Preprints of the 19th World Congress
The International Federation of Automatic Control
Cape Town, South Africa. August 24-29, 2014

In-orbit Calibration of Attitude Determination Systems for Land-survey Micro-satellites ⋆

Ye. Somov ∗, **C. Hacizade** †, **S. Butyrin** ∗

∗ *Samara State Technical University*
Department of Guidance, Navigation and Control
244 Molodogvardeyskaya Str. Samara 443100 Russia
e_somov@mail.ru butyrinsa@mail.ru

† *Istanbul Technical University*
Aeronautics and Astronautics Faculty
Maslak Istanbul 34469 Turkey
cingiz@itu.edu.tr

Abstract: Contemporary land-survey micro-satellites have general mass up to 100 kg and are placed on the orbit altitudes from 600 up to 800 km. For such spacecraft some principle problems on attitude determination, in-flight calibration and alignment are considered and elaborated methods are presented for their solving. We present discrete algorithms for calibration and alignment of the low cost strap-down inertial navigation systems with correction by signals from multi-head Sun sensor, magnetic sensor and the GPS/GLONASS satellites.

Keywords: spacecraft, attitude determination, in-flight calibration

1. INTRODUCTION

We consider a strapdown inertial navigation system (SINS) for attitude determination of a land-survey maneuvering micro-satellite (Fig. 1). For a satellite SINS, the requirement for the accuracy of attitude determination with small financial expenditures on practical implementation is about 0.05 deg. In this situation, SINS correction by the signals from the sun-magnetic system (SMS) and/or the GPS/GLONASS satellites is most promising.

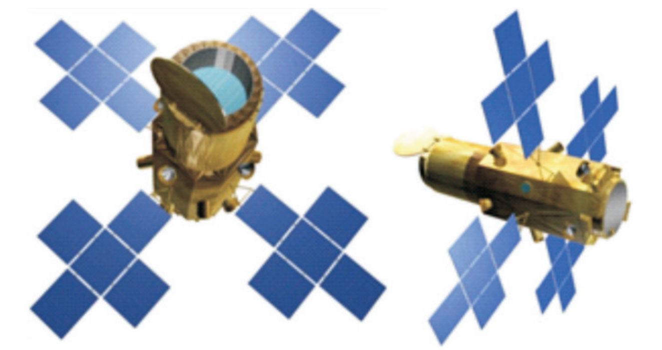

Fig. 1. The land-survey micro-satellite, two views

The SINS considered contains an inertial measurement unit (IMU) based on the gyro sensors of spacecraft (SC) angular position quasi-coordinates, the SMS correction based on a Sun sensor (SS) with a set of optical heads (Rufino and Grassi, 2009) and a magnetic sensor (MS), all devices are fixed rigidly on the satellite body.

⋆ This work was supported by TUBITAK (The Scientific and Technological Research Council of Turkey, Grant 113E595), RFBR (Russian Foundation for Basic Research, Grant 14-08-91373), and also by Division on EMMCP of the RAS (Program 14)

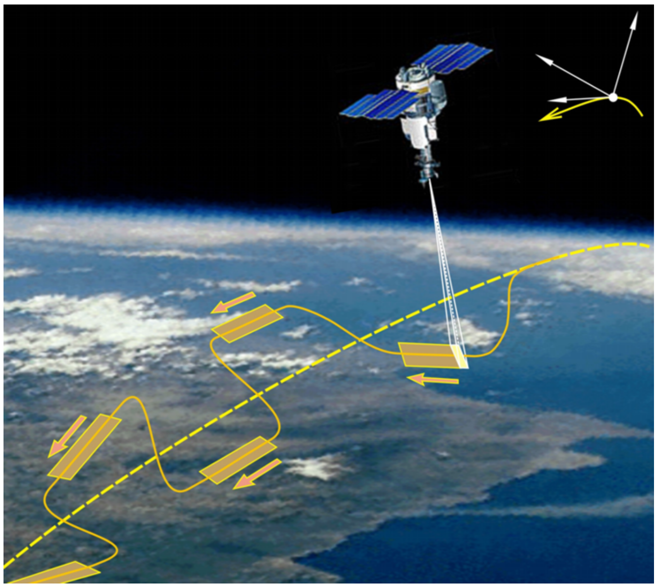

Fig. 2. Scheme of the LEO satellite scanning observation

At the GPS/GLONASS-based SC attitude determination the primary measurements are the between-station single-difference carrier phases by the navigation antenna cluster (NAC) which reflect the projections of a baseline vectors on a line-of-sight vector by a corresponding navigation satellite.

In the paper, we consider the low Earth orbit (LEO) land-survey micro-satellites and study the problem of SINS in-flight calibration at a long-term forecasting of the orbital motion of a micro-satellite, when its position is known with accuracy of up to 30 m along the orbit and up to 10 m both in the lateral direction and altitude. Programmed angular

