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Simultaneous State and Fault Estimation for Descriptor Systems using an Augmented PD Observer

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Abstract: This paper proposes an augmented Proportional plus Derivative (PD) state estimator to achieve simultaneous system state and fault estimation of descriptor systems. Descriptor systems without model uncertainty are initially considered, followed by a discussion of a general situation where the system model is subject to model uncertainty, external disturbance or sensor noise. The $\boldsymbol{H}_\infty$ performance is adopted to improve the estimator robustness subject to disturbance, sensor noise or model uncertainty. The estimator gains are obtained via an LMI approach. An example is studied to show the usefulness and effectiveness of the proposed approach.

Keywords: Descriptor systems, PD observers, system state and fault estimation, robust estimation, augmented state estimation.

1. INTRODUCTION

State estimation observers for dynamic systems have attracted a lot of attention due to their important role in state estimate feedback control, fault detection, fault estimation and fault-tolerant control (Chen and Patton, 1999, Blanke et al., 2006, Ding et al., 2011). From the stand point of active fault tolerant control (AFTC), the problem of simultaneous state and fault estimation is very attractive as it can provide the required state and fault information within one design, as long as robustness and boundedness conditions are satisfied.

During the last four decades, various effective methods, such as the augmented state observer (ASO) (Patton and Klinkhieo, 2009) and its transformations (Gao and Ding, 2007, Gao et al., 2007a), the unknown input observer (UIO) (Chen and Patton, 1999), the eigenstructure assignment approach (Shi and Patton, September 2012, Patton and Chen, 2000), sliding model observers (Edwards et al., 2000, Floquet et al., 2007), adaptive techniques (Zhang et al., 2008, Zhang et al., 2009) and learning methods based on neural networks (Talebi and Khorasani, 2012), etc., have been developed to take into account the effects that faults have on the FDD/FTC problems.

Descriptor systems, which are also referred to as singular systems, differential algebraic systems, generalized dynamical systems, implicit systems and even semi-state systems, often appear in power systems, electrical networks, social economic systems and chemical engineering and other application areas (Dai, 1989). The ASO has received significant attention because of its simplicity and the potential for simultaneously estimation of system states and faults. The traditional ASO is conservative as it is assumed the faults vary slowly. In the descriptor system context, a proportional multi-integral (PMI) observer, where the original system is multi-augmented and the augmentation times depend on the fault signal property, is proposed in (Gao et al., 2007b) to improve the performance of the traditional ASO. Generally speaking, if the q^{th} derivative of the fault signal is bounded, the original system would be augmented to an $n + qk$ order system, where n is the order of the original system and k is the number of faults.

It is well known that Proportional-Integral-Derivative (PID) controllers and observers in various combinations have been widely used for industrial processes. In particular, Proportional-Derivative (PD) observers (Zhang, 2011, Ting et al., 2011, Ren and Zhang, 2010, Duan and Patton, 1997, Chong et al., 2005, Gao, 2005), which introduce another design freedom, may provide an opportunity for generating estimates with good sensitivity properties. It is very attractive in observer design to explore the potential of derivatives as more design freedom can be used to improve the design performance. The derivatives of the system outputs are typically considered in the estimator design to achieve fast fault estimation (Gao and Ding, 2007, Zhang et al., 2009). To get fast estimation, a proportional multi-integral derivative (PMID) estimator is proposed in (Gao and Ding, 2007). The PMID estimator introduced the derivatives of the estimation error to get faster estimation.

This paper proposes an augmented PD observer which is designed using Linear Matrix Inequalities (LMIs) to estimate system states and fault simultaneously. With some modification, the output derivatives do not appear in the proposed PD observer which is then suitable for practical application. The proposed descriptor system augmented PD observer can be easily extended to a multi-augmented PD observer if the fault signals have fast variations. The remainder of this paper is organized as follows. Section 2 gives a general description of the augmented PD observer design problem. Section 3 discusses an augmented PD estimator for an ideal case in which the system model parameters are assumed to be known precisely known. The general case including robustness to uncertainty is discussed in Section 4. An effective wind speed estimation problem is studied in Section 5 and conclusions are drawn in Section 6.

