

## Stabilising and optimising a primary closed-loop milling circuit feeding a flotation circuit using StarCS RNMPC

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**Abstract:** Most milling circuits can be well approximated using linear process models, but the mill itself tends to exhibit strong non-linear behavior. To overcome this, each section of the milling circuit is typically controlled using a linear controller and the non-linearities are handled by non-linear capable supervisory layer, such as fuzzy logic or expert rules. Mintek developed the StarCS RNMPC that can use both linear and non-linear models to enable the entire milling circuit to be handled by one controller. MPC is inherently optimising and can therefore provide stabilisation as well as optimisation of the milling circuit, while adhering to all process constraints. This paper shows a simulation of a milling circuit that is stabilised and optimised using a single StarCS RNMPC controller with linear and non-linear prediction models as well as other unique features, such as cascaded CV-to-CV prediction models.

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### 1. INTRODUCTION

The non-linear behaviour of mills makes milling circuits difficult to optimise. Generally, milling control modules are implemented on the various sections of the milling circuit that exhibit linear responses. These typically comprise the mill discharge and mill feed sections. The mill itself exhibit non-linear behaviour and therefore requires non-linear control (Smith, et al., 2000). This requires the control strategy to be split up between the various sections of the milling circuit, such as feed, mill and discharge, requiring some supervisory layer to coordinate the various sections.

Mintek has developed a Robust Non-linear Model Predictive Control (RNMPC) module that, as the name suggests, uses non-linear models for control purposes. It also has a further advantage of tolerating a measure of plant-model mismatch without becoming unstable. The non-linear capability of the RNMPC enables the linear and non-linear behaviour of the milling circuit to be handled by a single controller. MPC controllers are inherently optimising and by controlling the whole circuit, it makes it easier to optimise the process while taking all process constraints into account without the need of a separate supervisory layer.

The RNMPC controller forms part of the control toolbox of StarCS, the control platform of Mintek. The philosophy for toolbox items is to move as much of the complexity into the tools as possible in order to make the job of the engineer as simple and efficient as possible. The controller, therefore, contains sophisticated technologies that will not always be needed, but is available for when a specific problem may require it. Future development will further increase the technologies incorporated into the controller in order to address even wider sets of problems.

The StarCS RNMPC has been successfully deployed at industrial plants around the world as a mill discharge controller; see for example Mantsho, et al. (2013). This paper investigates the option of deploying the RNMPC on the complete milling circuit.

### 2. STARCS RNMPC FOUNDATIONS

The StarCS RNMPC controller is based on work done by Coetzee, et al. (2010), where it was shown that good performance can be obtained by using RNMPC on a milling circuit, but that computation time was making it impractical for industrial application. The technique in Coetzee, et al. (2010) further lacked a feedback mechanism for real-world problems and required full state feedback, which is not possible in most cases.

The computation time issue was addressed by reducing the nonlinear problem internally using various techniques in order to reduce the computational burden. This resulted in less time being spent by the computer to solve the problem.

Feedback is provided by matching the RNMPC algorithm with an unscented Kalman filter implementation that can handle highly non-linear models without becoming inaccurate or unstable (Wan & Van der Merwe, 2000).

The RNMPC is further extended to use soft constraints for CVs (controlled variables), define time-delays in the process models, minimise MV (manipulated variable) movement, optimisation (minimisation or maximisation) of both MVs and CVs and the ability to put one or more MVs into manual that is needed for practical industrial application.