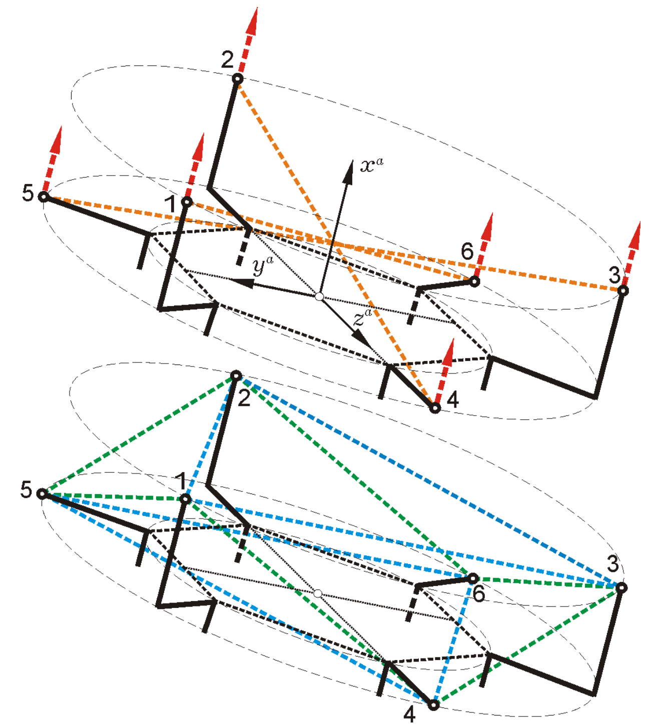

Fig. 3. Principle baselines for the attitude determination

motion of the maneuvering land-survey micro-satellite is presented by a sequence of time intervals for a target application – courses (CRs) and intervals of rotational maneuvers (RMs) (Fig. 2) with variable direction of vector $\boldsymbol{\omega}$, the module of which may be up to 0.5 deg/sec.

We yet have been considered some problems on optimizing a symmetric arrangement of the NAC (Somov et al., 2012). Here we suggest new configuration based on 6 antennas with parallel principle axes (red lines) but with another arrangement on the SC body, Fig. 3. These antennas are *pulled out* from the SC structure and then they are *fixed* to the structure. The configuration ensures 15 large-size baselines (3 largest values by orange color, 6 middle values by blue color and 6 standard values by green color in Fig. 3) and therefore it have best accuracy of the SC attitude determination.

Operating MS as primary sensor is a common method for achieving attitude information in small satellite missions (Psiaki et al., 1990). However, this sensor is not error free because of the biases, scaling errors and angular misalignments. The attitude accuracy requirements demand compensation of such MS errors. In literature there are several methods for estimating the MS bias in case of lack of attitude knowledge (Alonso and Shuster, 2002a,b,c, 2003; Crassidis et al., 2005), including so-called two-step algorithm. Two-step like methods can be used for guessing an initial estimation for Kalman type filter – both the Extended and Unscented Kalman filters (Soken and Hajiyev, 2011a,b). Those a priori estimations for the filter may be then corrected on an expanded state vector including attitude parameters and biases.

For agile land-survey micro-satellite an attitude determination system (ADS) is needed with the robust properties. Best low cost ADS has the form of SMS and is based on cluster of MS and multi-head SS (Wertz, 1978), when the MS accuracy is $3\sigma^{\mathrm{m}} \approx 0.05$ deg and the SS have the accuracy $3\sigma^{\mathrm{s}} \approx 30$ arc sec.

The problem of the SC attitude determination using signals of the GPS/GLONASS navigation systems has been studied by many authors owing to its advantages such as long-term stable accuracy, low cost and low power consumption. In developing the SC attitude estimation algorithms, the GPS/GLONASS vector observations are used in the principle approach. The basic idea of the vector method is to convert the carrier phase measurements into vector observations, which are further used to form a so-called Wahba's problem and next its solving by well-known QUEST algorithm. The ADS is based on the radio-measurements and have the accuracy $3\sigma^{\mathrm{r}} \approx 0.15$ deg when typical NAC baseline's size is ≈ 1.5 m and its onboard calibration is implemented with accuracy of up to 0.2 mm.

2. MODELS AND THE PROBLEM STATEMENT

The problems of the SINS signal processing are connected with integration of kinematic equations in using the information only on the quasi-coordinate increment vector obtained by the IMU at availability of noises, calibration (identification and compensation for the IMU bias $\mathbf{b}^{\mathrm{g}}$ and variation m of the measure scale factor by the angular rate vector $\boldsymbol{\omega}$) and alignment – identification and compensation of errors on a mutual angular position of the IMU and the SMS reference frames by its signals with the main period T_o. As kinematic parameters, many authors applied the quaternion $\boldsymbol{\Lambda} = (\lambda_0, \boldsymbol{\lambda})$ where $\boldsymbol{\lambda} = \{\lambda_1, \lambda_2, \lambda_3\}$, the orientation matrix $\mathbf{C}$, the Euler vector $\boldsymbol{\phi} = \mathbf{e}\theta$, the vector of terminal rotation $\boldsymbol{\theta} = 2\mathbf{e}\,\mathrm{tg}(\theta/2)$ etc. Moreover, for the SC low angular motions with little variation of angle θ during the discrete period T_o, the integration of the kinematic relations for vector $\boldsymbol{\phi}(t)$ with calculation of values $\Lambda_k \equiv \Lambda(t_k)$ were carried out by the scheme:

$$\delta\boldsymbol{\phi}_k = \mathbf{i}_k^{\omega} = \int_{t_k}^{t_{k+1}} \boldsymbol{\omega}(\tau)d\tau \equiv \mathbf{Int}(t_k, T_o, \boldsymbol{\omega}(t));$$
$$\boldsymbol{\phi}_k + \delta\boldsymbol{\phi}_k = \boldsymbol{\phi}_{k+1} \Rightarrow \mathbf{C}_{k+1} \Rightarrow \boldsymbol{\Lambda}_{k+1},$$

where $\delta\boldsymbol{\phi}_k = \delta\theta_k \mathbf{e}_k$, $t_{k+1} = t_k + T_o$, $k \in \mathbb{N}_0 \equiv [0, 1, 2, ...)$.

