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Robust Sliding Mode Trajectory Tracking Controller for a Nonholonomic Spherical Mobile Robot

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Abstract: Based on dynamical modeling, robust trajectory tracking control of a spherical mobile robot is proposed. The spherical robot is composed of a spherical shell and three independent rotors which act as the inner driver mechanism. Owing to rolling without slipping assumption, the robot is subjected to two nonholonomic constraints. The state space representation of the system is developed using dynamical equations of the robot's motion. As the main contribution, a dynamical model based SMC (sliding mode controller) is designed for position control of the robot under parameters uncertainty and unmodeled dynamics. To decrease the chattering phenomena originated by the sign function, the well-known boundary layer technique is imposed on the SMC. The control gains are determined through using Lyapanov's direct method in such a way that the robustness and to zero convergence of the controller's tracking error are guaranteed. Wide range computer simulations are performed to show the significant tracking performance of the proposed SMC in particular against parameters uncertainty and white Gaussian noises. The simulation results show the significant performance of the designed nonlinear control system in trajectory tracking control of the spherical robot even in the presence of parameters uncertainty and unmodeled dynamics.

Keywords: Spherical robot, Nonholonomic System, Sliding mode control, Boundary Layer, Parameters' uncertainty.

1. INTRODUCTION

A spherical mobile robot composed of a spherical shell and an inner driver mechanism is a new kind of mobile robots, which its applications have increased in past two decades. The driver mechanism is installed inside the spherical shell to generate the robot's motion. This structure provides a stable locomotion and some other advantages rather than the traditional types of mobile robots, Suomela et al. (2006).

Several types of inner driver mechanism have been proposed for spherical robots in recently documented researches which all of them generate the robot's motion either by changing the gravity center of the spherical shell or by changing the angular momentum of the robot. A mobile vehicle, Bicchi et al. (1997), a wheeled mass, Halme et al. (1996), four unbalanced masses, Javadi (2002) and a two DOF pendulum, Zhan et al. (2006) are examples of proposed driver mechanisms, which generate the robot's motion by changing the gravity center of the robot. On the other hand, two rotors, Bhattacharya (2000), three DOF gyro, Otani et al. (2006) and three perpendicular rotors, Azizi (2013) have been designed as inner driver mechanism in preceding studies that generate the robot's motion based on the angular momentum conservation principle.

Although, some investigations have been developed on spherical robots recently, motion control of these robots is still one of the major problem in robotic researches. Since it is assumed that the spherical shell rolls on the ground surface without any slipping, its motion is subjected to two nonholonomic constraints. On the other hand, according to Brocket's theorem (1983), the stabilization of the equilibrium points of the nonholonomic systems through time invariant state-feedback is not possible. By the way, for the control of the spherical robot, Zhao et al. have derived the dynamical model of the spherical robot merely for straight line motions and therefore, a PID controller has been proposed for the robot's motion on straight line trajectories. The control of the spherical robot by uses of a pendulum as a control actuator has been developed by feedback linearization method for straight line trajectories by Liu et al. (2008). Furthermore, trajectory tracking control of the spherical robot on straight paths has been investigated using SMC method, Liu et al. (2012), adaptive hierarchical sliding mode approach, Yue (2013); and also using combined adaptive neuro-fuzzy and SMC method, Kayacan et al. (2013).

The control problem of the spherical robot on curvilinear trajectories has been studied using kinematical and simplified dynamical model of the robot and some simplifying assumptions. Cai et al. have designed a two-state trajectory tracking control system for the kinematical model of the spherical robot based on shunting model of neurodynamics and Lyapunov's direct method. A SMC has been developed for the linearized model of the spherical robot without considering the dynamical effects of the inner mechanism by

Lui et al. Kayacan et al. (2012) have used fuzzy control approach to control the spherical robot's motion based on decoupled dynamical model and by neglecting the transversal and longitudinal rotation of the robot. Through neglecting the rotation of the spherical shell around the vertical axis and based on the dynamical model, simplified tracking control of the spherical robot in horizontal plane has been investigated using: real-time fuzzy guidance method by Cai (2012), back-stepping based trajectory tracking by Zhan (2008) and constant velocity PD sliding mode controller by Zheng et al. (2011).

According to the above mentioned literature review, the trajectory tracking control in 2-dimentional plane is a major problem with the spherical kind of nonholonomic mobile robots that should be solved completely. Although, several linear and nonlinear control strategies based on the kinematical model, linearized model or simplified dynamical model of the spherical robot have been introduced in the literature, these methods are not practically feasible considering highly complicated nonlinear structure of the robot's mathematical model, Kayacan et al. (2013). On the other hand, few researchers have focused on design of the nonlinear control systems based on full dynamical model of the robot without simplifying assumptions. By the way, the robustness of the designed nonlinear controller against parameters' uncertainty and noisy measurements is significant in practical applications, which have not been considered in preceding research works.

