

Extremum-Seeking for Adaptation of Urban Traffic Signal Control

Ronny Kutadinata* Will Moase* Chris Manzie*
Lele Zhang** Tim Garoni**

* *Department of Mechanical Engineering, The University of
Melbourne, Parkville, Victoria 3010 Australia (email:
r.kutadinata@student.unimelb.edu.au; moasew@unimelb.edu.au;
manziec@unimelb.edu.au).*

** *School of Mathematical Sciences, Monash University, Clayton,
Victoria 3800 Australia (e-mail: lele.zhang@monash.edu;
tim.garoni@monash.edu).*

Abstract:

This paper investigates the application of an extremum-seeking scheme to fine-tune the parameters of a “local” urban traffic signal control strategy to improve its performance. A set of conditions that the local traffic controller has to satisfy for stability of the closed-loop system are introduced. Employing a self-organised traffic light scheme as the local controller, it is shown that this strategy satisfies the aforementioned conditions, and its optimality is dependent on the selection of a certain parameter. It is then demonstrated that, by employing an ES scheme, it is possible to always regulate this parameter close to its optimum given any traffic conditions.

Keywords: Extremum-seeking, adaptive control, traffic control.

1. INTRODUCTION

Extremum-seeking (ES) is a non-model based steady-state optimisation scheme for dynamical plants. An ES controller regulates the input of a dynamical plant to the value that optimises the steady-state output of the plant, without requiring knowledge of the underlying dynamics. In order to achieve this, ES requires several components, namely the dither signals, gradient estimator, and the optimiser. There are many ways to realise these components, the simplest of which is in Tan et al. [2006].

The components of a typical ES controller are separated into three time-scales, with the plant being the fastest, the dither and gradient estimator acting on the medium time-scale, and the optimiser being the slowest. In order to achieve the desired behaviour of ES, the input to the plant is perturbed by a small dither signal. This perturbation enables the gradient estimator to estimate the local slope of the steady-state input-output map. The optimiser, typically a gradient descent scheme, acts based on the estimated slope.

A variety of different ES schemes (Spall [1992], Krstić and Wang [2000], Chichka et al. [2006], Srinivasan [2007],

Moase et al. [2010], Liu and Krstić [2010]) provide practical benefits compared to the simple ES scheme of Tan et al. [2006], although the working concepts are essentially the same. One of these variants uses a Luenberger observer as the gradient estimator (Banaszuk et al. [2004], Moase and Manzie [2012]). The observer-based scheme has the advantage of being able to extract the in-phase and quadrature components of the dither, rather than just the dither itself. Thus, the gradient of the steady-state map can be estimated without relying on the average behaviour of the gradient estimator. Hence, this study focuses on the observer-based ES scheme.

In this paper, an application of ES for urban traffic signal control is presented. The difficulties of choosing an optimal urban traffic signal setting are twofold: the high number of control inputs and the hard-to-model dynamics. Firstly, a traffic network consists of a large number of intersections, each with several control variables. When considering the traffic of large cities, the number of variables can reach into the thousands. Thus, finding an optimal signal setting is a very difficult task. Secondly, the dynamics of traffic are hard to model or predict. Although a simplified model based on average dynamics is sometimes used (Diakaki et al. [2002], Lin et al. [2012]), one cannot guarantee that the control strategy is optimal. In this scenario, the use of an extremum-seeking scheme might be beneficial since it has the potential to seek the optimum without requiring *a priori* knowledge of the system.

However, as mentioned previously, an ES scheme is a steady-state optimisation scheme and acts at a slower time-scale to the plant. Thus, it does not handle transient response very well. It is then unreasonable to expect

* This work was partially supported under Australian Research Council's Future Fellowships funding scheme FT100100538 and FT100100494. T.G. and L.Z. acknowledge the financial support of the Roads Corporation of Victoria (VicRoads) and the Australian Research Council's Linkage Projects funding scheme (project number LP120100258). This research was undertaken with the assistance of resources provided at the NCI National Facility through the National Computational Merit Allocation Scheme supported by the Australian Government.

the ES controller to directly optimise the traffic signals. It is proposed that a “local” traffic signal controller is employed, which is then optimised by using an ES scheme. Although Krstić and Wang [2000] and Killingsworth and Krstić [2005] have presented similar concepts, this paper extends the class of systems to address plants with noise applied to the states.