Here in the SINS algorithms, the measured information is applied at intermediate points with a period T_q multiple of the main sampling period T_o; polynomial approximation is used and integration of the kinematic equation for the vector of modified Rodrigues parameters (hereafter, simply, the Rodrigues vector) is carried out. The quaternion $\boldsymbol{\Lambda}$ is connected with the Rodrigues vector $\boldsymbol{\sigma} = \mathbf{e}\,\mathrm{tg}(\theta/4)$ by the straight $\boldsymbol{\sigma} = \boldsymbol{\lambda}/(1 + \lambda_0)$ $(\boldsymbol{\Lambda} \Rightarrow \boldsymbol{\sigma})$ and the reverse $\boldsymbol{\lambda} = 2\boldsymbol{\sigma}/(1+\sigma^2)$; $\lambda_0 = (1-\sigma^2)/(1+\sigma^2)$ $(\boldsymbol{\sigma} \Rightarrow \boldsymbol{\Lambda})$ relations. For vector $\boldsymbol{\sigma}$, the kinematic equations have the form:

$$\dot{\boldsymbol{\sigma}} = \mathbf{F}^{\sigma}(\boldsymbol{\sigma}, \boldsymbol{\omega}) \equiv \tfrac{1}{4}(1 - \sigma^2)\boldsymbol{\omega} + \tfrac{1}{2}\boldsymbol{\sigma} \times \boldsymbol{\omega} + \tfrac{1}{2}\boldsymbol{\sigma}\langle\boldsymbol{\sigma}, \boldsymbol{\omega}\rangle;$$
$$\boldsymbol{\omega} = 4[(1 - \sigma^2)\dot{\boldsymbol{\sigma}} - 2(\boldsymbol{\sigma} \times \dot{\boldsymbol{\sigma}}) + 2\boldsymbol{\sigma}\langle\dot{\boldsymbol{\sigma}}, \boldsymbol{\sigma}\rangle]/(1 + \sigma^2)^2.$$

We have introduced the inertial basis $\mathbf{I}$; basis $\mathbf{B}$ and the body reference frame (BRF) connected with the SC body; standard orbital basis $\mathbf{O}$ and the orbit reference frame (ORF); the sensor basis $\mathbf{S}$ (by a telescope); virtual basis $\mathbf{A}$ which is calculated by processing an accessible measurement information from any ADS, and the IMU virtual basis $\mathbf{G}$, which is computed by processing the measurement information from the integrating gyros. The BRF attitude with respect to basis $\mathbf{I}$ is defined by quaternion $\mathbf{\Lambda}$ and with respect to the ORF – by angles of yaw $\phi_1 = \psi$, roll $\phi_2 = \gamma$ and pitch $\phi_3 = \theta$ in the sequence $31'2''$.

For simplicity, assume that basis $\mathbf{B}$ is coaxial to basis $\mathbf{S}$. We also assume that the measurement information is processed in the IMU with a frequency of about 3 kHz and, as a result, the measured values of the quasi-coordinate increment vector $\mathbf{i}^{\mathrm{g}\,\omega}_{\mathrm{m}\,s}$, $s \in \mathbb{N}_0$ enter from the IMU with a period $T_q \ll T_o$, and the quaternion measured values $\mathbf{\Lambda}^{\mathrm{a}}_{\mathrm{m}\,k}$ enter from any ADS (SMS and/or NAC):

$$\begin{gathered}\mathbf{i}^{\mathrm{g}\,\omega}_{\mathrm{m}\,s} = \mathbf{Int}(t_s, T_q, \boldsymbol{\omega}^{\mathrm{g}}_{\mathrm{m}}(t)) + \boldsymbol{\delta}^{\mathrm{n}}_s;\\ \boldsymbol{\omega}^{\mathrm{g}}_{\mathrm{m}}(t) \equiv (1+m)\mathbf{S}^{\Delta}(\boldsymbol{\omega}(t) + \mathbf{b}^{\mathrm{g}});\ \mathbf{\Lambda}^{\mathrm{a}}_{\mathrm{m}\,k} = \mathbf{\Lambda}_k \circ \mathbf{\Lambda}^{\mathrm{n}}_k.\end{gathered} \tag{1}$$

Here $\boldsymbol{\omega}^{\mathrm{g}}_{\mathrm{m}}(t)$ is the measured vector of SC angular rate vector in base $\mathbf{G}$ taking into account the unknown small and slow variations of the IMU bias vector $\mathbf{b}^{\mathrm{g}} = \mathbf{b}^{\mathrm{g}}(t)$ on an angular rate; orthogonal matrix $\mathbf{S}^{\Delta}(t)$ describes errors on a mutual angular position of the IMU and ADS reference frames, moreover matrix $\mathbf{S}^{\Delta} \approx \mathbf{I}_3 + [\boldsymbol{\Delta}\times]$ where vector $\boldsymbol{\Delta} = \{\Delta_x, \Delta_y, \Delta_z\}$ presents the alignment error; scalar function $m = m(t)$ presents an unknown slow variation of the IMU scale factor. We take into consideration the Gaussian noises $\boldsymbol{\delta}^{\mathrm{n}}_s$ with RMS σ^{b} and $\mathbf{\Lambda}^{\mathrm{n}}_k$ with RMS σ^{a} in the IMU and ADS output signals, accordingly. We also assume small variation of the IMU scale factor, for example, $|m(t)| \le 0.01$, when the relation $1 - m^2 \cong 1$ is satisfied.