In this paper, the spherical robot comprising of three independent rotors is investigated. Using the dynamical model of the spherical robot without any simplifying assumptions derived by Azizi et al. (2013), the second order mathematical model of the robot is obtained in the standard affine form. A nonlinear SMC is designed for trajectory tracking control of the robot and the boundary layer technique is used to remove the chattering phenomena, which is originated by discontinuous switching control term in the neighbour of the sliding surface. The convergence of tracking error to zero and the robustness of the proposed controller are proved by Lyapanov's direct method. Besides, wide range computer simulations are performed to assess the tracking performance and robustness of SMC against the parameters uncertainty and unmodeled dynamics.

2. SPHERICAL ROBOT MODELING

The schematic model of the spherical robot with three independent rotors as the inner driver mechanism is shown in Fig. 1. The robot is composed of a spherical shell, three rotors and some counter weights to balance the rotors' weight. The driving rotors are connected to the inner surface of the spherical shell and rotate by use of three revolute actuators. Furthermore, it is assumed that the counter weights and other instruments are installed inside the spherical shell in such a way that the gravity center of the robot coincides with the geometric center of the spherical shell. In this case, based on the angular momentum conservation principle, the robot's motion could be controlled by the three considered actuators.

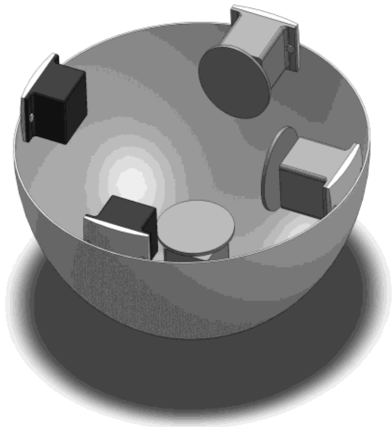

Fig. 1. The construction of the spherical robot

2.1 Kinematic Model

To determine the robot's position and configuration, three coordinate frames are considered. Frame {1} is an inertial reference frame. The origin of frame {2} is fixed to the geometric center of the spherical shell and the its axes remain parallel to the axes of frame {1}. The body frame {3} is fixed to the spherical shell and coincides to frame {2} if the rotation angles of the robot are zero values. By defining x and y as the components of 2-dimentional position vector of the spherical shell with respect to frame {1} and q_0, q_1, q_2, q_3 as the components of a unit quaternion to represent the orientation of the frame {3} with respect to frame {2}, the kinematical equations of the robot are obtained as follow (Azizi et al.):

$$ {}^2_3q = {}^1_3q = q_0 + q_1\mathbf{i} + q_2\mathbf{j} + q_3\mathbf{k} \tag{1} $$

$$ \left\| {}^2_3q \right\|^2 = q_0^2 + q_1^2 + q_2^2 + q_3^2 = 1 \tag{2} $$

$$ \begin{bmatrix} {}^1\omega_{3x} \\ {}^1\omega_{3y} \\ {}^1\omega_{3z} \end{bmatrix} = \begin{bmatrix} -2q_1 & 2q_0 & -2q_3 & 2q_2 \\ -2q_2 & 2q_3 & 2q_0 & -2q_1 \\ -2q_3 & -2q_2 & 2q_1 & 2q_0 \end{bmatrix} \begin{bmatrix} \dot{q}_0 \\ \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix} \tag{3} $$

$$ \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = 2R_s \begin{bmatrix} -q_2 & q_3 & q_0 & -q_1 \\ q_1 & -q_0 & q_3 & -q_2 \end{bmatrix} \begin{bmatrix} \dot{q}_0 \\ \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix} \tag{4} $$

Where ${}^1\vec{\omega}_3$ is the angular velocity vector of frame {3} with respect to frame {1} and R_S is the radius of the spherical shell. Equations (2) and (4) denote one algebraic and two differential constraints between the considered position and rotational variables. These two differential constraints (4) are indeed non-integrable equations, which make the robot a nonholonomic system.

2.2 Dynamic Model

Dynamical equations of the robot have been obtained using Kane's method, Azizi et al. (2013), and are rewritten in the following form.

$$
\begin{aligned}
& f_i(q_0,q_1,q_2,q_3,\dot q_0,\dot q_1,\dot q_2,\dot q_3,\ddot q_0,\ddot q_1,\ddot q_2,\ddot q_3,\\
&\quad \Omega_x,\Omega_y,\Omega_z,\dot\Omega_x,\dot\Omega_y,\dot\Omega_z)=0 \qquad (i=1,2,3)\\
& f_4\left(q_0,q_1,q_2,q_3,\ddot q_0,\ddot q_1,\ddot q_2,\ddot q_3,\dot\Omega_x,T_x\right)=0\\
& f_5\left(q_0,q_1,q_2,q_3,\ddot q_0,\ddot q_1,\ddot q_2,\ddot q_3,\dot\Omega_y,T_y\right)=0\\
& f_6\left(q_0,q_1,q_2,q_3,\ddot q_0,\ddot q_1,\ddot q_2,\ddot q_3,\dot\Omega_z,T_z\right)=0
\end{aligned}
\tag{5}
$$

