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LPV Formation Control of Non-Holonomic Multi-Agent Systems ⋆

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Abstract: This paper studies a formation control problem for nonlinear multi-agent systems; specifically non-holonomic vehicles represented as linear parameter-varying (LPV) models are of interest here. The objective of this study is twofold. First, we demonstrate the applicability of a novel approach to distributed control for LPV decomposable systems (Hoffmann et al., 2013). Second, we investigate different LPV representations of non-holonomic vehicles. We consider a group of agents in a leader-follower configuration which communicate through a directed time-varying but diagonalizable interconnection topology, where follower vehicles must achieve a desired formation and track the path determined by the leader agent. In addition, the leader vehicle is equipped with an LPV flatness-based controller to track a reference trajectory. The problem is formulated in terms of linear fractional transformations (LFT) for LPV systems with the objective of minimizing the closed-loop induced $\mathcal{L}_2$ gain. Simulation results with a formation of non-holonomic discs illustrate the proposed approach.

Keywords: Distributed Control, Switching Networks, Mobile Robots, Nonlinear Control, LPV.

1. INTRODUCTION

Swarm robotics has attracted considerable attention from the control engineering community. Possible applications areas are terrain recognition, disaster area exploration, zone surveillance etc. At the same time, decentralized cooperative vehicle control is currently an active research area with many recent advances; and it is of interest here, since the framework it offers, matches at many levels the real scenario of a group of vehicles acting together.

Mobile robots are typically subject to non-holonomic constraints, e.g. wheeled mobile robots, fixed-wing aircraft etc. Here we illustrate a novel apporach to formation control for such robots with the example of wheeled mobile robots, represented by a non-holonomic disk. Difficulty arises from its underactuated nature, which is a direct consequence of its non-holonomic characteristic. Different control techniques have shown to perform fairly well in different scenarios, either individually (e.g. Lee et al. (2001), Chen et al. (2009), Shojaei and Shahri (2012)) or as a group (e.g. Zhai et al. (2010), Sadowska and Huijberts (2013), Liu and Jiang (2013)). However, most of the techniques proposed in the literature are purely non-linear, where the problem often is to find a suitable Lyapunov function, which is not a straightforward task. As a consequence, no performance guarantees are offered.

In the light of LPV techniques, e.g. Scherer (2001), which today are mature and have proved to be reliable in practical applications, e.g. Gonzalez et al. (2013), it seems natural to consider the LPV framework for this non-linear problem. Few approaches have cast non-holonomic systems (not necessarily mobile robots) into LPV models, e.g. Shih and Jeng (1999), dos Reis et al. (2005), Andreo et al. (2009). The strategy presented in this paper makes use of different LPV models, which despite of the conservatism they inherently carry, are attractive for several reasons.

Recently, a novel approach has been proposed in the framework of distributed control for decomposable LPV systems (Hoffmann et al., 2013). This methodology can be used to tackle distributed control problems for non-linear systems modeled as LPV-LFT systems, where stability and performance guarantees are ensured for switching and undirected topologies. Thus, strictly speaking these policies are not applicable to directed topologies; this issue will be discussed. This methodology is exploited here and implemented in simulation studies.

To the authors' knowledge, only a single LPV formation control method for non-holonomic vehicles has been proposed (see Yang et al. (2006)). In there, a polytopic LPV model is obtained via a linearisation in discrete time whereas the control problem is solved with a predictive control technique. Here, the formation control problem is tackled without linearising, in continuous time and by employing LPV-LFT models.

The paper is organized as follows. In Section 2, LPV representations are derived for a non-holonomic vehicle. In Section 3, a brief review of distributed control for decomposable LPV systems is provided, based on that a formation controller is designed. An LPV flatness-based controller for the leader vehicle is presented in 4. Simulation results are shown in Section 5. Finally, Section 6 concludes the paper.

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1.1 Notation

Standard notation is used throughout the paper. A diagonalizable time-varying Laplacian matrix, representing the interconnection topology of the multi-agent system, is denoted $\mathscr{L}(t)\in\mathbb{D}_N$, where $\mathbb{D}_N$ is the subset of diagonalizable real matrices of dimension $N\times N$ and its eigenvalues are $\lambda_i(t)\in\boldsymbol{\lambda}=[\underline{\lambda},\bar{\lambda}]\,\forall t$ and $i\in[1,N]$. An identity matrix of dimension n is denoted by I_n. The notation $M>0$ ($M<0$) means a matrix is positive (negative) definite. Time dependence is often dropped, e.g. $\theta=\theta(t)$. An upper LFT is denoted by $\Delta\star\left[\begin{smallmatrix}M_{11} & M_{12}\\ M_{21} & M_{22}\end{smallmatrix}\right]=M_{22}+M_{21}\Delta(I-M_{11}\Delta)^{-1}M_{12}$.

