Preprints of the 19th World Congress
The International Federation of Automatic Control
Cape Town, South Africa. August 24-29, 2014

Synchronization of Unicycle Robots with Proximity Communication Networks ⋆

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Abstract: In this paper synchronization of both the orientation and velocity for a group of unicycle robots is studied. It is assumed that a robot can only detect and obtain information from those robots that lie in the proximity (within certain distance). A minimum dwell time is imposed on the updating of neighborhood relation in the controller in order to avoid introducing chattering in the closed-loop system that may be caused by abrupt changes of the relation, which as a consequence implies that the neighbor information will be updated only at discrete time instants in the control. In the paper a distributed feedback control law is designed for each agent, and a sufficient condition for uniformly and independently distributed initial states is provided for reaching the synchronization, which depends on the neighborhood radius, the maximum initial translational velocity and the dwell time.

Keywords: synchronization, unicycle, sampled-data nonlinear systems, hybrid systems, distributed control

1. INTRODUCTION

Over the last decade, cooperative control of multiple mobile robots has attracted increasing attention of researchers in control and robotics fields. Compared with a single agent, the multi-agent system has some advantages in fulfilling a a common task, among which synchronization can be regarded as a basic one and has been wildly investigated, see [1]-[4] among many others. Actually, it is closely related to different engineering applications, such as rendezvous problem[5], parallel computing[6], distributed optimization [7] and formation control[8].

In the synchronization study, a typical model is proposed by Vicsek and his colleagues in 1995 in [9], where the simulation results show that the system exhibits synchronization behavior under some conditions. Due to the simplicity and importance of this model, great efforts have been paid on the synchronization of the Vicsek model. Jadbabaie et al. [1] are the first to study this model, and they show that the headings of all agents reach synchronization if the neighbor graphs satisfy the connectivity assumption in a bounded time intervals. Then, a key question is : how can we guarantee the connectivity of neighbor graphs? To solve this, Cucker and Smale [10] modified the local interaction between agents into global interaction, and gave sufficient conditions imposed on the initial states only. Recently, in [11] and [12] a random framework is introduced, and theoretical analysis for the original Vicsek model is given where the prior connectivity assumption can be removed.

As a related topic, formation control of a group of mobile robots is also widely investigated. Generally speaking, three approaches for the control design are considered in the literature: leader-follower method (cf. [13]), virtual structure method (cf. [14]) and behavior-based method (cf. [15]). To simplify the analysis, quite often in literature either the dynamics of the individual robots is described as a single/double integrator or/and the communication graph satisfies an a priori given connectivity. How to analyze a multi-robot nonlinear system without prior connectivity assumption is an important and still unresolved issue.

Stimulated by the studies on multi-agent systems, in this paper we aim to investigate synchronization of both the orientation and velocity for a group of nonholonomic unicycle robots connected by proximity graphs that are induced by the distance between the agents. This can lead to abrupt changes of the neighbor relation, which in turn may cause chattering if the feedback control is not properly designed. Thus, a minimum dwell time on updating the neighborhood relation is imposed in our control design. Using partly sampled local information of the agents, we design a distributed discrete-time updated control law (angular velocity and translational acceleration) for each agent, which in conjunction with the continuous-time unicycle model yields a sampled-data hybrid multi-agent system. To the best of our knowledge, this paper is the first attempt to study and analyze the synchronization

⋆ This work was partly supported by the National Natural Science Foundation of China under grant Nos. 61273221 and 61203142, the National Key Basic Research Program of China (973 program) under grant no. 2014CB845301/2/3 and partly by the Project-sponsored by SRF for ROCS, SEM.

problem for such a sampled-data nonlinear system. By analyzing the dynamics of the system and estimating the statistical characteristics of the initial neighbor relation, we provide a sufficient condition for the synchronization of unicycle robots without relying on any prior connectivity assumption on the neighbor graphs.

