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# Robust $H\infty$ Power and Rate Control for Uncertain Wireless Networks with Time-Varying State and Input Delays

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**Abstract:** This paper is concerned with power and rate control for wireless networks. In the control theoretic framework, a new power and rate control system model is derived for uncertain wireless networks with time-varying delays in both state and control input. A robust power and rate controller is presented, which is designed based on the $H_\infty$ control approach with linear matrix inequality (LMI). Simulation results verify the effectiveness of the proposed algorithm.

## 1. INTRODUCTION

Time delays have critical effects on the performance of power and rate control in wireless networks, which will reduce the quality of service (QoS), or even lead to system instability. Therefore, power and rate control for time-delayed wireless networks have received considerable attention for the last decade.

Power control algorithms for wireless networks with constant time delays have been presented based on time delay compensation (TDC) by Gunnarsson, Gustafsson and Blom (2001), and based on a hidden Markov model by Zhang and Pathirana (2013). Yang and Chen (2010) presented a power control algorithm using a multiple-mode Smith prediction filter (MMSPF) to deal with time-varying delay. Based on Lyapunov-Razumikhin functions, Lestas (2012) has proven that, if a feasible power allocation exists, then the power control system is asymptotically stable for arbitrary time varying delays. However, all the proposed results did not consider the rate control.

Power and rate control algorithms for wireless networks with time delays have been presented based on a MMSPF by Chen, Yang and Li (2008); and based on a high order model by Moller, Jonsson, Blomgren and Gunnarsson (2011). Subramanian and Sayed (2005) presented a robust power and rate control algorithm based on a state-delayed state space model; Kong, Zhang, Zhang and Zhang (2007) extended the results of Subramanian and Sayed (2005) to input delay and presented a predictive control algorithm. However, the time delays were assumed to be known and constant in the design procedure. In fact, the time delay is often unknown even time-varying in real network environments.

Power and rate control algorithms for wireless networks with time-varying state delay were presented via adaptive control in Han, Sun, Shi and Bi (2013) and via robust $H_\infty$ control in Han, Sun and Liu (2013). However, only the time delay in rate control was considered in Subramanian and Sayed (2005), Han, Sun, Shi and Bi (2013), and Han, Sun and Liu (2013), while the time delay in power control was not considered. In fact, the power control is more sensitive to time delay than the rate control. Additionally, only the state time delay was considered in these papers, but the input delay was not considered. To the authors' best knowledge, research on power and rate control for wireless networks with time-varying delays in both state and control input has not been investigated, which motivates the work of this paper.

This paper considers the power and rate control for wireless networks with time-varying delays in both state and input, not only in the rate control but also in the power control. Firstly, we focus on the modelling and analysis of the power and rate control dynamics, and derive the system model for wireless networks with time-varying delays in Section 2. And then, we present a robust power and rate controller in Section 3. Simulation results are given in Section 4. Conclusion remarks and related future work are discussed in Section 5.

*Notation:* In this paper, $A^T$ and $A^{-1}$ denote the transpose and the inverse of a matrix $A$, respectively; $R^{n\times m}$ denotes the set of all $n\times m$ real matrices; $A > 0$ ($A < 0$) means that $A$ is symmetric positive definite (negative definite); $I$ is an appropriately dimensioned identity matrix; diag$\{\cdot\ \cdot\ \cdot\}$ denotes a block-diagonal matrix; and the symmetric terms in a symmetric matrix are denoted by *, e.g.,

$$\begin{bmatrix} X & Y \\ * & Z \end{bmatrix} = \begin{bmatrix} X & Y \\ Y^T & Z \end{bmatrix}.$$

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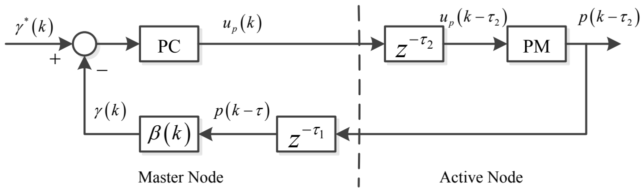


Figure 1. Time delays in power control

## 2. SYSTEM MODEL AND PROBLEM FORMULATION

Consider a wireless network with one node as a master node and others as active nodes which will communicate with each other only through the master node.

Time delays are mainly caused by the signal processing at the master and active nodes and signal transmission through the wireless channel in the network. In the following, we will analyze the effects of time delay in power control. The effects in rate control can be easily straight forwarded.