1.2 Problem Description

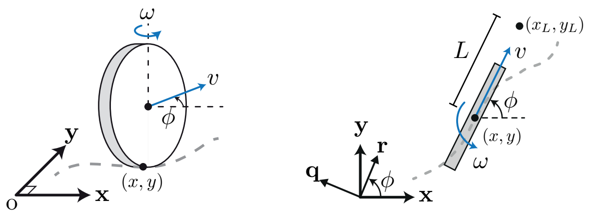
Consider a leader-follower multi-agent setting, where N non-holonomic agents communicate through a time-varying, communication graph with diagonalizable Laplacian $\mathscr{L}(t)$. Agents are labeled with $i=1\ldots N$, where $i=1$ stands for the leader agent and $i=2\ldots N$ for the follower agents. The problem is twofold. (Problem 1) Follower agents must converge to, and attain a geometric formation specified by $p_{\mathrm{f}}=[x_{\mathrm{f}_1}\,y_{\mathrm{f}_1}\ldots x_{\mathrm{f}_N}\,y_{\mathrm{f}_N}]^{\mathrm{T}}$, from any initial position and in the presence of disturbances. (Problem 2) Given a reference path $\Gamma_{\mathrm{r}}=[x_{\mathrm{r}}(t)\;y_{\mathrm{r}}(t)\;\phi_{\mathrm{r}}(t)]^{\mathrm{T}}$, the leader agent must track this path.

2. LPV REPRESENTATIONS OF A NON-HOLONOMIC VEHICLE

As an illustrative example of mobile robots subject to non-holonomic constraints, this section describes the kinematic model and possible LPV representations of a non-holonomic disk. It is well known, e.g. Lee et al. (2001), that a mobile robot can be represented by the kinematic equations of a non-holonomic disk (Fig. 1a)

$$\dot{x}=v\cos\phi\qquad\dot{y}=v\sin\phi\qquad\dot{\phi}=\omega,\tag{1}$$

where the states (x,y,ϕ) represent the center position and orientation of the disk on the Cartesian plane, while the inputs $(v,\,\omega)$ stand for the forward and angular velocities.



(a) Schematic. (b) Top view with distance L.

Fig. 1. A non-holonomic rolling disk.

The trajectories along the state space of the disk satisfy the non-holonomic constraint

$$\dot{x}\sin\phi-\dot{y}\cos\phi=0,\tag{2}$$

which basically states that motion is only allowed along the direction of its forward velocity, i.e. lateral motion is impossible for system (1). Note that if disturbances are introduced, such a constraint can be violated.

The underactuated nature of the plant is a direct consequence of the non-holonomic constraint (2). For this class of systems, the number of inputs equals the number of degrees-of-freedom minus the number of non-holonomic constraints, i.e. $2=3-1$. It is important to mention that despite of this constraint the whole state space is still reachable, only the motion to reach a point is restricted.

2.1 LPV Representation (Follower Agents)

The non-linear equations (1) can be directly represented by an LPV model just by employing ϕ as the scheduling signal. However, a key coupling is lost, since the input ω does not influence any more the position (x,y), i.e. v simply becomes a parameter-dependent input signal. Moreover, since the trigonometric terms in an LFT representation are not rational, ϕ can not be used a scheduling parameter. By employing the trigonometric terms as LFT parameters, conservatism is introduced (as the parameter space covers a complete square, instead of the unit circle contour), which easily renders the associated LMIs infeasible. As will be shown later, a reduced range of ϕ can alleviate this problem.

A slightly modified representation of system (1) will be used (Lawton et al., 2003). The difference lies in the fact that, instead of (x,y), a point (x_L,y_L) ahead of the disk, separated by a fixed and known distance L, is to be maneuvered (Fig. 1b). The kinematic equations associated with this point are:

$$\begin{aligned}\dot{x}_L&=v\cos\phi-\omega L\sin\phi\\ \dot{y}_L&=v\sin\phi+\omega L\cos\phi\\ \dot{\phi}&=\omega.\end{aligned}\tag{3}$$

Notice that the states (x_L,y_L) are now influenced directly by the angular velocity ω. The complexity of the system has been reduced. As an example, imagine it is desired to slightly move the point of interest towards the left. System (1) requires both input signals to achieve such a behaviour, while system (3) requires only ω to be slightly positive.

The non-holonomic behavior is still present; here given by the non-integrable constraint

$$\dot{x}_L\sin\phi-\dot{y}_L\cos\phi+L\omega=0.\tag{4}$$

System (3) still contains trigonometric terms, which, as mentioned before, can lead to infeasibility. Therefore the coordinate transformation

$$T_\phi=\begin{bmatrix}\cos\phi & \sin\phi & 0\\ -\sin\phi & \cos\phi & 0\\ 0 & 0 & 1\end{bmatrix}\tag{5}$$

is introduced. Transformation (5) provides a more appropriate representation. It rotates the reference system **x-y** counter-clockwise by a magnitude of ϕ radians to generate **r-q** (Fig. 1b). After it is applied to system (3), it yields

$$\begin{aligned}\dot{r}&=v+\omega q\\ \dot{q}&=\omega L-\omega r\\ \dot{\phi}&=\omega\,.\end{aligned}\tag{6}$$

The transformed system shows a clear advantage over the original one. The initial difficulties are not present any more, i.e. the couplings of the transformed system can easily be preserved by an adequate LPV representation and the affine dependence on the variables allows a non-conservative LFT representation of the system.