2. PROBLEM FORMULATION

2.1 Some notations and Preliminaries

First, we introduce some notations in graph theory and matrix analysis, see [16]-[18] for details. For an undirected graph $G = \{V, E\}$, where $V = \{1, 2, \cdots, n\}$ and $(i, j) \in E \subseteq V \times V$ means that there is an undirected edge between vertices i and j. The adjacency matrix $A = [a_{ij}]$ is a $0-1$ matrix where $a_{ij} = 1$ if there is an edge between i and j and otherwise $a_{ij} = 0$. The degree of the vertex i is defined as the number of neighbors, i.e., $d_i = \sum_j a_{ij}$, and $d_{\max} = \max_i d_i$ and $d_{\min} = \min_i d_i$. The degree matrix D and the Laplacian matrix L are defined as $D = diag(d_1, d_2, \cdots d_n)$ and $L = D - A$ respectively. The normalized Laplacian is defined as $\mathcal{L} = D^{-1/2} L D^{-1/2}$. Obviously $\mathcal{L}$ is a non-negative definite matrix and we denote its eigenvalues as $0 = \lambda_0 \le \lambda_1 \le \cdots \le \lambda_{n-1}$. A useful property is that the graph G is connected if and only if the multiplicity of the zero eigenvalue of $\mathcal{L}$ equals to one. Another property of $\mathcal{L}$ is $\|\mathcal{L}\| \le 2$. A square matrix $A = (a_{ij})_{n \times n}$ is called stochastic, if $a_{ij} \ge 0$ and $\sum_{j=1}^n a_{ij} = 1, \forall i$.

2.2 Unicycles robots

In this paper, we consider n nonholonomic unicycle mobile robots, which are labeled as $1, 2, \cdots, n$. We denote the set of the robots as $V = \{1, 2, \cdots, n\}$. Let $(x_i(t), y_i(t))^\tau$ and $\theta_i(t)$ denote the position of the center and the orientation of the ith mobile robot at time t, respectively. The evolution of such a robot is described by the following differential equation:

$$\begin{cases} \dot{x}_i(t) = v_i(t) \cos \theta_i(t) \\ \dot{y}_i(t) = v_i(t) \sin \theta_i(t) \\ \dot{\theta}_i(t) = \omega_i(t) \\ \dot{v}_i(t) = u_i(t) \end{cases}, \quad i \in V, \tag{1}$$

where $u_i(t)$, $v_i(t)$ and $\omega_i(t)$ denote the acceleration, translational and rotational velocity of the robot i at time t belonging to the interval $[0, \infty)$. for the ith mobile robot, what we can control is the acceleration $u_i(t)$ and the rotational velocity $\omega_i(t)$. Note that we have chosen here not to model the physical forces and torques that in practice affect both the translational and angular motion, while assuming that we can control directly the translational acceleration.

We assume that at each time instant, all agents can only sense the relative translational speed and relative orientation of its neighbors, i.e., for the robot i, it can receive the following information:

$$\{v_j(t) - v_i(t),\ \theta_j(t) - \theta_i(t),\ j \in \mathcal{N}_i(t)\}, \tag{2}$$

where the neighbors are defined via the distance between agents,

$$\mathcal{N}_i(t) = \left\{ j : \sqrt{(x_i(t) - x_j(t))^2 + (y_i(t) - y_j(t))^2} \le r_n \right\},$$

with r_n being the interaction radius depending on n. We use $n_i(t)$ to denote the cardinality of the set $\mathcal{N}_i(t)$.

The objective of this paper is to design control law of $u_i(t)$ and $\omega_i(t)$ based on the local information (2), such that the orientations and velocities become synchronized, that is, there exists a common vector (v, θ), for all $i \in V$,

$$\lim_{t\to\infty} v_i(t) = v, \quad \lim_{t\to\infty} \theta_i(t) = \theta.$$

2.3 Main results

From the definition of neighbor set (3), we see that the continuous time variables $\mathcal{N}_i(t)$ and $n_i(t)$ may lead to chattering because the neighbor relations might change abruptly when the positions of robots change. Similar to [1], we introduce a dwell time in the control design, which means that the neighbor relations of the agents are only updated at discrete time instants $t_0(= 0), t_1, t_2, \cdots$. To simplify the analysis, we assume that the dwell time is the same and denoted by τ, i.e., $t_{k+1} - t_k = \tau, k = 0, 1, \cdots$. In this paper, we will adopt the control law of the following form:

$$\begin{cases} u_i(t) = \dfrac{1}{\tau n_i(t_k)} \displaystyle\sum_{j \in \mathcal{N}_i(t_k)} (v_j(t_k) - v_i(t_k)) \\ \omega_i(t) = \dfrac{1}{n_i(t_k)} \displaystyle\sum_{j \in \mathcal{N}_i(t_k)} (\theta_j(t) - \theta_i(t)) \end{cases}, \tag{3}$$

when $t \in [t_k, t_{k+1}], k = 0, 1, \cdots$.

