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Adaptive stabilization of stochastic nonholonomic systems with uncertain parameters and time-varying coefficients

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Abstract: This paper investigates the adaptive state-feedback stabilization of stochastic nonholonomic systems which have both uncertain parameters and time-varying coefficients. The state-scaling and backstepping techniques are exploited in the design of controllers. The adaptive state-feedback stabilizing controllers and switching control strategy are proposed so that the closed-loop system can be stabilized in probability. In the end, two simulation examples are provided to illustrate effectiveness of controllers.

Keywords: stochastic nonholonomic systems; state-feedback stabilization; backstepping; switching control strategy.

1. INTRODUCTION

Consider the following stochastic nonholonomic nonlinear systems described by

$$\mathrm{d}x_0 = d_0(t)u_0\mathrm{d}t + f_0(x_0)\mathrm{d}t + g_0^T(x_0)\mathrm{d}\omega, \tag{1.1}$$

$$\left.\begin{aligned} \mathrm{d}x_i &= d_i(t)u_0x_{i+1}\mathrm{d}t + f_i(x_0,\overline{x}_i,\theta)\mathrm{d}t \\ &\quad + g_i^T(x_0,\overline{x}_i,\theta)\mathrm{d}\omega, i = 1,\cdots,n-1, \\ \mathrm{d}x_n &= d_n(t)u\mathrm{d}t + f_n(x_0,\overline{x}_n,\theta)\mathrm{d}t + g_n^T(x_0,\overline{x}_n,\theta)\mathrm{d}\omega \end{aligned}\right\} \tag{1.2}$$

where u_0 and $u \in \mathbb{R}$ are control inputs, $x_0 \in \mathbb{R}$ and $(x_1,\cdots,x_n)^T \in \mathbb{R}^n$ are system states, $\overline{x}_i = (x_1,\cdots,x_i)^T$, $\theta \in \mathbb{R}^m$ is a constant vector of uncertain parameters, $f_0(x_0) : \mathbb{R} \to \mathbb{R}$ and $f_i(x_0,\overline{x}_i,\theta) : \mathbb{R}^{i+1}\times\mathbb{R}^m \to \mathbb{R}$, $1 \le i \le n$, are smooth functions and nonlinear drifts with $f_0(0) = 0$ and $f_i(0,0,\theta) = 0$, $g_0(x_0) : \mathbb{R} \to \mathbb{R}^r$, $g_i(x_0,\overline{x}_i,\theta) : \mathbb{R}^{i+1} \times \mathbb{R}^m \to \mathbb{R}^r$, $1 \le i \le n$, are smooth functions with $g_0(0) = 0$ and $g_i(0,0,\theta) = 0$, $d_i(t) : \mathbb{R}^+ \to \mathbb{R}$, $0 \le i \le n$, are unknown uncertain time-varying control coefficients with known sign, and $\omega \in \mathbb{R}^r$ is an r-dimensional independent standard Wiener process defined on a complete probability space $(\Omega,\mathcal{F},P)$ with Ω being a sample space, $\mathcal{F}$ being a filtration, and P being a probability measure.

The stabilization of nonholonomic control systems has achieved remarkable development during the two decades. After the results in (Arnold, 1988), three methods are used to stabilization of nonholonomic systems: discontinuous time-invariant stabilization(Astolfi, 1996; Ge, 2003), smooth time-varying stabilization(Jiang, 1996; Tian, 2002; Hu, 2004), hybrid stabilization(Luo, 2000). Now much progress has been made in stability of stochastic differential equations(SDE). Especially, when backstepping designs were firstly introduced, stochastic nonlinear control had experienced a breakthrough(Krstic, 1998; Pan, 1999). Based on the quartic Lyapunov function, asymptotical stabilization control in the large of the open-loop system without any nonholonomic constraint was discussed(Deng, 2001). Further research was developed by the recent work (Wu, 2007; Tian, 2007).

It is known that stochastic disturbances are frequently encountered in nonholonomic systems, especially, nonholonomic mobile robots. There were some results which considered the problem of stabilization for stochastic nonholonomic systems(Wu, 2012; Gao, 2012). The problem of stabilization was discussed in (Wang, 2006) with no drift terms and (Zhao, 2011; Zhang, 2013) in which the first equation was ordinary differential equation. Y. Liu et al. studied the output feedback stabilization (Liu, 2011). Compared with the systems in (Zhang, 2013), there some new features in this paper: one is that the first equation contains drifts, the other is the uncertain parameters not only exist in nonlinear drifts but in the terms before $\mathrm{d}\omega$. These new features will lead to difficulty to the design controllers. Although, we have discussed the similar problem for simple three order stochastic nonholonomic in (**?**), there are many estimation parameters. How to design adaptive state-feedback stabilizing controllers for stochastic nonholonomic systems with uncertain parameters and time-varying coefficients simultaneously using less estimation parameters.

The purpose of this paper is to design stabilizing controllers for stochastic nonholonomic systems with unknown parameters and time-varying coefficients. The main idea of this paper is highlighted as follows:

adaptive state-feedback stabilizing controllers are designed for stochastic nonholonomic systems with unknown parameters and time-varying coefficients, simultaneously by an estimation parameter.

The paper is organized as follows: Section 2 begins with the mathematical preliminaries. In Sections 3, adaptive state-feedback backstepping controllers are designed. In Sections 4, a switching control strategy for the original system is discussed. Finally, two simulation examples are given to show the effectiveness of controllers in Section 5.