As depicted in Figure 1 for the power control loop, let $\tau_1$ denote the time delay in uplink channel (including the signal processing delay at the master node and the signal transmission delay from the active node to the master node), $\tau_2$ denote the time delay in downlink channel (including the signal processing delay at the active node and the signal transmission delay from the master node to the active node). The total round trip delay in the network is $\tau = \tau_1 + \tau_2$. Practical value of the round-trip delay is between 2 and 4 at the sampling rate of 800 Hz (Gunnarsson, Gustafsson, and Blom, 2001; Zhang and Pathirana, 2013).

In the uplink power controller, at time instant *k*, the master node sends a power control update command $u_p(k)$ to power multiplier (PM) at the active node. Then PM generates a transmit power $p(k)$ according to $u_p(k)$. Because of the downlink time delay $\tau_2$, the transmit power at the active node should be $p(k-\tau_2)$. It is then transmitted to the master node with the uplink time delay $\tau_1$. That is, the transmitted power at the active node is $p(k-\tau_2)$, while the received power at the master node is $p(k-\tau_2-\tau_1) = p(k-\tau)$, and the received signal-to-interference ratios (SIR) $\gamma(k)$ is related to the received power $p(k-\tau)$. Therefore, the SIR for node *i* at time *k* can be expressed as

$$\gamma_i(k) = \frac{G_{ii}(k)p_i(k-\tau_i)}{\sum_{\substack{j=1\\ j\neq i}}^{n} G_{ij}(k)p_j(k-\tau_j) + \sigma_i^2}, \tag{1}$$

where $\gamma_i(k)$ is the actual SIR, $G_{ij}(k)$ is the channel gain, $p_j(k)$ is the transmit power, *n* is the number of nodes using the same channel, $\sigma_i^2$ is the noise power, and $\tau_j$ is the round trip time delay. Define

$$\beta_i(k) = \frac{G_{ii}(k)}{\sum_{\substack{j=1\\ j\neq i}}^{n} G_{ij}(k)p_j(k-\tau_j) + \sigma_i^2}, \tag{2}$$

then

$$\gamma_i(k) = \beta_i(k)p_i(k-\tau_i), \tag{3}$$

or, in dB scale

$$\bar{\gamma}_i(k) = \bar{\beta}_i(k) + \bar{p}_i(k-\tau_i), \tag{4}$$

where $\bar{\gamma}_i(k) = \ln\gamma_i(k)$, $\bar{\beta}_i(k) = \ln\beta_i(k)$, and $\bar{p}_i(k) = \ln p_i(k)$. Subramanian and Sayed (2005) introduced the random walk model for $\bar{\beta}_i(k)$ as follows

$$\bar{\beta}_i(k+1) = \bar{\beta}_i(k) + n_i(k), \tag{5}$$

where $n_i(k)$ is a zero-mean disturbance with variance $\sigma_n^2$.

The *power control* algorithm (in dB scale) is given by

$$\bar{p}_i(k+1) = \bar{p}_i(k) + \alpha_i[\bar{\gamma}_i^*(k) - \bar{\gamma}_i(k)], \tag{6}$$

where $\alpha_i$ is a step-size factor, $\gamma_i^*(k)$ is the desired SIR, and $\bar{\gamma}_i^*(k) = \ln\gamma_i^*(k)$.

In order to make the actual SIR $\gamma_i(k)$ track the desired SIR $\gamma_i^*(k)$, as in Subramanian and Sayed (2005), we add a power control $u_{ip}(k)$ in (6) given by

$$\bar{p}_i(k+1) = \bar{p}_i(k) + \alpha_i[\bar{\gamma}_i^*(k) - \bar{\gamma}_i(k)] + b_{ip}u_{ip}(k), \tag{7}$$

for a given factor $b_{ip}$ and control $u_{ip}(k)$ to be determined.

From (4), (5), and (7), we have

$$\begin{aligned}\bar{\gamma}_i(k+1) = {} & \bar{\gamma}(k) - \alpha_i\bar{\gamma}_i(k-\tau_i) + \alpha_i\bar{\gamma}_i^*(k-\tau_i) \\ & + b_{ip}u_{ip}(k-\tau_i) + n_i(k).\end{aligned} \tag{8}$$

*Remark:* The power and rate control algorithms, presented in Subramanian and Sayed (2005), Han, Sun, Shi and Bi (2013), and Han, Sun and Liu (2013), just considered the time delay in rate control, while the time delay in power control was not considered, i.e., in the case $\tau_i = 0$ in (1). Because the power control is more sensitive to time delay than the rate control, the effect of time delay in power control must be considered. Furthermore, the time delay in control input was not considered in those algorithms, i.e., in the case $\tau_i = 0$ of $u_{ip}(k-\tau_i)$ in (8).

