Preprints of the 19th World Congress
The International Federation of Automatic Control
Cape Town, South Africa. August 24-29, 2014

IDA-PBC control for the coupled plasma poloidal magnetic flux and heat radial diffusion equations in tokamaks

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Abstract: The IDA-PBC control of plasma dynamics in a tokamak is investigated. It is based on a model made of the two coupled PDEs of resistive diffusion for the magnetic poloidal flux and of radial thermal diffusion. The used Thermal-Magneto-Hydro-Dynamics (TMHD) couplings are the Lorentz forces (with non-uniform resistivity) and the bootstrap current. The control model is obtained with the coupling of the two finite dimensional approximations obtained from the two diffusion models, using two geometric reduction schemes. A feedforward control is used to ensure the compatibility with the actuator physical ability. Then, an IDA-PBC (Interconnection and Damping Assignment - Passivity Based Control) controller is proposed for the coupled model to improve the system stabilization and convergence speed. The obtained numerical results are validated against the simulation data obtained from the RAPTOR (RApid Plasma Transport simulatOR) code for the TCV (Tokamak of Configuration Variable at CRPP, EPFL, Lausanne, Switzerland) tokamak real-time control system.
Keywords: *distributed parameters systems, plasma control, Tokamak, Port-Controlled Hamiltonian systems, IDA-PBC control*

1. INTRODUCTION

Before building a fully functional plant for nuclear fusion, the ITER project has to challenge many difficult issues. One of them is to reach some specific internal radial profiles of the plasma in order to avoid magneto-hydrodynamic (MHD) instabilities and to improve the plasma confinement. One of the most important of these characteristic profiles is the 1D plasma safety factor radial profile q (cf.Blum [1989]): its feedback control becomes the subject of many studies such as Argomedo et al. [2012], Gaye et al. [2011], or Vu et al. [2013b] which make use of the 1D resistive diffusion equation of the magnetic flux in the plasma (cf.Blum [1989]) as a control model. This model also accounts for the plasma resistivity variations and the bootstrap current described in Wesson [2004] (a magnetohydrodynamic coupling effect which produces an extra current density). Both of these effects are large and very sensitive to the plasma temperature (i.e. the electronic temperature). When this dependence is considered, scaling laws are usually used to determine the system parameters (resistivity and bootstrap current), see Boyer et al. [2013] or Moreau et al. [2003]. However, to the best of our knowledge, there is no work proposing the design of feedback controls using both the plasma resistive diffusion equation and the plasma thermal equation (here roughly modeled using a heat transport equation), as well as the corresponding interdomain couplings and actuators in both magnetic and thermal domains to achieve a better security factor profile regulation.

In this paper, we present first two discrete PCH (Port-Control Hamiltonian) models, one for the resistive diffusion of the magnetic poloidal flux and the other for the thermal diffusion. They are obtained from 3D electromagnetic and entropy balance equations Vu and L.Lefèvre [2013], using a 3D to 1D geometric reduction method and a symplectic discretization scheme as developed in Vu et al. [2013a] for the resistive diffusion equation. A thermal diffusion 0D model is obtained using the same approach. The geometric reduction and symplectic discretization schemes both preserve the qualitative spectrum properties of the original 3D model. The finite dimensional PCH models have the same invariants (for instance total energy) and model structure as the infinite-dimensional ones. The Thermo-magneto-hydrodynamics couplings between the two models will be discussed, reduced and discretized using the same approach to provide a simplified control model.

Then an IDA-PBC controller will be designed. It aims at reaching a predefined safety factor q, using three actuators: the voltage at the plasma boundary (the loop voltage V_{loop}), the distributed non inductive current-drive J_{ext}, and the external heating source S_{heat}. The considered distributed controls J_{ext} and S_{heat} have predefined shapes (a Gaussian form in the studied case) and the considered control signals are only the total external current power P_{ext} and heating power P_{heat}. The consequence is that the system is a finite rank input-output control systems with both boundary and distributed control actions. After discretization, the finite dimensional coupled control model may thus be considered as an underactuated system in the sense that the number of actuators is less than the number of system states (more details on underactuated PCH systems could be found in Ortega and Spong [2002]).

Therefore, only three reference points of the safety factor profile will be assigned with the help of the three available actuators mentioned here above (V_{loop}, P_{ext} and P_{heat}). However, the corresponding full radial profile reference for the safety factor (hence for the state variables of our model) is required for our IDA-PBC state feedback design. Therefore, this profile will be computed and used for the feedforward design of the control by taking into account at the same time the actuation constraints and the TMHD couplings. This procedure leads to an achievable steady state for the feedback design and, on the other hand, transforms the feedback design into a linearized IDA-PBC feedback control problem.