The modelling interface of the RNMPC allows for freeform ordinary differential equations (ODEs) to be defined in

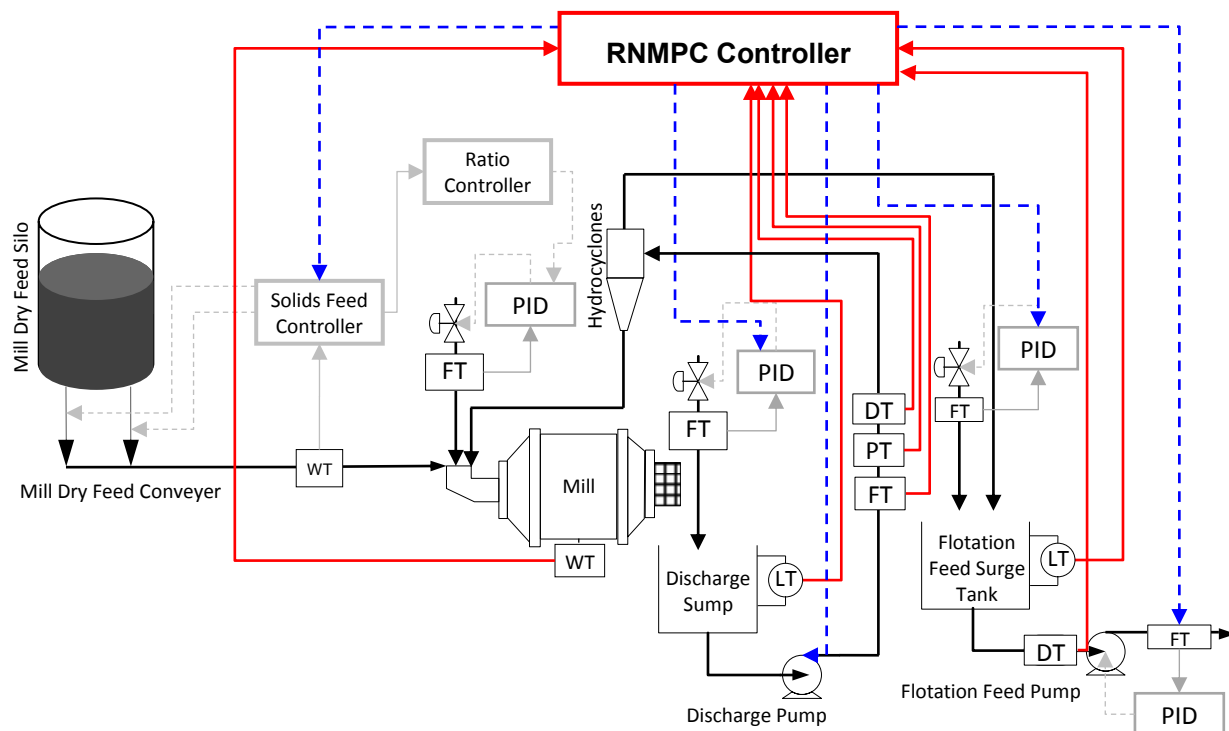


Figure 1. The milling circuit with PID and StarCS solids feed control local loops and RNMPC. The main CVs are in red lines and the main MVs are in dashed blue lines. Auxiliary controllers are in grey with CVs solids lines and MVs dashed lines.

MATLAB like syntax. This allows for complex models with intermediate calculation steps to be easily defined.

### 3. SIMULATION PROBLEM

An optimisation strategy using the StarCS RNMPC module has been developed for a complete milling circuit, from feeders through to the flotation feed surge tank.

The mill in this milling simulation is fed by two vibratory feeders discharging onto a feed belt. A weightometer on the feed belt measures the solids feedrate to the mill. The mill inlet water is ratioed to the solids feedrate. The mill discharges into a discharge sump. The cyclone feed density can be controlled by adding sump dilution water to the discharge sump. The sump level is controlled primarily by the discharge sump pump. The cyclone feed flowrate and pressure that is affected by the sump pump is measured. The mill discharge is partially recycled using a cluster of hydrocyclone classifiers that circulate the underflow containing the oversized material back to the mill for further grinding and passes the overflow containing the in-specification material to the flotation feed surge tank. The surge tank has discharge flowmeter and densitometer to the flotation roughers (see Figure 1). The density to the roughers can be further diluted by adding water to the surge tank.