Where $\Omega_x,\Omega_y,\Omega_z$ are angular velocities of the rotors with respect to the spherical shell; T_x,T_y,T_z are actuators' torque inputs of control system and $f_1(\cdot)$ to $f_6(\cdot)$ are nonlinear scalar terms. These terms are highly complicated and are not shown here due to the space limitation (see Azizi et al.). In (5), there exist none of the components of position vector x and y. Therefore, using (5), second time derivative of (2) and first time derivative of (4), the mathematical model of the spherical robot in terms of all the kinematical variables, could be written in the following standard form.

$$
\begin{aligned}
&\ddot x = -2R_S\left(\ddot q_0 q_2 - \ddot q_1 q_3 - \ddot q_2 q_0 + \ddot q_3 q_1\right)\\
&\ddot y = 2R_S\left(\ddot q_0 q_1 - \ddot q_1 q_0 + \ddot q_2 q_3 - \ddot q_3 q_2\right)\\
&\left\|{}_3^2\dot q\right\|^2 + q_0\ddot q_0 + q_1\ddot q_1 + q_2\ddot q_2 + q_3\ddot q_3 = 0
\end{aligned}
\tag{6}
$$

$$\mathbf{q} = \left[x,y,q_0,q_1,q_2,q_3,\theta_x,\theta_y,\theta_z\right]^T \tag{7}$$

$$
\begin{aligned}
&\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{V}(\mathbf{q},\dot{\mathbf{q}}) + \mathbf{G}(\mathbf{q}) = \boldsymbol{\tau}\\
&\boldsymbol{\tau} = \left[0,0,0,0,0,T_x,T_y,T_z,0\right]^T
\end{aligned}
\tag{8}
$$

Where $\theta_x,\theta_y,\theta_z$ are rotors' angular displacement with respect to the spherical shell, $\mathbf{q}\in R^9$ denote the state variable vector, $\mathbf{M}(\mathbf{q})\in R^{9\times 9}$ is the mass matrix of the robot, $\mathbf{V}(\mathbf{q},\dot{\mathbf{q}})\in R^9$ is the vector of Coriolis and centrifugal forces, $\mathbf{G}(\mathbf{q})\in R^9$ is the vector of gravitational effects and $\boldsymbol{\tau}\in R^9$ is the vector of impressed torque. Since $\mathbf{M}(\mathbf{q})$ is a nonsingular matrix, the robot mathematical model could be rewritten in the following affine form by multiplying both side of (8) by $\mathbf{M}^{-1}$.

$$\ddot{\mathbf{q}} = \mathbf{f}(\mathbf{q},\dot{\mathbf{q}}) + \mathbf{g}(\mathbf{q})\mathbf{u} \tag{9}$$

Where $\mathbf{f}(\mathbf{q},\dot{\mathbf{q}})$ is a (9×1) vector, $\mathbf{g}(\mathbf{q})$ is a (9×3) distribution matrix and $\mathbf{u}$ is the 3-dimentional torque vector.

3. CONTROL SYSTEM DESIGN

Considering the parameters uncertainty, measurement noises and disturbance torques which affect a real spherical robot, (8) is completed in the following form, Keighobadi (2012).

$$
\begin{aligned}
&(\mathbf{M}+\Delta\mathbf{M})\ddot{\mathbf{q}} + \mathbf{V}(\mathbf{q},\dot{\mathbf{q}}) + \Delta\mathbf{V}(\mathbf{q},\dot{\mathbf{q}})\\
&+\mathbf{G}(\mathbf{q}) + \Delta\mathbf{G}(\mathbf{q}) + \mathbf{D} = \boldsymbol{\tau}
\end{aligned}
\tag{10}
$$

Where $\mathbf{M},\mathbf{V}$ and $\mathbf{G}$ stand for the estimated values of mass matrix, centrifugal and Coriolis force vector and gravitational force vector, respectively. $\Delta\mathbf{M},\Delta\mathbf{V}$ and $\Delta\mathbf{G}$ denote the effects of parameters uncertainty and unmodeled dynamics which are considered unknown and bounded values and the exogenous input vector, $\mathbf{D}$ stands for disturbances and the measurements noise effects. According to (10), the affine model (9) is completed as:

$$\ddot{\mathbf{q}} = \mathbf{f}(\mathbf{q},\dot{\mathbf{q}}) + \Delta\mathbf{f}(\mathbf{q},\dot{\mathbf{q}}) + \left(\mathbf{g}(\mathbf{q}) + \Delta\mathbf{g}(\mathbf{q})\right).\mathbf{u} \tag{11}$$

Where $\Delta\mathbf{f}(\mathbf{q},\dot{\mathbf{q}})$ and $\Delta\mathbf{g}(\mathbf{q})$ are unknown but 2-norm bounded terms of state variables.