At this stage, attention is restricted to the position (r,q), i.e. the first 2 equations in (6); orientation ϕ is disregarded.

The reasoning is as follows; since the objective is to achieve a geometric formation, only consensus in (r, q) is required. Additionally, if a leader agent is to be followed (non-static scenario), the disks will orientate themselves towards the leader, just by attaining the formation relative to the moving leader, i.e. consensus in ϕ is unavoidable. This will become clear in Section 5.

An LPV system $D(\theta)$ representing the (r, q) dynamics is

$$\begin{bmatrix} \dot{r} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} 0 & \omega \\ -\omega & 0 \end{bmatrix} \begin{bmatrix} r \\ q \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & L \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix} \tag{7}$$

A standard LFT-LPV model of system (7) can be derived by choosing ω as the scheduling parameter, i.e. $\theta = \omega$, and $\omega \in [-5, 5]$. This representation will only be used to solve Problem 1. Also, notice the importance of $L \neq 0$, else controllability of q is degraded.

2.2 LPV Error Dynamics (Leader Agent)

The leader agent is supposed to track a reference trajectory, thus considering the error dynamics of the system will be helpful. A linearisation of such an error model will be used to design an LPV controller, which brings the leader back to its desired path when disturbances are present.

Consider system (1) and a reference disk governed by

$$\dot{x}_\mathrm{r} = v_\mathrm{r} \cos\phi_\mathrm{r} \qquad \dot{y}_\mathrm{r} = v_\mathrm{r} \sin\phi_\mathrm{r} \qquad \dot{\phi}_\mathrm{r} = \omega_\mathrm{r}. \tag{8}$$

Error signals for states and inputs are

$$e_x = x - x_\mathrm{r} \quad e_y = y - y_\mathrm{r} \quad e_\phi = \phi - \phi_\mathrm{r} \quad e_v = v - v_\mathrm{r} \quad e_\omega = \omega - \omega_\mathrm{r},$$

with $e = [e_x \, e_y \, e_\phi]^\mathrm{T}$, $e_u = [e_v \, e_\omega]^\mathrm{T}$ the error dynamics are $\dot{e} = f(e, u_e, v_\mathrm{r}, \phi_\mathrm{r})$, where

$$\begin{aligned} \dot{e}_x &= (e_v + v_\mathrm{r}) \cos(e_\phi + \phi_\mathrm{r}) - v_\mathrm{r} \cos\phi_\mathrm{r} \\ \dot{e}_y &= (e_v + v_\mathrm{r}) \sin(e_\phi + \phi_\mathrm{r}) - v_\mathrm{r} \sin\phi_\mathrm{r} \\ \dot{e}_\phi &= \omega - \omega_\mathrm{r} = e_\omega. \end{aligned}$$

An LPV model $H(\theta)$ could be derived from these nonlinear equations, which describes the full error dynamics. However, a Jacobian linearisation is preferred (about $e = 0$, $e_u = 0$). This yields a state space representation

$$\dot{e} \approx A(t)e + B(t)e_u,$$

where $A(t) = \left.\frac{\partial f}{\partial e}\right|_{\substack{e=0 \\ e_u=0}}$ and $B(t) = \left.\frac{\partial f}{\partial e_u}\right|_{\substack{e=0 \\ e_u=0}}$, leading to

$$\begin{bmatrix} \dot{e}_x \\ \dot{e}_y \\ \dot{e}_\phi \end{bmatrix} \approx \begin{bmatrix} 0 & 0 & -v_\mathrm{r} \sin\phi_\mathrm{r} \\ 0 & 0 & v_\mathrm{r} \cos\phi_\mathrm{r} \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} e_x \\ e_y \\ e_\phi \end{bmatrix} + \begin{bmatrix} \cos\phi_\mathrm{r} & 0 \\ \sin\phi_\mathrm{r} & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} e_v \\ e_\omega \end{bmatrix}. \tag{9}$$

Since it is intended to bring the disk to the path Γ_r, the linearized error dynamics of system (9) will be stabilized by an LPV controller, that drives the error between the actual state and the reference state back to 0.

The model $H(\theta)$ in equation (9) can be cast into an standard LPV-LFT system by employing $\theta = [\cos\phi_\mathrm{r} \; \sin\phi_\mathrm{r} \; v_\mathrm{r}]^\mathrm{T}$ as the scheduling parameters. A controller can be designed to stabilize the system about reference trajectories, which are only restricted to the specified parameter space. The assumed range of the parameters is given by $\phi_r \in {}^{\pi}/_{10}[1, 9]$ and $v_r \in [-1, 1]$.