2. THE PROBLEM STATEMENT

In the implementation of model-based estimation and control on real engineering systems, disturbances and uncertainty always exist and attention should be paid to the corresponding robust design problem. In this work, a descriptor system with disturbance and uncertainty is considered in the following form:

$$\left.\begin{array}{l} E\dot{x}(t) = Ax(t) + \mathrm{B}u(t) + F_f f(t) + Rd(t) \\ y(t) = \mathrm{C}x(t) \end{array}\right\} \quad (1)$$

where $x(t) \in R^n, y(t), u(t),$ and $f(t) \in R^p$, are system states, outputs, known inputs and fault signals, respectively. The vector $d(t)$, denotes the combined system uncertainty and external disturbances. E, A, B, C, F_f, R are matrices with compatible dimensions. The time subscript (t) is omitted in the remainder of this paper without causing confusion.

The following conditions are assumed:

A1) $rank\left(\begin{bmatrix} sE-A \\ C \end{bmatrix}\right) = n, s > 0$

A2) $rank\left(\begin{bmatrix} E \\ C \end{bmatrix}\right) = n$

A3) $rank\begin{bmatrix} A & F_f \\ C & 0 \end{bmatrix} = n + p$

Lemma 1: (Ren and Zhang, 2010) Given matrix E, U, V, L with appropriate dimensions, we have

$$\max_L\{rank(E+ULV)\} = min\left\{rank([E \quad U]), rank\left(\begin{bmatrix} E \\ V \end{bmatrix}\right)\right\}$$

Definition:(Dai, 1989) System (1) is said to be completely detectable if assumptions A1) and A2) hold simultaneously.

Lemma 2: If system (1) is completely detectable, then there exists an L such that $E + LC$ is nonsingular.

Proof: Suppose that for all L $rank(E + ULV) < n$.

By Lemma 1, we have:

$$\mathrm{n} > \max_L\{rank(E+LC)\} = min\left\{rank([E \quad I]), rank\left(\begin{bmatrix} E \\ C \end{bmatrix}\right)\right\} = rank\left(\begin{bmatrix} E \\ C \end{bmatrix}\right) = n$$

which is a contradiction. So there is an L such that $E + LC$ is nonsingular.

In this work an augmented PD estimator is proposed for slowly-varying faulst in the following form:

$$\left.\begin{array}{l} E\dot{\hat{x}} = A\hat{x} + \mathrm{B}u + F_f\hat{f} + L_x(y-\hat{y}) + L_{xd}(\dot{y}-\dot{\hat{y}}) \\ \hat{y} = \mathrm{C}\hat{x} \\ \dot{\hat{f}} = L_f(y-\hat{y}) + L_{fd}(\dot{y}-\dot{\hat{y}}) \end{array}\right\} \quad (2)$$

where $\hat{x}, \hat{y}$ and $\hat{f}$ are estimates of the system states, outputs and augmented signals, respectively. The gain matrices L_x, L_{xd}, L_f, L_{fd} are to be determined.

The original system can be augmented first if f varies rapidly as introduced in (Gao et al., 2007b). For the case when f varies rapidly and the q^{th} derivative of the signal f, i.e. $f^{(q)}$, varies slowly, the original system can be augmented as:

$$\left.\begin{array}{c} E\dot{x} = Ax + Bu + F_f f \\ \dot{f} = \delta_1 \\ \dot{\delta}_1 = \delta_2 \\ \dots \\ \dot{\delta}_{q-1} = f^{(q)} \\ y = \mathrm{C}x \end{array}\right\} \quad (3)$$

which can be reorganized in matrix form as:

$$\begin{bmatrix} E & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 1 \end{bmatrix}\begin{bmatrix} \dot{x} \\ \dot{f} \\ \dot{\delta}_1 \\ \dots \\ \dot{\delta}_{q-1} \end{bmatrix} = \begin{bmatrix} A & F_f & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ 0 & 0 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 0 \end{bmatrix}\begin{bmatrix} x \\ f \\ \delta_1 \\ \dots \\ \delta_{q-1} \end{bmatrix} + \begin{bmatrix} \mathrm{B} \\ 0 \\ 0 \\ \dots \\ 0 \end{bmatrix}u + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \dots \\ 1 \end{bmatrix}f^{(q)} \quad (4)$$

$$y = [\mathrm{C} \quad 0 \quad 0 \quad \dots \quad 0][x \quad f \quad \delta_1 \quad \dots \quad \delta_{q-1}]^T$$