Remark 1. What the control laws suggest is that we need to measure the relative orientation over the whole interval $[t_k, t_{k+1}]$ even if some agent j has left the neighborhood during the period. This can perhaps be justified in practice that sensors for measuring orientation such as vision has a longer range. Nevertheless, our future work will aim at relaxing this requirement.

The relationship of neighbors is completely determined by the position of all agents, and we will use graphs to describe it. As mentioned earlier, the graphs only change at discrete time $t_0, t_1, \cdots$, so we will use a graph sequence $G(t_k) = \{V, E(t_k)\}, k = 0, 1, \cdots$ to represent the interaction between robots, where the vertex set is composed of all robots, and the edge set at discrete-time t_k is defined as follows:

$$E(t_k) = \{(i, j) : d_{ij}(t_k) \le r_n\}.$$

The graphs formed in this way are undirected.

Remark 2. Different from almost all studies on consensus of unicycles, the neighbor graphs in this paper are distanced-induced. Notice that the neighbor graphs will affect the orientation and translational speed, and in turn the orientation and translational speed affect the neighbor graphs. This makes the theoretical analysis quite hard.

For the graph $G(t_k)$ defined above, the degree of the node i is denoted as $d_i(t_k)$, and the minimum degree and maximum degree are denoted as $d_{\min}(t_k)$ and $d_{\max}(t_k)$ respectively. The adjacency matrix, degree matrix, and the

Laplacian of $G(t_k)$ are denoted as $A(t_k), D(t_k)$ and $L(t_k)$, and the normalized Laplacian is denoted as $\mathcal{L}(t_k)$. Denote the eigenvalue of $\mathcal{L}(t_k)$ as $0 = \lambda_0(t_k) \leq \lambda_1(t_k) \leq \cdots \leq \lambda_{n-1}(t_k)$, and the spectral gap of graph $G(t_k)$ is defined as $\overline{\lambda}(t_k) = \max\{|1-\lambda_1(t_k)|, |1-\lambda_{n-1}(t_k)|\}$.

Substituting (3) into (1), the closed-loop dynamical system becomes the following sampled-data hybrid system:

$$\begin{cases} \dot{x}_i(t) = v_i(t_k)\cos\theta_i(t) \\ \dot{y}_i(t) = v_i(t_k)\sin\theta_i(t) \\ \dot{\theta}_i(t) = \dfrac{1}{n_i(t_k)} \displaystyle\sum_{j\in\mathcal{N}_i(t_k)} (\theta_j(t) - \theta_i(t)) \\ \dot{v}_i(t) = \dfrac{1}{\tau n_i(t_k)} \displaystyle\sum_{j\in\mathcal{N}_i(t_k)} (v_j(t_k) - v_i(t_k)) \end{cases}, \qquad (4)$$

when $t \in [t_k, t_{k+1}], k = 0, 1, 2, \cdots$. By the last equation of (4), we have

$$\begin{aligned} v_i(t) &= v_i(t_k) + (t-t_k)u_i(t_k) \\ &= \left(1 - \frac{t-t_k}{\tau}\right) v_i(t_k) + \frac{t-t_k}{\tau n_i(t_k)} \sum_{j\in\mathcal{N}_i(t_k)} v_j(t_k). \end{aligned} \qquad (5)$$

Thus,

$$\max_{i,j} |v_i(t) - v_j(t)| \leq \max_{i,j} |v_i(t_k) - v_j(t_k)|;$$

$$v_i(t) \leq \max_{1\leq i\leq n} v_i(t_k), t \in [t_k, t_{k+1}); \qquad (6)$$

$$v_i(t_{k+1}) = \frac{1}{n_i(t_k)} \sum_{j\in\mathcal{N}_i(t_k)} v_j(t_k) \qquad (7)$$

Denote $v(t) = [v_1(t), v_2(t), \cdots, v_n(t)]'$ and $\theta(t) = [\theta_1(t), \theta_2(t), \cdots, \theta_n(t)]'$, then we have

$$v(t_{k+1}) = Q(t_k)v(t_k), \qquad (8)$$

$$\dot{\theta}(t) = -P(t_k)\theta(t), \quad t \in [t_k, t_{k+1}), \qquad (9)$$

where $Q(t_k) = D^{-1}(t_k)A(t_k)$ and $P(t_k) = D^{-1}(t_k)L(t_k)$.