The proposed control design is validated in simulations using the RAPTOR (RApid Plasma Transport simulatOR) code, developed at the Centre de Recherches en Physique des Plasmas, Ecole Polytechnique Fédérale de Lausanne (CRPP/EPFL), Switzerland. RAPTOR is a 1D tokamak transport code specially designed for rapid execution compatible with needs for real-time execution or for use in nonlinear optimization schemes, Felici et al. [2011], Felici and Sauter [2012].

The paper is organized as follows. Section 2 presents two discrete PDE models in PCH formulation for resistive and thermal diffusion and the couplings between these two domains. In section 3, the coupled PCH system for the control model is figured out. The feedforward control is determined including the TMHD couplings, then the system parameters are linearized for the IDA-PBC control presented in section 4. Some numerical results are illustrated for the proposed method in section 5. Section 6 closes the paper with a brief conclusion and some perspectives.

2. PCH FORMULATION FOR TOKAMAK SYSTEM

A TMHD 3D model for Tokamak's plasma dynamics is presented in Vu and L.Lefèvre [2013]. This model uses the mass, entropy, momentum and electromagnetic balance equations. With the help of the PCH formulation for distributed parameter systems and Stokes-Dirac interconnection structures (cf. van der Schaft [2005]), balance and closure (constitutive) equations are organized in a physically coherent port-based model.

The assumptions of axisymmetry and quasi-static equilibrium Blum [1989] may be used to reduced the 3D to the 1D PCH model. Then the symplectic geometric discretization methodology presented in Vu et al. [2013a] transforms the 1D to 0D PCH model which preserves the energetic and spectral properties of the actual 1D (and 3D) model.

In this section, we present two 0D models obtained after this reduction procedure. The first one is obtained from the Maxwell field equations, considering an ohmic diffusion closure equation. It has been proved to be a symplectic reduction of the usual 1D resistive diffusion equation. Details can be found in Vu et al. [2013a]. The second one accounts for the thermal diffusion equation. It is obtained, using the same approach, from the entropy balance equation with a simple "Fourier" closure equation for the heat flow.

2.1 Resistive diffusion equation

The following time-dependent dissipative Port Hamiltonian system (cf. Vu et al. [2013a]) is considered for the resistive diffusion equation:

$$\begin{pmatrix} \partial_t \mathbf{d} \\ \partial_t \mathbf{b} \end{pmatrix} = \left[\begin{pmatrix} 0 & -J_1 \\ -J_2 & 0 \end{pmatrix} - \begin{pmatrix} R^{-1} & 0 \\ 0 & 0 \end{pmatrix} \right] \begin{pmatrix} \partial_{\mathbf{d}} \mathbb{H}_{EM} \\ \partial_{\mathbf{b}} \mathbb{H}_{EM} \end{pmatrix} - \begin{pmatrix} \mathbf{J}_{bs} + \mathbf{J}_{ext} \\ -J_4 V_{loop} \end{pmatrix} \tag{1}$$

where $\mathbf{d}$, $\mathbf{b}$, $\mathbf{J}_{ext}$, $\mathbf{J}_{bs} \in \mathbb{R}^{N\times 1}$ are respectively the time-varying coefficients of the electric and magnetic field (D, B), the external current densities source J_{ext} and the bootstrap current J_{bs}. The external current source is equal to $J_{ext} = f_{ext}(z)\, P_{ext}$ where $f_{ext} \in \mathbb{R}^N$ indicates the form of this current source depending on the normalized plasma radius $z \in [0, 1]$. The sum $J_{bs} + J_{ext}$ gives the non-inductive plasma current J_{ni}. The loop voltage V_{loop} is the boundary control. The matrices J_1, $J_2 \in \mathbb{R}^{N\times N}$ with $J_1 = -J_2^T$ represent the spatial derivation ∂_z in the new finite dimension system, $J_4 \in \mathbb{R}^{N\times 1}$ is related to the boundary coefficient, and $R \in \mathbb{R}^{N\times N}$ is a dissipation matrix which is determined by the resistivity η.

The electromagnetic energy smooth function $\mathbb{H}_{EM}$ is defined as:

$$\mathbb{H}_{EM} = \frac{1}{2}\left(\mathbf{d}^T G_{el} \mathbf{d} + \mathbf{b}^T G_{mg} \mathbf{b}\right)$$

where matrices G_{el} and $G_{mg} \in \mathbb{R}^{N\times N}$ are simply reduced to the electric and magnetic permeability, $\frac{1}{\epsilon_0 C_3}$ and $\frac{C_2}{\mu_0}$ in the simple anisotropic case. C_2, C_3 are the coordinate coefficients defined in Blum [1989].

