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Adaptive Neural Network Control of Uncertain State-Constrained Nonlinear Systems ⋆

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Abstract: In this paper, an adaptive neural network control scheme is developed for a class of uncertain nonlinear systems in strict-feedback form subject to the constraints on the full states and unknown system drift dynamics. The Integral Barrier Lyapunov Functionals (iBLF) are utilized to handle the state constraints directly, leading to the relax of the feasibility conditions compared with pure tracking errors based Barrier Lyapunov Function. The radial basis function neural networks (RBFNN) are adopted to approximate and compensate for the unknown continuous packaged functions composed of the unknown system nonlinearities. Novel adapting parameters are constructed to estimate the unknown bounds in neural networks approximation in real time. Based on backstepping design and Lyapunov synthesis, we show that the developed control scheme can guarantee that all signals are semi-globally uniformly ultimately bounded (SGUUB), all states remain in the predefined constrained state space and system output converges to a small neighborhood of the desired trajectory. A practical three-order example is provided to demonstrate the performance of proposed methods.

Keywords: Integral Barrier Lyapunov functionals; state constraints; neural networks

1. INTRODUCTION

Control of nonlinear systems with constraints on states and inputs has gained an increasing research attention due to its practical needs and theoretical challenges. Meanwhile, due to the modeling errors, unmodeled dynamics and external disturbance, or a combination of these unknowns etc., it is difficult to obtain an accurate system model for control design. This paper aims at solving the trajectory tracking problem for strict-feedback nonlinear systems subject to both full state constraints and unknown system drift dynamics.

Numerous methods have been proposed to address the control problems of linear and nonlinear constrained systems, including the invariance control in Bayer et al. (2011), Model Predictive Control in Mayne et al. (2000), non-overshooting control in Krstic and Bement (2006), extremum seeking control in DeHaan and Guay (2005) and error transformation in Do (2010), .etc. Motivated by the spirit of reshaping a control Lyapunov function using barrier function, Barrier Lyapunov Functions (BLF) have been developed to guarantee the constraints satisfaction. Based on the Lyapunov stability theorem and BLF's property of growing to infinity at some limits, the BLF based design methodology is to guarantee the boundedness of BLF in the closed-loop system, hence the stability of closed-loop system and constraint satisfaction can be ensured. BLF-based control design has been used to solve for the Brunovsky form constrained systems in Ngo et al. (2005), strict-feedback form output-constrained systems in Tee et al. (2009b); Tang et al. (2013); Meng et al. (2012), output-feedback form output-constrained systems in Ren et al. (2010) and strict-feedback form state-constrained system in Tee and Ge (2012), as well as the application in electrostatic parallel plate microactuatorsin Tee et al. (2009a) . Although previous works have tackled the issue of nonlinear systems with unknown dynamics and constraints, these results are either only applicable to output-feedback systems in Ren et al. (2010) or require the bounds of neural network approximation errors known for control design in Meng et al. (2012), and only the mode-based control design has been proposed for state-constrained nonlinear systems in Tee and Ge (2012). In all, the problem for strict-feedback systems subject to unknown system nonlinearities and full state constraints is currently unsolved and also challenging due to coupling difficulties from unknown system dynamics and state constraints.

On the other hand, several kinds of approximators, such as fuzzy logics and neural networks, have been proved as the general tools modeling any continuous nonlinear functions to any desired accuracy over a compact set in Wang (1992); Chen and Chen (1995) and many results have been obtained for different classes of systems by developing the stable adaptive neural network control and adaptive fuzzy system control for nonlinear systems with unknown dynamics, for example in Ge et al. (1998, 2002); Li et al. (2010); Tao et al. (2011).

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To address the previous unsolved problem, in this paper, we incorporate the integral Barrier Lyapunov functionals into the adaptive neural network scheme to handle the full state constraints, for which a conservative mapping of original constrains to dynamic error space constraints is avoided, and the system drift nonlinearities can be relaxed to be unknown; And the bounds of NN approximation errors, NN weight estimation errors and radial basis functions are not necessarily to be known for control design in the proposed scheme by constructing novel adapting parameters to estimate these unknown bounds online; The closed-loop system is proved to SGUUB, all the constraints on states are guaranteed provided the feasibility conditions are satisfied, and system output can stay arbitrarily close to the desired trajectory.