Our analysis is proceeded under the following assumption on the initial position of all agents:

Assumption 3. At the initial time instant, all agents are uniformly and independently distributed in the (normalized) unit square $[0,1]^2$.

The main result of this paper can be stated as follows:

Theorem 4. Assume that the initial translational velocities of all agents are non-negative and have an upper bound v_n, and the chosen dwell time satisfies $\tau \leq \frac{1}{6}$. If the neighborhood radius r_n and v_n satisfy the following condition:

$$\sqrt[4]{\frac{\log n}{n}} \ll r_n \ll 1, \quad v_n = O\left(\frac{r_n^5}{\log n}\right),$$

then for any initial headings, the system (4) will reach synchronization in orientation and velocity almost surely for large n.

Remark 5. If the dwell time is different at different discrete-time instants, but has a common upper bound that satisfies the condition of the theorem, then the result of the theorem still holds.

3. SOME KEY LEMMAS

3.1 Analysis of the translational velocity

By (8), we see that the translational velocity $v(t)$ has the following form:

$$\begin{aligned} v(t_{k+1}) &= Q(t_k)v(t_k) \\ &= D^{-1/2}(t_k)(I - \mathcal{L}(t_k))D^{1/2}(t_k)v(t_k), \end{aligned} \qquad (10)$$

where $\mathcal{L}(t_k)$ is the normalized Laplacian of the graph $G(t_k)$. For $v(t)$, we have the following lemma:

Lemma 6. [11] Let $\{G(t_k), k \geq 0\}$ be a sequence of time-varying undirected graphs, with the corresponding characteristic quantities $\{\mathcal{L}(t_k), Q(t_k), d_{\min}(t_k), d_{\max}(t_k), \overline{\lambda}(t_k), k \geq 0\}$ and $v(t_k)$ evolves according to equation (10). If $\|Q(t_k) - Q(t_0)\| \leq \epsilon$ with $\epsilon \geq 0$, then

$$\delta v(t_k) \leq \sqrt{2}\kappa(\overline{\lambda}(t_0) + \kappa(t_0)\epsilon)^k \|v(t_0)\|,$$

where $\delta v(t_k) = \max_{1\leq i,j\leq n} |v_i(t_k) - v_j(t_k)|$, $\kappa(t_0) = \sqrt{\frac{d_{\max}(t_0)}{d_{\min}(t_0)}}$ and $\overline{\lambda}(t_0)$ is the spectral gap of graph $G(t_0)$.

3.2 Analysis of the orientation

For $t \in [t_k, t_{k+1})$, the matrix $P(t)$ will not change. So the solution of (9) can be written as

$$\begin{aligned} \theta(t) &= \exp\{-P(t_k)(t-t_k)\}\theta(t_k) \\ &= \exp\{-D^{-1/2}(t_k)\mathcal{L}(t_k)D^{1/2}(t_k)(t-t_k)\}\theta(t_k), \\ &\qquad t \in [t_k, t_{k+1}). \end{aligned} \qquad (11)$$

We can deduce the following lemma concerning the matrix $\exp\{-P(t_k)\tau\}$:

Lemma 7. For any $k \geq 0$ and $\tau \geq 0$, the matrix $\exp\{-P(t_k)\tau\}$ is a stochastic matrix.