3. FORMATION CONTROL (PROBLEM 1)

3.1 Distributed Control of Decomposable LPV Systems

In this section we provide a very brief review of recent results on distributed control for decomposable LPV systems, on which the approach proposed here is based. For a complete description, readers are referred to Hoffmann et al. (2013).

In the framework of decomposable systems (Massioni and Verhaegen, 2008), a matrix $\breve{M} \in \mathbb{R}^{Nm \times Nn}$ is said to be decomposable (d), if it can be represented as $\breve{M} = I_N \otimes M^\mathrm{d} + \mathscr{L} \otimes M^\mathrm{i}$, where $\mathscr{L}$ is an interconnection (i) matrix. We call an LPV system decomposable, if its system matrices are decomposable, w.r.t. the same interconnection matrix.

Note that multi-agent systems are a special case of the decomposable systems framework. In this sense, our multi-agent setting can easily be represented as a decomposable LPV system just by considering either the output/input matrices of the plant/controller as decomposable matrices, respectively. That is, either agents exchange information at the output stage or controllers exchange information at the input stage. Then, it is only necessary to set as 0 the corresponding interconnection matrices, e.g. $A^\mathrm{Ki} = 0$ in $\breve{A}^\mathrm{K} = I_N \otimes A^\mathrm{Kd} + \mathscr{L} \otimes A^\mathrm{Ki}$ (see below the general case).

Consider N identical LPV systems and associated controllers that communicate through $\mathscr{L}(t)$, and together form the decomposable LPV systems $\mathcal{P}(\Theta)$ and $\mathcal{K}(\Theta)$, respectively,

$$\underbrace{\begin{bmatrix} \dot{x}^\mathrm{P} \\ \hdashline z_\Theta^\mathrm{P} \\ z \\ y^\mathrm{P} \end{bmatrix} = \left[\begin{array}{c:ccc} \breve{A}^\mathrm{P} & \breve{B}_\Theta^\mathrm{P} & \breve{B}_\mathrm{p}^\mathrm{P} & \breve{B}_u^\mathrm{P} \\ \hdashline \breve{C}_\Theta^\mathrm{P} & \breve{D}_{\Theta\Theta}^\mathrm{P} & \breve{D}_{\Theta\mathrm{p}}^\mathrm{P} & \breve{D}_{\Theta u}^\mathrm{P} \\ \breve{C}_\mathrm{p}^\mathrm{P} & \breve{D}_{\mathrm{p}\Theta}^\mathrm{P} & \breve{D}_{\mathrm{pp}}^\mathrm{P} & \breve{D}_{\mathrm{p}u}^\mathrm{P} \\ \breve{C}_y^\mathrm{P} & \breve{D}_{y\Theta}^\mathrm{P} & \breve{D}_{y\mathrm{p}}^\mathrm{P} & \breve{D}_{yu}^\mathrm{P} \end{array}\right] \begin{bmatrix} x^\mathrm{P} \\ \hdashline w_\Theta^\mathrm{P} \\ w \\ u \end{bmatrix} \atop w_\Theta^\mathrm{P} = \Theta^\mathrm{P} z_\Theta^\mathrm{P}}_{\mathcal{P}(\Theta)}, \quad \underbrace{\begin{bmatrix} \dot{x}^\mathrm{K} \\ \hdashline z_\Theta^\mathrm{K} \\ u \end{bmatrix} = \left[\begin{array}{c:cc} \breve{A}^\mathrm{K} & \breve{B}_\Theta^\mathrm{K} & \breve{B}_y^\mathrm{K} \\ \hdashline \breve{C}_\Theta^\mathrm{K} & \breve{D}_{\Theta\Theta}^\mathrm{K} & \breve{D}_{\Theta y}^\mathrm{K} \\ \breve{C}_u^\mathrm{K} & \breve{D}_{u\Theta}^\mathrm{K} & \breve{D}_{uy}^\mathrm{K} \end{array}\right] \begin{bmatrix} x^\mathrm{K} \\ \hdashline w_\Theta^\mathrm{K} \\ y^\mathrm{P} \end{bmatrix} \atop w_\Theta^\mathrm{K} = \Theta^\mathrm{K} z_\Theta^\mathrm{K}}_{\mathcal{K}(\Theta)},$$

where all signals and matrices are of appropriate dimensions. The plant parameter block $\Theta^\mathrm{P} = \mathrm{diag}(\Theta_1^\mathrm{P} \ldots \Theta_N^\mathrm{P})$ contains the scheduling parameters $\theta_i(t)$ according to $\Theta_i^\mathrm{P} = (\theta_{i1} I_{r_1} \ldots \theta_{in_\theta} I_{r_{n_\theta}})$, where $\theta_i = [\theta_{i1} \ldots \theta_{in_\theta}]^\mathrm{T} \in \boldsymbol{\theta} \subset \mathbb{R}^{n_\theta} \forall t$, and r_i denotes the individual parameter repetition. Note that all N subsystems in the weighted plant $\mathcal{P}(\Theta)$ are assumed to have identical dynamics, while their local scheduling vectors θ_i can take different values.