The problem consists in developing algorithms for obtaining the quaternion estimation $\hat{\mathbf{\Lambda}}_l$, $l \in \mathbb{N}_0$, with any period $T_p = t_{l+1} - t_l$ multiple to a period T_o, in a general case, with a fixed delay T_d with respect to the time moments t_k, and also discrete algorithms for the SINS calibration and alignment with the derivation of estimates $\hat{\mathbf{b}}^{\mathrm{g}}_k$, $\hat{\mathbf{S}}^{\Delta}_k$ and $\hat{m}_k$ at rotation maneuvers of land-survey micro-satellite with a variable direction of its angular rate vector.

3. ALGORITHMS FOR IN-FLIGHT CALIBRATION

The problems on identification of "alignment" matrix $\mathbf{S}^{\Delta}_k$ and variation of a measure scale factor m are the most complicated ones. This is due to a multiplicative character of the interconnected parametric disturbances indicated.

Suggested principal ideas are that: there is needed to define the $\hat{\boldsymbol{\Delta}}$ and $\hat{m}$ estimations only on the *whole* for virtual bases $\mathbf{A}$ and $\mathbf{G}$ with respect to main base $\mathbf{S} = \mathbf{B}$, without concrete details on errors of individual onboard measuring devices and to integrate the kinematic equations with a small computing drift, an idea is being developed to use the measured information in the intermediate points with a period T_q that is multiple of the main sampling period T_{o}; identification of the vector value $\mathbf{b}^{\mathrm{g}}$ is ensured by discrete Luenberger observer.

Solution of the problems is obtained by simultaneous implementation of two sets of onboard computing algorithms.

3.1 The ADS in-flight calibration

The first set of algorithms realizes alignment of ADS calibration with respect to the sensor basis $\mathbf{S}$ which is connected with onboard telescope.

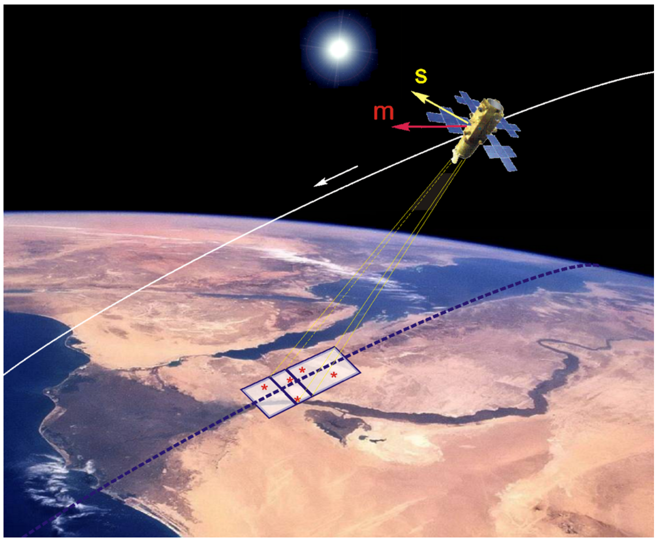

Fig. 4. Scheme of the SMS calibration

As SMS correction, its calibration is carried out in the process of the SC regular guidance onto the terrestrial bench marks (Somov and Butyrin, 2011, 2012). At a scanning observation of the bench marks in small neighbourhood of the nadir direction (Fig. 4) a "moving window" with fixed frequency is implemented for accumulation of the electronic image charge packets along columns of the CCD matrix.

We define the sequence of the SC attitude quaternion values with exact binding to the time moments t_s by the following technique. All sequence of discerned marked objects onto the photo is divided into the groups (frames), so that each frame would contain a given odd number n of marked objects, according to the following rules:

- objects are arranged in the order of increasing the time moments t_s of their registration without omission;
- each next frame includes only one additional bench mark.

Each i-th frame "is tied" to a time moment t^m_i according to its central marked object with number i. Then two sets of the unit directions on marked objects are defined for each frame: the set of units in base $\mathbf{S}$ by the marked objects' relative coordinates in the CCD matrix and the set of the calculated units on the same marked objects into basis $\mathbf{I}$. In a result, one can obtain sequences of values by the SC attitude quaternion with respect to basis $\mathbf{I}$.

The error on determination of turn around a telescope axis is significantly worse, that result is explained by the small telescope's field-of-view, e.g. the insufficient measuring base. The elaborated technique for a more accurate definition of basis $\mathbf{S}$ position with respect to the inertial basis $\mathbf{I}$ is based on widening the measuring basis by an increasing the terrestrial marked objects' number in a frame up to 100.

Simultaneously the "designed" values are computed by unit $\mathbf{s}$ of the Sun direction and by unit $\mathbf{m}$ of the Earth magnetic induction vector, see Fig. 4, for fixed time moments. The values of the same units are also computed by physical signals that are obtained from the Sun and magnetic