2. PRELIMINARIES

The following notations will be used throughout the paper. $\mathbb{R}^+$ denotes the set of all nonnegative real numbers, $\mathbb{R}^n$ denotes the real n-dimensional space. For a given vector or matrix X, X^T and $|X|$ denote its transpose and the Euclidean norm, respectively, $\mathrm{Tr}\{X\}$ denotes its trace when X is square, $|X|_F = \sqrt{\mathrm{Tr}\{XX^T\}}$.

Consider the following stochastic nonlinear system

$$\mathrm{d}x = f(t,x)\mathrm{d}t + g(t,x)\mathrm{d}\omega, \quad x(0) \in \mathbb{R}^n. \tag{2}$$

Define a differential operator $\mathcal{L}$:

$$\mathcal{L}V(x) = \frac{\partial V}{\partial x} f(t,x) + \frac{1}{2}\mathrm{Tr}\Big\{g^T(t,x)\frac{\partial^2 V}{\partial x^2} g(t,x)\Big\}, \tag{3}$$

where $x \in \mathbb{R}^n$ is the state, the Borel measurable functions $f : \mathbb{R}^+ \times \mathbb{R}^n \to \mathbb{R}^n$ and $g : \mathbb{R}^+ \times \mathbb{R}^n \to \mathbb{R}^{n\times r}$ are locally Lipschitz in x, $f(t,0) = 0$, $g(t,0) = 0$, and $\omega \in \mathbb{R}^r$ is an r-dimensional independent standard Wiener process defined on the complete probability space $(\Omega, \mathcal{F}, P)$.

The following definitions and lemmas will be used throughout the paper.

Definition 1. (Deng, 2001). The equilibrium $x = 0$ of system (2) is

• globally stable in probability if for $\forall \varepsilon > 0$, there exists a class $\mathcal{K}$ function $\gamma(\cdot)$ such that

$$P\{|x(t)| < \gamma(|x_0|)\} \geq 1 - \varepsilon, \quad \forall t \geq 0, \quad x(0) \in \mathbb{R}^n \setminus \{0\},$$

• globally asymptotically stable in probability if it is globally stable in probability and

$$P\{\lim_{t\to\infty} |x(t)| = 0\} = 1, \quad \forall x(0) \in \mathbb{R}^n.$$

Lemma 2. (Xie, 2009). Considering the stochastic system (2), if there exist a $\mathcal{C}^2$ function $V(x)$, class $\mathcal{K}_\infty$ functions $\alpha_1(\cdot)$ and $\alpha_2(\cdot)$, constants $c_1 > 0$, $c_2 \geq 0$, and a nonnegative function $W(x)$ such that

$$\begin{cases} \alpha_1(|x|) \leq V(x) \leq \alpha_2(|x|), \\ \mathcal{L}V(x) = \dfrac{\partial V}{\partial x} f + \dfrac{1}{2}\mathrm{Tr}\Big\{g^T \dfrac{\partial^2 V}{\partial x^2} g\Big\} \leq -c_1 W(x) + c_2, \end{cases}$$

then for each $x(0) \in \mathbb{R}^n$

i). There exists an almost surely unique solution on $[0, \infty)$ for (2),

ii). When $c_2 = 0$, $f(t, 0, \theta) = 0$, $g(t, 0, \theta) = 0$ and $W(x)$ is continuous, then the equilibrium $x = 0$ is globally stable in probability and $P\{\lim_{t\to\infty} W(x(t)) = 0\} = 1$.

Lemma 3. (Lin, 2002). Let x and y be real variables. Then, for any positive integers m, n and any real number $\varepsilon > 0$, the following inequality holds:

$$|x|^m|y|^n \leq \frac{m}{m+n}\varepsilon|x|^{m+n} + \frac{n}{m+n}\varepsilon^{-\frac{m}{n}}|y|^{m+n}.$$

Lemma 4. (Lin, 2000). For any vector-valued continuous function $f(x,y)$, where $x \in \mathbb{R}^m$, $y \in \mathbb{R}^n$, there are smooth scalar functions $a(x) \geq 1$ and $b(x) \geq 1$ such that

$$|f(x,y)| \leq a(x)b(y).$$

3. ADAPTIVE STATE-FEEDBACK STABILIZATION

For system (1), the following assumptions and remark are needed.

Assumption 5. For smooth functions $f_0(\cdot)$, $g_0(\cdot)$, there exist known constant m_1 and constant vector $m_2 \in \mathbb{R}^r$, such that

$$f_0(x_0) = m_1 x_0, \quad g_0(x_0) = m_2 x_0. \tag{4}$$

Assumption 6. For smooth functions $f_i(\cdot)$ and $g_i(\cdot)$, $i = 1, \cdots, n$, there exist known non-negative smooth functions $\overline{\gamma}_i : \mathbb{R}^{i+1} \times \mathbb{R}^m \to \mathbb{R}^+$ and $\overline{\xi}_i : \mathbb{R}^{i+1} \times \mathbb{R}^m \to \mathbb{R}^+$ such that for any $x_0, \overline{x}_i$ and θ:

$$|f_i(x_0, \overline{x}_i, \theta)| \leq (|x_1| + \cdots + |x_i|)\overline{\gamma}_i(x_0, \overline{x}_i, \theta),$$
$$|g_i(x_0, \overline{x}_i, \theta)| \leq (|x_1| + \cdots + |x_i|)\overline{\xi}_i(x_0, \overline{x}_i, \theta).$$