The interconnection $\mathscr{L}(t)$ in $\mathcal{P}(\Theta)$ is inherited by $\mathcal{K}(\Theta)$. The closed-loop system $\mathcal{M}(\Theta)$ follows from the interconnection (under certain assumptions, (Hoffmann et al., 2013)) between $\mathcal{P}(\Theta)$ and $\mathcal{K}(\Theta)$, and is represented as

$$\begin{bmatrix} \dot{\xi} \\ \hdashline z_\Theta \\ z \end{bmatrix} = \left[\begin{array}{c:cc} \breve{A} & \breve{B}_\Theta & \breve{B}_\mathrm{p} \\ \hdashline \breve{C}_\Theta & \breve{D}_{\Theta\Theta} & \breve{D}_{\Theta\mathrm{p}} \\ \breve{C}_\mathrm{p} & \breve{D}_{\mathrm{p}\Theta} & \breve{D}_{\mathrm{pp}} \end{array}\right] \begin{bmatrix} \xi \\ \hdashline w_\Theta \\ w \end{bmatrix}, \qquad w_\Theta = \Theta z_\Theta \tag{10}$$

where again all signals and matrices are of appropriate dimensions. The block $\Theta = \mathrm{diag}(\Theta_1 \ldots \Theta_N)$ contains plant and controller parameters, where $\Theta_i = \mathrm{diag}\,(\Theta_i^\mathrm{P}, \Theta_i^\mathrm{K})$. It is not assumed $\Theta_i^\mathrm{P} = \Theta_i^\mathrm{K}$, however Θ_i^K depends also on θ_i.

After some transformations (Hoffmann et al., 2013) are applied to system (10), and since $\mathscr{L}(t)$ is symmetric (thus diagonalizable), it is possible to decompose (10) into N subsystems of the form $G_i(\theta_i) = \Theta_i \star \begin{bmatrix} G_{11}^\lambda & G_{12}^\lambda \\ G_{21}^\lambda & G_{22}^\lambda \end{bmatrix}$, with

$$\begin{bmatrix} G_{11}^\lambda & G_{12}^\lambda \\ G_{21}^\lambda & G_{22}^\lambda \end{bmatrix} = \begin{bmatrix} G_{11}^\mathrm{d} & G_{12}^\mathrm{d} \\ G_{21}^\mathrm{d} & G_{22}^\mathrm{d} \end{bmatrix} + \lambda_i \begin{bmatrix} G_{11}^\mathrm{i} & G_{12}^\mathrm{i} \\ G_{21}^\mathrm{i} & G_{22}^\mathrm{i} \end{bmatrix}, \tag{11}$$

where λ_i is the i-th eigenvalue of $\mathscr{L}(t)$. Thus, $\mathcal{M}(\Theta)$ can be cast as a single subsystem with LFT dependence on

Θ_i, and affine dependence on λ_i. The following theorem gives analysis conditions for stability, performance and robustness (Hoffmann et al., 2013).

Theorem 1. System (10) is stable with induced $\mathcal{L}_2$ gain less than γ, $\forall\theta_i \in \boldsymbol{\theta}$ and for symmetric switching interaction topologies represented by $\mathscr{L}(t)$, if there exist $X > 0$, and $\Pi = \Pi^{\mathrm{T}}$ that satisfy

$$\begin{bmatrix} * \\ \hdashline * \end{bmatrix}^{\mathrm{T}} \left[\begin{array}{c:cc} \Pi & 0 & 0 \\ \hdashline 0 & \begin{smallmatrix} 0 & X \\ X & 0 \end{smallmatrix} & 0 \\ 0 & 0 & \begin{smallmatrix} -\gamma & 0 \\ 0 & 1/\gamma \end{smallmatrix} \end{array}\right] \begin{bmatrix} G_{11}^{\lambda} & G_{12}^{\lambda} \\ I & 0 \\ \hdashline G_{21}^{\lambda} & G_{22}^{\lambda} \end{bmatrix} < 0, \quad \forall\lambda_i \in \boldsymbol{\lambda}$$

$$[\,*\,]^{\mathrm{T}}\,\Pi \begin{bmatrix} I \\ \Theta_i \end{bmatrix} > 0, \qquad \forall\theta_i \in \boldsymbol{\theta}.$$

Proof. See Hoffmann et al. (2013). □

Theorem 1 represents an extended and condensed version of the bounded real lemma for decomposable parameter-dependent systems where the full block S-procedure (Scherer, 2001) is exploited. Since the complexity of the problem is reduced to a single-agent level, the controller synthesis conditions follow from standard LPV techniques. To render the problem finite-dimensional, the multiplier Π must be structured, typically as a D or D-G scaling, e.g. Wu and Dong (2006).

3.2 Distributed Controller Design

This section presents a solution to Problem 1. Since Problem 1 and Problem 2 are independent, to this end it is not assumed that the leader is moving.