In the following part, only the slowly-varying fault case is discussed, which can be easily extended to a multi-augmented PD observer problem for the case when the fault signals have fast variations, as discussed above. Comparing the proposed estimator with existing estimators, some interesting points can be found from the point of view of observer structure. If $L_{xd} = 0$ and if L_{fd} can be set as $L_{fd} = kL_d$, then the proposed estimator reduces to the so-called *fast adaptive fault estimator* proposed in (Zhang et al., 2009). If $L_{fd} = 0$, the proposed estimator reduces to the zero-integral estimator proposed in (Gao and Ding, 2007, Gao et al., 2007a) which is designed via orthogonal decomposition or by solving a Lyapunov equation.

In the system description of Eq. (1) no sensor noise or faults from the sensors are included. Actually, the sensor noise or sensor faults can be realized in the form of actuator faults or via external disturbance. The augmented system is described as follows. Consider a system with sensor fault or sensor noise as:

$$\left.\begin{array}{l} E\dot{x} = Ax + \mathrm{B}u + F_f f + Rd \\ y = Cx + Nf \end{array}\right\} \quad (5)$$

An augmented descriptor system is constructed as follows:

$$E_a\dot{x}_a = A_a x_a + F_{fa} f + B_a u + R_a d \quad (6)$$

$$y = C_a x_a \quad (7)$$

where

$$x_a = \begin{bmatrix} x \\ f \end{bmatrix}, E_a = \begin{bmatrix} E & 0 \\ 0 & 0 \end{bmatrix}, A_a = \begin{bmatrix} A & 0 \\ 0 & -I \end{bmatrix}, F_{fa} = \begin{bmatrix} F_f \\ I \end{bmatrix},$$

$$B_a = \begin{bmatrix} B \\ 0 \end{bmatrix}, R_a = \begin{bmatrix} R \\ 0 \end{bmatrix}, C_a = [C \quad N]$$

An ideal case without disturbance considered is discussed first in Section 3 for the sake of simplicity. And the general case is discussed in Section 4.

3. AUGMENTED PD OBSERVER: THE IDEAL CASE

Consider an LTI system in the following form:

$$\left.\begin{aligned} E\dot{x} &= Ax + \mathrm{B}u + F_f f \\ y &= \mathrm{C}x \end{aligned}\right\} \tag{8}$$

where x, y, u and f are system states, outputs, known inputs and fault signals, respectively. A, B, C, F_f are matrices with comparable dimensions.

Define $e_x = x - \hat{x}$ and $e_f = f - \hat{f}$. If $\boldsymbol{f}$ varies slowly, which means it is reasonable to set $\dot{f} = 0$, it comes

$$\dot{e}_f = -\dot{\hat{f}} = -L_f(y - \hat{y}) - L_d(\dot{y} - \dot{\hat{y}}) \tag{9}$$

Then an error state system is obtained as:

$$\left.\begin{aligned} E\dot{e}_x &= (A - L_x\mathrm{C})e_x + F_f e_f - L_{xd}C\dot{e}_x \\ \dot{e}_f &= -L_f(y - \hat{y}) - L_d(\dot{y} - \dot{\hat{y}}) \end{aligned}\right\} \tag{10}$$

which can be rewritten as:

$$E_o\begin{bmatrix}\dot{e}_x\\ \dot{e}_f\end{bmatrix} = A_o\begin{bmatrix}e_x\\ e_f\end{bmatrix} \tag{11}$$

where

$$A_o = \begin{bmatrix}A - L_x\mathrm{C} & F_f\\ -L_f\mathrm{C} & 0\end{bmatrix} = A_a - L_pC_a, A_a = \begin{bmatrix}A & F_f\\ 0 & 0\end{bmatrix} \tag{12}$$

$$E_o = E_a + \begin{bmatrix}L_dC & 0\\ L_{fd}C & 0\end{bmatrix} = E_a + L_dC_a, E_a = \begin{bmatrix}E & 0\\ 0 & I\end{bmatrix} \tag{13}$$

$$L_p = \begin{bmatrix}L_x\\ L_f\end{bmatrix}, L_d = \begin{bmatrix}L_{xd}\\ L_{fd}\end{bmatrix}, C_a = [C \quad 0] \tag{14}$$

Lemma 3: The augmented system (E_a, A_a, C_a) is observable with assumptions A1), A2) and A3).