Remark 7. From Lemma 3, there exist positive smooth functions $\gamma_i(x_0, \overline{x}_i)$, $c_i(\theta)$, $\xi_i(x_0, \overline{x}_i)$ and $e_i(\theta)$, $i = 1, \cdots, n$, such that

$$|f_i(x_0, \overline{x}_i, \theta)| \leq (|x_1| + \cdots + |x_i|)\gamma_i(x_0, \overline{x}_i)c_i(\theta),$$
$$|g_i(x_0, \overline{x}_i, \theta)| \leq (|x_1| + \cdots + |x_i|)\xi_i(x_0, \overline{x}_i)e_i(\theta).$$

Assumption 8. Without loss of generality, the sign of $d_i(t)$ is assumed to be positive, and for $i = 0$ and any $t \in \mathbb{R}^+$, there exist known positive constants λ_0 and μ_0, for $i = 1, \cdots, n$ and any $t \in \mathbb{R}^+$, there exist known positive constants λ_i and unknown positive constant μ, such that

$$0 < \lambda_0 \leq d_0(t) \leq \mu_0, 0 < \lambda_i \leq d_i(t) \leq \mu.$$

In the following two subsections, we will consider the system (1) under the condition of $x_0(0) \neq 0$ and the case of $x_0(0) = 0$ will be discussed in the Section 4.

3.1 The first state stabilization

Let us consider the subsystem (1.1) in stochastic nonholonomic nonlinear systems (1). In order to guarantee that x_0 converges to zero, one can take u_0 as follows:

$$u_0 = -\eta_0 x_0, \eta_0 = \frac{1}{\lambda_0}\{\lambda + \mathrm{sgn}(m_1)m_1 + \frac{3}{2}|m_2 m_2^T|_F\}, \tag{5}$$

where λ is a design parameter and $\mathrm{sgn}(\cdot)$ denotes sign function. If we employ a Lyapunov function of the form $V_0(x_0) = \frac{1}{4}x_0^4$. From (1.1), Assumption 8, (3) and (5), one can obtain $\mathcal{L}V_0 \leq -\lambda x_0^4$. From above analysis, we have the following Theorem.

Theorem 9. If Assumptions 5 holds, for constant m_1, λ_0, constant vector m_2, and the design parameter $\lambda > 0$, the smooth controller u_0 satisfy (5), then

i). the closed-loop system composed by (1.1) and (5) has an almost surely unique solution on $[0, \infty)$ for all $x_0(0)$, which is bounded in probability.

ii). the equilibrium $x_0 = 0$ of the closed-loop system composed by (1.1) and (5) is globally asymptotically stable in probability.

Substituting (5) into the subsystem (1.1), it is easy to obtain that

$$\mathrm{d}x_0 = -\eta_0 d_0(t)x_0\mathrm{d}t + f_0(x_0)\mathrm{d}t + g_0^T(x_0)\mathrm{d}\omega. \quad (6)$$

Proposition 10. For any initial state $x_0(t_0) \neq 0$, the solution of (6), that is, the solution of the closed system composed by (1.1) and (5), will never reach the zero, which avoids the uncontrollability of the subsystem (1.2).

Proof. From Lemma 2.3((Mao, 1997), P.93), one gets Proposition 10 holds.

3.2 Other states stabilization

In order to design a smooth adaptive state-feedback controller, the following state-input scaling transformation is needed

$$z_i = x_i/u_0^{n-i}, \quad 1 \le i \le n. \quad (7)$$

Remark 11. For the initial state $x_0(t_0) \neq 0$, from Proposition 10, one can obtain that the transformation (7) is meaningful.

Under the new z-coordinate, the second subsystem (1.2) can be transformed into

$$\begin{aligned} \mathrm{d}z_i &= d_i(t)z_{i+1}\mathrm{d}t + \phi_i\mathrm{d}t + \psi_i^T\mathrm{d}\omega, \\ & i = 1,\cdots,n-1, \\ \mathrm{d}z_n &= d_n(t)u\mathrm{d}t + \phi_n\mathrm{d}t + \psi_n^T\mathrm{d}\omega, \end{aligned} \quad (8)$$

where

$$\begin{aligned} \phi_i &= \frac{f_i}{u_0^{n-i}} + \eta_0(n-i)d_0(t)z_i - (n-i)z_i\frac{f_0}{x_0} \\ &+\frac{1}{2}(n-i)(n-i+1)z_i\frac{g_0^Tg_0}{x_0^2} + \eta_0(n-i)\frac{g_i^Tg_0}{u_0^{n-i+1}}, \\ \psi_i &= \frac{g_i}{u_0^{n-i}} - (n-i)z_i\frac{g_0}{x_0}. \end{aligned}$$

To invoke the backstepping method, the error variables ε_i are given by

$$\varepsilon_1 = z_1, \varepsilon_i = z_i - z_i^*(x_0, \overline{z}_{i-1}, \widehat{\Theta}), i = 2,\cdots,n, \quad (9)$$

where $z_i^*(i = 2,\cdots,n)$ are virtual smooth controllers, $\widehat{\Theta}$ denotes the estimate of Θ and

$$\Theta = \max\Big\{1, \mu, c_i(\theta), e_i(\theta), e_i^2(\theta)\Big\}. \quad (10)$$