The RNMPC uses a combination of a non-linear model for the mill load, linear models for the discharge and surge tank and linear cascaded models between the mill discharge and surge tank as well as mill feed and mill discharge (see Figure 3, Table 2 and Table 3 in Appendix A). The cascaded models can easily be extended to cascade of the non-linear mill load model to the mill discharge. The simulation models for the milling circuit is described in Coetzee, et al. (2010) and

further discussed and validated by le Roux, et al. (2013). The mill model for this simulation was based on plant results obtained by van der Westhuizen & Powell (2006) and does not match the model in the previous two references.

Typical model predictive controllers map MVs to CVs (e.g. sump feed water to primary rougher feed density) or measured disturbances (DVs) to CVs (e.g. spillage water to sump level). This has the downside of only rejecting unmeasured disturbances once they have moved to the modelled CV, which can be far downstream. The use of cascaded models in the RNMPC that map CVs to CVs (e.g. the cyclone feed density to primary rougher feed density model) will provide a direct correlation between the two variables and has the advantage of capturing upstream disturbances faster. For example, an unmeasured spillage water disturbance will first affect cyclone feed density and only after a while rougher feed density. By capturing the disturbance at the cyclone feed density, it can be rejected before it has a material impact on the rougher feed density. Under normal conditions, a sump feed water to cyclone feed density and sump feed water to rougher feed density model will be required, mapping the cyclone feed density to rougher feed density using a supervisory layer that can calculate a cyclone feed density setpoint that will be used to obtain the required rougher feed density as well as enable the MPC to reject discharge sump disturbances. With the RNMPC, this is all done implicitly.

The control strategy for the milling circuit is as follows:

- Control rougher feed density to a strict setpoint (large weight in the objective function).

- Control surge tank level to a weak setpoint (low weight in the objective function) allowing flowrate variations to the roughers to be minimised.
- High priority on the rougher feed flowrate stability to minimise changes to the flowrate (high weight on MV movement in the objective function).
- Minimise cyclone feed density to maximise circulating load and maximise cyclone classification performance.
- Maximise solids feedrate to the mill in order to maximise production.

#### 4. RESULTS

In this section there are four scenarios that will be analysed. For all four scenarios the control strategy is as described in Section 3. The setpoint for the rougher feed density is set at 1.32 relative specific gravity (RSG). The sump and surge tank has weak setpoints at 50 % and 70 % respectively. Initially, the flowrate limits to the roughers are as defined in Table 1. The pump speed and cyclone feed flowrate is not shown in Figure 2 due to space limitations, because cyclone feed pressure is a good proxy for both, but both are included in the controller.

**Table 1. Controlled and manipulated variable limits.**

Variable (CVs and MVs)	Units	MIN	MAX
Mill Load	Vol %	20	35
Sump Level	%	30	70
Cyclone Feed Density	RSG	1.4	1.9
Cyclone Feed Flowrate	m <sup>3</sup> /h	600	1600
Cyclone Feed Pressure	psi	40	75
Surge Tank Level	%	45	85
Primary Rougher Feed Density	RSG	1.28	1.45
Solids Feedrate	tons/h	0	450
Sump Feed Water Flowrate	m <sup>3</sup> /h	0	350
Sump Discharge pump	%	75	100
Rougher Feed Flowrate	m <sup>3</sup> /h	300	1300
Surge Tank Water Flowrate	m <sup>3</sup> /h	0	200

In the first scenario, the controller is started up to stabilise the process with a fixed feedrate of 360 tons/hour and to minimise cyclone feed density. There is also a spillage disturbance of 50 m<sup>3</sup>/hr during this period.

During the second scenario, the controller is further given authority to maximise the solids feedrate to the mill.

During the third scenario, the maximum rougher feed flowrate is reduced from 1300 m<sup>3</sup>/hr to 800 m<sup>3</sup>/hr in order to simulate a scenario where longer residence times are required to improve flotation performance.

During the fourth and last scenario, a spillage water disturbance of 50 m<sup>3</sup>/hr is again introduced to show the disturbance rejection performance of the controller.

##### 4.1 Start-up scenario

During the start-up scenario, indicated by 1 in Figure 2, the controller stabilises the process by moving the rougher

density to setpoint. The cyclone feed density is minimised to 1.78 RSG and it is confirmed by no water being added to the surge tank.