Consider N disks of type (7). Absolute position information is assumed in system (7), but it becomes relative after it passes through $\mathscr{L}(t)$. That is, availability of global information is not strictly required, since only relative information is to be used. In this sense, each disk has attached a reference system to its center position, and, due to the transformation T_ϕ, it is orientated towards its direction of motion. From this reference system the position of the neighbour disks is seen.

Since the leader agent does not receive information from other agents (otherwise it is not a leader), the distributed formation controller will have no effect on the leader. Agents are supposed to achieve the formation specified by the formation reference p_{f} (given with respect to **x**-**y**). Since the leader agent does not receive information, the formation is relative to it, thus w.l.o.g. $x_{\mathrm{f}1} = 0$, $y_{\mathrm{f}1} = 0$. The objective is to obtain a distributed LPV controller which performs this task.

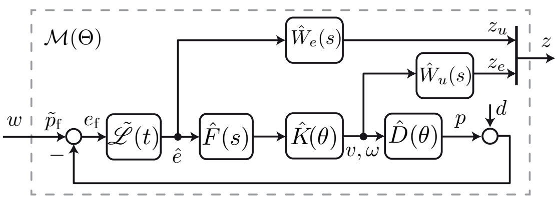


Fig. 2. Formation control problem configuration.

The overall system used for the formation control is shown in Fig. 2, where a mixed sensitivity approach is proposed to tune the formation error and reduce the control effort. Where $p = [r_1\,q_1 \dots r_N\,q_N]^{\mathrm{T}}$ and $\tilde{p}_{\mathrm{f}}$ is the transformed formation reference. The interconnection error signal passes then through a filter stage (first-order low pass filter with cutoff frequency at 100 rad/s) that is necessary to ensure solvability of the problem (Hoffmann et al., 2013). The objective of this filter is to move the interconnection $\mathscr{L}$ to the states.

All the hatted systems in Fig. 2 are in the form of, e.g. $\hat{F}(s) = I_N \otimes F(s)$, where $F(s)$ represents a single subsystem, i.e. interconnection between subsystems is only present trough $\mathscr{L}(t)$ (as is typical in multi-agent systems). An augmented laplacian $\tilde{\mathscr{L}}(t) = \mathscr{L}(t) \otimes I_2$ is defined to match the signal dimensions.

Theorem (1) provides analysis conditions for the closed loop system $\mathcal{M}(\theta)$ at a single-agent level, thus it can be seen as a standard LPV-LFT problem. Based on that, the methodology in Scherer (2001) was used to synthesize a controller at the single-agent level, which becomes distributed when it is placed at the network level. Notice that synthesis of this robust control scheme against changes in the topology needs to consider only the two extremes of the polytope formed by $\boldsymbol{\lambda}$.

Theorem 1 covers only symmetric switching topologies, thus rigorously speaking guarantees of stability and performance are lost for directed topologies, which is the case for a leader-follower setting. However, as will be shown in Section 5 the formulation approach has been concluded to be reliable, possibly due to the inherited conservatism.

The following filters were used to tune the sensitivity and control sensitivity

$$W_e = \mathrm{diag}(W_1, W_1) \quad W_u = \mathrm{diag}(W_2, W_2)$$
$$W_1 = \frac{s/3 + 0.1}{s + 0.001} \quad W_2 = \frac{s/5 + 2}{s + 1 \times 10^4}.$$

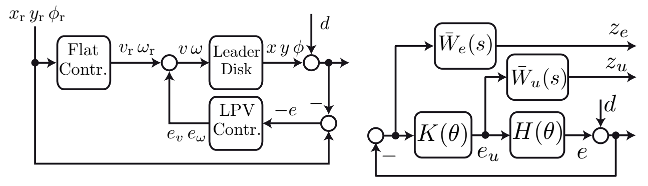
A constant quadratic Lyapunov function is obtained by minimizing the induced $\mathcal{L}_2$ norm of the mapping from w to z. The achieved bound was of $\gamma = 36.0$.

4. TRAJECTORY TRACKING (PROBLEM 2)

Problem 2 is considered in this section (trajectory tracking problem). Given a disk modeled as in Eq. (1), and a path defined by $\Gamma_{\mathrm{r}} = [x_{\mathrm{r}}(t)\ y_{\mathrm{r}}(t)\ \phi_{\mathrm{r}}(t)]^{\mathrm{T}}$, it is desired that the actual disk trajectory converges to the given reference, i.e.

$$\lim_{t\to\infty} (x(t), y(t)) = (x_{\mathrm{r}}(t), y_{\mathrm{r}}(t))$$

In order to solve the above problem, the configuration in Fig. 3a is proposed, where an LPV and flat control combination is used.



(a) Flat + LPV control.

(b) S/KS mixed sensitivity.

Fig. 3. Control strategy for the leader agent.

The objective of the LPV controller is to ensure robustness against disturbances. On the other hand, the flat controller is intended to provide the appropriate inputs to track the reference path.