Then, by Itô differentiation formula, we have

$$\begin{cases} \mathrm{d}\varepsilon_1 = d_1(t)z_2\mathrm{d}t + \phi_1\mathrm{d}t + \psi_1^T\mathrm{d}\omega, \\ \mathrm{d}\varepsilon_i = d_i(t)z_{i+1}\mathrm{d}t + F_i\mathrm{d}t + G_i^T\mathrm{d}\omega, \\ \qquad i = 2,\cdots,n-1, \\ \mathrm{d}\varepsilon_n = d_n(t)u\mathrm{d}t + F_n\mathrm{d}t + G_n^T\mathrm{d}\omega, \end{cases} \quad (11)$$

where $\overline{z}_i = (z_1,\cdots,z_i)^T$, $z = \overline{z}_n$ and

$$\begin{aligned} G_i &= G_i(t, x_0, \overline{z}_i, \widehat{\Theta}) = \frac{g_i}{u_0^{n-i}} - (n-i)z_i\frac{g_0}{x_0} - \frac{\partial z_i^*}{\partial x_0}g_0(x_0) \\ &-\sum_{k=1}^{i-1}\frac{\partial z_i^*}{\partial z_k}\Big\{\frac{g_k}{u_0^{n-k}} - (n-k)z_k\frac{g_0}{x_0}\Big\}, \quad i = 1,\cdots,n. \end{aligned}$$

$$\begin{aligned} F_i &= F_i(t, x_0, \overline{z}_i, \widehat{\Theta}) \\ &= f_i/u_0^{n-i} + \eta_0(n-i)d_0(t)z_i - (n-i)z_if_0/x_0 \\ &+\frac{1}{2}(n-i)(n-i+1)z_i\frac{g_0^Tg_0}{x_0^2} + \eta_0(n-i)\frac{g_i^Tg_0}{u_0^{n-i+1}} \\ &-\frac{\partial z_i^*}{\partial x_0}d_0(t)u_0 - \frac{\partial z_i^*}{\partial x_0}f_0(x_0) - \frac{1}{2}\frac{\partial^2 z_i^*}{\partial x_0^2}g_0^Tg_0 \\ &-\sum_{k=1}^{i-1}\frac{\partial z_i^*}{\partial z_k}\Big\{d_k(t)z_{k+1} + \frac{f_k}{u_0^{n-k}} + \eta_0(n-k)d_0(t)z_k \\ &-(n-k)z_k\frac{f_0}{x_0} + \frac{1}{2}(n-k)(n-k+1)z_k\frac{g_0^Tg_0}{x_0^2} \\ &+\eta_0(n-k)\frac{g_k^Tg_0}{u_0^{n-k+1}}\Big\} \\ &-\frac{1}{2}\sum_{k=1}^{i-1}\frac{\partial^2 z_i^*}{\partial x_0\partial z_k}\Big\{\frac{g_k^T}{u_0^{n-k}} - (n-k)z_k\frac{g_0^T}{x_0}\Big\}g_0 \\ &-\frac{1}{2}\sum_{j,k=1}^{i-1}\frac{\partial^2 z_i^*}{\partial z_j\partial z_k}\Big\{[\frac{g_k^T}{u_0^{n-j}} - (n-j)z_j\frac{g_0^T}{x_0}]\cdot \\ &\qquad [\frac{g_k}{u_0^{n-k}} - (n-k)z_k\frac{g_0}{x_0}]\Big\} - \frac{\partial z_i^*}{\partial\widehat{\Theta}}\dot{\widehat{\Theta}}, \end{aligned}$$

By using the Assumption 6, (7), (8) and (9), we have the following proposition.

Proposition 12. For smooth functions $f_i(\cdot)$ and $g_i(\cdot)$, $i = 2,\cdots,n$, there exist known non-negative smooth functions $\gamma_{ij} : \mathbb{R}^{i+1}\times\mathbb{R}^m \to \mathbb{R}^+$, $j = 1,2,3,4,5$, such that for any $x_0, \overline{x}_i$ and θ:

$$\begin{aligned} &|\frac{f_i}{u_0^{n-i}}| \le \Theta\gamma_{i1}(x_0,\overline{x}_i)\sum_{k=1}^{i}|\varepsilon_k|, |\frac{g_i}{u_0^{n-i}}| \\ &\le \Theta\gamma_{i2}(x_0,\overline{x}_i)\sum_{k=1}^{i}|\varepsilon_k|, \\ &|\frac{g_i^T}{u_0^{n-i}} - (n-i)z_i\frac{g_0^T}{x_0} - \frac{\partial z_i^*}{\partial x_0}g_0(x_0) \\ &-\sum_{k=1}^{i-1}\frac{\partial z_i^*}{\partial z_k}\Big\{\frac{g_k^T}{u_0^{n-k}} - (n-k)z_k\frac{g_0^T}{x_0}\Big\}| \\ &\le \Theta\gamma_{i3}(x_0,\overline{x}_i)\sum_{k=1}^{i}|\varepsilon_k|, \\ &-\frac{1}{2}\sum_{k=1}^{i}\frac{\partial^2 z_i^*}{\partial x_0\partial z_k}\Big\{\frac{g_k^T}{u_0^{n-k}} - (n-k)z_k\frac{g_0^T}{x_0}\Big\}g_0| \\ &\le \Theta\gamma_{i4}(x_0,\overline{x}_i)\sum_{k=1}^{i}|\varepsilon_k|, \\ &|-\frac{1}{2}\sum_{j,k=1}^{i}\frac{\partial^2 z_i^*}{\partial z_j\partial z_k}\Big\{[\frac{g_k^T}{u_0^{n-j}} - (n-j)z_j\frac{g_0^T}{x_0}]\cdot \\ &[\frac{g_k^T}{u_0^{n-k}} - (n-k)z_k\frac{g_0^T}{x_0}]^T\Big\}| \le \Theta\gamma_{i5}(x_0,\overline{x}_i)\sum_{k=1}^{i}|\varepsilon_k|. \end{aligned}$$

Now we design the adaptive backstepping controller of the system (11).