Proof: $rank\left(\begin{bmatrix}sE_a - A_a\\ C_a\end{bmatrix}\right) = rank\left(\begin{bmatrix}s\mathrm{E} - \mathrm{A} & F_f\\ 0 & sI\\ C & N\end{bmatrix}\right)$

$$= \begin{cases} rank\begin{bmatrix}A & F_f\\ C & 0\end{bmatrix} = n + p & s = 0\\ rank\left(\begin{bmatrix}sE - A\\ C\end{bmatrix}\right) + rank(F_f) = n + p & s \neq 0\end{cases} = n + q$$

$$rank\left(\begin{bmatrix}E_a\\ C_a\end{bmatrix}\right) = rank\left(\begin{bmatrix}\mathrm{E} & 0\\ 0 & I\\ C & 0\end{bmatrix}\right) = rank\left(\begin{bmatrix}E\\ C\end{bmatrix}\right) + p = n + p$$

Hence, the augmented system is completely observable.

Therefore, by **Lemma 2**, there is an L_d that E_o is invertible. Now define

$$A_{od} = E_o^{-1}A_o \tag{15}$$

The augmented system is transformed to

$$\begin{bmatrix}\dot{e}_x\\ \dot{e}_f\end{bmatrix} = A_{od}\begin{bmatrix}e_x\\ e_f\end{bmatrix}$$

Subject to the above proposed augmented PD estimator system and the error system, the following theorem is stated.

Theorem 1: For slowly varying fault signals, the estimator is stable and the estimation of states and faults of system (1) converge to the real values if there is a symmetric matrix $P_1 > 0$, and matrices P_2, P_3, L_p, and L_d satisfying the following condition:

$$\begin{bmatrix}P_2 + P_2^T & P_1^TA_o - P_2^TE_o + P_3\\ * & -P_3^TE_o - E_o^TP_3\end{bmatrix} < 0 \tag{16}$$

where A_o and E_o are defined as in (12), (13), and (14). $*$ denotes the symmetric part of a matrix.

Proof: (Necessity) As the eigenvalues of $E_o^{-T}A_o$ are the same as those of $A_oE_o^{-1}$ (Horn and Johnson, 1986), the stability of the observer error system can be guaranteed by assigning the eigenvalues of $A_oE_o^{-1}$ to the left hand panel. Hence, based on Lyapunov theory, the observer error system is stable if and only if there exists a matrix $P_1 > 0$ satisfying the following inequality:

$$P_1A_oE_o^{-1} + E_o^{-T}A_o^TP_1 < 0$$

which can be pre and post-multiplied by E_o^T and E_o respectively, and the following inequalities can be obtained:

$$A_o^TP_1E_o + E_o^TP_1A_o < 0 \tag{17}$$

One can always find a P_2 such that:

$$P_2 + P_2^T < 0 \tag{18}$$

Set $P_3 = -P_1A_o - P_2E_o$ then (17) and (18) can be rewritten as:

$$\begin{bmatrix}P_2 + P_2^T & P_1A_o + P_2E_o + P_3\\ * & A_o^TP_1E_o + E_o^TP_1A_o\end{bmatrix} < 0 \tag{19}$$

Pre and post-multiplying (19) by $\begin{bmatrix}I & 0\\ -E_o^T & I\end{bmatrix}$ and its transpose respectively, leads to (16).

(Sufficiency) Assume the (16) holds. It is clear that E_o is non-singular. From above proof one can see that (18) holds, which implies that (17) and (19) hold completing the proof. ■

The importance of the above theorem is the separation of E_o and A_o, P_1. The solubility of above theorem is considered in the following theorem which transfers the inequality (16) to an LMI.