3.1 Trajectory tracking control system

To design a robust trajectory tracking control system of the spherical mobile robot, the output vector is considered as:

$$\mathbf{H} = \begin{bmatrix} x & y & \phi\end{bmatrix}^T = \begin{bmatrix} x & y & \int {}_3^2\omega_z dt\end{bmatrix}^T \tag{12}$$

Where ϕ denotes time integral of the robot's angular velocity around the vertical axis, z. The second time derivative of the output vector is obtained as:

$$\ddot{\mathbf{H}} = \begin{bmatrix} \ddot x & \ddot y & -\ddot q_0 q_3 - \ddot q_1 q_2 + \ddot q_2 q_1 + \ddot q_3 q_0\end{bmatrix}^T \tag{13}$$

Through substituting $\ddot x,\ddot y$ and ${}_3^2\alpha_z$ from (11) in (13), the control inputs $\mathbf{u}$ appears. Therefore, the system (9) is of relative degree two. So, Equation (13) could be written in the following form:

$$\ddot{\mathbf{H}} = \begin{bmatrix} \ddot x \\ \ddot y \\ {}_3^2\alpha_z\end{bmatrix} = \mathbf{H_1}(\mathbf{q},\dot{\mathbf{q}}) + \mathbf{H_2}(\mathbf{q})\mathbf{u} \tag{14}$$

Where $\mathbf{H_1}\in R^3, \mathbf{H}_2\in R^{3\times 3}$ are obtained from (9). To design the SMC, the sliding surfaces $\mathbf{S}(t)=0$ is considered as follows.

$$\mathbf{S}(t) = \begin{bmatrix} \dot{\tilde x} + \lambda_1\tilde x \\ \dot{\tilde y} + \lambda_2\tilde y \\ \dot{\tilde\phi} + \lambda_3\tilde\phi\end{bmatrix} \tag{15}$$

Where $\tilde{\cdot}$ stands for the tracking error of the corresponding output variable and the sliding surface parameters, $\lambda_1,\lambda_2,\lambda_3$ are strictly positive and fixed values. The time derivative of the sliding surface vector is obtained as:

$$\dot{\mathbf{S}}(t) = \begin{bmatrix} \ddot x - \ddot x_d + \lambda_1\dot{\tilde x} \\ \ddot y - \ddot y_d + \lambda_2\dot{\tilde y} \\ {}_3^2\alpha_z - {}_3^2\alpha_{zd} + \lambda_3\dot{\tilde\varphi}\end{bmatrix} = \ddot{\mathbf{H}}\text{-}\ddot{\mathbf{H}}_{\mathbf{d}} + \boldsymbol{\Lambda}\dot{\tilde{\mathbf{H}}} \tag{16}$$

Where $\mathbf{H}_d$ is the vector of the desired outputs and $\boldsymbol{\Lambda}$ is a diagonal matrix of $\lambda_1,\lambda_2,\lambda_3$. Substituting (14) in (16) leads to:

$$\dot{\mathbf{S}}(t) = \mathbf{H_1}(\mathbf{q},\dot{\mathbf{q}}) + \mathbf{H_2}(\mathbf{q})\mathbf{u} - \ddot{\mathbf{H}}_{\mathbf{d}} + \boldsymbol{\Lambda}\dot{\tilde{\mathbf{H}}} \tag{17}$$

To achieve prefect trajectory tracking, $\dot{\mathbf{S}}(t)$ should remain zero during the robot's motion. Therefore, considering $\dot{\mathrm{S}}(t)=0$

in (17) and using the nominal model of the robot (9), the following equivalent control law is obtained.

$$\mathbf{u_{eq}} = \mathbf{H_2^{-1}}(\mathbf{q})\left[\ddot{\mathbf{H}}_\mathbf{d} - \Lambda\dot{\tilde{\mathbf{H}}} - \mathbf{H_1}(\mathbf{q},\dot{\mathbf{q}})\right] \tag{18}$$

Furthermore, the tracking error should reach the sliding surface in finite time and move along it to the origin. Therefore, the whole control input vector is assumed as follow.

$$\mathbf{u} = \mathbf{u_{eq}} + \mathbf{u_s} \tag{19}$$

Where, $\mathbf{u_s}$ is determined by guaranteeing the global stability of the SMC against the uncertainties and unmodeled dynamics. Therefore, the following positive definite function is considered as a Lyapanov function candidate.

$$V(t) = \frac{1}{2}\mathbf{S^T S} \tag{20}$$

The time derivative of (20) along the system trajectories yields:

$$\dot{V}(t) = \mathbf{S^T\dot{S}} = \mathbf{S^T}\,[\,\mathbf{H_1}(\mathbf{q},\dot{\mathbf{q}}) + \Delta\mathbf{H_1}(\mathbf{q},\dot{\mathbf{q}}) + (\mathbf{H_2}(\mathbf{q}) + \Delta\mathbf{H_2}(\mathbf{q}))(\mathbf{u_{eq}} + \mathbf{u_s}) - \ddot{\mathbf{H}}_\mathbf{d} + \Lambda\dot{\tilde{\mathbf{H}}}] \tag{21}$$