Step 1: Define the 1st Lyapunov function $V_1(x_0, z_1, \widetilde{\Theta}) = V_0 + \frac{1}{4}\varepsilon_1^4 + \frac{1}{2}\widetilde{\Theta}^2$, where $\widetilde{\Theta} = \Theta - \widehat{\Theta}$ are the parameter estimation errors. According to Assumption 6, (8) and (9), there exist nonnegative smooth functions $\gamma_{11}(x_0, z_1)$, $c_1(\theta)$, $e_1(\theta)$, $\gamma_{12}(x_0, z_1)$ and $\gamma_{13}(x_0, z_1)$, adding and subtracting the term $\Theta\bar{c}_1\varepsilon_1^4 + \bar{\bar{c}}_1\varepsilon_1^4$ on the right-hand side of $\mathcal{L}V_1$, Supposing that $z_2^*(x_0, z_1, \widehat{\Theta}) = -\alpha_1(x_0, z_1, \widehat{\Theta})\varepsilon_1$, where $\alpha_1(\cdot) \geq 0$ is a smooth function to be chosen. Thus, by Assumption 8, (3) and (9), we have

$$\begin{aligned}\mathcal{L}V_1 \leq& -\lambda x_0^4 - \Theta\bar{c}_1\varepsilon_1^4 - \bar{\bar{c}}_1\varepsilon_1^4 + d_1(t)(z_2 - z_2^*)\varepsilon_1^3 + \lambda_1\varepsilon_1^3 z_2^* \\ &+\widetilde{\Theta}\Big\{\tau_1 - \dot{\widehat{\Theta}}\Big\} + \Big\{\sqrt{1+\widehat{\Theta}^2}H_{11} + H_{12}\Big\}\varepsilon_1^4, \end{aligned} \tag{12}$$

where $\tau_1 = H_{11}\varepsilon_1^4$, $H_{11} = \bar{c}_1 + \gamma_{11} + (n-1)\gamma_{12}m_1 + \frac{3}{2}\gamma_{13}^2$, $H_{12} = \bar{\bar{c}}_1 + (n-1)\Big\{\eta_0\mu_0 + m_1 + \frac{1}{2}n|m_2|^2\Big\}$. Choosing the virtual smooth control $z_2^* = -\alpha_1(x_0, z_1, \widehat{\Theta})\varepsilon_1$, $\alpha_1(x_0, z_1, \widehat{\Theta}) = \frac{1}{\lambda_1}\Big(\sqrt{1+\widehat{\Theta}^2}H_{11} + H_{12}\Big)$, one gets

$$\begin{aligned}\mathcal{L}V_1 \leq& -\lambda x_0^4 - \Theta\bar{c}_1\varepsilon_1^4 - \bar{\bar{c}}_1\varepsilon_1^4 + d_1(t)\varepsilon_1^3(z_2 - z_2^*) \\ &+\widetilde{\Theta}\{\tau_1 - \dot{\widehat{\Theta}}\}.\end{aligned} \tag{13}$$

Step i, $2 \leq i \leq n$: Suppose that the design steps from 1 to $i-1$ have been finished. The smooth virtual control z_j^* and the tuning function τ_{j-1} for Step $j-1 (j = 2, \cdots, i)$ have been chosen as $z_j^*(x_0, \bar{z}_{j-1}, \widehat{\Theta}) = -\alpha_{j-1}(x_0, \bar{z}_{j-1}, \widehat{\Theta})\varepsilon_{j-1}, \tau_{j-1} = \tau_{j-2} + H_{j-1,1}\varepsilon_{j-1}^4$, where α_{j-1}, δ_{j-1} and $H_{j-1,1}$ are smooth functions. If we choose the $(i-1)$th Lyapunov candidate function as $V_{i-1}(x_0, \bar{\varepsilon}_{i-1}, \widetilde{\Theta}) = V_{i-2}(x_0, \bar{\varepsilon}_{i-2}, \widetilde{\Theta}) + \frac{1}{4}\varepsilon_{i-1}^4$, one gets

$$\begin{aligned}\mathcal{L}V_{i-1} =& -(\lambda - \sum_{k=2}^{i-1}\beta_k)x_0^4 - \Theta\sum_{j=1}^{i-1}(\bar{c}_j - \sum_{k=j+1}^{i-1}\bar{c}_{kj})\varepsilon_j^4 \\ &-\sum_{j=1}^{i-1}(\bar{\bar{c}}_j - \sum_{k=j+1}^{i-1}\bar{\bar{c}}_{kj})\varepsilon_j^4 + d_{i-1}(t)\varepsilon_{i-1}^3(z_i - z_i^*) \\ &+(\widetilde{\Theta} + \sum_{k=2}^{i-1}\varepsilon_k^3\frac{\partial z_k^*}{\partial\widehat{\Theta}})\Big\{\tau_{i-1} - \dot{\widehat{\Theta}}\Big\},\end{aligned} \tag{14}$$

where $\bar{\varepsilon}_i = (\varepsilon_1, \cdots, \varepsilon_i)^T$. In the following, we will prove that (14) also holds for i. Define the ith Lyapunov candidate function $V_i(x_0, \bar{\varepsilon}_i, \widetilde{\Theta}) = V_{i-1}(x_0, \bar{\varepsilon}_{i-1}, \widetilde{\Theta}) + \frac{1}{4}\varepsilon_i^4$. From Assumption 8, Lemma 3, (3), (9)-(11), Proposition 2 and adding and subtracting the term $\Theta\bar{c}_i\varepsilon_i^4 + \bar{\bar{c}}_i\varepsilon_i^4$ on the right-hand side of $\mathcal{L}V_i$, one gets