2.2 Thermal diffusion equation

From Vu and L.Lefèvre [2013], the 3D thermal model can be rewritten with the covariant form as:

$$\begin{pmatrix} T D_t \mathbf{s} \\ F \end{pmatrix} = \begin{pmatrix} 0 & -d \\ -d & 0 \end{pmatrix} \begin{pmatrix} T \\ f_q \end{pmatrix} + \begin{pmatrix} S \\ 0 \end{pmatrix} \tag{2}$$

where d, $\mathbf{s}$, f_q, F, and T denote respectively the external derivative, the entropy, the heat flux, the thermal force and the average plasma temperature. The thermal source S includes the Joule effect $\eta J_{tot}\,(J_{tot} - J_{ni})$ where J_{tot} is the total plasma current, the external heating source $f_{heat}(z) P_{heat}$ with spatial distribution $f_{heat}(z)$ and total heating power P_{heat}, and the radiation losses [1] which are neglected in this work.

The same reduction procedure as for the electromagnetic domain is applied to the thermal domain. First, the assumptions of axisymmetry and quasi-static plasma equilibrium lead the system presented in (2) to a 1D model. Then the same symplectic geometric discretization method as previously used gives a 0D model. Note that one can choose different number N of approximation base in two previous models. However, we set them the same to simplify the

[1] such as Bremsstrahlung and cyclotron radiation (cf. Wesson [2004])

calculus and moreover, we don't have particular requires in term of approximation precision for two models:

$$\begin{pmatrix} \partial_t \mathbf{e}_{ex} \\ 0 \end{pmatrix} = \left[\begin{pmatrix} 0 & J_{T1} \\ J_{T2} & 0 \end{pmatrix} - \begin{pmatrix} 0 & 0 \\ 0 & R_T^{-1} \end{pmatrix} \right] \begin{pmatrix} G_T \mathbf{e}_{ex} \\ \mathbf{f}_q \end{pmatrix} + \begin{pmatrix} \overline{S} \\ J_{T4} T_1 \end{pmatrix}$$
$$\mathbb{H}_T = \frac{1}{2} \mathbf{e}_{ex}^T G_T \mathbf{e}_{ex} \tag{3}$$

where $\mathbf{e}_{ex}$, $\mathbf{f}_q \in \mathbb{R}^{N \times 1}$ are the time dependent coefficients of respectively $TD_t\mathbf{s}$ and the heat flux f_q. The matrices J_{T1}, $J_{T2} \in \mathbb{R}^{N \times N}$ with $J_{T1} = -J_{T2}^T$ are obtained from the reduction of the spatial derivation operator ∂_z in the chosen finite dimensional spatial approximation bases. $G_T \in \mathbb{R}^{N \times N}$ represents the constitutive relation between e_{ex} and T, and $R_T \in \mathbb{R}^{N \times N}$ is the thermal resistivity which depends on the thermal diffusion coefficient χ. This coefficient represents the relation between the thermal force F and the heat flux f_q. $J_{T4} \in \mathbb{R}^{N \times 1}$ is related to the boundary coefficient, T_1 is the fixed value of the average temperature at the boundary. $\mathbb{H}_T$ is the thermal energy.

Notice that the PCH model for thermal diffusion in (2) is given in implicit form only, since there's only one balance equation for entropy is used. The second equation is considered the thermal force, which shows up the thermal dissipation in order to drive the model into the standard PCH form.

2.3 Thermo-MagnetoHydroDynamics coupling

The plasma dynamics in tokamaks are clearly separated into different time scales (cf.Blum [1989]). The particle diffusion time constant, τ_n, and the heat diffusion for electrons and ions, respectively τ_e, τ_i, are of the order of a millisecond, while the time constant for the current density and magnetic field radial diffusion is of the order of a hundred of milliseconds for the TCV tokamak, for instance. Hence, the plasma temperature T steady state profile in the thermal diffusion is established far faster than the magnetic field steady state profile in the resistive diffusion. This observation allows us to decouple the two-PDE-solvers (for the head and flux diffusion equations). The interdomain coupling in the resistive term $\eta(T_e)$ and in the source term $J_{bs}(T_e, \partial_z T_e)$ appearing in the resistive diffusion model may then be computed using the expressions given in Sauter et al. [1999] once T_e, the electronic temperature, is determined using the fastest solvers for the heat equation. On the other hand, the coupling appearing in the thermal diffusion coefficient $\chi(\partial_z T, B)$ may be determined by using the analytic expression which depends mainly on $\partial_z T$ and B (induction field coordinates) as in Erba et al. [1998].

Remark 1. The electronic temperature T_e is deduced from the average temperature T with the assumption of a linear relation between electronic and ionic temperatures (respectively τ_e, τ_i) and densities (respectively n_e, n_i) :

$$\begin{cases} T_i &= \alpha_{T_i} T_e \\ n_i &= \alpha_i n_e \\ T &= \dfrac{n_e T_e + n_i T_i}{n_e + n_i} = \dfrac{1 + \alpha_i \alpha_{T_i}}{1 + \alpha_i} T_e \end{cases}$$

where α_i and α_{Ti} are the ratios defined in Witrant et al. [2007].