There have been many traffic signal control strategies developed throughout the last few decades. A traffic signal control strategy can either be open-loop (Allsop [1976], Gartner et al. [1991], Robertson [1969], Alvarez and Poznyak [2010]) or closed-loop (Lowrie [1982], Boillot et al. [1992], Diakaki et al. [2002], Tettamanti and Varga [2010], Lin et al. [2012]), with the latter gaining more popularity in the last decade.

One example of a closed-loop traffic signal control is self-organising traffic lights (SOTL). SOTL is a control strategy which utilises a set of rules to be employed by each intersection independently. Although there are several variants (Nakatsuji and Kaku [1991], Chiu and Chand [1993], Lammer and Helbing [2008]), one variant was first presented by Gershenson [2005] for Manhattan-like lattices, which only consists of two signal phases. This was recently generalised by de Gier et al. [2011], Gershenson and Rosenblueth [2012] and Zhang et al. [2013] to handle multiple signal phases, and to consider the traffic condition downstream of the intersection. One interesting observation by de Gier et al. [2011] is that the performance of the SOTL scheme is dependent upon one of its parameters: the “threshold” value for each intersection. The mean travel time through a SOTL controlled network was shown to exhibit dependence on the threshold value, such that there is an optimal threshold value which yields the minimum mean travel time. However, the relationship between these two quantities is not explicitly known, and it can vary for different traffic conditions. Thus, manually selecting the threshold value will potentially result in non-optimal performance.

The aim of this paper is to study the use of extremum-seeking schemes in optimising the selection of the threshold used in SOTL for urban traffic signal control. The main contributions of this paper are: providing conditions under which an ES scheme can be applied to adaptation of a local control with noisy plants; outlining a preliminary stability result; and performing simulations which show the performance of the ES scheme in various traffic scenarios.

2. THEORETICAL RESULT

Consider a nonlinear system,

$$\dot{x} = f(x, \theta, \xi(t)), \quad (1a)$$

$$y = g(x), \quad (1b)$$

where $x \in X \subset \mathbb{R}^{N_x}$ is the plant’s state; $\theta \in \Theta^{N_\theta}$ is the control input to the plant, where $\Theta \subseteq \mathbb{R}$; $y \in \mathbb{R}$ is the plant’s output; $\xi(t) \in \mathcal{V}_\xi \subset \mathbb{R}^{N_\xi}$ is a disturbance term; and $g : \mathbb{R}^{N_x} \rightarrow \mathbb{R}$ is a continuous function.

The plant is locally controlled by,

$$\dot{\kappa} = l(x, \kappa, u), \quad (2a)$$

$$\theta = m(x, \kappa, u), \quad (2b)$$

where $\kappa \in \mathbb{R}^{N_\kappa}$ is the controller’s state and $u \in [u_{min}, u_{max}] \subseteq \mathbb{R}$ is the controller’s parameter.

Assumption 1. The local controller (2) are designed such that there exist a steady-state input-output (IO) map $J : \mathbb{R} \rightarrow \mathbb{R}$, a disturbance measuring function $\sigma : \mathcal{V}_f \rightarrow \mathbb{R}_{\geq 0}$, and $\beta_f \in \mathcal{KL}$ such that given a constant u , the trajectory of the plant satisfies

$$|g(x(t)) - J(u)| \leq \max\{\beta_f(|g(x(0)) - J(u)|, t), \|\sigma(\xi)\|_\infty\}. \quad (3)$$

Assumption 2. There exist $u^* \in [u_{min}, u_{max}]$ and $K > 0$ such that: $J'(u^* + \tilde{u})$ is continuous and bounded for all $\tilde{u} \in [u_{min} - u^*, u_{max} - u^*]$; $J'(u^* + \tilde{u})/\tilde{u} \leq K$ for all $\tilde{u} \in [u_{min} - u^*, u_{max} - u^*] - \{0\}$.

Remark 1. These two assumptions state that the local controller stabilises the plant, and its steady-state performance is governed by a parameter u . Thus, the aim of the extremum-seeking is to tune this parameter u such that the performance is optimised.

Assumption 3. For any $\xi \in \mathcal{V}_f$, $\sigma(\xi)$ is bounded.