$$\begin{aligned}\mathcal{L}V_i \leq& -(\lambda - \sum_{k=2}^{i}\beta_k)x_0^4 - \Theta\sum_{j=1}^{i}(\bar{c}_j - \sum_{k=j+1}^{i}\bar{c}_{kj})\varepsilon_j^4 \\ &-\sum_{j=1}^{i}(\bar{\bar{c}}_j - \sum_{k=j+1}^{i}\bar{\bar{c}}_{kj})\varepsilon_j^4 + d_i(t)\varepsilon_i^3(z_{i+1} - z_{i+1}^*) \\ &+\lambda_i\varepsilon_i^3 z_{i+1}^* + (\widetilde{\Theta} + \sum_{k=2}^{i}\varepsilon_k^3\frac{\partial z_k^*}{\partial\widehat{\Theta}})\Big\{\tau_i - \dot{\widehat{\Theta}}\Big\} + E,\end{aligned} \tag{15}$$

where

$$\beta_i = (1/4)\eta_0\mu_0 + 3/8, E = \{\sqrt{1+\widehat{\Theta}^2}H_{i1} + H_{i2}\}\varepsilon_i^4, \tag{16}$$

$$\begin{aligned}\bar{c}_{i1} =& 1 + (1/4)(n-i) + (1/4)(i-1) + (1/4)\eta_0\mu_0(n-2) \\ &+(1/4)(n-2) + (1/8)(n-1)(n-2) + (3/4)i \\ &+(1/8)(i-1)(2n-i),\end{aligned}$$

$$\begin{aligned}H_{i1} =& \bar{c}_i + \frac{1}{4} + \gamma_{i1} + (n-i)\Big(\gamma_{i2}|m_2| + \frac{3}{4}\sum_{k=1}^{i-1}(|m_2|\gamma_{i2})^{\frac{4}{3}}\Big) \\ &+\sum_{k=1}^{i-1}\frac{3}{4}\gamma_{i1}^{\frac{4}{3}} + \sum_{k=1}^{i-2}\frac{3}{4}\sqrt{1+(A)^2}^{\frac{4}{3}} + \sqrt{1+(B)^2} \\ &+\sum_{k=1}^{i-1}\frac{3}{4}\sqrt{1+(A)^2}^{\frac{4}{3}} + \sum_{k=1}^{i-1}\frac{3}{4}\gamma_{i5}^{\frac{4}{3}} + \frac{3}{4}i\gamma_{i3}^4 \\ &+\sum_{k=1}^{i-1}(n-k)\sum_{j=1}^{k}\frac{3}{4}\sqrt{1+(\gamma_{k2}|m_2|A)^2}^{\frac{4}{3}} \\ &+\sum_{k=1}^{i-1}\sum_{j=1}^{k}\frac{3}{4}\sqrt{1+(A)^2}^{\frac{4}{3}} + \sum_{k=1}^{i-1}\frac{3}{4}\gamma_{i4}^{\frac{4}{3}} + \frac{3}{2}i\gamma_{i3}^2,\end{aligned} \tag{17}$$

$$\begin{aligned}H_{i2} =& \bar{\bar{c}}_i + \eta_0\mu_0(n-i)(1 + (3/4)\alpha_{i-1}^{4/3}) \\ &+(n-i)(m_1 + (3/4)(m_1\alpha_{i-1})^{4/3}) \\ &+\frac{1}{2}(n-i)(n-i+1)\Big(|m_2|^2 + \frac{3}{4}(|m_2|^2\alpha_{i-1})^{\frac{4}{3}}\Big) \\ &+\frac{3}{4}\eta_0\mu_0\sqrt{1+(D)^2}^{\frac{4}{3}} + \frac{3}{4}m_1^{\frac{4}{3}}\sqrt{1+(D)^2}^{\frac{4}{3}} \\ &+\frac{3}{8}\sqrt{1+(|m_2|^2 x_0\frac{\partial^2 z_i^*}{\partial x_0^2})^2}^{\frac{4}{3}} \\ &+\frac{3}{4}\eta_0\mu_0\sum_{k=1}^{i-1}(n-k)\{\sqrt{1+(A)^2}^{\frac{4}{3}} + \sqrt{1+(\alpha_{k-1}A)^2}^{\frac{4}{3}}\} \\ &+\frac{3}{4}\sum_{k=1}^{i-1}(n-k)\sqrt{1+(m_1A)^2}^{\frac{4}{3}} \\ &+\frac{3}{4}\sum_{k=1}^{i-1}(n-k)\sqrt{1+(m_1\alpha_{k-1}A)^2}^{\frac{4}{3}} \\ &+\frac{3}{8}\sum_{k=1}^{i-1}(n-k)(n-k+1)\sqrt{1+(\alpha_{k-1}|m_2|^2A)^2}^{\frac{4}{3}} \\ &+\frac{3}{8}\sum_{k=1}^{i-1}(n-k)(n-k+1)\sqrt{1+(|m_2|^2A)^2}^{\frac{4}{3}} \\ &+\sum_{k=1}^{i-1}\frac{3}{4}\sqrt{1+(\varepsilon_k^3H_{k1}C)^2}^{\frac{4}{3}} + \sqrt{1+(\varepsilon_i^3H_{i1}C)^2} \\ &+\sum_{k=2}^{i-1}\sqrt{1+(\varepsilon_k^3H_{i1}C)^2},\end{aligned} \tag{18}$$