The *rate control* algorithm is given by (Subramanian and Sayed, 2005)

$$f_i(k+1) = f_i(k) + \mu[d(k) - c_1(k)f_i(k) - c_2(k)f_i(k-\tau_i)], \tag{9}$$

where $f_i(k)$ is the flow rate at node *i*, $\mu > 0$ is a step-size factor, $c_1(k)$ and $c_2(k)$ are congestion factors, $d(k)$ is a zero-mean random variable with variance $\sigma_d^2$, and $\tau_i$ is the round trip delay time.

From Shannon's capacity formula, we obtain

$$f_i(k) = \frac{1}{2}\log_2[1 + \gamma_i^*(k)]. \tag{10}$$

Usually $\gamma_i^*(k) >> 1$, then $f_i(k)$ is proportional to $\log_2\gamma_i^*(k)$. Therefore, the desired SIR can be obtained as follows (in dB scale)

$$\bar{\gamma}_i^*(k+1) = [1-\mu c_1(k)]\bar{\gamma}_i^*(k) - \mu c_2(k)\bar{\gamma}_i^*(k-\tau_i) + \bar{\mu} d(k), \quad (11)$$

where $\bar{\mu} = 20\mu / \log_2(10)$.

Like that in power control, as in Subramanian and Sayed (2005), and considering the round trip time delay, we add a rate control $u_{if}(k)$ in (11) as follows

$$\bar{\gamma}_i^*(k+1) = [1-\mu c_1(k)]\bar{\gamma}_i^*(k) - \mu c_2(k)\bar{\gamma}_i^*(k-\tau_i) + b_{if}u_{if}(k-\tau_i) + \bar{\mu} d(k), \quad (12)$$

For simplicity, we drop the node index $i$ in the following. Define

$$x(k) = \begin{bmatrix} \bar{\gamma}(k) \\ \bar{\gamma}^*(k) \end{bmatrix}, \quad (13)$$

then from (8) and (12), we have

$$x(k+1) = \begin{bmatrix} 1 & 0 \\ 0 & 1-\mu c_1(k) \end{bmatrix} x(k) + \begin{bmatrix} -\alpha & \alpha \\ 0 & -\mu c_2(k) \end{bmatrix} x(k-\tau) + \begin{bmatrix} b_p & 0 \\ 0 & b_f \end{bmatrix} u(k-\tau) + \begin{bmatrix} n(k) \\ \bar{\mu} d(k) \end{bmatrix}, \quad (14)$$

or

$$x(k+1) = A(k)x(k) + A_d(k)x(k-\tau) + Bu(k-\tau) + \omega(k), \quad (15)$$

with

$$A(k) = \begin{bmatrix} 1 & 0 \\ 0 & 1-\mu c_1(k) \end{bmatrix}, \quad A_d(k) = \begin{bmatrix} -\alpha & \alpha \\ 0 & -\mu c_2(k) \end{bmatrix}, \quad (16)$$

$$B = \begin{bmatrix} b_p & 0 \\ 0 & b_f \end{bmatrix}, \quad \omega(k) = \begin{bmatrix} n(k) \\ \bar{\mu} d(k) \end{bmatrix}, \quad (17)$$

where $\tau$ is the round trip time delay, $\omega(k)$ is a zero-mean random vector with covariance matrix

$$\bar{\omega} = E\{\omega(k)\omega^T(k)\} = \begin{bmatrix} \sigma_n^2 & 0 \\ 0 & \bar{\mu}^2\sigma_d^2 \end{bmatrix}. \quad (18)$$