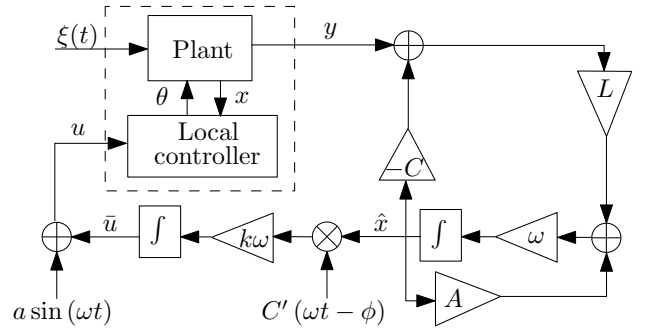


Fig. 1. Observer-based ES scheme applied to a plant with local control.

Applying the considered ES scheme results in the following full closed-loop system:

$$\dot{x} = f(x, m(x, \kappa, \bar{u} + a \sin(\omega t)), \xi(t)), \quad (4a)$$

$$\dot{\kappa} = l(x, \kappa, \bar{u} + a \sin(\omega t)), \quad (4b)$$

$$\dot{\hat{x}} = \omega A \hat{x} + \omega L(g(x) - C \hat{x}), \quad (4c)$$

$$\dot{\bar{u}} = \kappa \omega C'(\omega t - \phi) \hat{x}. \quad (4d)$$

where $L \in \mathbb{R}^3$ is the observer gain, $C = [1 \ 1 \ 0]$, $C'(\cdot) = [0 \ \sin(\cdot) \ \cos(\cdot)]$, and

$$A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}. \quad (5)$$

Assumption 4. L is chosen such that $(A - LC)$ is Hurwitz.

In addition, the following continuity assumption on the plant and local control dynamics is required. For notational compactness, define: $x_s := (\hat{x}, \bar{u})$, $x_f := (x, \kappa)$ and

$$\begin{bmatrix} f(x, m(x, \kappa, \bar{u}), \xi) \\ l(x, \kappa, \bar{u}) \end{bmatrix} =: F_f(x_f, x_s, \xi). \quad (6)$$

Assumption 5. There exists $L > 0$, such that for each $\rho > 0$, there exists $\omega^* > 0$ such that for all $\xi \in \mathcal{V}_f$, $x_{s,1}, x_{s,2} \in (\mathbb{R}^3, [u_{min}, u_{max}])$, $x_{f,1}, x_{f,2} \in (X, \mathbb{R}^{N_\kappa})$, $\omega \in (0, \omega^*]$, if $\|x_{s,1} - x_{s,2}\| \leq \omega^*$, then it implies

$$\|F_f(x_{f,1}, x_{s,1}, \xi) - F_f(x_{f,2}, x_{s,2}, \xi)\| \leq L \|x_{f,1} - x_{f,2}\| + \rho \quad (7)$$

Theorem 1. Under Assumptions 1–5, for any $\delta > 0$ and $r_u \in \max\{u_{max} - u^*, u^* - u_{min}\}$, there exist $(a^*, k^*, c) \in$

$\mathbb{R}_{>0}^3$ such that for all $(a, k) \in (0, a^*] \times (0, k^*]$, there exists $\omega^* > 0$ such that for all $\omega \in (0, \omega^*]$, the trajectory of $\bar{u}(t)$ with any initial conditions $\bar{u}(0) \in \{\bar{u} : \bar{u} \in (u_{min}, u_{max}), |\bar{u} - u^*| \leq r_u\}$ will asymptotically converge to a $(2\pi/\omega)$ -periodic solution satisfying

$$\limsup_{\tau \rightarrow \infty} |u(t) - u^*| \leq ca + \delta. \quad (8)$$

Proof. Only a sketch of the proof is provided due to space limitation. The proof follows two steps. First, the stability of the ES scheme acting on the steady-state map $J(u)$ is proven using the same approach to [Moase and Manzie, 2012, Section III.A]. Then, the stability of the full closed-loop system follows from [Teel et al., 2003, Theorem 1], which enables the ES to act upon plants with noise.

3. SELF-ORGANISING TRAFFIC LIGHT

3.1 SOTL overview

At an intersection, all of the possible traffic flows are grouped into several “phases”. The flows that belong to the same phase will get green lights simultaneously. SOTL controls the traffic lights at each intersection by deciding which phase is the “busiest”, and switching the green light accordingly.