Where $\Delta\mathbf{H_1}(\mathbf{q},\dot{\mathbf{q}}), \Delta\mathbf{H_2}(\mathbf{q})$ are determined according to (11). Substituting $\mathbf{u_{eq}}$ from (18) in (21) leads to:

$$\dot{V}(t) = \mathbf{S^T}\{\Delta\mathbf{H_1}(\mathbf{q},\dot{\mathbf{q}}) + \Delta\mathbf{H_2}(\mathbf{q})\mathbf{H_2^{-1}}(\mathbf{q})\times \left[\ddot{\mathbf{H}}_\mathbf{d} - \Lambda\dot{\tilde{\mathbf{H}}} - \mathbf{H_1}(\mathbf{q},\dot{\mathbf{q}})\right] + (\mathbf{H_2}(\mathbf{q}) + \Delta\mathbf{H_2}(\mathbf{q}))\mathbf{u_s}\} \tag{22}$$

Using 2-norm operator $\|\cdot\|$, on (22) results in.

$$\begin{aligned}\dot{V}(t) = \mathbf{S^T\dot{S}} \le &\|\mathbf{S^T}\|\cdot\|\Delta\mathbf{H_1}(\mathbf{q},\dot{\mathbf{q}})\| + \|\mathbf{S^T}\|\cdot\|\Delta\mathbf{H_2}(\mathbf{q})\|\cdot\|\mathbf{H_2^{-1}}(\mathbf{q})\|\|\ddot{\mathbf{H}}_\mathbf{d}\| \\ &+ \|\mathbf{S^T}\|\cdot\|\Delta\mathbf{H_2}(\mathbf{q})\|\cdot\|\mathbf{H_2^{-1}}(\mathbf{q})\|\cdot\|\Lambda\dot{\tilde{\mathbf{H}}}\| + \\ &\|\mathbf{S^T}\|\cdot\|\Delta\mathbf{H_2}(\mathbf{q})\|\cdot\|\mathbf{H_2^{-1}}(\mathbf{q})\|\cdot\|\mathbf{H_1}(\mathbf{q},\dot{\mathbf{q}})\| + \mathbf{S^T}(\mathbf{H_2}(\mathbf{q}) + \Delta\mathbf{H_2}(\mathbf{q}))\mathbf{u_s}\end{aligned} \tag{23}$$

To achieve the global asymptotic stability, $\dot{V}(t)$ must be negative. Therefore $\mathbf{u_s}$ is proposed as follow.

$$\mathbf{u_s} = (\mathbf{H_2}(\mathbf{q}) + \Delta\mathbf{H_2}(\mathbf{q}))^{-1}\begin{bmatrix} -K_1\,\mathrm{sgn}(S_1) \\ -K_2\,\mathrm{sgn}(S_2) \\ -K_3\,\mathrm{sgn}(S_3)\end{bmatrix} \tag{24}$$

Where K_1, K_2 and K_3 are positive gains which are determined such that $\dot{V}(t)$ become negative. Using (24) in (23) leads to:

$$\begin{aligned}\dot{V}(t) = \mathbf{S^T\dot{S}} \le &\|\mathbf{S^T}\|\cdot\|\Delta\mathbf{H_1}(\mathbf{q},\dot{\mathbf{q}})\| + \\ &\|\mathbf{S^T}\|\cdot\|\Delta\mathbf{H_2}(\mathbf{q})\|\cdot\|\mathbf{H_2^{-1}}(\mathbf{q})\|\|\ddot{\mathbf{H}}_\mathbf{d}\| + \\ &\|\mathbf{S^T}\|\cdot\|\Delta\mathbf{H_2}(\mathbf{q})\|\cdot\|\mathbf{H_2^{-1}}(\mathbf{q})\|\cdot\|\Lambda\dot{\tilde{\mathbf{H}}}\| + \\ &\|\mathbf{S^T}\|\cdot\|\Delta\mathbf{H_2}(\mathbf{q})\|\cdot\|\mathbf{H_2^{-1}}(\mathbf{q})\|\cdot\|\mathbf{H_1}(\mathbf{q},\dot{\mathbf{q}})\| \\ &- K_1|S_1| - K_2|S_2| - K_3|S_3| < 0\end{aligned} \tag{25}$$

Using the fact that $\|\mathbf{S}\| \le |S_1| + |S_2| + |S_3|$ in (25), K_1, K_2 and K_3 are obtained according to the upper bounds of the parameters' uncertainty and unmodeled dynamics to guarantee the stability of the proposed control system.