The sump dilution water is pushed up to 320 m<sup>3</sup>/hr in order to minimise density, but cannot get all the way to its upper constraint of 350 m<sup>3</sup>/hr, because the surge tank water is completely closed leaving only sump dilution water and mill solids feedrate affecting rougher feed density. Additionally the mill solids feedrate is at maximum and therefore adding more sump dilution water without a corresponding increase in mill solids feedrate would result in the rougher density moving from setpoint.

When the spillage water disturbance is introduced, the sump dilution water flowrate is dropped by 50 m<sup>3</sup>/hr to compensate and operates at 270 m<sup>3</sup>/hr. When the spillage water disturbance is removed, the sump dilution water returns to 320 m<sup>3</sup>/hr.

##### 4.2 Solids feedrate maximisation scenario

During the solids feedrate maximisation scenario, indicated by 2 in Figure 2, the feedrate increases to its upper limit of 450 tons/hour. This corresponds to an increase in mill load that has an upper limit of 35 % volume. Practically, mill mass will be used as a proxy for mill volume.

Using the mill load limit is not ideal, as the maximum mill throughput as a function of mill load is available from the non-linear mill model and the optimisation should ideally use this information directly.

The circulating load also contribute to the mill loading and the RNMPC starts to reduce sump dilution water to 280 m<sup>3</sup>/hr in order to free capacity in the mill to reach its primary objective of maximising feedrate. This results in the cyclone feed density increasing from 1.78 RSG to 1.83 RSG that requires the controller to start adding dilution water to the surge tank to maintain the rougher feed density at setpoint.

The rougher feed flowrate increases to 1000 m<sup>3</sup>/hr and rougher density is maintained at 1.32 RSG.

##### 4.3 Rougher feed flowrate limitation scenario

During this scenario, indicated by 3 in Figure 2, the rougher feed flowrate limit is reduced from 1300 m<sup>3</sup>/hr to 800 m<sup>3</sup>/hr. The controller immediately drop water addition to the surge tank to zero and reduces water addition to the sump to reduce the volumes in the circuit until it had time to reduce solids feedrate to compensate.

Both the surge capacity in the sump and surge tank is utilised to buffer the excess volumes until feedrate was reduced to target, but it had the side effect of causing a small increase in rougher feed density because of the sudden drop in water around the circuit to maintain all the process variables inside the operating regions.

Once the solids feedrate reduced to normal levels, the controller immediately starts maximising feedrate and minimising cyclone feed density. The feedrate settles out