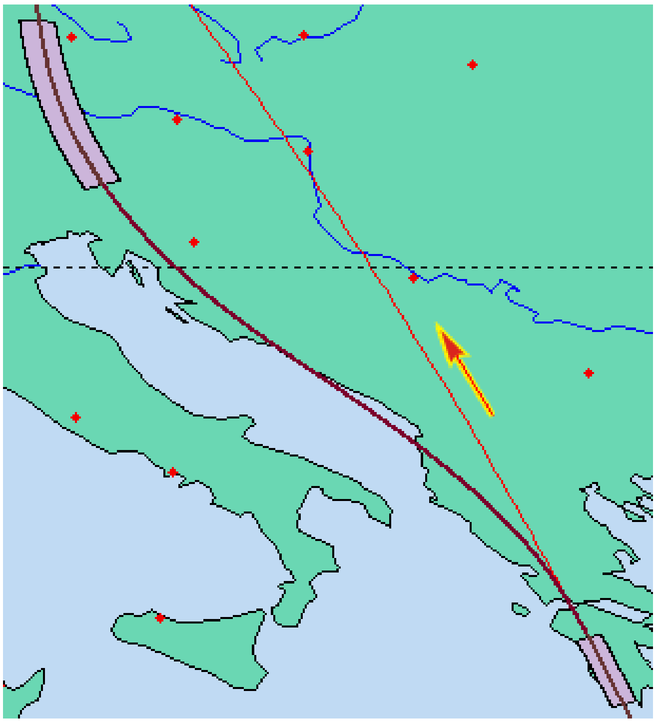

Fig. 5. Courses and rotation maneuver images on a map

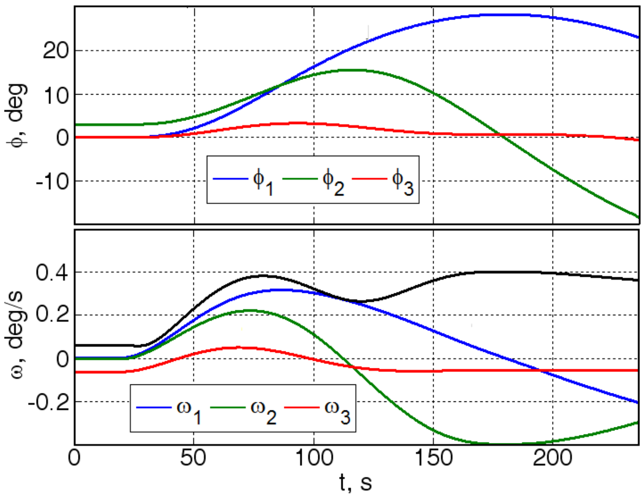

Fig. 6. The attitude guidance program at land-survey

sensors. Next, for the time moments, the angular positions are computed by well-known TRIAD algorithm (Markley, 2002, 2008) for the "designed" basis and "measured" basis in the inertial basis $\mathbf{I}$; statistical processing is carried out and one can obtain an estimation of constant quaternion, which presents the error in their mutual angular position.

As for ADS in the form of NAC, its in-flight calibration is carried out by the same way, but here TRIAD algorithm is not applied and main onboard computations are based on the QUEST algorithm.

3.2 The IMU in-flight calibration

The second set of algorithms ensures calibration of the IMU with respect to a bias and a scale factor. These algorithms also ensure an alignment of mutual angular position of the IMU basis $\mathbf{I}$ and the ADS basis $\mathbf{A}$.

We assume formation of estimates $\hat{\mathbf{b}}^{\mathrm{g}}_k$, $\hat{\mathbf{S}}^{\Delta}_k$ and $\hat{m}_k$ which are *constant* during a period T_o. Moreover, estimate $\hat{\mathbf{b}}^{\mathrm{g}}_k$ is renewed at every time moment t_k, and estimates $\hat{\mathbf{S}}^{\Delta}_k$ and $\hat{m}_k$ are formed off-line, i.e., based on processing of an accessible measurement information that was accumulated over long time intervals. At discrete filtering of the measured values of vector $\mathbf{i}^{\mathrm{g}\,\omega}_{\mathrm{m}\,s}$ in (1), the accumulated set of the values $\mathbf{i}^{\mathrm{g}\,\omega}_{\mathrm{m}\,s}$ over the k-th time interval are used for suppression of the IMU noise $\boldsymbol{\delta}^{\mathrm{n}}_s$.

The method of least squares is applied for approximation of the set by 4-th degree polynomial $\tilde{\mathbf{i}}^{\mathrm{g}\,\omega}_k(\tau)$ with a local time $\tau = t - k\,T_o \in [0, T_o]$. At compensation for bias $\mathbf{b}^{\mathrm{g}}$ effect on this time interval, one can obtain a vector polynomial estimation $\check{\mathbf{i}}^{\mathrm{g}\,\omega}_k(\tau) = \tilde{\mathbf{i}}^{\mathrm{g}\,\omega}_k(\tau) - \hat{\mathbf{b}}^{\mathrm{g}}_k\tau$ of the quasi-coordinate increments and presentation of the vectors

$$\check{\mathbf{i}}^{\mathrm{g}\,\omega}_s = \check{\mathbf{i}}^{\mathrm{g}\,\omega}_k(t_s - kT_o) \text{ and } \check{\mathbf{i}}^{\mathrm{g}\,\omega}_{k+1} = \check{\mathbf{i}}^{\mathrm{g}\,\omega}_k(T_o) = \sum_{s=0}^{r-1} \check{\mathbf{i}}^{\mathrm{g}\,\omega}_s$$

in basis $\mathbf{G}$, where $r = \mathrm{E}[s/k]$ and $\mathrm{E}[\cdot]$ is the operator for separation of whole part of a number.

In basis $\mathbf{A}$, taking into account the small error $m(t)$ values, the estimation of the quasi-coordinate increment vector $\check{\mathbf{i}}^{\omega}_s$, $s \in \mathbb{N}_0$, is formed by the important relation

$$\check{\mathbf{i}}^{\omega}_s = (1 - \hat{m}_k)(\hat{\mathbf{S}}^{\Delta}_k)^{\mathrm{t}}\, \check{\mathbf{i}}^{\mathrm{g}\,\omega}_s$$

with a period T_q and a vector polynomial $\check{\mathbf{i}}^{\omega}_s(\tau)\forall\tau \in [0, T_o]$ estimate with compensation for the bias $\mathbf{b}^{\mathrm{g}}$ effect is obtained by the procedure for the polynomial $\check{\mathbf{i}}^{\mathrm{g}\,\omega}_s(\tau)$; moreover, the vector $\check{\mathbf{i}}^{\omega}_{k+1} = \check{\mathbf{i}}^{\omega}_k(T_o)$.