Theorem 2: There is a stable PD observer of system (2) if there are matrices P_2, P_3, Y_1, and Y_2 and a positive symmetric matrix P_1, satisfying the following condition:

$$\begin{bmatrix}P_2 + P_2^T & P_1A_a - P_2^TE_a + P_3 + Y_1^TC_a\\ * & -P_3^TE_a - E_a^TP_3 + Y_2^TC_a^T + C_aY_2\end{bmatrix} < 0 \tag{20}$$

where A_o and E_o are defined as in (12), (13), and (14). Then the PD observer gains can be calculated as:

$$[L_x \quad L_f] = L_p^T = -[Y_1 \quad Y_2]\begin{bmatrix}P_1^{-1}\\ -P_3^{-1}P_2P_1^{-1}\end{bmatrix} \tag{21}$$

$$[L_{xd} \quad L_{fd}] = L_d^T = -[Y_1 \quad Y_2]\begin{bmatrix}P_1^{-1}\\ P_3^{-1}\end{bmatrix} \tag{22}$$

Proof: (Sufficiency): The inequality (20) can be rewritten as:

$$S + W < 0 \tag{23}$$

where

$$S = \begin{bmatrix}P_2 + P_2^T & P_1A_a - P_2^TE_a + P_3\\ * & -P_3^TE_a - E_a^TP_3\end{bmatrix} \tag{24}$$

$$W = \begin{bmatrix}0\\ C_a^T\end{bmatrix}[Y_1 \quad Y_2] + [Y_1 \quad Y_2]^T\begin{bmatrix}0\\ C_a^T\end{bmatrix}^T \tag{25}$$

As (21) and (22):

$$\begin{bmatrix} -L_p^T & -L_d^T \end{bmatrix} = \begin{bmatrix} Y_1 & Y_2 \end{bmatrix} \begin{bmatrix} P_1 & 0 \\ P_2 & P_3 \end{bmatrix}^{-1} \quad (26)$$

which is equivalent to:

$$\begin{bmatrix} Y_1 & Y_2 \end{bmatrix} = \begin{bmatrix} -L_p^T & -L_d^T \end{bmatrix} \begin{bmatrix} P_1 & 0 \\ P_2 & P_3 \end{bmatrix} \quad (27)$$

Substituting (26) into (25), Eq. (23) leads to (16).

(N**ecessity**) Eq. (15) can be rewritten as:

$$S + \begin{bmatrix} 0 \\ C_a \end{bmatrix} \begin{bmatrix} -L_p^T & -L_d^T \end{bmatrix} \begin{bmatrix} P_1 & 0 \\ P_2 & P_3 \end{bmatrix} + * < 0 \quad (28)$$

where S is the same as in (24). Let $[Y_1 \quad Y_2]$ to be set as (27), it follows that (26) is satisfied. Substituting (26) into (28), leads to (20). ■

Remark 1: It can be seen that the derivatives of the outputs exist in the proposed PD observer. This is obviously not desirable because noise or uncertainty always exist in practical systems. This provides the motivation for modifying the proposed observer as follows. Let $z(t) = \hat{x} - L_d(y - \hat{y})$:

$$\left.\begin{aligned} \dot{z} &= A\hat{x} + Bu + F_f\hat{f} + L(y - \hat{y}) \\ \hat{x} &= E_o^{-1}z + E_o^{-1}L_d y \\ \hat{y} &= C\hat{x} \\ \hat{f} &= L_f \int_0^t (y - \hat{y})dt + L_{fd}(y - \hat{y}) \end{aligned}\right\} \quad (29)$$

where E_o is defined as in (13).

The output time derivatives do not appear in the modified PD observer and only the original coefficient matrices are utilized; therefore the modified PD observer is reliable for practical application.

Remark 2: To achieve the so-called *fast fault and state estimation*, a well-known α stability can be considered by assigning the closed-loop poles of the desired estimator to the left-hand complex plane of $x < -\alpha$, $\alpha > 0$. Without proof, the following LMI is proposeS:

$$\begin{bmatrix} P_2 + P_2^T & P_1A_a - P_2^TE_a + P_3 + Y_1^TC_a & P_1 \\ * & -P_3^TE_a - E_a^TP_3 + C_a^TY_2 + Y_2^TC_a & 0 \\ * & * & -\frac{1}{2\alpha}P_1 \end{bmatrix} < 0 \quad (30)$$