The “busyness” of a phase is measured by a function referred to as the “threshold function”. Let \mathcal{P} denote an arbitrary phase and $\kappa_{\mathcal{P}}(t)$ denote the threshold function of \mathcal{P} at time t . Note that notation used for the threshold function is the same as the one used for controller’s state, κ , because the threshold function can be considered as the states of the controller SOTL. This will be explained in more detail later.

Then, \mathcal{P} is a candidate for the next active phase when its threshold function $\kappa_{\mathcal{P}}(t)$ exceeds a threshold value, $\kappa_{\mathcal{P}}(t) > u$. The threshold function of a phase depends on both the amount of traffic demand and how long it has been idle. Specifically,

$$\kappa_{\mathcal{P}}(t) = q(d_{\mathcal{P}}(t), \tau_{\mathcal{P}}(t)), \quad (9)$$

where $d_{\mathcal{P}}(t)$ is the total number of vehicles waiting for phase \mathcal{P} to be active; $\tau_{\mathcal{P}}(t)$ is the idle time of phase \mathcal{P} ; and q is the threshold function to be designed.

As stated previously, this work attempts to optimise the selection of the SOTL threshold, u , by using ES. A more detailed description of SOTL is outlined by Zhang et al. [2013], including the threshold function used.

3.2 Performance metric

Although typically the performance metric in traffic control is vehicle flow rate, in steady-state the flow would be equal to the incoming traffic flow into the network (vehicle conservation) which is fixed. Thus, it is better to minimise the travel time instead, which is approximately equivalent to maximising speed. Specifically, let $v_i(t)$ be the speed of vehicle i at time t and $\mathcal{H}(t)$ be the set of the vehicles that are in the network at time t . Then, with $T_a = 1500$ seconds,

$$y(t) = \frac{1}{T_a} \int_{t-T_a+1}^t \sum_{i \in \mathcal{H}(t)} v_i(t). \quad (10)$$

In practice, measuring $y(t)$ in real time might be impossible. However, it is possible to use another performance metric, which might be equivalent to the proposed metric. In fact, any performance metric can be used as long as, coupled with the local controller (2), it satisfies the outlined assumptions.

4. SIMULATIONS

4.1 Traffic Network

The network studied in this paper is a square grid network consisting 16 intersections, each equipped with one SOTL controller. Furthermore, at each time step, a vehicle is inserted into the network through each inbound boundary road with probability α . Similarly, when a vehicle reaches the end of an outbound boundary road, it has β probability of exiting the network, simulating queue build-up due to congestion outside the network. α and β can be varied to simulate various traffic conditions. Lastly, there are four phases which are present at each intersection: a north/south phase, an east/west turning phase, an east/west phase and a north/south turning phase. Each vehicle, at each intersection, makes a random turning decision with a predetermined probability.

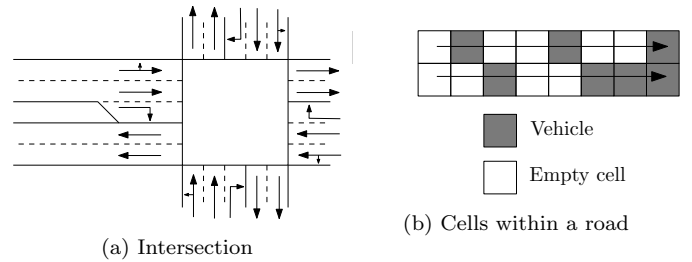


Fig. 2. The illustration of the traffic model used.

In relation to (1) and (2), traffic is often represented with flow equations. Using vehicle conservation, the dynamics of the plant’s state x , which is the queue length in the network, is represented by the nonlinear flow rate f , which captures all of the events in the traffic, including but not limited to: intersection queuing, lane changing and turning. Furthermore, $\xi(t)$ represents the inbound and outbound traffic. The traffic light setting, θ , is controlled by SOTL. The dynamics of the controller’s state κ are governed by a function l with input from the queue length x , itself κ (related to idle time), and the SOTL threshold u . Then, θ is determined by a local feedback control m based on the plant’s and controller’s state.

The simulations performed in this study utilise a cellular automata model, which was comprehensively described by de Gier et al. [2011]. In this model, each vehicle is simulated individually. This implies that almost all events in traffic are represented. Thus, this simulation model is deemed sufficient to capture the traffic dynamics as in (1).