Since the congestion factors $c_1(k)$ and $c_2(k)$ are usually not known exactly, we consider uncertainties in $c_1(k)$ and $c_2(k)$ as follows (Subramanian and Sayed, 2005)

$$c_1(k) = c_1 + HF(k)\bar{d}, \quad c_2(k) = c_2 + HF(k)\bar{d}_d, \quad (19)$$

where $F(k)$ is a zero mean random noise with variance $\sigma_F^2$, $H$, $\bar{d}$ and $\bar{d}_d$ are known scalars, and $c_1$ and $c_2$ are unknown but bounded as

$$c_{1,l} \le c_1 \le c_{1,u}, \quad c_{2,l} \le c_2 \le c_{2,u}. \quad (20)$$

Therefore, we rewrite $A(k)$ and $A_d(k)$ in (15) as follows

$$A(k) = A + \Delta A(k), \quad A_d(k) = A_d + \Delta A_d(k), \quad (21)$$

where

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1-\mu c_1 \end{bmatrix}, \quad A_d = \begin{bmatrix} -\alpha & \alpha \\ 0 & -\mu c_2 \end{bmatrix}, \quad (22)$$

and

$$\Delta A(k) = HF(k)D, \quad \Delta A_d(k) = HF(k)D_d, \quad (23)$$

with

$$D = \begin{bmatrix} 0 & 0 \\ 0 & -\mu\bar{d} \end{bmatrix}, \quad D_d = \begin{bmatrix} 0 & 0 \\ 0 & -\mu\bar{d}_d \end{bmatrix}. \quad (24)$$

The goal of power and rate control is to drive the actual SIR obtained from (8) by the power control rule towards the desired SIR obtained from (11) by the rate control rule for wireless networks with time-varying state and input delays and uncertainties $\Delta A(k)$ and $\Delta A_d(k)$. The controlled output can be chosen as $z(k) = Cx(k)$ and $C = [1 \quad -1]$, which yields

$$z(k) = \bar{\gamma}(k) - \bar{\gamma}^*(k). \quad (25)$$

Then the power and rate control system with time-varying state and input delays and uncertainties can be rewritten as

$$\begin{aligned} x(k+1) &= (A + \Delta A(k))x(k) + (A_d + \Delta A_d(k))x(k-\tau_k) \\ &\quad + Bu(k-\tau_k) + \omega(k), \\ z(k) &= Cx(k), \\ x(k) &= 0, \quad k \in [-\tau_M, 0]. \end{aligned} \quad (26)$$

where $\tau_k$ is a round-trip time-varying delay and satisfies $0 \le \tau_m \le \tau_k \le \tau_M$, where $\tau_m$ and $\tau_M$ are known lower and upper delay bounds.

The problem of this paper is to design a state feedback power and rate controller such that the resulting closed-loop system is asymptotically stable with $\omega(k) = 0$ and satisfies

$$J = \sum_{k=0}^{\infty} \left\{ z^T(k)z(k) - \upsilon^2\omega^T(k)\omega(k) \right\} < 0, \forall \omega(k) \ne 0, \quad (27)$$

for any $\upsilon > 0$. If such conditions are satisfied, the system (26) is said to be with an $H_\infty$ performance index less than $\upsilon$.

Before ending this section, we introduce the following lemma which is useful to prove our results.

*Lemma 3.1:* (Xie, 1996) Given matrices $S = S^T$, $H$, $D$ and $V = V^T > 0$ with appropriate dimensions,

$$S + HF(k)D + D^T F^T(k) H^T < 0, \quad (28)$$

for all $F(k)$ satisfying $F^T(k)F(k) \le I$ if and only if there exists a scalar $\varepsilon > 0$ such that

$$S + \frac{1}{\varepsilon} HH^T + \varepsilon D^T V D < 0. \quad (29)$$

## 3. ROBUST $H_\infty$ POWER AND RATE CONTROL

For convenience of analysis, we firstly consider the following power and rate control system without any uncertainty

$$\begin{aligned} x(k+1) &= Ax(k) + A_d x(k-\tau_k) + Bu(k-\tau_k) + \omega(k), \\ z(k) &= Cx(k), \\ x(k) &= 0, \quad k \in [-\tau_M, 0]. \end{aligned} \quad (30)$$