Identification of IMU bias $\mathbf{b}^{\mathrm{g}}$ is carried out with a period T_o by the Luenberger discrete observer. Preliminary estimation $\breve{\boldsymbol{\omega}}_k(\tau)$ of the angular rate vector $\boldsymbol{\omega}(t)$ on k-th interval of a time $t \in [t_k, t_{k+1}]$ is carried out by evident differentiation of the polynomial $\check{\mathbf{i}}^{\omega}_k(\tau)$ which results in analytic dependence $\breve{\boldsymbol{\omega}}_k(\tau)$.

At the time interval a preliminary estimation of the SC attitude is attained by integration of the vector differential equation $\dot{\breve{\boldsymbol{\sigma}}}_k(\tau) = \mathbf{F}^{\sigma}(\breve{\boldsymbol{\sigma}}_k(\tau), \breve{\boldsymbol{\omega}}_k(\tau))$ using well-known ODE45 method (Shampine, 1986) with a forming of a preliminary estimation of the Rodrigues vector $\breve{\boldsymbol{\sigma}}_k(\tau)$. For the vector equation initial condition is formed by ADS signals in the time moment t_0 only (at the SINS switch on), at another cases the initial conditions are calculated by signals of the Luenberger observer.

Assume that at the time moment $t = t_k$ we have the SMS measurement information on the SC attitude in the form of quaternion $\boldsymbol{\Lambda}^{\mathrm{a}}_{\mathrm{m}\,k}$, the correcting vector $\Delta\mathbf{p}_k(g^{\mathrm{o}}_2, \mathbf{Q}_k)$ and quaternion $\Delta\mathbf{P}_k(g^{\mathrm{o}}_1, \mathbf{Q}_k)$ are formed, where

$$\mathbf{Q}_k \equiv (q_{0k}, \mathbf{q}_k) \equiv (C_{\frac{\varphi_k}{2}}, \mathbf{e}^q_k\, S_{\frac{\varphi_k}{2}}) \equiv \mathbf{Q}_k(\mathbf{e}^q_k, \varphi_k) = \tilde{\boldsymbol{\Lambda}}^{\mathrm{a}}_{\mathrm{m}\,k} \circ \hat{\boldsymbol{\Lambda}}_k.$$

At the same time moment t_k by a transformation $\hat{\Lambda}_k \Rightarrow \breve{\boldsymbol{\sigma}}_k$ the initial condition $\breve{\boldsymbol{\sigma}}_k(0) \equiv \breve{\boldsymbol{\sigma}}_k$ is defined for calculation of the Rodrigues vector estimate $\breve{\boldsymbol{\sigma}}_k(\tau)$ on the k-th interval by numerical integration of the differential equation. After such integration one can obtain the Rodrigues vector value $\breve{\boldsymbol{\sigma}}_{k+1} = \breve{\boldsymbol{\sigma}}_k(T_o)$ for the local time moment $\tau = T_o$. The

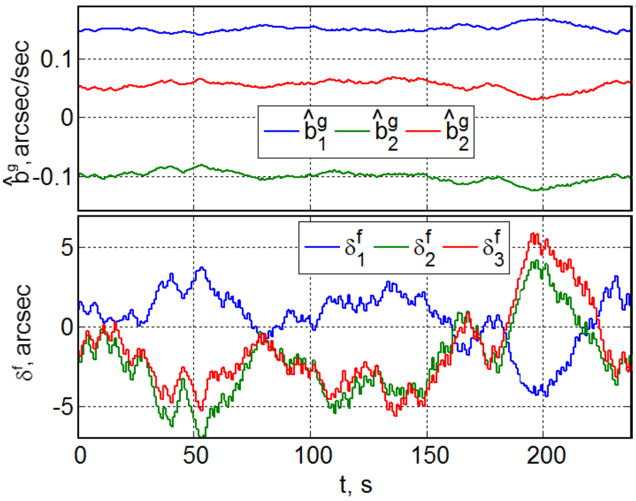

Fig. 7. Errors of estimating for the SMS correction

quaternion value $\check{\mathbf{R}}_k$ is calculated by a transformation $\check{\boldsymbol{\sigma}}_{k+1} \Rightarrow \check{\mathbf{R}}_k$. The developed discrete Luenberger observer has the form:

$$\hat{\boldsymbol{\Lambda}}_{k+1} = \check{\mathbf{R}}_k \circ \Delta\mathbf{P}_k(g_1^{\mathrm{o}}, \mathbf{Q}_k); \quad \hat{\mathbf{b}}^{\mathrm{g}}_{k+1} = \hat{\mathbf{b}}^{\mathrm{g}}_k + \Delta\mathbf{p}_k(g_2^{\mathrm{o}}, \mathbf{Q}_k);$$
$$\Delta\mathbf{P}_{k+1} = \mathbf{Q}_{k+1}(\mathbf{e}^q_{k+1}, g_1^{\mathrm{o}}\varphi_{k+1}); \quad \Delta\mathbf{p}_{k+1} = 4g_2^{\mathrm{o}}\boldsymbol{\sigma}^q_{k+1},$$

where both the quaternion and vector relations are applied, moreover the Rodrigues vector $\boldsymbol{\sigma}^q_{k+1}$ is defined analytically on the quaternion value $\mathbf{Q}_{k+1}$ and the observer scalar coefficients $g_1^{\mathrm{o}}, g_2^{\mathrm{o}}$ are calculated by analytic relations.