3.1 Smoothing the control inputs

Although the switching control law (24) and (31) are robust against the parameters uncertainty and unmodeled dynamics, the chattering phenomena may also occur due to using discontinuous sign function in the sliding control. Chattering phenomenon may lead to high frequency control inputs and resonance the vibrations of the system's components and therefore should be removed. To remove the chattering phenomena the boundary layer technique, is used in this paper. Therefore, in the boundary layer of thickness ε around the origin, the sign terms are replaced by linear continuous terms as demonstrated in Fig. 2. In this way, the smoother control inputs are obtained and the chattering phenomena are reduced. The thickness of the boundary layer affects the tracking performance and the robustness of the controller against uncertainties. Therefore, the thickness of the boundary layer is designed in such a way that the robustness of the SMC is considerable and the control torques are sufficiently smooth. Therefore, $\mathbf{u}_s$ could be rewritten as follow:

$$\mathbf{u_s} = (\mathbf{H_2}(\mathbf{q}) + \Delta\mathbf{H_2}(\mathbf{q}))^{-1}\begin{bmatrix} -K_1 sat(S_1/\varepsilon) \\ -K_2 sat(S_2/\varepsilon) \\ -K_3 sat(S_3/\varepsilon)\end{bmatrix} \tag{26}$$

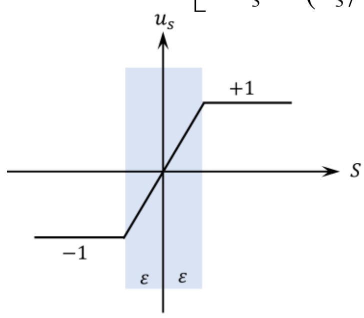

Fig. 2. Interpolated control inputs inside the boundary layer

4. COMPUTER SIMULATION

The performance of the designed trajectory tracking control system against the parameters' uncertainty and unmodeled dynamics is investigated by computer simulations. The nominal values of the robot's physical parameters are used according to table 1. It is assumed that the mass, the radius and the moment of inertia of the spherical shell are not exactly known and therefore, the corresponding nominal values together with upper bound of their uncertainties are used in the SMC system. Furthermore, to consider the effects of unmodeled dynamics and measurements noises, the measured values of the simulated state variables are gathered with Gaussian white noise.

To assess the trajectory tracking performance and robustness of the SMC against parameter uncertainty and measurements noises, a circular reference trajectory in x-y plane is considered as follow.

$$x_d = 2\cos(.2\pi t)\ ;\ y_d = 2\sin(.2\pi t) \tag{26}$$

Table 1: Nominal values of the robot's parameters

Parameter	Nominal Value	Uncertainty
M_S	$9\,kg$	$\Delta M_S = 10\% M_S$
R_S	$.55m$	$\Delta R_S = 20\% R_S$
$l_i \quad (i=1,2,3)$	$.35m$	-
I_S	$\begin{bmatrix} 1.815 & 0 & 0 \\ 0 & 1.815 & 0 \\ 0 & 0 & 1.815 \end{bmatrix} kg.m^2$	$\Delta I_S = 10\% I_S$
$M_{ri} \; (i=1,2,3)$	$1\,kg$	-
I_r	$\begin{bmatrix} 0.05 & 0 & 0 \\ 0 & 0.05 & 0 \\ 0 & 0 & 0.1 \end{bmatrix} kg.m^2$	-

It is assumed that, the middle ring of the spherical shell should roll along the desired reference path. Therefore, the desired trajectory of angular velocity of the spherical shell around the vertical axis is considered as:

$$ {}_3^2\omega_{zd} = \frac{V_S}{R_C} = \frac{\sqrt{\dot{x}_d^2 + \dot{y}_d^2}}{R_C} = 0.2\pi \quad ; \quad {}_3^2\alpha_{zd} = 0 \qquad (27) $$

Where V_S and R_C denote the velocity of the geometric center of the spherical shell and the curvature radius of the trajectory, respectively. Considering the following initial off-tracks in the state variables and using suitable controller gains as $\begin{bmatrix} K_1 & K_2 & K_3 \end{bmatrix} = \begin{bmatrix} 3 & 3 & 3 \end{bmatrix}$, the thickness of boundary layer as, $\varepsilon = .4$ and $\mathbf{\Lambda} = diag(2,2,2)$, the simulation results are obtained as shown in Figs. 3 to 8.

$$ \begin{aligned} &\left[x, y, r_1, r_2, r_3, r_4, \theta_x, \theta_y, \theta_z\right] = [0,0,0,0,0,1,0,0,0] \\ &\left[\dot{x}, \dot{y}, \dot{r}_1, \dot{r}_2, \dot{r}_3, \dot{r}_4, \Omega_x, \Omega_y, \Omega_z\right] = [0,0,0,0,0,0,0,0,0] \end{aligned} \qquad (28) $$