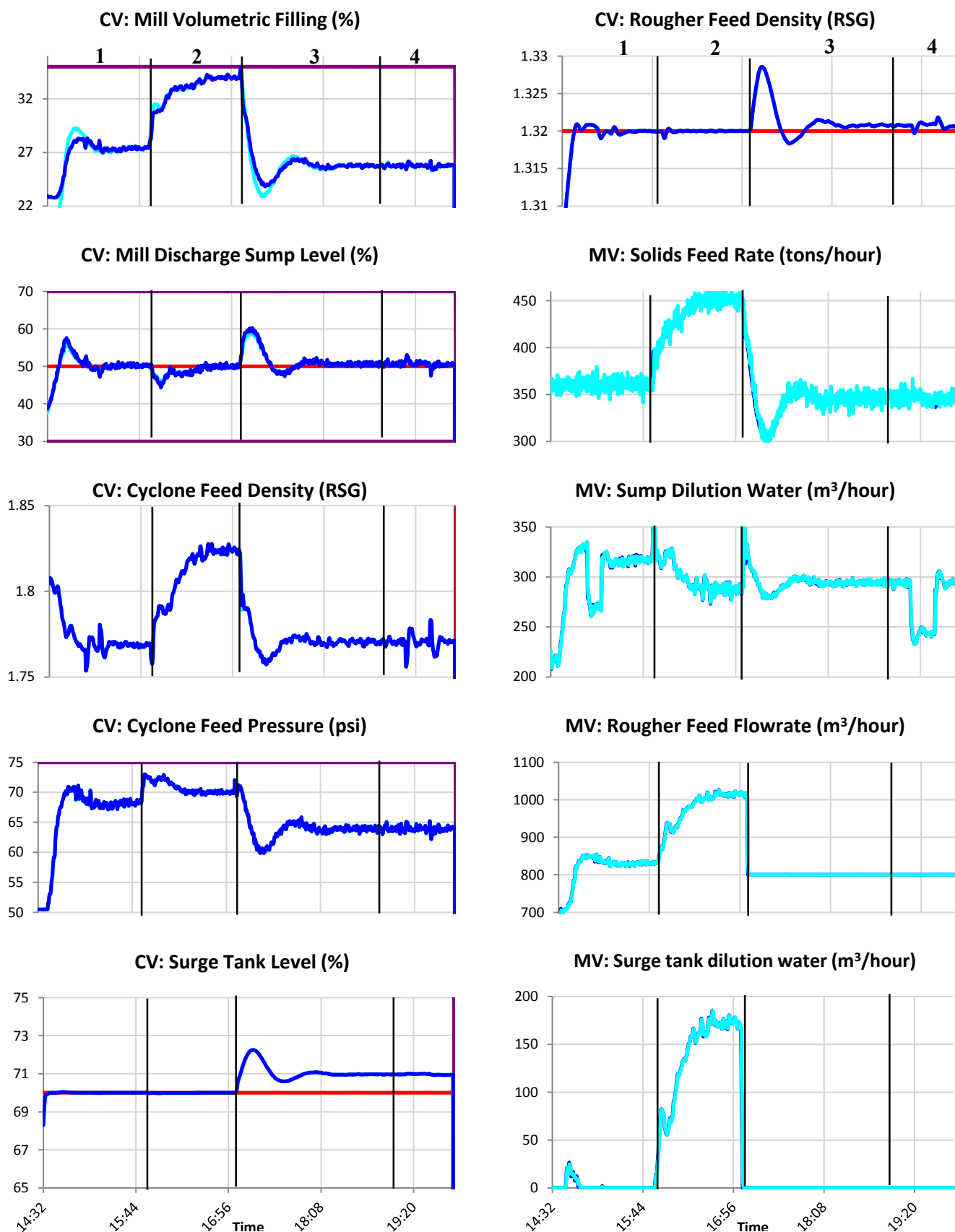


Figure 2. Simulation results of RN MPC controlling the milling circuit. Time axis is only shown on bottom graphs. Units for y-axis are shown next to graph title. Legend for CVs: ■ Measurement, ■ Kalman filter estimate, ■ Setpoint, ■ Constraint. Legend for MVs: ■ RN MPC Setpoint, ■ Actual MV value.

at 350 tons/hour and cyclone feed density reaches 1.78 RSG, which is the minimum because no water is added to the surge tank. In this case, the controller could not reduce water on the sump to increase feedrate as it done in the previous scenario and then compensate using surge tank water, because the combined volume of solids and water necessary to reach the rougher feed density target would have exceeded the allowed 800 m<sup>3</sup>/hr to the roughers. The controller could have added water to the sump and the surge tank to get the necessary rougher density, but the minimisation objective on cyclone feed density causes the controller to add all the water to the sump in order to minimise cyclone feed density after maximising the mill solids feedrate that will still result in a final volume that adheres to the rougher feed flowrate constraint. The drop in total volume is also visible from the low mill volumetric filling as new feed and circulating load had to be reduced to meet the new downstream limit.

#### 4.4 Spillage disturbance with rougher flowrate limit scenario

During the last scenario, indicated by 4 in Figure 2, where the process is operating at a critical downstream constraint, the spillage water disturbance is picked up on cyclone feed density and rejected before it could have an effect on the rougher density and cause an increase in volume to the surge tank. The RNMPC used the dilution water to correct for the sump level disturbance first and then corrected for cyclone feed density and did not use pump speed, as can be seen by looking at cyclone feed pressure as a proxy for pump speed in Figure 2, to bleed off the excess volume.