3. CONTROL MODEL

3.1 The coupled TMHD model in PCH form

The aggregation of the resistive diffusion model and the heat transport model results in the finite dimensional TMHD model:

$$\begin{pmatrix} \partial_t \mathbf{d} \\ \partial_t \mathbf{b} \\ \partial_t \mathbf{e}_{ex} \\ 0 \end{pmatrix} =$$
$$\left(\begin{bmatrix} -\begin{pmatrix} 0 & J_1 \\ J_2 & 0 \end{pmatrix} & 0 \\ 0 & \begin{pmatrix} 0 & J_{T1} \\ J_{T2} & 0 \end{pmatrix} \end{bmatrix} - \begin{bmatrix} \begin{pmatrix} R^{-1} & 0 \\ 0 & 0 \end{pmatrix} & 0 \\ 0 & \begin{pmatrix} 0 & 0 \\ 0 & R_T^{-1} \end{pmatrix} \end{bmatrix} \right)$$
$$\times \begin{pmatrix} G_{el} \mathbf{d} \\ G_{mg} \mathbf{b} \\ G_T \mathbf{e}_{ex} \\ \mathbf{f}_q \end{pmatrix} + \begin{pmatrix} -f_{ext} P_{ext} - \mathbf{J}_{bs} \\ J_4 V_{loop} \\ \overline{S} \\ J_{T4} T_1 \end{pmatrix} \tag{4}$$

which is written in the PCH form:

$$\begin{pmatrix} \dot{x} \\ 0 \end{pmatrix} = [\mathcal{J}(x) - \mathcal{R}(x)] \frac{\partial \mathbb{H}}{\partial x}(x) + gu \tag{5}$$

where $\mathcal{J} = -\mathcal{J}^T$ is a skew-symmetric interconnection matrix defining a corresponding Dirac structure while $\mathcal{R} = \mathcal{R}^T \geqslant 0$ is a symmetric positive semi-definite dissipation matrix which is nonlinearly depending on the state variables. The total energy function or Hamiltonian is simply the sum of electromagnetic and thermal energy: $\mathbb{H} = \mathbb{H}_{EM} + \mathbb{H}_T$. As a consequence of this port-Hamiltonian representation for the TMHD model, the IDA-PBC approach for nonlinear control may be applied to the whole interconnected model as it was similarly in Vu et al. [2013b] for the electromagnetic part of it (equivalent to the resistive diffusion model).

The control of the interconnected system (resistive diffusion and thermal diffusion equations) is expected to take advantage of the explicit state space representation of the TMHD coupling analysis to improve the control performance through a better parameter estimations for the resistivity R (via a good approximation of $\eta(T_e)$ and of the bootstrap current $J_{bs}(T_e, \partial_z T_e)$), as well as for the thermal resistivity R_T (via the diffusion coefficient $\chi(\partial_z T, B)$).

Besides, the control actions are assumed to satisfy specific shapes (radial distribution) via the functions f_{ext} and f_{heat} (namely Gaussian distributions in the studied case). Therefore control variables are the scalar total external current power P_{ext} and heating power P_{heat}. As discussed in Vu et al. [2013b], this implies that only a limited set of equilibrium states $x_d = (\mathbf{d}_d, \mathbf{b}_d, \mathbf{e}_{ex})^T$ are reachable. Therefore a feedforward control will be designed first which leads to a reachable steady state for which closed loop convergence of the feedback diffusion system may be obtained via an IDA-PBC controller. In a previous work Vu et al. [2013b] a feedforward control $(P_{ext}, V_{loop})_d^T$ has been proposed to achieve regulation for two reference points of the safety factor profile: at the center and at the boundary. Here, using the interconnected TMHD model will allows us to add a third reference point of q by the use of the new control action P_{heat}.

3.2 Steady state generation

The steady state x_d of (4) satisfies:

$$\begin{cases} J_1 G_{mg}\mathbf{b}_d + R^{-1}G_{el}\mathbf{d}_d + f_{ext}P_{ext} + \mathbf{J}_{bs} & = 0 \\ -J_2 G_{el}\mathbf{d}_d + J_4 V_{loop.d} & = 0 \\ J_{T1}\mathbf{f}_{qd} + \overline{S} & = 0 \\ J_{T2}G_T\mathbf{e}_{ex} - R_T^{-1}\mathbf{f}_{qd} + J_{T4}T_1 & = 0 \end{cases} \tag{6}$$

The following points could be noticed:

- The input signal T_1 is fixed constant
- The source term $\overline{S}$ includes the Joule effect $S_{Joule} = \eta J_{tot}\left(J_{tot} - J_{ni}\right)$ (where J_{tot} is the total current density) and the external heating source S_{heat} which is controlled by the heating power P_{heat}.
- J_{bs} is one of the MHD couplings. It is assumed that it may be deduced from the temperature T, the temperature gradient $\partial_z T$ and the induction field (via the safety factor q) profiles:

$$J_{bs} = q\left(B_1 T + B_2 \partial_z T\right) \tag{7}$$

  where B_1 and B_2 are the constants determined in Witrant et al. [2007] in the case of a steady state particle density n. With the assumption that the thermal steady state for T, $\partial_z T$ are quickly established, we can deduce the discrete bootstrap current $\mathbf{J}_{bs}$ as:

$$\begin{aligned} \mathbf{J}_{bs} &= \frac{1}{\mathbf{b}}\left(B_1\mathbf{e}_{ex} + B_2 R_T^{-1}\mathbf{f}_q\right) \\ &= \frac{1}{\mathbf{b}}\left(B1\left(J_{T2}G_T\right)^{-1} + B_2\right)\left(J_{T1}R_T\right)^{-1}\overline{S} \\ &= \frac{1}{\mathbf{b}}\left(B_1\left(J_{T2}G_T\right)^{-1} + B_2\right)\left(J_{T1}R_T\right)^{-1}\left(S_{Joule} + S_{heat}\right) \end{aligned} \tag{8}$$

  The Joule effect S_{Joule} may be considered as a measurable output assuming that the total current and external non-inductive current are known). S_{heat} is given by the analytic expression $f_{heat}P_{heat}$ where f_{heat} is a chosen (known) Gaussian function of z which is a characteristic of the used actuator.
- $\eta(T)$, the resistivity coefficient, may be linearized from the analytic expression given in Sauter et al. [1999]:

$$\eta(T) = C_\eta(\mathbf{b})\,T^{-3/2} \tag{9}$$

  and is integrated in the online computation of the resistivity dissipation matrix $R(\eta(T)) \sim T^{3/2}$,
- $\chi(\partial_z T, B)$, the thermal diffusion coefficient, whose analytic expression given in Witrant et al. [2007], is approximated as:

$$\chi(\partial_z T, B) = C_\chi(\mathbf{b})\,\partial_z T \tag{10}$$

Due to the different orders of magnitude between T and $\mathbf{b}$ and to the fact that only small variations of the magnetic field are considered, the dependence of the bootstrap current, the plasma resistivity and the thermal diffusion coefficient with the magnetic field may be neglected. This allows a linearization for the feedforward computation, using, for the computation of these quantities, the measurement of the magnetic field instead of the foreseen reference $\mathbf{b_d}$. It has to be noticed however that doing so, the stabilization of the state feedback control will be reached only locally, when the requested references will be close enough from the system initial state values.

Finally, the feedforward is deduced from the steady state equation (6) using the relation between the safety factor q and the magnetic state $\mathbf{b}$:

$$w^f(z)\,\mathbf{b}(t) = \frac{2\pi\left(B_{\phi 0}a^2 z\right)}{q(z)}$$

where $w^f(z)$ are the spatial approximation basis functions used for magnetic field, $B_{\phi 0}$ is the toroidal magnetic field intensity at the plasma center $z = 0$, and a is the small radius of plasma torus. The obtained feedforward u_d is:

$$\begin{pmatrix} P_{ext} \\ (P_{heat})^{3/2} V_{loop} \\ P_{heat} \end{pmatrix}_d = \left(\begin{pmatrix} w^f(z_1) \\ w^f(z_2) \\ w^f(z_3) \end{pmatrix} C\right)^{-1} \times \left[\begin{array}{c} 2\pi\left(B_{\phi 0}a^2\right)\left(\frac{z_1}{q_{1d}}, \frac{z_2}{q_{2d}}, \frac{z_3}{q_{3d}}\right)^T \\ + \begin{pmatrix} w^f(z_1) \\ w^f(z_2) \\ w^f(z_3) \end{pmatrix}\left(J_1 G_{mg}\right)^{-1} C_{bs.d}S_{Joule} \end{array}\right] \tag{11}$$

where $C = -\left(J_1 G_{mg}\right)^{-1}\left(f_{ext} \quad C_R J_2^{-1} J_4 \quad C_{bs} f_{ext}\right)$; $C_R = R/\left(P_{heat}\right)_d^{3/2}$,and z_1, z_2, z_3 are the three positions of the three references q_{1d}, q_{2d}, q_{3d} for the safety factor.

The feedforward control is thus obtained from the steady state for the system obtained by the linearization at each time step of the non-linear parameters R, R_T, G_T and J_{bs}. The feedback control is then required not only to increase the convergence speed but also to overcome the errors caused by the linearization assumption.

4. IDA-PBC CLOSED LOOP CONTROL

The main idea of this IDA-PBC method is to choose an appropriate feedback control law $u(x)$ so that the original system (5) is pulled back to a reference system with a set of desired properties. A brief reminder of IDA-PBC design methodology from Ortega et al. [2002] is given hereafter.