2. PROBLEM FORMULATION AND PRELIMINARIES

2.1 Plant Dynamics

Consider the following nonlinear system in strict feedback form Krstic et al. (1995)

$$\begin{aligned}\dot{x}_i(t) &= f_i\left(\bar{x}_i(t)\right) + g_i\left(\bar{x}_i(t)\right) x_{i+1}(t),\\ &\quad i = 1, 2, \ldots, n-1\\ \dot{x}_n(t) &= f_n(\bar{x}_n(t)) + g_n(\bar{x}_n(t))u(t)\\ y(t) &= x_1(t), t \in R^+ \end{aligned}\tag{1}$$

where $\bar{x}_i(t) \triangleq [x_1(t), x_2(t), \ldots x_i(t)]^T \in \mathbb{R}^i, i = 1, \ldots, n$, $u \in \mathbb{R}$ and $y \in \mathbb{R}$ are the system states, control input and system output, respectively; $f_i(\cdot) : \mathbb{R}^i \to \mathbb{R}$ are the unknown system drift dynamics and $g_i(\cdot) : \mathbb{R}^i \to \mathbb{R}$ represent the control coefficients and are assumed as known. The system states $x_i(t)$ are required to satisfy the constraints as follows

$$|x_i(t)| < k_{c_i}, \quad \forall t \geq 0, \quad i = 1, \ldots, n. \tag{2}$$

where k_{c_i} are positive constants, which represent the predefined constraints on the states and the constrained state space is denoted as the set $\chi := \{x \in \mathbb{R}^n : |x_1(t)| < k_{c_1}, \ldots, |x_n(t)| < k_{c_n}, t \geq 0\} \subset \mathbb{R}^n$.

In this paper, the control objective is to enforce the system output $y(t)$ track a desired trajectory $y_d(t)$ meanwhile all signals in the closed-loop system remain bounded, and the state constraints are not violated.

Assumption 2.1. Tee and Ge (2012) For any $k_{c_1} > 0$, there exits positive constants $A_0, Y_i, i = 1, \ldots, n$, such that the desired trajectory y_d and its time derivative satisfy $|y_d(t)| \leq A_0 < k_{c_1}, |y_d^{(i)}(t)| < Y_i, \forall t \geq 0$ and $i = 1, \ldots, n$. We also denote $\bar{y}_{d_i} \triangleq [y_d, y_d^{(1)}, \ldots, y_d^{(i)}] \in R^{i+1}$.

Assumption 2.2. The signs of $g_i(\bar{x}_i)$, $i = 1, 2, \ldots, n$, are known, and there exists positive constants $g_{i,\min}$ and $g_{i,\max}$ such that $0 < g_{i,\min} \leq |g_i(\bar{x}_i)| \leq g_{i,\max} < \infty$ for $|x_j| < k_{c_j}, j = 1, 2 \ldots, i$. Without loss of generality, we further assume that the $g_i(\bar{x}_i)$ are all positive for $|x_j| < k_{c_j}, j = 1, 2 \ldots, i$.

2.2 Radial Basis Function Neural Networks

In this paper, the continuous function $h(Z) : R^q \to R$ is approximated as

$$h_{nn}(Z) = W^T S(Z), \tag{3}$$

where $Z \in \Omega_z \subset R^q$ and $W = [w_1, w_2, \ldots, w_l]^T \in \mathbb{R}^l$ are the NN input vector and weight vector, respectively, the NN node number $l > 1$; and $S(Z) = [s_1(Z), \ldots, s_l(Z)]^T$, with $s_i(Z)$ being chosen as the commonly used Gaussian functions form.

It has been proved that $W^T S(Z)$ can approximate any continuous $h(Z)$ over a compact set $\Omega_Z \subset R^q$ to any desired accuracy as by choosing l sufficiently larege

$$h(Z) = W^{*T} S(Z) + \epsilon(Z), \tag{4}$$

where W^* is the ideal constant weight vector and $\epsilon(Z)$ is the unknown approximation error and bounded over the compact set. The ideal weight vector W^* is defined as

$$W^* := \arg \min_{W \in R^l} \{ \sup_{Z \in \Omega_z} |h(Z) - W^T S(Z)|\}. \tag{5}$$

Assumption 2.3. For a given smooth function $h(Z)$, there exist ideal unknown constant weight vector W^* such that $|\epsilon| \leq \epsilon_n^*$ with unknown positive constant ϵ_n^* for all $Z \in \Omega_z$. The radial basis function is also bounded as $\|S(Z)\| \leq s_n^*$ with unknown positive constant s_n^* for all $Z \in \Omega_z$.

Remark 2.1. Although we utilize the RBFNN in the control design, it can be replaced by other linearly parameterized approximators, such as fuzzy logic system, splines, wavelet networks and high-order NNs. We refer to Farrel and Polycarpou (2006) for interested readers on a unified framework of approximation-based control.

3. CONTROL DESIGN FOR FIRST-ORDER SYSTEM

This paper considers the following integral Barrier Lyapunov Functionals in Tee and Ge (2012)

$$V_{z_i}(t) = \int_0^{z_i} \frac{\sigma k_{c_i}^2}{k_{c_i}^2 - (\sigma + \alpha_{i-1})^2} d\sigma, \quad i = 1, \ldots, n, \tag{6}$$

where $z_i = x_i - \alpha_{i-1}, \alpha_0 = y_d$ and $\alpha_1, \ldots, \alpha_{n-1}$ are continuously differentiable functions satisfying $|\alpha_i| \leq A_i < k_{c_{i+1}}$ for positive constants $A_i, i = 0, 1, \ldots, n-1$. The functionals (6) are positive definite, continuously differentiable and satisfy the decresent condition in the set $|x_i| < k_{c_i}$,