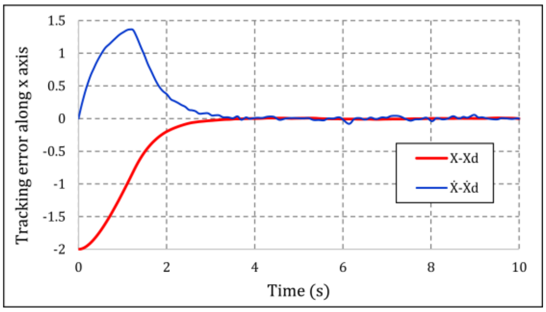

Fig. 3. Tracking error along x direction

5. CONCLUSION

In this paper, robust trajectory tracking control of a nonholonomic spherical mobile robot with three independent actuators has been investigated. The state space model of the robot has been obtained using the dynamical equations of robot and the two nonholonomic kinematical constraints without any simplifying assumptions. For purpose of trajectory tracking control of the robot, nonlinear SMC has been designed. Using the dynamical modelling as the design base of the nonlinear SMC, all the inertial, Coriolis and centrifugal effects are considered in the control actions. Using Lyapanov's direct method in design and analysis of SMC, the global stability of the control systems and the convergence of tracking errors to zero have been obtained. Furthermore, by use of boundary layer technique, the discontinuous switching terms of the SMC have been replaced by linear continuous terms to decrease the chattering effects.

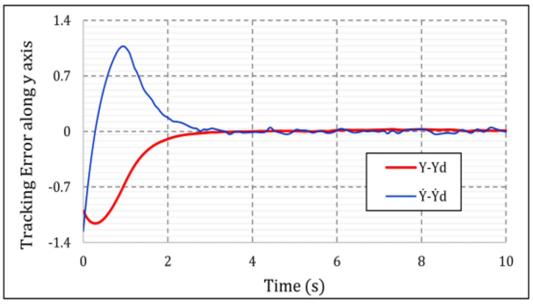

Fig. 4. Tracking error along y direction

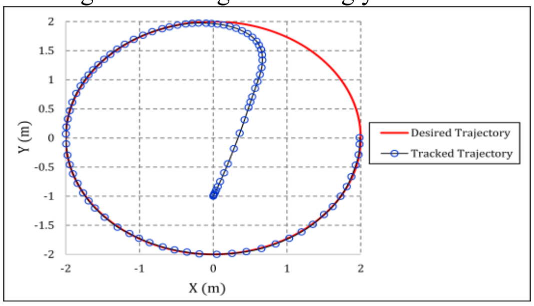

Fig. 5. Robot circular trajectory in x-y plane

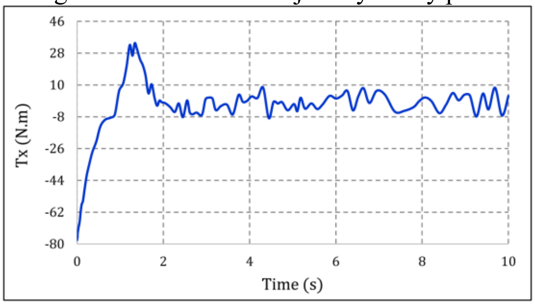

Fig. 6. Control torque, T_x for tracking of circular trajectory

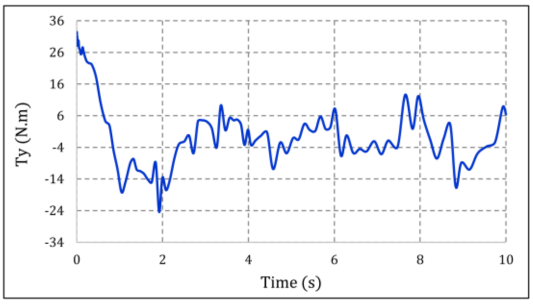

Fig. 7. Control torque, T_y for tracking of circular trajectory

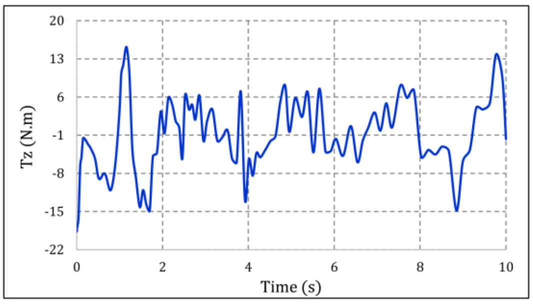

Fig. 8. Control torque, T_z for tracking of circular trajectory

Through computer simulations, the trajectory tracking performance of the proposed SMC has been shown. Owing the global stability and the robust nature of the SMC, the accurate tracking performance has been obtained through considering large position and velocity off-tracks, large parameters uncertainty and 3% white Gaussian noises on measured state variables of the robot. The results show that the robot could track the desired position and velocity trajectory as well as the angular velocity of shell around the vertical axis in the existence of parameters' uncertainty and noisy measurements. Therefore, due to the stability and the robustness, the proposed SMC systems could be implemented on the spherical robot in real world applications. Furthermore, the capability of the designed SMC in control of the angular velocity of the robot around the vertical axis could be extended to achieve full trajectory tracking together with complete attitude control of the robot in feature works.