4.1 Design methodology

Let us design a closed loop reference system:

$$\dot{x} = \left[\mathcal{J}_d(x) - \mathcal{R}_d(x)\right]\frac{\partial \mathbb{H}_d}{\partial x}(x) \tag{12}$$

with $\mathcal{J}_d = -\mathcal{J}_d^T$, $\mathcal{R}_d = \mathcal{R}_d^T \geqslant 0$ and a strict local minimum x_d for the closed loop Hamiltonian $\mathbb{H}_d$. This minimum x_d is a locally stable equilibrium since:

$$\dot{\mathbb{H}}_d = -\left(\partial_x \mathbb{H}_d\right)^T \mathcal{R}_d\left(\partial_x \mathbb{H}_d\right) \ \leq 0 \tag{13}$$

The static state feedback is then chosen such that the closed loop system match this reference PCH system, using "tuning parameters" $\mathcal{J}_a(x), \mathcal{R}_a(x), \mathbb{H}_a(x)$ such that $\mathcal{J}_d(x) = \mathcal{J}(x) + \mathcal{J}_a(x)$, $\mathcal{R}_d(x) = \mathcal{R}(x) + \mathcal{R}_a(x)$ and $\mathbb{H}_d(x) = \mathbb{H}(x) + \mathbb{H}_a(x)$ has the minimum at x_d. This leads to a matching equation for the equivalence of (5) and (12:

$$\left(\mathcal{J} - \mathcal{R}\right)\frac{\partial \mathbb{H}}{\partial x} + gu = \left(\mathcal{J}_d - \mathcal{R}_d\right)\frac{\partial \mathbb{H}_d}{\partial x} \tag{14}$$

The following conditions are required for the solution:

i) *(Integrability)*

$$\frac{\partial^2 \mathbb{H}_d}{\partial x^2}(x) = \left[\frac{\partial^2 \mathbb{H}_d}{\partial x^2}(x)\right]^T \tag{15}$$

ii) *(Equilibrium assignment)*

$$\frac{\partial \mathbb{H}_d}{\partial x}(x_d) = 0 \tag{16}$$

iii) *(Lyapunov stability)*

$$\frac{\partial^2 \mathbb{H}_d}{\partial x^2}(x_d) > 0 \tag{17}$$

This general design methodology preserves many degrees of freedom since the controller is set only once $\mathcal{J}_a$, $\mathcal{R}_a$

and $\mathbb{H}_a$ have been chosen. We propose a particular design methodology for our system in the next subsection.

4.2 Control tuning

The interconnected system naturally converge to its equilibrium thanks to the two dissipations represented by the dissipation matrices R and R_T. We decide to preserve the interconnection structure $\mathcal{J}$ of the original system ($\mathcal{J}_a = 0$), hence not modifying the existing interrelation. Our control design is to set $\mathcal{R}_a$ constant such that $\mathcal{R}_d = \mathcal{R}_d^T > 0$ (dissipation rate), and then to determine $\mathbb{H}_d$ and the feedback signal δu with the help of the matching equation (14) and the conditions in (15-17).

Let $X = x - x_d$ denotes the distance from the equilibrium of (4). The matching equation reads then:

$$(\mathcal{J} - \mathcal{R}) \frac{\partial \mathbb{H}}{\partial X} + g\delta u = (\mathcal{J}_d - \mathcal{R}_d) \frac{\partial \mathbb{H}_d}{\partial X} \tag{18}$$

$\partial_X \mathbb{H}_d$ may thus be obtained as the solution of a "linear" PDE obtained by multiplying the matching equation (18) to the left annihilator of g (i.e. such that $g^{\perp} g = 0$):

$$0 = \left(g^{\perp} g\right) \delta u \;=\; g^{\perp} \left((\mathcal{J}_d - \mathcal{R}_d) \frac{\partial \mathbb{H}_d}{\partial X} - (\mathcal{J} - \mathcal{R}) \frac{\partial \mathbb{H}}{\partial X} \right) \tag{19}$$

The robustness of the controller with respect to two kinds of uncertainties is studied. First ones are uncertainties on the system resistivity $\mathcal{R}$ resulting from poor estimations for the plasma resistivity η and the thermal diffusion coefficient χ. Second ones are uncertainties related to the linearization assumption made in the derivation the feedforward control and in the approximation of the bootstrap current $\mathbf{J}_{bs}$. The robustness analysis detail is reffed to Vu et al. [2013b].

Brief, with a choice of $\mathcal{R}_a$ such that $\mathcal{R}_d$ is sufficiently large, we can handle these uncertainties. Of course, the actuator power saturation will prevent us to compensate very large perturbations. Besides, the designed controller being basically a proportional controller, the choice of large values for the proportional gain may create undesired oscillations and instability for the closed loop system system.