### 5. CONCLUSIONS

The StarCS RNMPC controller shows great promise for combining the linear and non-linear elements of a typical milling circuit into one controller. This will simplify and improve the optimisation of the milling circuit, while adhering to all process constraints without the need for a separate supervisory layer.

The RNMPC controller also managed to keep execution times below 1 second with the controller only needing to run every 10 seconds indicating that the controller will be able to run this scenario in real-time on an Intel Core i5-750 at 2.66 GHz or better.

The cascaded CV-to-CV models allowed the RNMPC to reject upstream disturbances that is visible on intermediate variables without having to generate explicit setpoints for the intermediate variables. It is also simpler to obtain the cascaded models for certain processes than obtaining MV-CV models, such as for sump dilution water to rougher feed density.

The maximisation of the feedrate and minimisation of the cyclone feed density was easily obtained while adhering to all process constraints. Ideally, the mill load should be explicitly optimised based on the non-linear model without requiring an upper constraint.

The StarCS RNMPC has been successfully deployed at industrial plants around the world as a mill discharge

controller. This simulation shows that the StarCS RNMPC is capable of optimising the whole circuit in future with some improvements to the real-time optimiser and by adding a feedback mechanism for the non-linear mill model to track milling conditions, such as feed ore characteristics and mill liner wear.

The strategy outlined in this paper opens up further integration between Mintek's milling and flotation controllers by allowing the flotation controllers to feedback requirements to the milling controller on density and flowrates to the roughers to improve overall flotation performance and the milling controller in turn optimising the milling circuit as a whole to achieve those target the most efficiently.

### 6. FUTURE WORK

One area that still needs improvement is the real-time optimiser that should optimise the mill load based on the non-linear model without the need to explicitly limit the load using constraints. This becomes important as the optimum load will change over time owing to changing feed ore conditions and liner wear.

To complement the improved real-time optimiser, the non-linear model parameters will need to be updated continuously to match the conditions of the real system by using some sort of feedback mechanism, such as closed-loop system identification with persistent excitation.

### 7. ACKNOWLEDGEMENT

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### Appendix A. RN MPC Model and Parameters

The model, as used in the StarCS RN MPC control module, is given verbatim in Figure 3 and is written in differential equation form. There are some intermediate variables for the nonlinear part, such as ZTv. The nomenclature used in the model of Figure 3 is given in Table 2 and the parameter values used in the model is given in Table 3.

**Table 2. RN MPC Model Nomenclature**

Symbol	Meaning
Xc[1..n]	Continuous state
dXc[1..n]	Continuous state time derivative
U[1..n], Uf[1..n]	Manipulated variable
Y[1..n]	Controlled variable
D[1..n]	Disturbance variable. In this model it does not represent actual measured disturbances and is only used to make the problem square for the Kalman filter.
TD(var,τ <sub>d</sub> )	Defines a time-delay of τ <sub>d</sub> seconds for variable var.
MFS, SFR	U[1] Mill solids feedrate setpoint
MillFilling	Xc[1], Y[1] Volumetric mill filling
ZTv	Fraction of volumetric filling from optimum volumetric filling.
Vmill	Total volume of mill
VTmax	Fraction volumetric filling for maximum feedrate
Tmax	Maximum mill feedrate
Ds	Density of ore
DTv	Curvature of throughput to load curve
SLEV	Xc[2], Y[2] Sump level
Sfw	U[2] Sump dilution water flowrate PID setpoint
Pump	U[3] Sump discharge pump speed
Cfd	Y[3], Xcfd Cyclone feed density
Cff	Xc[4], Y[4] Cyclone feed flowrate
Cfp	Xc[5], Y[5] Cyclone feed pressure
Rfd	Y[7] Rougher feed density
Rff	U[4] Rougher feed flowrate PID setpoint
SuLEV	Xc[6], Y[6] Surge Tank Level
SuFW	U[5] Surge tank dilution water flowrate PID setpoint
	Xc[3] First order response of cyclone feed density to sump feed water.
	Xc[11] First order response of cyclone feed density to mill solids feedrate.
	Xc[12] First order response of cyclone feed density to sump discharge pump speed (due to circulating load effect)
	Xc[9] First order effect of mill solids feedrate increasing mill discharge that contribute to integrating sump level, Xc[2].
	Xc[10] First order effect of sump discharge pump increasing circulating load and affecting integrating sump level, Xc[2].
	Xc[7] First order effect of cyclone feed density on rougher feed density.
	Xc[8] First order effect of surge tank water addition on rougher feed density.