Flat control (see Fliess et al. (1995)) is relatively simple when proper flat outputs are found. Since there are only two control inputs, only two flat outputs can be found for this rolling disk. Here are used

$$z_1 = x, \qquad z_2 = y.$$

It is straightforward to show that the following input signals track any given reference for the above flat outputs

$$v_{\mathrm{r}} = \sqrt{\dot{x}_{\mathrm{r}}^2 + \dot{y}_{\mathrm{r}}^2}, \qquad \omega_{\mathrm{r}} = \tan^{-1}(\dot{y}_{\mathrm{r}}/\dot{x}_{\mathrm{r}}). \qquad (12)$$

It is important to mention that the computation of reference signals depends on several factors, such as initial and final position and velocity, actuator saturation violations, etc. The computation of adequate reference signals $x_{\mathrm{r}}(t)$ and $y_{\mathrm{r}}(t)$ is out of the scope of this note. It is assumed that proper reference signals are provided.

The control methodology used to synthesize the LPV controller is well-known and can be found in Scherer (2001). A classical S/KS mixed sensitivity design is employed (Fig. 3b) by means of a constant Lyapunov function. The induced $\mathcal{L}_2$ performance obtained is $\gamma = 57.1$. The weighting filter used to shape the sensitivity and control sensitivity functions are

$$\bar{W}_e = \mathrm{diag}(W_{e_x}, W_{e_y}, W_{e_\phi}) \quad \bar{W}s_u = \mathrm{diag}(W_{e_v}, W_{e_\omega})$$

$$W_{e_x} = \frac{s/5 + 0.01}{s + 1\times10^{-4}} \; W_{e_y} = \frac{s/5 + 0.01}{s + 1\times10^{-3}} \; W_{e_\phi} = \frac{s/5 + 0.01}{s + 0.008}$$

$$W_{e_v} = 20\frac{s + 100}{s + 1\times10^{5}} \; W_{e_\omega} = 200\frac{s + 100}{s + 1\times10^{6}}$$

5. SIMULATION RESULTS

In this example, a network of $N = 5$ agents is studied. The agents communicate according to the different topologies shown in Fig. 4, where an arrow indicates flow of information from tail to head. That is, *head*-agent receives position of *tail*-agent. The topology switches randomly every 4 seconds. Although the communication matrix is not symmetric it is always assumed to be diagonalizable, which is the case for (a) to (e). This can be controlled in simulations, but in real applications one must rely on this assumption. However, experiments have shown successful results for non-diagonalizable interaction topologies, which suggest the approach has some degree of conservatism. New theoretical results have been obtained in this direction (Hoffmann et al., 2014).

A standard Laplacian is used to represent the interconnections in Fig. 4, where $\boldsymbol{\lambda} = [0, 5]\ \forall t$. For formation control purposes we use the distance $L = 0.1$, but for reference tracking, the central position of the leader is used.

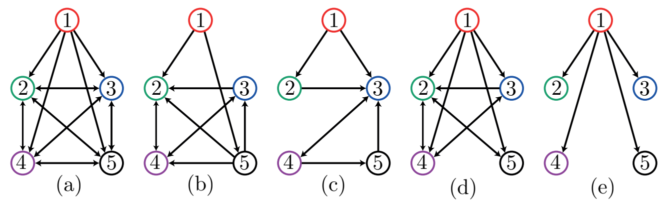


Fig. 4. Leader-follower topologies with 5 vehicles.

The follower agents initially are provided with a square formation reference, which changes at $t = 120$ s to a line formation reference, given by $p_{\mathrm{f}_1} = {}^1\!/_2[0\,0, \text{-}1\,1, 1\,\text{-}1, 1\,1, \text{-}1\,\text{-}1]^{\mathrm{T}}$ and $p_{\mathrm{f}_2} = {}^1\!/_4[0\,0, 0\,1, 0\,2, 0\,\text{-}2, 0\,\text{-}1]^{\mathrm{T}}$, respectively. The intended path to be followed by the leader agent, as well as by the followers, is supposed to be defined to lay in the parameter range of the scheduling parameters of the leader vehicle. However, in Fig. 5 one can see how those parameters evolve for a curved path, and clearly ϕ_{r} violates the assumption given in Section 4. Symmetry is exploited and when $\phi_{\mathrm{r}} < 0$, a rotation of 180 deg is performed, which leads again ϕ_{r} to range in the assumed parameter space.

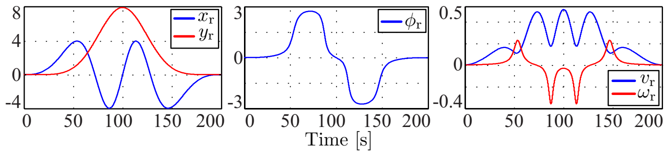


Fig. 5. Path parameters of trajectory to track.