6. REFRENCES

A. Bicchi, A. Balluchi, D. Prattichizzo, A. Gorelli, (1997). introducing the "SPHERICLE": An experimental testbed for research and teaching in nonholonomy, *IEEE International Conference on Robotics and Automation,* pp. 2620-2625.

A. Halme, T. Schönberg, Y. Wang, (1996). Motion control of a spherical mobile robot, *International Workshop on Advanced Motion Control (AMC),* pp. 259-264.

A.H. Javadi, P. Mojabi, (2002). Introducing august: a novel strategy for an omnidirectional spherical rolling robot, *IEEE International Conference on Robotics and Automation,* pp. 3527-3533.

B. Liu, M. Yue, R. Liu, (2012). Motion control of an underactuated spherical robot: A hierarchical sliding-mode approach with disturbance estimation, *IEEE International Conference on Mechatronics and Automation (ICMA),* pp. 1804-1809.

B. Zhao, M. Li, H. Yu, H. Hu, L. Sun, (2010). Dynamics and motion control of a two pendulums driven spherical robot, *IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)*, pp. 147-153.

D. Liu, H. Sun, Q. Jia, (2008). A family of spherical mobile robot: Driving ahead motion control by feedback linearization, *IEEE International Symposium on Systems and Control in Aerospace and Astronautics (ISSCAA),* pp. 1-6.

D. Liu, H. Sun, Q. Jia, (2008). Stabilization and path following of a spherical robot, *IEEE International Conference on Robotics, Automation and Mechatronics (RAM),* pp. 676-682.

E. Kayacan, E. Kayacan, H. Ramon, W. Saeys, (2013). Adaptive neuro-fuzzy control of a spherical rolling robot using sliding-mode-control-theory-based online learning algorithm. *IEEE TRANSACTIONS ON CYBERNETICS,* 43(1), pp. 170-179.

E. Kayacan, Z.Y. Bayraktaroglu, W. Saeys, (2012). Modeling and control of a spherical rolling robot: a decoupled dynamics approach. *Robotica,* 30(4), pp. 671-680.

J. Keighobadi, Y. Mohamadi, (2012). Fuzzy robust trajectory tracking control of WMRs. In: *S. I. Ao, O. Castillo, X. Huang, Eds., Intelligent Control and Innovative Computing.* Singapore: Springer, pp. 77-90.

J. Suomela, T. Ylikorpi, (2006). Ball shaped robots: An historical overview and recent. *Field and Service Robotics,* 25(6), pp. 343– 354.

M. Yue, B. Liu, (2013). Adaptive control of an underactuated spherical robot with a dynamic stable equilibrium point using hierarchical sliding mode approach. *International journal of adaptive control and signal processing,* DOI: 10.1002/acs.2413.

M. Zheng, Q. Zhan, J. Liu, Y. Cai, (2011). Control of a spherical robot: Path following based on nonholonomic kinematics and dynamics. *Chinese Journal of Aeronautics,* 24(3), pp. 337-345.

M.R. Azizi, D. Naderi, (2013). Dynamic modeling and trajectory planning for a mobile spherical robot with a 3DOF inner mechanism. *Mechanism and Machine theory,* Volume 64, pp. 251-261.

Q. Zhan, T. Zhou, M. Chen, S. Cai, (2006). Dynamic trajectory planning of a spherical mobile robot, *IEEE International Conference on Robotics, Automation & Mechatronics (RAM),* pp. 1-6.

Q. Zhan, Z. Liu, Y. Cai, 2008. A back-stepping based trajectory tracking controller for a non-chained nonholonomic spherical robot. *Chinese Journal of Aeronautics,* 21(5), pp. 472-480.

R.W. Brockett, (1983). In: *Brockett, R.W., Milman, R.S., Sussmann, H.J. (Eds.), Asymptotic stability and feedback stabilization.* Boston: Dzflerential, pp. 181-191.

S. Bhattacharya, S.K. Agrawal, (2000). Spherical rolling robot: A design and motion planning studies. *IEEE Transactions on Robotics and Automation,* 16(6), pp. 835-839.

T. Otani, T. Urakubo, S. Maekawa, H. Tamaki, Y. Tada, (2006). Position and Attitude Control of a Spherical Rolling Robot Equipped with a Gyro, *IEEE International Workshop on Advanced Motion Control (AMC)*, pp. 416-421.

Y. Cai, Q. Zhan, C. Yan, (2012). Two-state trajectory tracking control of a spherical robot using neurodynamics. *Robotica,* 30(2), pp. 195-203.

Y. Cai, Q. Zhan, X. Xi, (2012). Path tracking control of a spherical mobile robot. *Mechanism and Machine Theory,* Volume 51, pp. 58- 73.