Proof: The matrix $\exp\{-P(t_k)\tau\}$ can be written as

$$\exp\{-P(t_k)\tau\} = \exp\{-\tau\}\exp\{D(t_k)^{-1}A(t_k)\tau\}.$$

Note that the entries of the matrix $D(t_k)^{-1}A(t_k)$ are non-negative, so are the entries of $\exp\{-P(t_k)\tau\}$ in the Taylor expansion. In addition to this,

$$\begin{aligned} &\exp\{-P(t_k)\tau\}\mathbf{1}_n \\ &= \{I + (-D(t_k)^{-1}L(t_k)\tau) + \frac{(-D(t_k)^{-1}L(t_k)\tau)^2}{2!} + \cdots\}\mathbf{1}_n \\ &= \mathbf{1}_n, \end{aligned}$$

where $\mathbf{1}_n = [1, 1, \cdots, 1]'$. This completes the proof of the lemma. ∎

To analyze the dynamical behavior of the system, we will first provide a preliminary result on the evolution of orientation for the case where the neighbor graphs does not change with time.

Lemma 8. Assume that the neighbor graphs keep unchanged, and denoted by G with the corresponding characteristic quantities $\{\mathcal{L}, D, d_{\min}, d_{\max}, \lambda_1\}$. Then we have

$$\max_{1\leq i,j\leq n} |\theta_i(t) - \theta_j(t)| \leq \sqrt{2}\kappa\exp\{-\lambda_1 t\}\|\theta(0)\|,$$

where κ is a constant defined as $\kappa = \sqrt{\frac{d_{\max}}{d_{min}}}$.

The proof is omitted due to space limitations. From the above lemma, it is easy to obtain the following corollary:

Corollary 9. If the neighbor graphs keep unchanged and are connected at all time instants, then we have

$$\max_{1\leq i,j\leq n}|\theta_i(t)-\theta_j(t)|\to 0.$$

Based on the above lemma, we can get the following result for the case where the neighbor graphs change with time:

Lemma 10. Let $G(t_k)$ denote the neighbor graph at time interval $[t_k,t_{k+1})$, and the corresponding degree, minimum degree, maximum degree, average matrix, degree matrix and the normalized Laplacian are denoted as $d_i(t_k), d_{\min}(t_k), d_{\max}(t_k), P(t_k), D(t_k)$ and $\mathcal{L}(t_k)$. If $\|P(t_k)-P(0)\|\leq\varepsilon$, then we have

$$\delta\theta(t)\leq$$
$$\sqrt{2}\kappa\left(\exp\{-\lambda_1(0)\tau\}+\frac{\kappa\varepsilon\tau}{1-\varepsilon\tau}\exp\{\|P(0)\|\tau\}\right)^{\lceil\frac{t}{\tau}\rceil}\|\theta(0)\|,$$

where $\delta\theta(t)=\max_{1\leq i,j\leq n}|\theta_i(t)-\theta_j(t)|$, and the notation $\lceil x\rceil$ denotes the largest integer no more than x.

The proof of the lemma is omitted due to space limitations.

Remark 11. From the above lemma, we see that if

$$\exp\{-\lambda_1\tau\}+\frac{\kappa\varepsilon\tau}{1-\varepsilon\tau}\exp\{\|P(0)\|\tau\}\leq\alpha<1 \tag{12}$$

holds, then all agents can move with the same heading eventually. To verify this inequality, we need to estimate the characteristics of the initial states and deal with the relationship between the positions and headings of all agents.

Lemma 12. [11] Let $\widehat{G}$ be a graph formed by changing the neighborhood of G. If the number of points changed in the neighborhood of the k-th $(1\leq k\leq n)$ node satisfies $R_k\leq R_{\max}<d_{\min}$, then

$$\begin{aligned}&\|D^{-1}L-\widehat{D}^{-1}\widehat{L}\|=\|D^{-1}A-\widehat{D}^{-1}\widehat{A}\|\\ &\leq\frac{R_{\max}}{d_{\min}}\cdot\frac{d_{\max}+d_{\min}}{d_{\min}-R_{\max}},\end{aligned}$$

where A,D,L and $\widehat{A},\widehat{D},\widehat{L}$ are the adjacency matrix, degree matrix and the Laplacian of G and $\widehat{G}$, respectively.