5. SIMULATION

For the simulation, the RAPTOR code with the configuration of TCV is used. Two ECCD/ECRH clusters [2] are used to generate both the non-inductive current and the external heating source. The first one is used as a co-current source and the second one as a counter-current source. Both clusters are also used as external heating sources and are configured to have the same profile shapes. Therefore distributed control actions have the forms:

$$J_{ext} = f_{ext} P_{ext} = f_{ext} \left(P_A - P_B\right)$$
$$S_{heat} = f_{heat} P_{heat} = f_{heat} \left(P_A + P_B\right)$$

In this work, we consider that all the states are measurable or computable from measurements. Three reference values for safety factor q are defined at the radial relative coordinates $z = 0.1$, 0.3, and 0.45. The feedforward calculus gives u_d, the reference profile q_d and the average temperature profile T_d, corresponding to the three references and taking into account actuators limits. The IDA-PBC control determines the feedback signal δu from (19) to correct the error X. The IDA-PBC parameters are designed as discussed in the previous subsection, with the choice of $\mathcal{J}_a = 0$ and

$$\mathcal{R}_a = \begin{bmatrix} \begin{pmatrix} 0 & 0 \\ 0 & R_{a1} \end{pmatrix} & 0 \\ 0 & \begin{pmatrix} R_{a2} & 0 \\ 0 & 0 \end{pmatrix} \end{bmatrix} \tag{20}$$

where the positive diagonal matrix R_{a1} and R_{a2} account for the dampings added in electromagnetic domain and thermal domain respectively. As a particular case one can set only three diagonal values for the matrices R_{a1} and R_{a2} which correspond to three chosen reference positions.

Figures 1 and 2 show the results obtained with feedforward and feedback controls. The controller starts at $t = 0.2s$ with the initial values $(P_{ext}, V_{loop}, P_{heat})_{init} = (0, -0.65V, 300kW)$, whereas at $z = (0.1, 0.25, 1)$ the reference q profile is set as $q_a = (0.85, 1, 6.3)$. Then at $t = 0.8s$, the reference is changed to $q_b = (1.05, 1.1, 7.5)$.

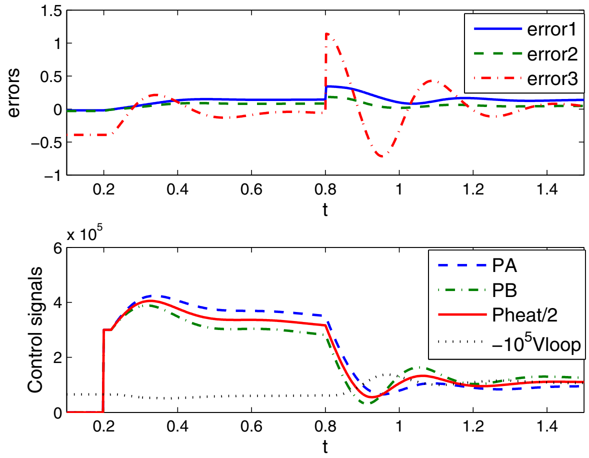

Fig. 1. Feedforward control: errors at three reference points (top) and feedforward control signals (bottom)

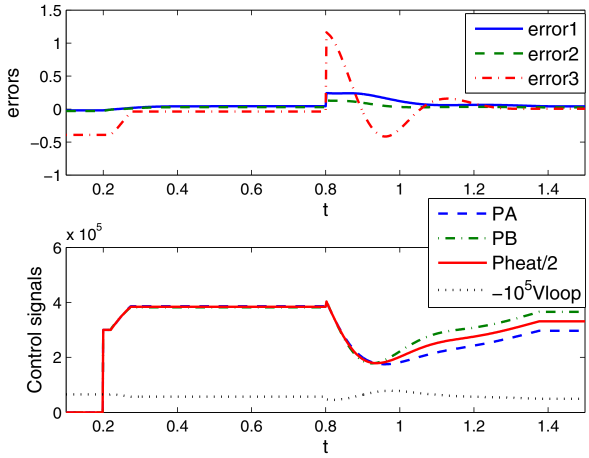

Fig. 2. Feedback control: errors at three reference points (top) and feedback control signals (bottom)

[2] The details for TCV actuators can be easily found in the website https://crppwww.epfl.ch/crpp_tcv.html

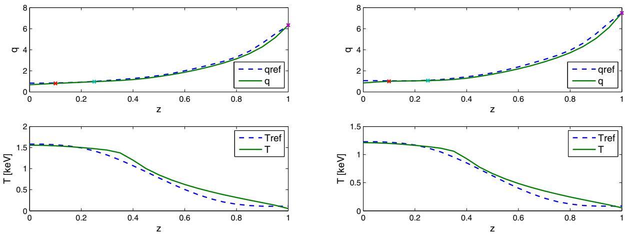

Fig. 3. q_a and T_a profiles at $t = 0.7s$ (left) and q_b and T_b profiles at $t = 1.5s$ (right)

The feedforward does bring the q profile to the reference values, but the actuator values as well as the q profile oscillate around the equilibrium due to the parameter linearization and approximation discussed in subsection 3.2. The feedback however makes effort to improve the result by continuing to react significantly on P_{heat}.