$$\begin{aligned}\bar{c}_{ik} &= (5/4) + (1/4)(n-i) + (1/4)(i-k) \\ &\quad +(1/4)(n-k-1) + (1/8)(n-k-1)(n-k) \\ &\quad +(1/8)(i-k)(2n-i+1-k) + (3/4)i \\ &\quad +(1/4)\eta_0\mu_0(n-k-1), k = 2, \cdots, i-2, \\ \bar{\bar{c}}_{ik} &= (1/4)\{1 + \eta_0\mu_0(n-k) + (n-k)\} \\ &\quad +(1/8)(n-k)(n-k+1), k = 1, \cdots, i-2,\end{aligned}$$

$$A = \frac{\partial z_i^*}{\partial z_k}, B = \frac{\partial z_i^*}{\partial z_{i-1}}, C = \frac{\partial z_i^*}{\partial \widehat{\Theta}}, D = \frac{\partial z_i^*}{\partial x_0}, \tag{19}$$

$$\begin{aligned}\bar{c}_{i,i-1} &= 9/4 + (1/4)(n-i) + (1/4)(n-i+1) + (3/4)i, \\ \bar{\bar{c}}_{i,i-1} &= (1/4) + (1/4)(n-i)\{\eta_0\mu_0 + 1 + (1/2)(n-i+1)\} \\ &\quad +(1/4)(n-i+1)\{\eta_0\mu_0 + 1 + (1/2)(n-i+2)\}.\end{aligned}$$

Choosing the virtual smooth control z_{i+1}^* as $z_{i+1}^*(x_0, \bar{z}_i, \widehat{\Theta}) = -\alpha_i(x_0, \bar{z}_i, \widehat{\Theta})\varepsilon_i$, $\alpha_i(x_0, \bar{z}_i, \widehat{\Theta}) = \frac{1}{\lambda_i}\Big(\sqrt{1+\widehat{\Theta}^2}H_{i1} + H_{i2}\Big)$, one can obtain

$$\begin{aligned}\mathcal{L}V_i &\le -(\lambda - \sum_{k=2}^{i}\beta_k)x_0^4 - \Theta\sum_{j=1}^{i}(\bar{c}_j - \sum_{k=j+1}^{i}\bar{c}_{kj})\varepsilon_j^4 \\ &\quad -\sum_{j=1}^{i}(\bar{\bar{c}}_j - \sum_{k=j+1}^{i}\bar{\bar{c}}_{kj})\varepsilon_j^4 + d_i(t)\varepsilon_i^3(z_{i+1} - z_{i+1}^*) \\ &\quad +(\widetilde{\Theta} + \sum_{k=2}^{i}\varepsilon_k^3\frac{\partial z_k^*}{\partial \widehat{\Theta}})\Big\{\tau_i - \dot{\widehat{\Theta}}\Big\}.\end{aligned} \tag{20}$$

In the end, when $i = n$, $z_{n+1} = z_{n+1}^* = u$ is the actual control, choose the actual controller

$$u(x_0, \bar{z}_n, \widehat{\Theta}) = -\alpha_n(x_0, \bar{z}_n, \widehat{\Theta})\varepsilon_n \tag{21}$$

and adaptive law for $\widehat{\Theta}$

$$\dot{\widehat{\Theta}} = \tau_n = \sum_{i=1}^{n} H_{i1}\varepsilon_i^4, \tag{22}$$

where $\alpha_n \ge 0$ are smooth functions. If we choose nth Lyapunov function $V_n(x_0, \varepsilon, \widetilde{\Theta}) = \frac{1}{4}x_0^4 + \frac{1}{4}\sum_{k=1}^{n}\varepsilon_n^4 + \frac{1}{2}\widetilde{\Theta}^2$, where $\varepsilon = \bar{\varepsilon}_n = (\varepsilon_1, \cdots, \varepsilon_n)^T$, one gets

$$\begin{aligned}\mathcal{L}V_n &\le -(\lambda - \sum_{k=2}^{n}\beta_k)x_0^4 - \Theta\sum_{k=1}^{n}(\bar{c}_k - \sum_{i=k+1}^{n}\bar{c}_{ik})\varepsilon_k^4 \\ &\quad -\sum_{k=1}^{n}(\bar{\bar{c}}_k - \sum_{i=k+1}^{n}\bar{\bar{c}}_{ik})\varepsilon_k^4.\end{aligned} \tag{23}$$

Theorem 13. If Assumptions 5, 6 and 8 hold and choose the control input u and updating law $\widehat{\Theta}$ as (17) and (18), respectively. Then
i) the closed-loop system composed by (11), (17) and (18) has an almost surely unique solution on $[0, \infty)$ for $\forall z(0)$ and $\widehat{\Theta}(0)$;
ii) the equilibrium $(z^T, \widetilde{\Theta}^T) = (0, 0)$ of the closed-loop system is globally stable in probability;
iii) for $\forall z(0)$ and $\widehat{\Theta}(0)$, $P\{\lim_{t\to\infty}|z(t)| = 0\} = 1$, $P\{\lim_{t\to\infty}\widehat{\Theta}(t)$ exists and is finite$\} = 1$.

Proof. By (19), one can obtain $\mathcal{L}V_n$ in (19) becomes the same form as (3.19) in (Deng, 2001). Using (19) and Lemma 2, and following the same procedure as in the proof of Theorem 3.1 in (Deng, 2001), one can easily prove Theorem 2.