3.3 Estimation of some characteristics concerning the initial states

The notation $\mathcal{R}_i$ is introduced to denote the change of neighbors of the robot $i(1\leq i\leq n)$:

$$\mathcal{R}_i=\{j:(1-\eta_n)r_n\leq d_{ij}(0)\leq(1+\eta_n)r_n\}, \tag{13}$$

where η_n can be taken as follows:

$$\eta_n=\frac{\pi r_n^2}{288\cdot 320}. \tag{14}$$

The cardinality of $\mathcal{R}_i$ will be denoted as R_i, and $R_{max}=\max_i R_i$.

Lemma 13. [11] For the initial random geometric graph G_0, if the interaction radius satisfies $\sqrt{\frac{\log n}{n}}\ll r_n\ll 1$, then the following results hold almost surely for all large n :

1) The maximum and minimum degrees satisfy

$$d_{\max}=n\pi r_n^2(1+o(1));\quad d_{min}(0)=\frac{n\pi r_n^2}{4}(1+o(1)).$$

2) The maximum number of agents in (13) satisfies

$$R_{max}\leq 4n\pi\eta r_n^2(1+o(1)). \tag{15}$$

3) The second smallest and the largest eigenvalues of the normalized Laplacian $\mathcal{L}(0)$ of the graph $G(0)$ satisfy

$$\begin{aligned}&\lambda_1(0)\geq\frac{\pi r_n^2}{144}(1+o(1));\\ &\lambda_{n-1}(0)\leq 2\left(1-\frac{1}{12\pi}(1+o(1))\right).\end{aligned}$$

Remark 14. Under Assumption 3, the following assertions hold almost surely for large n:

(1) The spectral gap of $\mathcal{L}(t_0)$ satisfies the following inequality:

$$\overline{\lambda}(0)\leq 1-\frac{\pi r_n^2}{144}(1+o(1)).$$

(2)The constant κ in Lemma 6 can be taken as $\kappa=2(1+o(1))$.

(3) The norm $\|P(0)\|$ in Lemma 10 satisfy the following equations:

$$\|P(0)\|\leq 4(1+o(1)).$$

Proposition 15. Assume the dwell time $\tau\leq\frac{1}{6}$. If the moving speed and the neighborhood radius satisfy the following conditions:

$$\sqrt[4]{\frac{\log n}{n}}\ll r_n\ll 1,\quad v_n\leq\frac{\eta_n r_n^3}{576(1+\tau)\log n},$$

then for $k\geq 0$, we have

$$|d_{ij}(t_k)-d_{ij}(0)|\leq\eta_n r_n; \tag{16}$$
$$\|P(t_k)-P(0)\|\leq 80\eta_n,\ \|Q(t_k)-Q(0)\|\leq 80\eta_n, \tag{17}$$

where η_n is taken as (14).

Proof: We will use mathematical induction for k. It is obvious the inequalities (16) and (17) hold for $k=0$.

Assume that (16) and (17) hold for all $k\leq k_0$. We will prove that (16) and (17) hold for $k=k_0+1$. By (1), we have

$$\begin{aligned}x_i(t_{k_0+1})-x_i(t_{k_0})&=\int_{k_0}^{t_{k_0+1}}v_i(t)\cos\theta_i(t)dt;\\ y_i(t_{k_0+1})-y_i(t_{k_0})&=\int_{t_{k_0}}^{t_{k_0+1}}v_i(t)\sin\theta_i(t)dt\end{aligned}$$

Denote $X_i(k)=(x_i(t_k),y_i(t_k))^\tau$. So, the distance between agents satisfies the following inequality:

$$|d_{ij}(t_{k_0+1})-d_{ij}(0)|\le\sum_{l=0}^{k_0}|d_{ij}(l+1)-d_{ij}(l)|$$
$$\le\sum_{l=0}^{k_0}\|X_i(l+1)-X_j(l+1)-X_i(l)+X_j(l)\|_2$$
$$\le\sum_{l=0}^{k_0}\left\{\left|\int_{t_l}^{t_{l+1}}[v_i(t)\cos\theta_i(t)-v_j(t)\cos\theta_j(t)]\,dt\right|\right.$$
$$\left.+\left|\int_{t_l}^{t_{l+1}}[v_i(t)\sin\theta_i(t)-v_j(t)\sin\theta_j(t)]\,dt\right|\right\}.\quad(18)$$