The controller structure is chosen as

$$u(k) = Kx(k) . \tag{31}$$

Then the closed-loop system is given by

$$x(k+1) = Ax(k) + (A_d + BK)x(k - \tau_k) + \omega(k) . \tag{32}$$

***Theorem 3.1:*** For given integers $\tau_m$ and $\tau_M$ , if there exist matrices $\overline{P} = \overline{P}^T > 0$ , $\overline{Q} = \overline{Q}^T > 0$ , $\overline{R} = \overline{R}^T > 0$ , $\overline{X}_{11} = \overline{X}_{11}^T > 0$ , $\overline{X}_{22} = \overline{X}_{22}^T > 0$ , $W > 0$ , $Z$ , $\overline{X}_{12}$ , $\overline{Y}_1$ , and $\overline{Y}_2$ satisfying the following LMI

$$\begin{bmatrix} \overline{X}_{11} & \overline{X}_{12} & \overline{Y}_1 \\ * & \overline{X}_{22} & \overline{Y}_2 \\ * & * & 2W - \overline{R} \end{bmatrix} \geq 0 , \tag{33}$$

$$\Phi = \begin{bmatrix} \Phi_{11} & \Phi_{12} & 0 & WA^T & W(A^T - I) & WC^T \\ * & \Phi_{22} & 0 & \Phi_{24} & \Phi_{25} & 0 \\ * & * & -I & I & I & 0 \\ * & * & * & -W & 0 & 0 \\ * & * & * & * & -\overline{R}/\tau_M & 0 \\ * & * & * & * & * & -\upsilon I \end{bmatrix} < 0 , \tag{34}$$

then the closed-loop system (32) is asymptotically stable and with an $H_\infty$ performance index less than $\upsilon$ . Furthermore, the state feedback power and rate control law is given by

$$u(k) = ZW^{-1}x(k) , \tag{35}$$

where

$$\Phi_{11} = (\tau_M - \tau_m + 1)\overline{Q} - W + \overline{Y}_1 + \overline{Y}_1^T + \tau_M \overline{X}_{11} ,$$

$$\Phi_{12} = -\overline{Y}_1 + \overline{Y}_2^T + \tau_M \overline{X}_{12} ,$$

$$\Phi_{22} = -\overline{Q} - \overline{Y}_2 - \overline{Y}_2^T + \tau_M \overline{X}_{22} ,$$

$$\Phi_{24} = \Phi_{25} = WA_d^T + Z^T B^T .$$

*Proof:* see Appendix.

Now we consider the power and rate control for wireless networks with uncertainties. By extending Theorem 3.1, we obtain a robust $H_\infty$ power and rate control for uncertain wireless networks (26).

Under the state feedback power and rate control (31), the closed-loop system can be written as

$$\begin{aligned} x(k+1) &= (A + \Delta A(k))x(k) + (A_d + \Delta A_d(k) + BK)x(k - \tau_k) + \omega(k) \\ z(k) &= Cx(k) . \end{aligned} \tag{36}$$

***Theorem 3.2:*** For given integers $\tau_m$ and $\tau_M$ , if there exist matrices $\overline{P} = \overline{P}^T > 0$ , $\overline{Q} = \overline{Q}^T > 0$ , $\overline{R} = \overline{R}^T > 0$ , $\overline{X}_{11} = \overline{X}_{11}^T > 0$ , $\overline{X}_{22} = \overline{X}_{22}^T > 0$ , $W > 0$ , $Z$ , $\overline{X}_{12}$ , $\overline{Y}_1$ , $\overline{Y}_2$ and a scalar $\lambda > 0$ satisfying the following LMI

$$\Pi = \begin{bmatrix} \Phi + \lambda \overline{D}^T \overline{D} & \overline{H}^T \\ * & -\lambda I \end{bmatrix} , \tag{37}$$

then the closed-loop system (36) is asymptotically stable and with an $H_\infty$ performance index less than $\upsilon$ . Furthermore, the state feedback control law is given by (35), where

$$\overline{D} = \begin{bmatrix} 0 & D & D_d & 0 & 0 & 0 & 0 \end{bmatrix} , \tag{38}$$

$$\overline{H} = \begin{bmatrix} -H^T T_1^T & -H^T T_2^T & 0 & 0 & 0 & 0 & 0 \end{bmatrix} . \tag{39}$$

*Proof:* Replacing $A$ and $A_d$ in (32) with $A + HF(k)D$ and $A_d + HF(k)D_d$ , respectively, we obtain the robust $H_\infty$ performance (34) for the uncertain system (26):

$$\Phi + \overline{H}^T F(k)\overline{D} + \overline{D}^T F^T(k)\overline{H} < 0 . \tag{40}$$

From Lemma 3.1, a necessary and sufficient condition that guarantees (40) is that there exists a scalar $\lambda > 0$ such that

$$\Phi + \frac{1}{\lambda}\overline{H}^T \overline{H} + \lambda \overline{D}^T \overline{D} < 0 . \tag{41}$$