4.2 Steady-state maps

In this study, $u_i = \bar{u}$ for all i (i.e. the same parameter is used at all intersections), leading to a one dimensional optimisation problem. Separating the u_i ’s would provide

scope for closed-loop improvement, but also complicates the optimisation.

It is necessary to investigate whether Assumptions 1 and 2 are satisfied for the combination of the traffic network and SOTL. In order to do this, the simulation is run several times using various values of constant u . It is found that indeed for each (α, β) , there exists a map $J_{\alpha, \beta}(u)$ and the output y converges to the vicinity of $J_{\alpha, \beta}(u)$ (Fig. 3).

In addition, it can be seen in Fig. 3a that β does not affect the IO map except when β is very low, and the IO map becomes relatively flat. This is to be expected from a low density system. In a high density regime (approximately $\alpha \geq 0.17$), a greater dependence on β is exhibited, yet its effect only becomes apparent at approximately $\beta < 0.3$, which is still fairly low. Thus, $\beta = 0.9$ is used for the remainder of this paper, since it does not significantly affect the result.

Moreover, note that the shape of the IO map changes significantly with α , and when approximately $\alpha \geq 0.18$, it becomes flat (Fig. 3b) due to the network being saturated. Thus, when $\alpha \geq 0.18$, there is little benefit in adaptively tuning the SOTL threshold u . However, if a different local controller was used, there may be benefit in using ES for adaptation of that controller's parameter(s).

Lastly, in this simulation a vehicle is only inserted into the network if there is space on the inbound boundary road. It is found that the number of vehicles being denied generation due to space limitation is also minimised at the optimum u^* , and is increasing further away from u^* . The increase is more pronounced as α is higher, except when α is at the very high density regime ($\alpha \geq 0.19$).

4.3 ES parameter selection

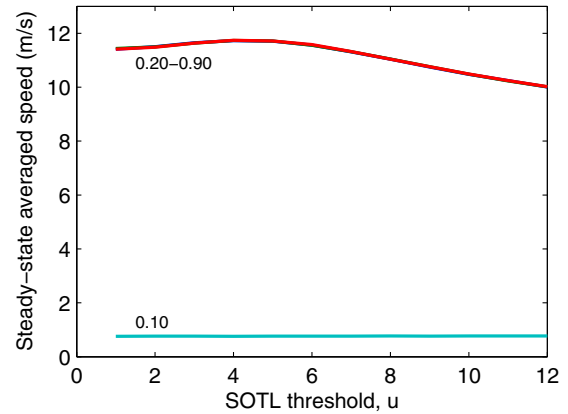
Following the guidelines provided in Theorem 1, the ES parameters are tuned to be: $a = 1$, $k = 0.2$, $\omega = 10^{-3}$ rad/s, $\phi = 55^\circ$, and $L = [0.237 \ 0.343 \ 0.167]^T$. The dither frequency ω is chosen such that the ES scheme is sufficiently slow that it is not too adversely affected by the plant's dynamics; the phase shift ϕ is chosen to partially compensate the effect of the plant's dynamics; and L is chosen based on the optimisation done by Moase and Manzie [2012].

4.4 Time-invariant demand

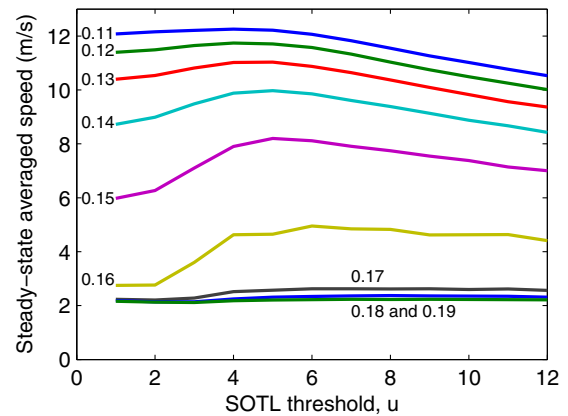
The traffic conditions considered are $\alpha = 0.13$ and 0.17 . Furthermore, for each traffic condition, two initial conditions for u are used, one from each side of the optimum.

As shown in Fig. 4, the ES scheme always converges to the vicinity of the optimum. Note that in the $\alpha = 0.17$ case, u converges to slightly different regions (on either side of the optimum) for each of the initial conditions tested. Although this error can be reduced by decreasing (a, k, ω) , the resulting output of the plant will be relatively unaffected due to J being almost flat around the optimum.