The profiles of q and T are also showed up in the figure 3 with the q_a target at $t = 0.7s$ and the q_b target at $t = 1.5s$. These profiles between the steady states defined by feedforward control and the one of RAPTOR don't totally match each other but at three reference positions for q profile.

6. CONCLUSION

In this paper, a static feedback law is presented for a set of two interconnected models of the resistive diffusion of the magnetic flux and of the thermal diffusion inside a Tokamak's plasma. The control methodology is IDA-PBC and the control law is derived from a PCH control model obtained from the geometric/symplectic discretization of the corresponding coupled PDEs. The actuator limitations are taken into account in the equilibrium computation for the feedforward control action. The temperature profile and its influence on the resistivity coefficients are integrated into the control law via the MagnetoHydroDynamics couplings. The controller has been tested in simulations made with the RAPTOR code. An convergence has been observed with the computed feedforward and feedback controls. The static error can be eliminate by the integral effect, which is one of the perspectives. In future works, we will analyze and optimize the performance increase which may be obtained by simultaneous control of the thermal and magnetic models. Besides, the matching idea will be used to perform boundary and distributed (finite rank) feedback control directly on the infinite dimension port Hamiltonian model.

REFERENCES

FB. Argomedo, C. Prieur, E. Witrant, and S. Brémond. A strict control lyapunov function for a diffusion equation with time-varying distributed coefficients. *accepted for IEEE Transactions on Automatic Control*, 2012.

J. Blum. *Numerical Simulation and Optimal Control in Plasma Physics*. Gauthier-Villars, 1989.

M.D. Boyer, J. Barton, E. Schuster, and et al. First-principles-driven model-based current profile control for the diii-d tokamak via lqi optimal control. *Plasma Physics and Controlled Fusion*, (55), 2013.

M. Erba, T. Aniel, V. Basiuk, A. Becoulet, and X. Litaudon. Validation of a new mixed bohm/gyro-bohm for electron and ion heat transport against the iter, tore supra and start database discharges. *Nuclear Fusion*, 6(7):2835–2839, July 1998.

F. Felici and O. Sauter. Non-linear model-based optimization of actuator trajectories for tokamak plasma profile control. *Plasma Physics and Controlled Fusion*, 54, 2012.

F. Felici, O. Sauter, S. Coda, B.P. Duval, T.P. Goodman, J.M. Moret, J.I. Paley, and the TCV Team. Real-time physics-model-based simulation of the current density profile in tokamak plasmas. *Nuclear Fusion*, 51, 2011.

O. Gaye, E. Moulay, S. Brémond, L. Autrique, R. Nouailletas, and Y. Orlov. Sliding mode stabilization of the current profile in tokamak plasmas. *CDC*, December 2011.

D. Moreau, F. Crisanti, X. Litaudon, and et al. Real-time control of the q-profile in jet for steady state advanced tokamak operation. *Nuclear Fusion*, 46:870–882, August 2003.

R. Ortega and Mark W. Spong. Stabilization of underactuated mechanical systems via interconnection and damping assignment. *Automatic Control*, 47:1218 – 1233, 2002.

R. Ortega, A.J. van der Schaft, B. Maschke, and G. Escobar. Interconnection and damping assignment: passivity-based control of port-controlled Hamiltonian systems. *Automatica*, 38(4):585–596, 2002.

O. Sauter, C. Angioni, and Y.R. Lin-Liu. Neoclassical conductivity and bootstrap current formulas for general axisymmetric equilibria and arbitrary collisionality regime. *Physic of Plasma*, 6(7):2835–2839, July 1999.

A.J. van der Schaft. Theory of port hamiltonian systems. *Network Modeling and Control of Physical Systems, DISC*, Chapter 1,2, April 2005.

Trang.N.M. Vu and L.Lefèvre. Material balance and closure equations for plasmas in tokamaks. *IFAC Workshop on Thermodynamic Foundations of Mathematical Systems Theory*, July 13-16 2013.

Trang.N.M. Vu, L. Lefèvre, R. Nouailletas, and S. Brémond. Geometric discretization for a plasma control model. *IFAC Joint conference: 5th Symposium on System Structure and Control; 11th Workshop on Time-Delay Systems, 6th Workshop on Fractional Differentiation and Its Applications, Grenoble, France*, February 4-6 2013a.

Trang.N.M. Vu, L. Lefèvre, R. Nouailletas, and S. Brémond. An ida-pbc approach for the control of 1d plasma profile in tokamaks. *52nd IEEE Conference on Decision and Control, Florence, Italia*, December 10-13 2013b.

J. Wesson. *Tokamaks. Third edition.* Oxford Science Publications, 2004.

E. Witrant, E. Joffrin, S. Brémond, G. Giruzzi, D. Mazon, O. Barana, and P. Moreau. A control-oriented model of the current profile on tokamak plasma. *Plasma Physics and Controlled Fusion*, 49:1075–1105, 2007.