Applying Schur complement formula, we can obtain that (41) is equivalent to (37). □

## 4. SIMULATION RESULTS

Consider a wireless network with the following channel gain (Subramanian and Sayed, 2005)

$$G_{ii}(k) = d_{ii}^{-\delta_i}(k) \cdot 10^{0.1\zeta_i(k)} .$$

The first term $d_{ii}^{-\delta_i}(k)$ denotes the path loss, where $d_{ii}(k)$ is the distance from the active node *i* to its master node, $\delta_i \in R$ is the path-loss exponent between 2 and 6; the second term $10^{0.1\zeta_i(k)}$ denotes the shadow effect (from building, terrain, or foliage), where $\zeta_i(k) \in R$ is a white Gaussian noise. The congestion factors $c_1(k)$ and $c_2(k)$ are chosen as random variables between 0 and 0.5. $n(k)$ and $d(k)$ are zero-mean with variance 0.01. $\overline{\mu} = 0.8$ , $\alpha = 0.2$ , and the round-trip delay is varied between 2 and 4.

Figure 2 shows that the proposed algorithm can effectively compensate for the time-varying delays and uncertainties, and has better performance than the traditional power and rate control algorithm which is shown in Figure 3.

## 5. CONCLUSION

This paper presents a robust $H_\infty$ power and rate control for uncertain wireless networks with time-varying state and input delays. The power and rate controller is designed based on a new system model and via $H_\infty$ control approach with LMI.

For convenience of analysis, this paper assumes that the time delay in power control is same as that in rate control. The different time delays in power control and in rate control will be considered in the future work.

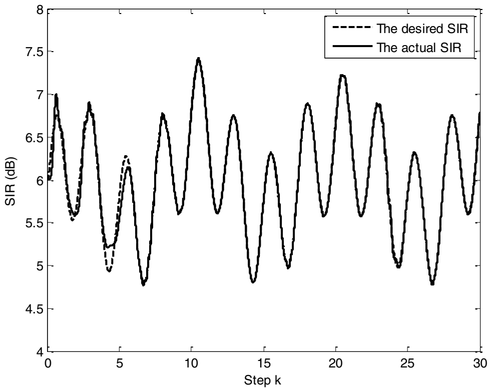


Figure 2. Performance of the proposed algorithm

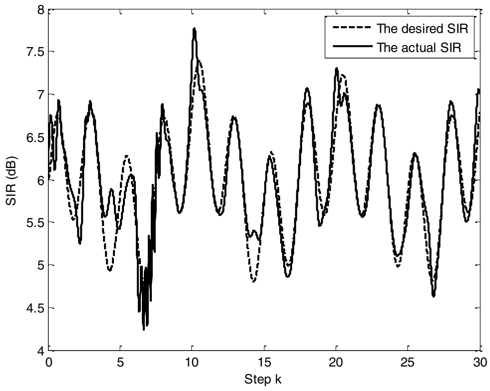


Figure 3. Performance of the traditional power and rate control algorithm

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APPENDIX. Proof of Theorem 3.1

Consider a Lyapunov-Krasovskii functional

$$V(k) = V_1(k) + V_2(k) + V_3(k) + V_4(k)\,, \tag{42}$$

where

$$V_1(k) = x^T(k)Px(k)\,, \tag{43}$$

$$V_2(k) = \sum_{i=k-\tau_k}^{k-1} x^T(i)Qx(i)\,, \tag{44}$$

$$V_3(k) = \sum_{i=-\tau_k+2}^{-\tau_m+1} \sum_{j=k+i-1}^{k-1} x^T(j)Qx(j)\,, \tag{45}$$

$$V_4(k) = \sum_{j=-\tau_M}^{-1} \sum_{i=k+j}^{k-1} e^T(i)Re(i)\,, \tag{46}$$

and $e(k) = x(k+1) - x(k)$ , $P > 0$ , $Q \geq 0$ and $R > 0$ are matrices to be determined. Define $\Delta V(k) = V(k+1) - V(k)$, then

$$\begin{aligned}\Delta V_1(k) &= x^T(k+1)Px(k+1) - x^T(k)Px(k)\\ &= [Ax(k) + (A_d + BK)x(k-\tau_k) + \omega(k)]^T P\\ &\quad \times[Ax(k) + (A_d + BK)x(k-\tau_k) + \omega(k)] - x^T(k)Px(k)\,,\end{aligned}$$