However, note that the convergence speed is low, taking approximately 35 and 85 hours for the output to converge for the first and second traffic conditions respectively. This is caused by the fact that the dynamics of the plant



(a) IO maps for various β when $\alpha = 0.12$.



(b) IO maps for various α when $\beta = 0.9$.

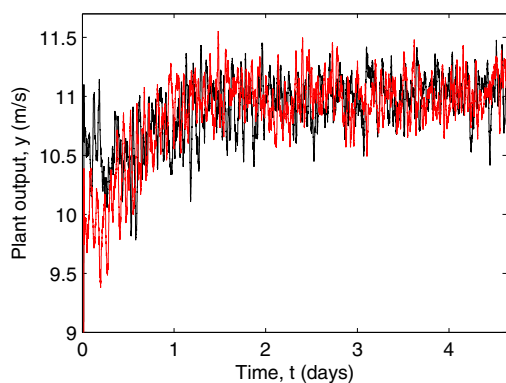
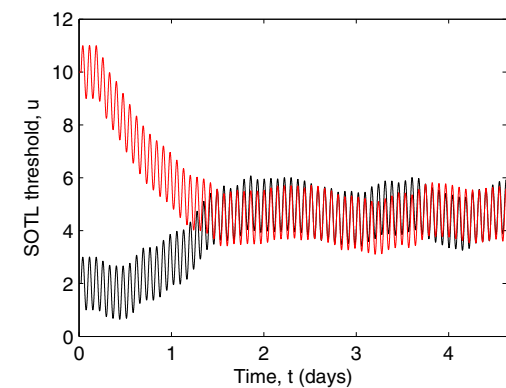
Fig. 3. The effect of SOTL threshold, α and β on the plant's steady state output.

take a long time to settle and the ES scheme must act on a time scale slower than that of the plant. However, note that in this study, the initial conditions of u are set deliberately far from u^* . A more educated selection of the initial condition might significantly reduce the convergence time. Furthermore, the slow convergence implies that for this combination of local controller and ES scheme, it is not possible to converge to the optimum in a day, considering that the demand will change within hours. In practice, it is likely that different sections of the day would be compartmentalised (e.g. different peak periods and off-peak), with tuning performed separated over many days.

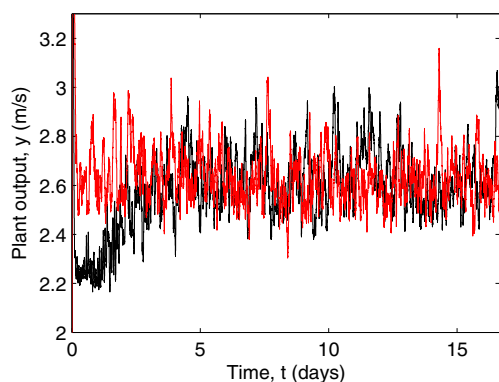
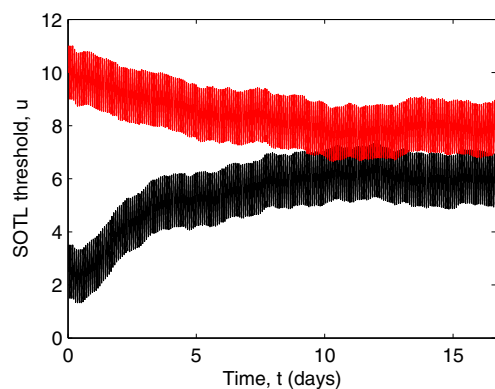
4.5 Step-change demand

In this subsection, the benefit of using the ES scheme is highlighted by applying boundary conditions with a step-change to the simulation. For the first half of this simulation, $\alpha = 0.13$ and $\alpha = 0.17$ for the second half. These values represent the change from fairly light to high inbound traffic volume. Furthermore, the result of using ES is compared with cases where u is kept constant. Five cases of constant u are used, $u = \{4, 5, 6, 7, 8\}$, within which lie the optima of the first and the second traffic condition. The initial condition for the ES case is $u = 4.5$.