For the first term of (18), we have

$$\left|\int_{t_l}^{t_{l+1}}[v_i(t)\cos\theta_i(t)-v_j(t)\cos\theta_j(t)]dt\right|$$
$$\le\int_{t_l}^{t_{l+1}}|(v_i(t)-v_j(t))\cos\theta_i(t)|dt+$$
$$\int_{t_l}^{t_{l+1}}|v_j(t)(\cos\theta_i(t)-\cos\theta_j(t))|dt$$
$$\le\tau\Big\{\max_{1\le i,j\le n}|v_i(t_l)-v_j(t_l)|$$
$$+\max_{1\le i\le n}v_i(t_l)\max_{1\le i,j\le n}|\cos\theta_i(t_l)-\cos\theta_j(t_l)|\Big\}.\quad(19)$$

Note that by Lemmas 10, 13, Remark 14 and the induction assumption, we have for $0\le l\le k_0+1$,

$$\max_{1\le i,j\le n}|\theta_i(t_l)-\theta_j(t_l)|\le$$
$$\sqrt{2}\left(\exp\{-\lambda_1(0)\tau\}+\frac{\kappa\varepsilon\tau}{1-\varepsilon\tau}\exp\{\|P(0)\|\tau\}\right)^l\kappa\|\theta(0)\|$$
$$\le 2\pi\sqrt{2n}\left(1-\frac{\pi\tau r_n^2(1+o(1))}{144}+\frac{\tau\pi r_n^2(1+2\tau)(1+o(1))}{576(1-2\tau)}\right)^l$$
$$\le 2\pi\sqrt{2n}\left(1-\frac{\pi\tau r_n^2(1+o(1))}{288}\right)^l,$$

where $\tau\le\frac{1}{6}$ is used in the last inequality.

Denote $l_0=\min\left\{l:2\pi\sqrt{2n}\left(1-\frac{\pi\tau r_n^2(1+o(1))}{288}\right)^l\le 1\right\}$, then we have

$$l_0\le\frac{-\log(2\pi\sqrt{2n})}{\log(1-\frac{\pi\tau r_n^2(1+o(1))}{288})}+1.$$

By this, we can derive that

$$\sum_{l=0}^{k_0}\max_{1\le i\le n}v_i(t_l)\max_{1\le i,j\le n}|\cos\theta_i(t_l)-\cos\theta_j(t_l)|$$
$$\le 2v_n\sum_{l=0}^{k_0}\max_{1\le i,j\le n}\left|\sin\frac{\theta_i(t_l)-\theta_j(t_l)}{2}\right|$$
$$\le 2v_n\left\{l_0+\sum_{l=s_0+1}^{k_0}\left(1-\frac{\pi\tau r_n^2(1+o(1))}{288}\right)^{l-l_0}\right\}$$
$$\le 2v_n\left\{\frac{-\log(\pi\sqrt{2n})}{\log(1-\frac{\pi\tau r_n^2(1+o(1))}{288})}+1+\frac{1}{\frac{\pi\tau r_n^2(1+o(1))}{288}}\right\}$$
$$\le\frac{2v_n\cdot 288}{\pi\tau r_n^2(1+o(1))}\left\{\log(2\pi\sqrt{2n})+2\right\}$$
$$=\frac{288v_n\log n}{\pi\tau r_n^2(1+o(1))}.\quad(20)$$

where the inequality $\log x\le x-1,\forall 0<x<1$ is used.

On the other hand, by Lemma 6 and induction assumption, we have for $0\le l\le k_0+1$,

$$\max_{1\le i,j\le n}|v_i(t_l)-v_j(t_l)|\le 2\sqrt{2n}v_n\left(1-\frac{\pi r_n^2(1+o(1))}{288}\right)^l.$$

Denote $s_0=\min\left\{l:2\sqrt{2n}v_n\left(1-\frac{\pi r_n^2(1+o(1))}{288}\right)^l\le 2v_n\right\}$, then

$$s_0\le\frac{-\log\sqrt{2n}}{\log\left(1-\frac{\pi r_n^2(1+o(1))}{288}\right)}+1.$$

