Walk
*ARW : Angular Random Walk

5.1 Case A: Rough terrain, small gravity anomaly

In Fig. 6, simulation result of the flight of Case A is introduced. The magenta circle line is the position fix result of proposed algorithm. Green and cyan circle lines represent the navigation result of terrain database only and gravity anomaly only mode. The rough terrain generally gives the best position correction performance without the help of an additional database. However, the performance degradation is occurred due to appearance of local flat terrain. In contrast, in the case of proposed algorithm is complemented this drawback. This means that the proposed algorithm is effective for preventing false correction due to rack of database uniqueness. The horizontal error mean and standard deviation is improved using multiple databases together as shown Table 1.

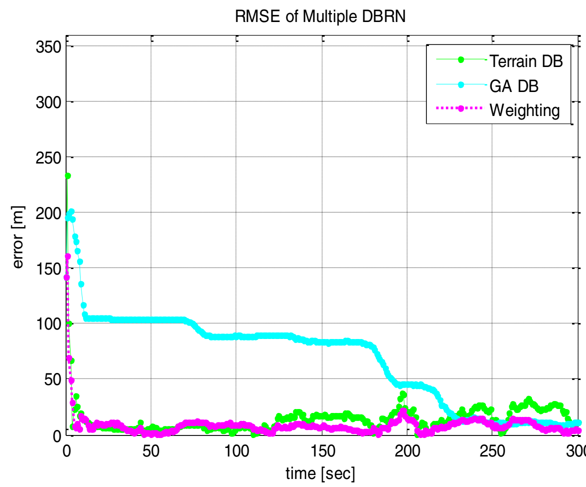

Fig. 6. Horizontal position error of Case A

Table 2. Navigation error of Case A

	Terrain DB only	Gravity anomaly DB only	Multiple DB
Horizontal error mean	14.23m	61.24m	8.29m
Horizontal error std.	15.03m	43.20m	10.06m

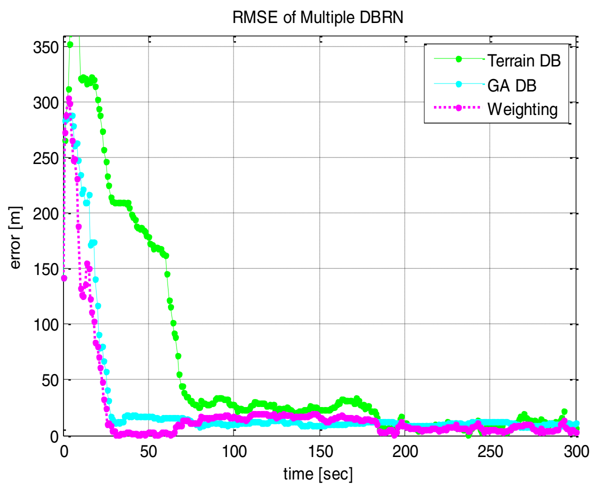

Fig. 7. Horizontal position error of Case B

Table 3. Navigation error of Case B

	Terrain DB only	Gravity anomaly DB only	Multiple DB
Horizontal error mean	58.17m	24.91m	21.74m
Horizontal error std.	93.27m	52.98m	48.67m

5.2 Case B: Flat terrain, medium gravity anomaly

In general, the flat terrain is not adequate for performing TRN. 'Flat' means that there are many candidate positions in the region of interest of PMF. Probabilities of each grid are getting small. The performance of TRN is more sensitive to terrain database noise and altimeter sensor noise on the flat terrain. It means PMF is difficult to find the correct position. In this case, we trust more the navigation information by gravity anomaly than terrain height is used one. This hybrid database referenced navigation result is more robust and precise. As shown Fig. 7, conventional TRN is converged very slowly caused by the beginning of the flat terrain. Terrain is not enough to perform TRN. However gravity anomaly information is useful at that position. As a result, gravity anomaly database applied navigation can be used to calculate more precise position.

On the other hands, it also is appeared false corrections because of the local area which has a small gravity anomaly. However, proposed multiple databases navigation algorithm makes more reliable navigation result. Proposed TRN has the position error of tens of meters. Clearly, the navigation performance of proposed multiple databases referenced navigation is superior to the conventional method.

6. CONCLUSION

The application of multiple databases referenced navigation algorithm which is implied weighting logic based on kurtosis has been described. The fundamental motivation in this application was the effectiveness in terms of performance of single database applied navigation, generally TRN, and reliability. Conventional TRN algorithm is sensitive to several noise sources and difficult to use locally flat terrain. Thus, we considered additional helps from gravity anomaly database to solve the problems of TRN algorithm. As the gravity anomaly database is applied, the performance of navigation can be confident. For a calculation of weighting factor, kurtosis of posteriori pdf is used because kurtosis has a proportional relationship with navigation accuracy. Using the size of kurtosis, we can effectively obtain the position correction information. It is derived from weighted each navigation result. And the horizontal position errors of proposed navigation algorithm are bounded in tens of meters. Horizontal error is less than single database used navigation results. Performance by the proposed algorithm can be achieved effectively.

ACKNOWLEDGE

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