Similarly, the damping ratio can be restricted to $\varsigma < \cos(\beta)$ if the following LMI is satisfied:

$$W \otimes \left(\begin{bmatrix} P_2^T & P_1A_a - P_2^TE_a + Y_1^TC_a \\ P_3^T & -P_3^TE_a + Y_2^TC_a \end{bmatrix} \right) + * < 0 \quad (31)$$

where ⊗ denotes Kronecker Product and:

$$W = \begin{bmatrix} \sin(\beta) & \cos(\beta) \\ -\cos(\beta) & \sin(\beta) \end{bmatrix}$$

The problem can be solved easily with tools from the MATLAB LMI TOOLBOX (Gahinet et al., 1995) as all constraints are LMIs.

4. ROBUST AUGMENTED PD OBSERVER DESIGN: THE GENERAL CASE

The general case of (1) is considered in this Section. Consider the system (1) with augmented PD estimator (2), and assume that f is slowly-varying, the error system is then:

$$E_o \begin{bmatrix} \dot{e}_x \\ \dot{e}_f \end{bmatrix} = A_o \begin{bmatrix} e_x \\ e_f \end{bmatrix} + \begin{bmatrix} R \\ 0 \end{bmatrix} \quad (32)$$

where A_o and E_o are defined as in (12),(13), and (14).

As E_o is invertible and with definition of:

$$A_{od} = E_o^{-1}A_o, R_{od} = E_o^{-1}R \quad (33)$$

The error system is transformed to:

$$\begin{bmatrix} \dot{e}_x \\ \dot{e}_f \end{bmatrix} = A_{od} \begin{bmatrix} e_x \\ e_f \end{bmatrix} + R_{od}d \quad (34)$$

Define a performance function as:

$$\hat{\imath} = C_{od} \begin{bmatrix} e_x \\ e_f \end{bmatrix} \quad (35)$$

where C_{od} is a weighting matrix. Based on the Bounded Real Lemma of (Gahinel and Apkarian, 1994), the error system is stable and $\|\xi\|_2 < \gamma\|d\|_2$ if and only if $\exists P > 0$. So that:

$$\begin{bmatrix} A_{od}^TP + PA_{od} & PR_{od} & C_{od}^T \\ E_{od}^TP & -\gamma & 0 \\ C_{od} & 0 & -\gamma \end{bmatrix} < 0 \quad (36)$$

which can be rewritten as:

$$\begin{bmatrix} P & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix} \left(\begin{bmatrix} E_o^{-1} & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix} \begin{bmatrix} A_o & R & 0 \\ 0 & -\frac{\gamma}{2} & 0 \\ C_{od} & 0 & -\frac{\gamma}{2} \end{bmatrix} \right) + * < 0 \quad (37)$$

As $det(sI - MN) = det(sI - NM)$, and MN and NM have the same eigenvalues where M and N have compatible dimensions, it follows that inequality (37) is equivalent to:

$$\begin{bmatrix} P & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix} \left(\begin{bmatrix} A_o & R & 0 \\ 0 & -\frac{\gamma}{2} & 0 \\ C_{od} & 0 & -\frac{\gamma}{2} \end{bmatrix} \begin{bmatrix} E_o^{-1} & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix} \right) + * < 0 \quad (38)$$

which leads to:

$$\begin{bmatrix} PA_oE_o^{-1} + E_o^{-T}A_o^TP & PR & E_o^{-T}C_{od}^T \\ R^TP & -\gamma & 0 \\ C_{od}E_o^{-1} & 0 & -\gamma \end{bmatrix} < 0 \quad (39)$$

Pre and post-multiplying (39) by $\begin{bmatrix} E_o^T & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix}$ and its transpose respectively, leads to:

$$\begin{bmatrix} E_o^TPA_o + A_o^TPE_o & E_o^TPR & C_{od}^T \\ R^TPE_o & -\gamma & 0 \\ C_{od} & 0 & -\gamma \end{bmatrix} < 0 \quad (40)$$

Theorem 3: For a system with faults as in (1), the estimator (2) is stable and satisfies $\|\xi\|_2 < \gamma\|d(t)\|_2$ if there exist a symmetric matrix P, and matrices L, and L_d such that the conditions (40) is satisfied.