So,

$$\sum_{l=0}^{k_0}\max_{1\le i,j\le n}|v_i(t_l)-v_j(t_l)|$$
$$\le 2v_nl_0+2v_n\sum_{l=l_0+1}^{k_0}\left(1-\frac{\pi r_n^2(1+o(1))}{288}\right)^{l-l_0}$$
$$\le 2v_n\left(\frac{-\log\sqrt{2n}}{\log\left(1-\frac{\pi r_n^2(1+o(1))}{288}\right)}+1+\frac{1}{\frac{\pi r_n^2(1+o(1))}{288}}\right)$$
$$=\frac{288v_n\log n}{\pi r_n^2}(1+o(1)).\quad(21)$$

By (19), (20) and (21), we have

$$\sum_{l=0}^{k_0}\left|\int_{t_l}^{t_{l+1}}[v_i(t_l)\cos\theta_i(t)-v_j(t_l)\cos\theta_j(t)]dt\right|$$
$$\le\frac{288v_n\log n}{\pi r_n^2}(1+\tau)(1+o(1)).$$

By the similar analysis, we can show

$$\sum_{l=0}^{k_0}\left|\int_{t_l}^{t_{l+1}}[v_i(t_l)\sin\theta_i(t)-v_j(t_l)\sin\theta_j(t)]dt\right|$$
$$\le\frac{288v_n\log n}{\pi r_n^2}(1+\tau)(1+o(1)).$$

By the assumption of the proposition, we have

$$|d_{ij}(t_{k_0+1}) - d_{ij}(t_{k_0})| \leq \eta_n r_n. \quad (22)$$

Now we will prove that (17) holds at time $k+1$. By (22), we see that for each agent, the number of neighbors changed at time t_{k+1} in comparison with the initial neighbors is bounded by $R_{\max}$ defined via (13). So by Lemma 12 and 13, we have

$$\begin{aligned}|P(t_{k+1} - P(0))\| &\leq \frac{R_{\max}}{d_{\min}(0)} \cdot \frac{d_{\max}(0) - d_{\min}(0)}{d_{\min}(0) - R_{\max}} \\ &\leq \frac{4n\pi\eta r_n^2}{\frac{1}{4}n\pi r_n^2} \cdot \frac{n\pi r_n^2 + \frac{1}{4}n\pi r_n^2}{\frac{1}{4}n\pi r_n^2 - 4n\pi\eta r_n^2}(1+o(1)) \\ &= 80\eta(1+o(1)).\end{aligned}$$

By the similar analysis, we can deduce $\|Q(t_k) - Q(0)\| \leq 80\eta$. This completes the proof of the proposition. ■

Proof of Theorem 4

By the translational velocity update equation (5), we know that $\max_{1\leq i\leq n} v_i(t_k)$ (resp. $\min_{1\leq i\leq n} v_i(t_k)$) is a non-increasing (resp. non-decreasing) sequence. So as $k \to \infty$, $\max_{1\leq i\leq n} v_i(t_k)$ and $\min_{1\leq i\leq n} v_i(t_k)$ have bounded limits . On the other hand, by Proposition 15, we have for all $k \geq 0$

$$\begin{aligned}\max_{1\leq i,j\leq n} |v_i(t_k) - v_j(t_k)| &\leq 2\sqrt{2n}v_n \left(1 - \frac{\pi r_n^2(1+o(1))}{288}\right)^k \\ &\to 0, \qquad \text{as } k \to \infty.\end{aligned}$$

It is easy to see that the translational velocity tends to the same value. Moreover, by Lemma 10 and Proposition 15, we can prove that for $1 \leq i \leq n$, the orientation $\theta_i(t)$ will also tend to the same value. This completes the proof of the theorem. ■

4. CONCLUDING REMARKS

This paper investigated synchronization of the orientation and velocity for a group of unicycle robots that are connected by distance-induced graphs, for which a dwell time is introduced to avoid issues that are caused by the abrupt change of neighbor graphs. The control laws are designed using only the local information each robot can obtain. By analyzing the dynamics of the system and using the estimation of some characteristics concerning the initial states, we provided a sufficient condition for reaching synchronization without relying on any prior connectivity assumption of the neighbor graphs. Some interesting problems deserve to be further investigated, for example, how to design a distributed control of robots using only sampled information, and how to intervene the unicycle robots such that the system exhibits more sophisticated emergence.

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