Agents start at challenging initial conditions (0,-2,0), (2,-1,π/2), (-3,-2,π), (3,-2,-π/2), and (-1,-2,-π/2). Moreover, disturbances are introduced during the evolution of the trajectory. At time $\hat{t} = 65$ s, agent 1 is affected by $d_y = -2\sigma(t-\hat{t})$, and at $\hat{t} = 150$ s, agent 4 is perturbed by $d_y = d_x = -2\sigma(t-\hat{t})$, where $\sigma(t)$ is the unit step function. Disturbances are low-pass filtered through $(0.01s+1)^{-1}$.

In Fig. 7 the simulation, which runs for 200 s, is depicted. Colors are taken from Fig. 4 and the black dot indicates (x_L, y_L), while the white dot depicts for (x, y). Some observations can be made. The effect of the disturbance on the leader is propagated to the follower through the interconnection, and in an attempt to keep the formation, they move in the direction of the disturbance, however the leader agent quickly returns to the path. On the other hand, the disturbance introduced to the follower agent 4 is not seen by agent 2. This indicates that at that moment topology (c) is active. Furthermore, as expected, proper orientation of the followers disks occurs just by attaining the formation relative to the leader.

In Fig. 6, we observe in the upper plot that the leader vehicle successfully converges to the desired path and effectively rejects disturbances. Additionally, in the lower plot we conclude that formation is effectively achieved. The small peaks are due to changes in the topology, whereas large peaks are caused by the reference change and disturbance introduction.

An animation is available at
`www.tuhh.de/~soam2066/others/Animation_IFAC14.pdf`

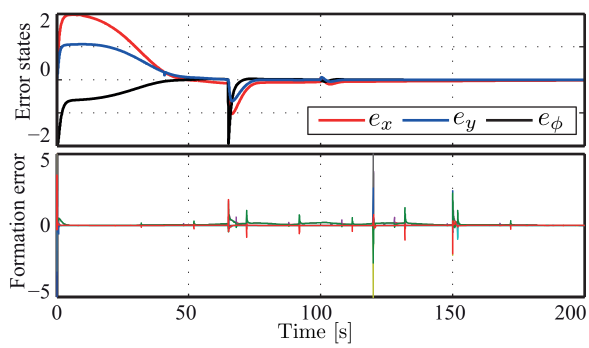


Fig. 6. Error signals: leader states and formation.

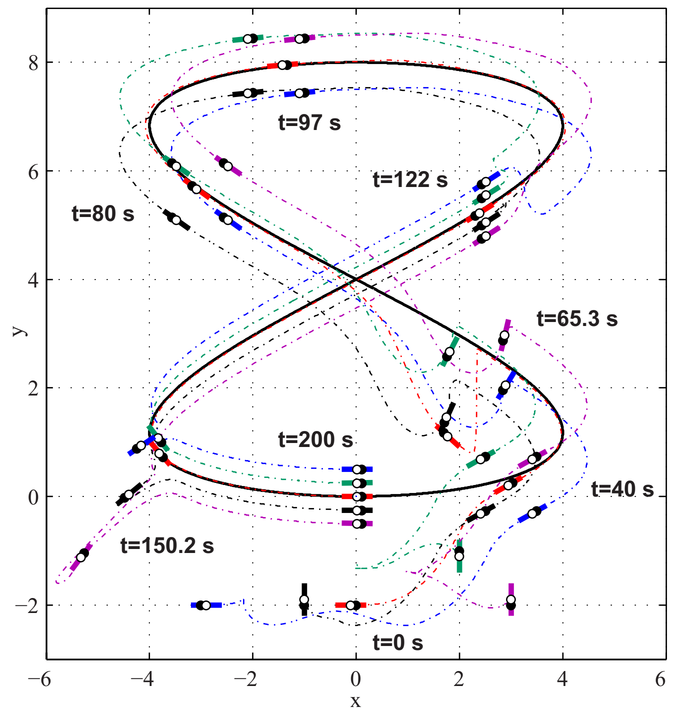


Fig. 7. Formation control and reference tracking (solid black line). Vehicles point towards the black circle.

6. CONCLUSIONS

In this paper a new control strategy for a formation control and path following problem of a group of non-holonomic vehicles is presented. The solution of the problem is proposed to be solved in two stages, by separating the leader from the followers. The main problems are analysed by considering different LPV models in LFT representation for the non-holonomic vehicles. The proposed scheme for formation control only requires a feedback signal containing a transformed vector position, although knowledge of the orientation is also required to perform such transformation. Simulations have shown the successful application of this approach where the overall system maintains the formation in the presence of disturbances and changes in the topology and tracks the desired reference path. The LPV distributed control synthesis method used, shows several attractive properties such as being a systematic design while ensuring a level of performance and stability as is typical in induced $\mathcal{L}_2$ control. Although the results are promising, stability and performance guarantees are strictly lost in a leader-follower setting, since the approach in Hoffmann et al. (2013) is valid only for undirected switching topologies. The approach proposed here has been extended in Mendez G. and Werner (2014) based on Hoffmann et al. (2014), where a second-order consensus problem is studied for directed and switching topologies and all guarantees are valid.

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