The simulation results are summarised in Table 1. The second column shows the averaged plant's output after



(a) $\alpha = 0.13$



(b) $\alpha = 0.17$

Fig. 4. Simulation results of the time-invariant traffic condition using ES. The red and black curves show the results when $u(0) = 10$ and $u(0) = 2$ respectively.

the initial major dynamic has settled down. This includes the transient period after α is switched from 0.13 to 0.17. The third column compares the performance degradation relative to the case with the highest averaged y , which is the ES case. In addition, the output is also averaged ignoring the transient (to give an approximate steady-state y) for both $\alpha = 0.13$ and $\alpha = 0.17$, and these values are shown in the last two columns.

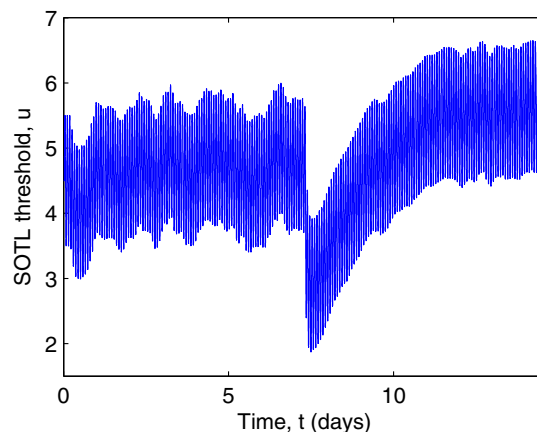


Fig. 5. The performance of the ES scheme in the step-change condition case.

Table 1. Simulation results for a step change in α .

u	Aveaged output, y (m/s)	Performance degradation relative to ES	Aveaged “steady-state” y (m/s)	
			$\alpha = 0.13$	$\alpha = 0.17$
ES	6.887	-	11.00	2.85
4	6.782	1.5%	11.00	2.48
5	6.828	0.9%	11.03	2.57
6	6.792	1.4%	10.87	2.65
7	6.659	3.3%	10.66	2.60
8	6.518	5.4%	10.37	2.60

From Fig. 5, it can be seen that the ES scheme is able to adapt to a change in the traffic condition such that for the first and the second half of the simulation, the ES converges to approximately 4.8 and 6 respectively. Moreover, from the last two columns, it can be seen that the ES successfully regulates y to the vicinity of its new optimum. Furthermore, the performance of the scheme is superior compared to the cases with constant u . The very small degradation when $\alpha = 0.13$ compared to the constant $u = 5$ case is caused by the dither of the ES.

Therefore, it has been shown that when faced with a change in traffic condition, the ES scheme can gain a small advantage by adapting the threshold to the vicinity of the new optimum. Moreover, by using ES, *a priori* knowledge of the location of the optimum is not required. Without online tuning, calibrating the optimal threshold for a given network would be a lengthy and expensive process which would be complicated by the fact that the optimal threshold depends upon the network conditions.

5. CONCLUSIONS AND FURTHER WORK

The benefits of using extremum-seeking to fine-tune a local traffic control strategy was demonstrated. A set of conditions that the local controller has to satisfy were presented. By using simulation, it was shown that the extremum-seeking scheme is able to adapt the parameter close to its optimum in various traffic situations. Some extensions for future work include using MISO and MIMO extremum-seeking (Kutadinata et al. [2012]); investigation using a more realistic traffic network; and finding more easily measured performance metrics for a traffic network.