In final stage the Rodrigues vector values $\boldsymbol{\sigma}_s$ are processed by recurrent discrete filter with a period T_p. As a result, one can obtain the Rodrigues vector values $\hat{\boldsymbol{\sigma}}_l$ which are applied for formation of the quaternion estimate$\hat{\boldsymbol{\Lambda}}_l$, $l \in \mathbb{N}_0$ with given period T_p using transformation $\hat{\boldsymbol{\sigma}}_l \Rightarrow \hat{\boldsymbol{\Lambda}}_l$.

3.3 The SINS alignment

The SINS alignment (calculation of matrix $\hat{\mathbf{S}}^{\Delta}$) and determination of estimate $\hat{m}$ for the error of the scale factor m are carried out off-line by comparing the angular rate vector values, which are recovered autonomously from the IMU signals (vectors $\hat{\boldsymbol{\omega}}^{\mathrm{g}}$) and from the ADS correction signals (vectors $\hat{\boldsymbol{\omega}}^{\mathrm{a}}$) at the same time moments. Moreover, sets of vectors $\hat{\boldsymbol{\omega}}^{\mathrm{g}}_l$ and $\hat{\boldsymbol{\omega}}^{\mathrm{a}}_l$ values are formed, fixed to the time moments t_l with a period T_p. For these vectors the values of modules $\hat{\omega}^{\mathrm{g}}_l = |\hat{\omega}^{\mathrm{g}}_l|$, $\hat{\omega}^{\mathrm{a}}_l = |\hat{\omega}^{\mathrm{a}}_l|$, units $\hat{\mathbf{e}}^{\mathrm{g}}_{\omega l} = \hat{\boldsymbol{\omega}}^{\mathrm{g}}_l/\hat{\omega}^{\mathrm{g}}_l$, $\hat{\mathbf{e}}^{\mathrm{a}}_{\omega l} = \hat{\boldsymbol{\omega}}^{\mathrm{a}}_l/\hat{\omega}^{\mathrm{a}}_l$ are calculated and for basis $\mathbf{A}$ and basis $\mathbf{G}$ the alignment problem is solved on the values of units $\hat{\mathbf{e}}^{\mathrm{a}}_{\omega l}$ in basis $\mathbf{A}$ and the units $\hat{\mathbf{e}}^{\mathrm{g}}_{\omega l}$ in basis $\mathbf{G}$ using QUEST algorithm.

For calibration of the scale factor error m, the set of values $m_l = 1 - \hat{\omega}^{\mathrm{m}}_l/\hat{\omega}^{\mathrm{a}}_l$ is calculated, the estimate $\hat{m}$ is obtained by processing this set by the method of least squares and this estimation is applied in the form of $\hat{m}_k$ until the next calibration is completed.

4. RESULTS OF THE COMPUTER VERIFICATION

The simulation results were obtained for LEO land-survey micro-satellite on the sun-synchronous orbit with altitude 600 km. In Fig. 5 one can see the SC trace (red line), first course at the trace scanning optoelectronic observation in the nadir direction, the line-of-sight track at the SC rotation maneuver and second course for next the trace scanning observation but with the initial line-of-sight angular deflection on 30 deg. at the roll channel. The SC attitude guidance program was calculated taking into account the restriction $|\boldsymbol{\omega}(t)| \le \omega^* = 0.4$ deg/s, obtained results are presented in Fig. 6.

Figure 6 presents the SC program angular motion in the orbit reference frame for angles $\phi_i(t) = \phi^p_i(t)$with the turning sequence $31'2''$, for components $\omega_i = \omega^p_i(t)$ and module of the program angular rate vector $\boldsymbol{\omega}$.

In the SINS simulation with the IMU correction by the SMS signals, we used the RMS values $\sigma^{\mathrm{m}} = 60$ arc sec, $\sigma^{\mathrm{s}} = 10$ arc sec and $\sigma^{\mathrm{b}} = 0.01$ arc sec. For the periods $T_o = 1$ s (frequency Hz) and $T_q = 0.03125$ s (32 Hz), the IMU test bias vector $\mathbf{b}^{\mathrm{g}} = \{0.15, -0.1, 0.05\}$ arc sec/s is recovered with the RMS value of about 5 %, see Fig. 7.

At the time moments t_l with the period $T_p = 0.0625$ s (16 Hz), the attitude estimation error is presented by vector $\boldsymbol{\delta}_l = 4\boldsymbol{\sigma}^{\delta}_l$, where the Rodrigues vector $\boldsymbol{\sigma}^{\delta}_l$ corresponds to quaternion $\boldsymbol{\Xi}^{\delta}(t_l) = \tilde{\boldsymbol{\Lambda}}^p(t_l) \circ \hat{\boldsymbol{\Lambda}}_l$ of the estimation error. The components of error vector $\boldsymbol{\delta}^{\mathrm{f}}(t)$ by the attitude estimation filtered with the period T_p are presented also in Fig. 7 in the form of digital signals. The developed procedures allow for SINS alignment with an accuracy of about 10 arc sec and the estimate $\hat{m}$ of error in the measurement scale factor is obtained with an accuracy of about 0.05 % for such IMU correction.