$$\begin{aligned}\Delta V_2(k) &= \sum_{i=k+1-\tau_{k+1}}^{k} x^T(i)Qx(i) - \sum_{i=k-\tau_k}^{k-1} x^T(i)Qx(i)\\ &\leq x^T(k)Qx(k) - x^T(k-\tau_k)Qx(k-\tau_k)\\ &\quad - \sum_{i=k+1-\tau_M}^{k-\tau_m} x^T(i)Qx(i)\,,\end{aligned}$$

$$\Delta V_3(k) = (\tau_M - \tau_m)x^T(k)Qx(k) - \sum_{i=k-\tau_M+1}^{k-\tau_m} x^T(i)Qx(i)\,,$$

$$\begin{aligned}\Delta V_4(k) &= [(A-I)x(k) + (A_d + BK)x(k-\tau_k) + \omega(k)]^T R\\ &\quad \times[(A-I)x(k) + (A_d + BK)x(k-\tau_k) + \omega(k)]\\ &\quad - \sum_{i=k-\tau_M}^{k-1} e^T(i)Re(i)\,.\end{aligned}$$

Using the similar method in Xiong and Lam (2006), we have

$$\begin{bmatrix} X_a & Y_a \\ * & R \end{bmatrix} \geq 0\,, \tag{47}$$

for any matrices $X_{11} = X_{11}^T \in R^{n\times n}$, $X_{12} \in R^{n\times n}$, $X_{22} = X_{22}^T \in R^{n\times n}$, $Y_1 \in R^{n\times n}$, and $Y_2 \in R^{n\times n}$, where

$$X_a = \begin{bmatrix} X_{11} & X_{12} \\ X_{12}^T & X_{22} \end{bmatrix} \quad \text{and} \quad Y_a = \begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix}.$$

Then

$$0 \le \sum_{i=k-\tau_k}^{k-1} \begin{bmatrix} \xi(k) \\ e(i) \end{bmatrix}^T \begin{bmatrix} X_a & Y_a \\ * & R \end{bmatrix} \begin{bmatrix} \xi(k) \\ e(k) \end{bmatrix}$$

$$= \tau_k \xi^T(k) X_a \xi(k) + 2\xi^T(k) Y_a \sum_{i=k-\tau_k}^{k-1} e(i) + \sum_{i=k-\tau_k}^{k-1} e^T(i) R e(i)$$

$$\le \xi^T(k) \left( \begin{bmatrix} Y_1 + Y_1^T & -Y_1 + Y_2^T \\ * & -Y_2 - Y_2^T \end{bmatrix} + \tau_M X_a \right) \xi(k) + \sum_{i=k-\tau_2}^{k-1} e^T(i) R e(i)$$

$$= \Gamma_a ,$$

where $\xi^T(k) = [x^T(k), x^T(k-\tau_k)]$. It implies

$$\Delta V(k) \le \Delta V_1(k) + \Delta V_2(k) + \Delta V_3(k) + \Delta V_4(k) + \Gamma_a$$
$$\le \chi^T(k) \Omega \chi(k) ,$$

where

$$\chi^T(k) = [x^T(k), x^T(k-\tau_k), \omega^T(k)] ,$$

$$\Omega = \begin{bmatrix} \Omega_{11} & \Omega_{12} & 0 \\ * & \Omega_{22} & 0 \\ * & * & 0 \end{bmatrix} + \begin{bmatrix} A^T \\ \bar{A}_d^T \\ I \end{bmatrix} P \begin{bmatrix} A^T \\ \bar{A}_d^T \\ I \end{bmatrix}^T + \begin{bmatrix} (A-I)^T \\ \bar{A}_d^T \\ I \end{bmatrix} (\tau_M R) \begin{bmatrix} (A-I)^T \\ \bar{A}_d^T \\ I \end{bmatrix}^T ,$$

$$\Omega_{11} = (\tau_M - \tau_m + 1)Q - P + Y_1 + Y_1^T + \tau_M X_{11} ,$$
$$\Omega_{12} = -Y_1 + Y_2^T + \tau_M X_{12} ,$$
$$\Omega_{22} = -Q - Y_2 - Y_2^T + \tau_M X_{22} ,$$
$$\bar{A}_d = A_d + BK .$$