4. SWITCHING CONTROL STABILITY

In section 3, we have considered the case of $x_0(t_0) \neq 0$. The controllers (5) and (17) for system (1) are given. Now we turn to the case of $x_0(t_0) = 0$. If the initial is zero, one can choose an open loop control $u_0 = -u_0^* \neq 0$. With the similar method in in Section V in (Wu, 2012), we have there exists $t_s^* > 0$ such that $|x_0(t_s^*)| \neq 0$, which can drive the state x_0 away from zero in a limited time. So, when $t \in [t_0, t_s^*)$, one can choose the control law $u_0 = -u_0^*$ and $u = u^*$ in order to drive the state x_0 away from zero. After that, at the time $t = t_s^*$, we switch the control inputs u_0 and u into (5) and (17) in $t \in [t_s^*, +\infty)$, respectively. Based on above analysis, we give the main results of this paper.

Theorem 14. Suppose that Assumptions 5, 6 and 8 hold. If the following switching control procedure is applied to the system (1),

i) When the initial state belongs to

$$\{(x_0(t_0), x_1(t_0), \cdots, x_n(t_0)) \in \mathbb{R}^{n+1} | x_0(t_0) \neq 0\},$$

controllers u_0 and u in form (5) and (17), respectively;

ii) When the initial state belongs to

$$\{(x_0(t_0), x_1(t_0), \cdots, x_n(t_0)) \in \mathbb{R}^{n+1} | x_0(t_0) = 0\},$$

If $t \in [t_0, t_s^*)$, one can choose the control law $u_0 = u_0^*$ and $u = u^*$; If $t \in [t_s^*, +\infty)$, at the time $t = t_s^*$, we switch the control inputs u_0 and u into (5) and (17), respectively. Then, for any initial conditions in the state space , system (1) will be asymptotically stabilized in probability at the equilibrium and specifically, the states are asymptotically regulated to zero in probability.

5. A SIMULATION EXAMPLE

Consider the following system

$$\begin{aligned}\mathrm{d}x_0 &= (0.525 + 0.375\sin t)u_0\mathrm{d}t + 0.5x_0\mathrm{d}t + 0.5x_0\mathrm{d}\omega, \\ \mathrm{d}x_1 &= (3 + 0.1\sin t)x_2u_0\mathrm{d}t + x_1\theta\mathrm{d}t + x_1\theta\mathrm{d}\omega, \\ \mathrm{d}x_2 &= (3 + 0.2\sin t)u\mathrm{d}t + x_2\theta^2\mathrm{d}\omega\end{aligned}$$

In simulation, choose $\theta = 1$, $\lambda_i(i = 0, 1, 2)$, μ_0 and μ to satisfy $0.15 = \lambda_0 \le d_0(t) \le \mu_0 = 0.9$, $\lambda_1 = 2.9 \le d_1(t) \le \mu$ and $\lambda_2 = 2.8 \le d_2(t) \le \mu$ which satisfy Assumption 3,$\eta_0 = 3.8$, $\bar{c}_1 = 1.8$, $\bar{\bar{c}}_1 = 0.8$, $\bar{c}_2 = 0.1$, $\bar{\bar{c}}_2 = 0.1$ and the initial values $x_0(0) = 0.15$, $x_1(0) = 0.057$, $x_2(0) = -0.13$, $\widehat{\Theta}(0) = 0.18$. Figure 1, 2 and 3 give the responses of the closed-loop system consisting of (1), (5) and (17).

6. CONCLUSIONS

This paper studies the adaptive state-feedback stabilization of stochastic nonholonomic systems with unknown parameters. By using the backstepping approach, a recursive adaptive state-feedback backstepping controllers is designed for stochastic nonholonomic systems with unknown parameters.

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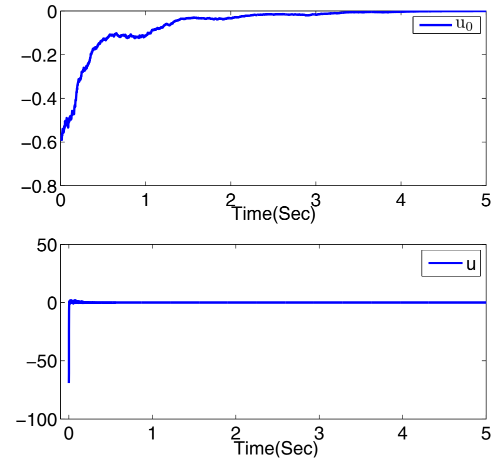

Fig.1. The responses of controllers u_0 and u.

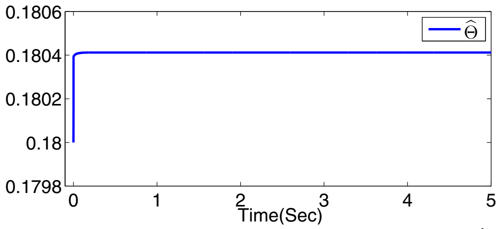

Fig.2. The responses of estimate parameter $\widehat{\Theta}$.

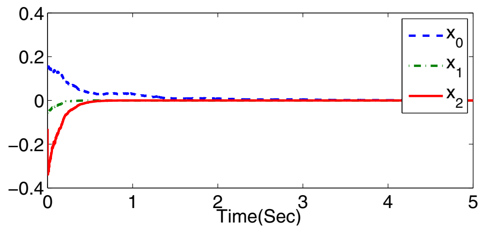

Fig.3. The responses of states x_0, x_1 and x_2.

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