As $A_o^TPE_o$ and its transpose are bilinear matrices which is hard to be solved, the following solvability theorem is proposed to separate E_o from P and A_o:

Theorem 4: For a system with faults as in (1), the estimator (2) is stable and satisfies $\|\xi\|_2 < \gamma\|d(t)\|_2$ if there exist a symmetric matrices P, and matrices L, and L_d so that the following conditions are satisfied.

$$\begin{bmatrix} P_2 + P_2^T & P_3 + P_1 A_o - P_2^T E_o & P_1 R & 0 \\ P_3^T + A_o^T P_1 - E_o^T P_2 & -E_o^T P_3 - P_3 E_o & 0 & C_{od}^T \\ R^T P_1 & 0 & -\gamma & 0 \\ 0 & C_{od} & 0 & -\gamma \end{bmatrix} < 0 \quad (41)$$

Proof: There is always one P_2 satisfying (18) and:

$$\begin{bmatrix} E_o^T P_1 A_o + A_o^T P_1 E_o & E_o^T P_1 R & C_{od}^T \\ R^T P_1 E_o & -\gamma - R^T P_1 (P_2 + P_2^T)^{-1} P_1 R & 0 \\ C_{od} & 0 & -\gamma \end{bmatrix} < 0 \quad (42)$$

According to the Schur complement Lemma, it follows that:

$$\begin{bmatrix} P_2 + P_2^T & 0 & P_1 R & 0 \\ 0 & E_o^T P_1 A_o + A_o^T P_1 E_o & E_o^T P_1 R & C_{od}^T \\ R^T P_1 & R^T P_1 E_o & -\gamma & 0 \\ 0 & C_{od} & 0 & -\gamma \end{bmatrix} < 0 \quad (43)$$

Let $P_3 = -P_1 A_o - P_2 E_o$ and pre and post-multiplying (43) by ℸ and its transpose respectively, where:

$$ℸ = \begin{bmatrix} I & 0 & 0 & 0 \\ -E_o^T & I & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \end{bmatrix}.$$

This leads to (41). ■

For completeness, the following theorem is proposed without proof as it can easily be obtained using the techniques developed in Section 3 and in this Section.

Theorem 5: For a system with faults as in (1), the estimator (2) is stable and satisfies $\|\xi\|_2 < \gamma\|d(t)\|_2$ if there exist matrices 換$_2$, P_3, Y_1, Y_2 and a positive symmetric matrix P_1, so that the following condition is satisfied.

$$\begin{bmatrix} P_2 + P_2^T & P_1 A_a - P_2^T E_a + P_3 + Y_1^T C_a & P_1 R & 0 \\ * & -P_3^T E_a - E_a^T P_3 + C_a^T Y_2 + Y_2^T C_a & 0 & C_{od}^T \\ * & * & -\gamma & 0 \\ * & * & * & -\gamma \end{bmatrix} < 0 \quad (44)$$

The PD observer gains are calculated by (21) and (22).

As all constraints are given in LMI form, according to the Theorems, sufficient conditions can be obtained for combined performance. For instance, the combination of Theorem 5 with Theorems 3 and 4 can provide robustness and desired time response performance.

5. Example

A numerical example is modified from (Ren and Zhang, 2010). Consider a descriptor system as:

$$E\dot{x} = Ax + Bu + F_f f + Rd$$
$$y = Cx$$

where

$$E = \begin{bmatrix} 0.5 & -2.5 & 0 \\ 3 & -3 & 4 \\ 2 & -1 & 3 \end{bmatrix}, A = \begin{bmatrix} -1 & 4.5 & -0.5 \\ -7 & 7 & -8 \\ -5 & 3 & -6 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}, \mathrm{F}_f = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, R = \begin{bmatrix} 0.1 \\ 0.1 \\ 0 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