REFERENCES

- R. B. Allsop. SIGCAP: A computer program for assessing the traffic capacity of signal-controlled road junctions. *Traffic engineering and control*, 17:338–341, 1976.
- I. Alvarez and A. Poznyak. Game theory applied to urban traffic control problem. In *Proceedings of the 2010 International Conference on Control Automation and Systems (ICCAS)*, pages 2164–2169, Oct. 2010.
- A. Banaszuk, K.B. Ariyur, M. Krstić, and C.A. Jacobson. An adaptive algorithm for control of combustion instability. *Automatica*, 40:1965–1972, 2004.
- F. Boillot, J. M. Blossville, J. B. Lesort, V. Motyka, M. Papageorgiou, and S. Sellam. Optimal signal control of urban traffic networks. In *Road Traffic Monitoring (IEE Conf. Pub. 355)*, pages 75–79, 1992.
- D. F. Chichka, J. L. Speyer, C. Fanti, and C. G. Park. Peak-seeking control for drag reduction in formation flight. *Journal of Guidance, Control, and Dynamics*, 29(5):1221–1230, 2006.
- S. Chiu and S. Chand. Self-organizing traffic control via fuzzy logic. In *Proceedings of the 32nd IEEE Conference on Decision and Control*, volume 2, pages 1897–1902, 1993.
- J. de Gier, T. Garoni, and O. Rojas. Traffic flow on realistic road networks with adaptive traffic lights. *Journal of Statistical Mechanics: Theory and Experiment*, page P04008, 2011.
- C. Diakaki, M. Papageorgiou, and K. Aboudolas. A multivariable regulator approach to traffic-responsive network-wide signal control. *Control Engineering Practice*, 10(2):183–195, 2002.
- N. H. Gartner, S. F. Assman, F. Lasaga, and D. L. Hou. A multi-band approach to arterial traffic signal optimization. *Transportation Research Part B: Methodological*, 25(1):55–74, 1991.
- C. Gershenson. Self-organizing traffic lights. *Complex System*, 16(1):29–53, 2005.
- C. Gershenson and D. A. Rosenblueth. Self-organizing traffic lights at multiple-street intersections. *Complexity*, 17(4):23–39, 2012.
- N. Killingsworth and M. Krstić. Auto-tuning of PID controllers via extremum seeking. In *Proceedings of the 2005 American Control Conference*, pages 2251–2256, June 2005.
- M. Krstić and H. H. Wang. Stability of extremum seeking feedback for general nonlinear dynamic systems. *Automatica*, 36(4):595–601, Apr. 2000.
- R. J. Kutadinata, W. Moase, and C. Manzie. Non-local stability of a Nash equilibrium seeking scheme with dither re-use. In *Proceedings of the 51st IEEE Conference on Decision and Control*, pages 6077–6082, 2012.
- S. Lammer and D. Helbing. Self-control of traffic lights and vehicle flows in urban road networks. *Journal of Statistical Mechanics: Theory and Experiment*, page P04019, 2008.
- S. Lin, B. De Schutter, X. Yugeng, and H. Hellendoorn. Efficient network-wide model-based predictive control for urban traffic networks. *Transportation Research Part C: Emerging Technologies*, 24:122–140, 2012.
- S.-J. Liu and M. Krstić. Stochastic averaging in continuous time and its applications to extremum seeking. *IEEE Transactions on Automatic Control*, 55(10):2235–2250, 2010.
- P. R. Lowrie. The Sydney co-ordinated adaptive traffic system - principles, methodology, algorithms. In *Proceedings of the IEE Conference on Road Traffic Signalling*, volume 207, pages 67–70, 1982.
- W. H. Moase and C. Manzie. Semi-global stability analysis of observer-based extremum-seeking for Hammerstein plants. *IEEE Transactions on Automatic Control*, 57(7):1685–1695, 2012.
- W.H. Moase, C. Manzie, and M.J. Brear. Newton-like extremum-seeking for the control of thermoacoustic instability. *IEEE Transactions on Automatic Control*, 55(9):2094–2105, Sept. 2010.
- T. Nakatsuji and T. Kaku. Development of a self-organizing traffic control system using neural network models. *Transportation Research Record*, 1324:137–145, 1991.
- D. I. Robertson. TRANSYT method for area traffic control. *Traffic engineering and control*, 11(6):276–281, 1969.
- J.C. Spall. Multivariate stochastic approximation using a simultaneous perturbation gradient approximation. *IEEE Transactions on Automatic Control*, 37(3):332–341, 1992.
- B. Srinivasan. Real-time optimization of dynamic systems using multiple units. *International Journal of Robust and Nonlinear Control*, 17(13):1183–1193, 2007.
- Y. Tan, D. Nešić, and I. Mareels. On non-local stability properties of extremum seeking control. *Automatica*, 42(6):889–903, 2006.
- A. Teel, L. Moreau, and D. Nešić. A unified framework for input-to-state stability in systems with two time scales. *IEEE Transactions on Automatic Control*, 48(9):1526–1543, Sep. 2003.
- T. Tettamanti and I. Varga. Distributed traffic control system based on model predictive control. *Periodica Polytechnica Civil Engineering*, 54(1):3–9, 2010.
- L. Zhang, T. Garoni, and J. de Gier. A comparative study of macroscopic fundamental diagrams of arterial road networks governed by adaptive traffic signal systems. *Transportation Research Part B: Methodological*, 49:1–23, 2013.