A sufficient condition on the stability of system (32) is given by $\Omega < 0$. Since

$$z^T(k) z(k) - \upsilon^2 \omega^T(k) \omega(k) = \chi^T(k) \begin{bmatrix} C^T C & 0 & 0 \\ * & 0 & 0 \\ * & * & -\upsilon^2 I \end{bmatrix} \chi(k) ,$$

then

$$\Delta V(k) + z^T(k) z(k) - \upsilon^2 \omega^T(k) \omega(k)$$
$$\le \chi^T(k) \left( \Omega + \begin{bmatrix} C^T C & 0 & 0 \\ * & 0 & 0 \\ * & * & -\upsilon^2 I \end{bmatrix} \right) \chi(k) .$$

If

$$\Omega + \begin{bmatrix} C^T C & 0 & 0 \\ * & 0 & 0 \\ * & * & -\upsilon^2 I \end{bmatrix} < 0 , \tag{48}$$

we have

$$\Delta V(k) + z^T(k) z(k) - \upsilon^2 \omega^T(k) \omega(k) < 0 .$$

When $\omega(k) = 0$, we have $\Delta V(k) < 0$, then the stability with $\omega(k) = 0$ is established. Summing up from $k = 0$ to $k = \infty$, we can get $V(\infty) - V(0) + J < 0$. Because $V(\infty) \ge 0$ and $V(0) = 0$, we have $J < 0$. Therefore, the closed loop system (32) is asymptotically stable and with an $H_\infty$ performance index less than $\upsilon$.

In the following, we will prove that (48) is equivalent to (33) and (34). Rewrite (48) as

$$\begin{bmatrix} \Lambda_{11} & \Omega_{12} & 0 \\ * & \Omega_{22} & 0 \\ * & * & -\upsilon^2 \end{bmatrix} + \begin{bmatrix} A^T \\ \bar{A}_d^T \\ I \end{bmatrix} P \begin{bmatrix} A^T \\ \bar{A}_d^T \\ I \end{bmatrix}^T + \begin{bmatrix} (A-I)^T \\ \bar{A}_d^T \\ I \end{bmatrix} (\tau_M R) \begin{bmatrix} (A-I)^T \\ \bar{A}_d^T \\ I \end{bmatrix}^T , \tag{49}$$

where $\Lambda_{11} = \Omega_{11} + C^T C$.

By using Schur complement, we can get that (49) is equivalent to the following inequality

$$\begin{bmatrix} \Omega_{11} & \Omega_{12} & 0 & A^T P & (A^T - I) & C^T \\ * & \Omega_{22} & 0 & \bar{A}_d^T P & \tau_M \bar{A}_d^T & 0 \\ * & * & -I & P & \tau_M I & 0 \\ * & * & * & -P & 0 & 0 \\ * & * & * & * & -\tau_M R^{-1} & 0 \\ * & * & * & * & * & -\upsilon I \end{bmatrix} < 0 , \tag{50}$$

In order to obtain the state feedback gain *K*, we pre- and post-multiply $diag\{W, W, I, W, I, I\}$ and $diag\{W, W, W\}$ with $W = P^{-1}$ to (47) and (50), respectively; and apply the change of variables such that $\bar{Q} = WQW$, $\bar{X}_{11} = WX_{11}W$, $\bar{X}_{12} = WX_{12}W$, $\bar{X}_{22} = WX_{22}W$, $\bar{Y}_1 = WY_1W$, $\bar{Y}_2 = WY_2W$, $\bar{R} = R^{-1}$, and $Z = KW$, we can get (34), (35) and

$$\begin{bmatrix} \bar{X}_{11} & \bar{X}_{12} & \bar{Y}_1 \\ * & \bar{X}_{22} & \bar{Y}_2 \\ * & * & WRW \end{bmatrix} \ge 0 . \tag{51}$$

Because of the term $WRW$, (51) is not LMI. It is well know that we can simply set $R = W^{-1}$ to obtain LMI, however, it will increase conservation. Note $W = W^T > 0$ and $R = R^T > 0$, then

$$[W - R^{-1}] R [W - R^{-1}]^T \ge 0 . \tag{52}$$

It implies

$$WRW \ge 2W - R^{-1} . \tag{53}$$

Replacing the term $WRW$ in (51) with $2W - R^{-1}$, we can get (33) immediately. Using Schur complements, we can obtain that (48) is equivalent to (33) and (34). This completes the proof. □