In this study, a one-step augmentation is carried out to illustrate the design procedure. Extra augmentation should be considered if the potential signals are fast-varying, following the steps given in Section 2. The MATLAB LMI Toolbox is used to solve (30) and (44), and set $\alpha = 1.5$, $C_z = [0 \quad 0 \quad 0 \quad .1]$ to reduce the disturbance influence to the estimation of f and the set $\gamma = 1$, a feasible solution is found as follows:

$$P_1 = \begin{bmatrix} 1.400 & -0.971 & 1.047 & -0.514 \\ -0.971 & 71.645 & -1.919 & 0.380 \\ 1.047 & -1.919 & 2.746 & -0.601 \\ -0.514 & 0.380 & -0.601 & 0.403 \end{bmatrix},$$

$$P_2 = \begin{bmatrix} -5.842 & 1.680 & -0.6245 & 0.606 \\ 1.915 & -4.274 & 3.207 & 0.074 \\ -0.807 & 3.454 & -6.836 & 0.516 \\ 0.426 & -0.097 & 0.457 & -5.855 \end{bmatrix},$$

$$P_3 = \begin{bmatrix} -0.430 & -5.761 & -1.438 & -0.509 \\ 2.743 & -3.545 & 4.134 & 0.289 \\ 2.461 & 0.323 & 3.733 & -0.533 \\ -0.152 & 0.2675 & -0.153 & 5.320 \end{bmatrix}$$

$$Y_1 = \begin{bmatrix} 0.468 & -1.089 & -0.950 & -0.290 \\ 1.276 & 0.044 & -0.591 & 0.307 \end{bmatrix},$$

$$Y_2 = \begin{bmatrix} -4.993 & -1.933 & 0.303 & 0.015 \\ 2.624 & -2.866 & -4.253 & 0.193 \end{bmatrix}$$

The PD gains are then computed as:

$$\begin{bmatrix} L_x \\ L_f \end{bmatrix}^T = L_p^T = \begin{bmatrix} 179.915 & -180.475 \\ 141.523 & -128.792 \\ 194.659 & -178.186 \\ 404.829 & -392.798 \end{bmatrix}$$

$$\begin{bmatrix} L_{xd} \\ L_{fd} \end{bmatrix}^T = L_d^T = \begin{bmatrix} 10.948 & -11.374 \\ -16.017 & 15.497 \\ 22.048 & -20.573 \\ 4.121 & -4.025 \end{bmatrix}$$

Simulations are implemented with zero inputs. The estimation is carried out with step fault signals and the results are given in Fig1-3.

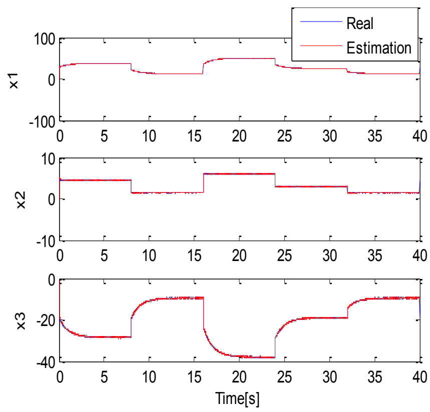

Fig. 1. Estimation of system states

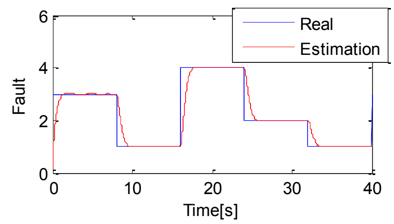

Fig. 2. Estimation of f

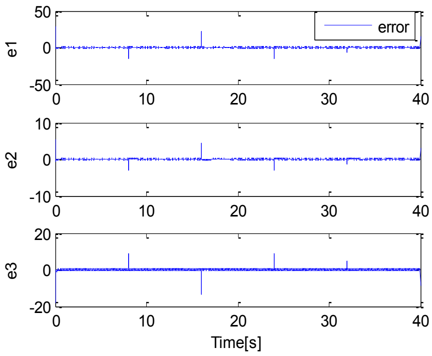

Fig. 3. Estimation errors

From Figs.1 & 3 it can be seen that both the estimates of the system states and that of fault converge fast in the transition period to the steady state in which the estimates are very close to the real signals.

6. CONCLUSION

In this paper, an augmented PD observer is proposed to estimate system state and fault simultaneously for descriptor systems. The proposed augmented PD observer is more general compared with existing ones. An ideal case is considered first and a more general case follows. For a system with disturbance or uncertainty, the H_∞ performance is considered to improve the observer robustness. An augmented version is proposed for systems with sensor fault or sensor noise.

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