As the GPS/GLONASS-based attitude determination for the SINS correction, we used the RMS value $\sigma^{\mathrm{r}} = 0.05$ deg on NAC accuracy with the same other data, obtained results are presented in Fig. 8

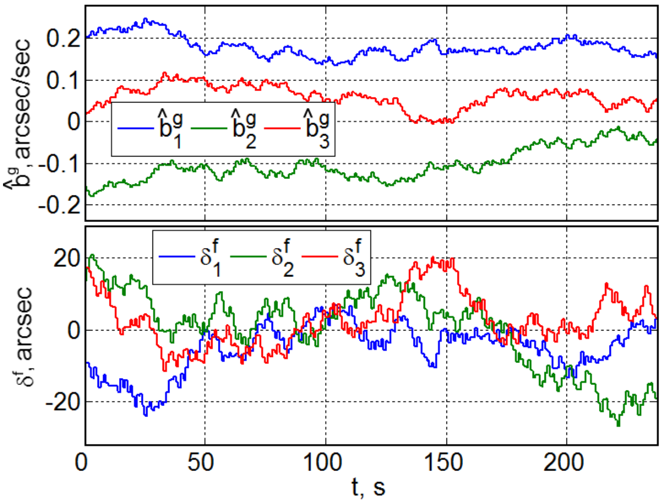

Fig. 8. Errors of estimating for the NAC correction

CONCLUSIONS

In progress of last paper Somov et al. (2013) we have presented original methods and multiple discrete algorithms for filtering, in-flight calibration and alignment of the strapdown inertial navigation systems with sun-magnetic and GPS/GLONASS-based external position correction for precise attitude determination of the LEO maneuvering land-survey micro-satellites.

We have suggested new configuration of the navigation antenna cluster based on 6 antennas with parallel principle axes, which are pulled out from the SC structure and then they are fixed to the structure. The configuration ensures 15 large-size baselines (Fig. 3) and therefore it have best accuracy of the SC attitude determination.

Nonlinear discrete Luenberger observer was developed where both the quaternion and vector relations are applied. Some presented numeric results prove an efficiency of suggested methods and onboard algorithms.

REFERENCES

Alonso, R. and Shuster, M. (2002a). Attitude-independent magnetometer-bias determination: A survey. *The Journal of the Astronautical Sciences*, 50(4), 453–475.

Alonso, R. and Shuster, M. (2002b). Attitude-independent magnetometer-bias determination: A survey. *The Journal of the Astronautical Sciences*, 50(4), 477–490.

Alonso, R. and Shuster, M. (2002c). Two step: A fast robust algorithm for attitude-independent magnetometer-bias determination. *The Journal of the Astronautical Sciences*, 50(4), 433–451.

Alonso, R. and Shuster, M. (2003). Centering and observability in attitude-independent magnetometer-bias determination. *The Journal of the Astronautical Sciences*, 51(2), 133–141.

Crassidis, J., Lai, K., and Harman, R. (2005). Real-time attitude-independent three-axis magnetometer calibration. *Journal of Guidance, Control, and Dynamics*, 28(1), 115–120.

Markley, F. (2002). Fast quaternion attitude estimation from two vector measurements. *Journal of Guidance, Control, and Dynamics*, 25(2), 411–414.

Markley, F. (2008). Optimal attitude matrix from two vector measurements. *Journal of Guidance, Control, and Dynamics*, 31(3), 765–768.

Psiaki, M., Martel, F., and Pal, P. (1990). Three-axis attitude determination via Kalman filtering of magnetometer data. *Journal of Guidance, Control, and Dynamics*, 13(3), 506–514.

Rufino, G. and Grassi, M. (2009). Multi-aperture CMOS Sun sensor for microsatellite attitude determination. *Sensors*, 9(6), 4503–4524.

Shampine, L. (1986). Some practical Runge-Kutta formulas. *Mathematics of Computation*, 46(173), 135–150.

Soken, H. and Hajiyev, C. (2011a). In-flight magnetometer calibration via unscented Kalman filter. In *Proceedings of 5th IEEE/AIAA International Conference on Recent Advances in Space Technologies http:// ieeexplore.ieee.org/xpl/articleDetails.jsp?arnumber=5966970*, 885–890. Istanbul.

Soken, H. and Hajiyev, C. (2011b). Reconfigurable UKF for in-flight magnetometer calibration and attitude parameter estimation. In *Proceedings of 18th IFAC Congress http://www.ifac-papersonline.net/Detailed/ 36919.html*, 741–746. Milan.

Somov, Y., Butyrin, S., Siguerdidjane, H., Hajiyev, C., and Fedosov, V. (2013). In-flight calibration of the attitude determination systems for information mini-satellites. In *Proceedings of 19th IFAC Symposium on Automatic Control in Aerospace http://www.ifac-papers online.net/Detailed/63251.html*, 393–398. Wuerzburg.

Somov, Y. and Butyrin, S. (2011). Geodesic tie of the space images by an integration of the navigating and observing information. In *Proceedings of 18th Saint Petersburg international conference on integrated navigation systems*, 319–322. Saint Petersburg.

Somov, Y. and Butyrin, S. (2012). In-flight alignment of a space telescope and a star tracker cluster at a scanning observation of the Earth marked objects. In *Proceedings of 19th Saint Petersburg international conference on integrated navigation systems*, 242–244. Saint Petersburg.

Somov, Y., Butyrin, S., and Somov, S. (2012). Optimizing arrangement of the onboard navigation antennas on accuracy of satellite attitude determination. In *Proceedings of 19th Saint Petersburg international conference on integrated navigational systems*, 335–337. Saint Petersburg.

Wertz, J. (ed.) (1978). *Spacecraft Attitude Determination and Control*. Kluwer, Dordrecht.