$$ZTv = (Xc[1]) / ((VTmax) * Vmill) - 1$$

$$Tmill = (Tmax) * (1 - (DTv) * ZTv ^ 2)$$

$$dXc[1] = (TD(U[1],60)*0.65+Xc[4]*0.65 - Tmill) / (TimeConstantMillFilling*Ds)$$

$$SFR = TD(Uf[1],60)$$

$$dXc[9] = (GainSlevMfs*SFR-Xc[9])/TimeConstantSlevMfs$$

$$dXc[10] = (GainSlevPumpCirculating*Uf[3]-Xc[10])/TimeConstantPumpCirculating$$

$$dXc[2] = (GainSlevSfw*Uf[2] + GainSlevPump*U[3] + Xc[9] + Xc[10])$$

$$dXc[11] = ((GainCfdMfs*TD(Uf[1],60)-Xc[11])/TimeConstantCfdMfs)$$

$$dXc[12] = (GainCfdPump*Uf[3]-Xc[12])/TimeConstantCfdPump$$

$$dXc[3] = ((GainCfdSfw*U[2]-Xc[3])/TimeConstantCfdSfw)$$

$$Xcfd = Xc[3] + Xc[11] + Xc[12]$$

$$dXc[4] = ((GainCffPump*(Uf[3]+D[1])-Xc[4])/TimeConstantCffPump)$$

$$dXc[5] = (((GainCfpPump)*(Uf[3]+D[2])-Xc[5])/TimeConstantCfpPump)$$

$$dXc[6] = (GainSurgeLevelRff*U[4] + GainSurgeLevelSuFw*Uf[5] + GainSurgeLevelCFF*Xc[4])$$

$$dXc[7] = (GainRfdCfd*Xcfd-Xc[7])/TimeConstantRfdCfd$$

$$dXc[8] = (GainRfdSuFw*U[5]-Xc[8])/TimeConstantRfdSuFw$$

$$Y[1] = Xc[1]$$

$$Y[2] = Xc[2]$$

$$Y[3] = Xcfd$$

$$Y[4] = Xc[4]$$

$$Y[5] = Xc[5]$$

$$Y[6] = Xc[6]$$

$$Y[7] = Xc[7] + Xc[8]$$

Figure 3. The model as defined for the StarCS RN MPC.

**Table 3. RN MPC Model Parameters**

Parameter Name	Parameter Value
GainSlevSfw	0.0011
GainCfdSfw	-0.0005
GainCffPump	13.3
GainSlevPump	-0.0147
GainCfpPump	1
GainSurgeLevelRff	-7.3E-05
GainSurgeLevelSuFw	0.000073
GainSurgeLevelCFF	3.78E-05
GainRfdCfd	1.16
GainRfdSuFw	-0.0006
TimeConstantCfdSfw	36
TimeConstantCffPump	1
TimeConstantCfpPump	1
TimeConstantRfdCfd	364
TimeConstantRfdSuFw	364.8
Vmill	100
VTmax	0.346
Tmax	950
Ds	2.9
DTv	1.2
TimeConstantMillFilling	400
GainSlevMfs	0.0065
GainSlevPumpCirculating	-0.0147
TimeConstantPumpCirculating	120
TimeConstantSlevMfs	200
GainCfdMfs	0.0004
TimeConstantCfdMfs	60
GainCfdPump	0.0042
TimeConstantCfdPump	36