To illustrate the design method, the first-order system is considered firstly

$$\dot{x}_1(t) = f_1(x_1(t)) + g_1(x_1(t))u(t), \tag{7}$$

where f_1 is the unknown smooth function. By taking the time derivative of V_{z_1}, it is not difficult to obtain the following ideal control for system (7),

$$u^*(t) = -\kappa_1 z_1 - h_1(Z_1), \tag{8}$$

where κ_1 is a positive constant and

$$h_1(Z_1) = \frac{1}{g_1}\left(f_1 - \frac{(k_{c_1}^2 - x_1^2)\dot{y}_d \rho_1}{k_{c_1}^2}\right), \tag{9}$$

$$\rho_1 = \frac{k_{c_1}}{2z_1} \log \frac{(k_{c_1} + z_1 + y_d)(k_{c_1} - y_d)}{(k_{c_1} - z_1 - y_d)(k_{c_1} + y_d))}, \tag{10}$$

$$Z_1 = [x_1, y_d, \dot{y}_d] \in \Omega_{z_1} \subset \mathbb{R}^3. \tag{11}$$

However, Under the condition that the function $f_1(x_1)$ is unknown, the desired feedback control u^* is not available due to the unknown smooth function $h_1(Z_1)$. In this paper, the appropriate controller by approximating unknown smooth function $h_1(Z_1)$ using RBFNN (3) is presented as

$$h_1(Z_1) = W_1^{*T} S_1(Z_1) + \epsilon_1, \tag{12}$$

where W_1^* is the unknown ideal weight vector defined in (5) and $|\epsilon_1| < \epsilon_{n_1}^*$ with $\epsilon_{n_1}^* > 0$. Hence, the NN controller for system (7) is designed as

$$u(t) = -\kappa_1 z_1 - \hat{W}_1^T S_1(Z_1) - \frac{\hat{p}_1}{g_1} \tanh\left(\frac{\frac{z_1 k_{c_1}^2}{k_{c_1}^2 - x_1^2}}{\delta_1}\right) \tag{13}$$

where $\hat{W}_1$ denotes the estimation of ideal weights vector W_1^*. Define the estimation errors $\tilde{W}_1 = \hat{W}_1 - W_1^*$, and $\|\tilde{W}_1\| \leq \epsilon_{w_1}^*$ with $\epsilon_{w_1}^*$ being a positive constant; $\hat{p}_1$ represents the parameter estimates of the grouped unknown bounds of $\epsilon_{n_1}^* + \epsilon_{w_1}^* s_{n_1}^*$, and define $\tilde{p}_1 = \hat{p}_1 - p_1^*$, p_1^* represents the ideal unknown parameters to be estimated. $\delta_1 > 0$ is a small positive constant. The update laws for $\hat{p}_1$ and $\hat{W}_1$ are designed as

$$\dot{\hat{p}}_1 = \frac{z_1 k_{c_1}^2}{k_{c_1}^2 - x_1^2} \tanh\left(\frac{\frac{z_1 k_{c_1}^2}{k_{c_1}^2 - x_1^2}}{\delta_1}\right) - \sigma_{p_1}\hat{p}_1, \tag{14}$$

$$\dot{\hat{W}}_1 = \Gamma_1(S_1(Z_1)z_1 - \sigma_{w_1}|z_1|\hat{W}_1), \tag{15}$$

where $\hat{p}_1(0) \geq 0$, $\sigma_{w_1}, \sigma_{p_1}, \Gamma_1 = \Gamma_1^T > 0$. From (14), it is easy to see that $\hat{p}(t) \geq 0, \forall t \geq 0$. Based on (15) and Huang et al. (2003), we have the following result on the boundedness of $\hat{W}_1$:

Lemma 3.1. Huang et al. (2003). Under the update law (15), the $\hat{W}_1(t)$ is semiglobally uniformly bounded in the compact set

$$\Omega_{w_1} = \{\hat{W}_1 | \|\hat{W}_1\| \leq \frac{s_{n_1}^*}{\sigma_{w_1}}\}, \tag{16}$$

where $\|S_1(Z_1)\| \leq s_{n_1}^*$, provided $\hat{W}_1(0) \in \Omega_{w_1}$.

Theorem 3.1. Consider the closed-loop system consisting of the first-order system (7), controller (13) and update laws (14) with (15), then for any bounded initial conditions $x_1(0) \in \chi_1 := \{x \in \mathbb{R}^n : |x_1(t)| < k_{c_1}, \forall t \geq 0\}$ and $\hat{W}_1(0) \in \Omega_{w_1}$, the tracking error $z_1(t)$ is bounded as $|z_1(t)| \leq \sqrt{2(V_1(0) + \frac{C_1}{\theta_1})}, \forall t \geq 0$, with $\theta_1 := \min\{\kappa_1 g_{1,\min}, \sigma_{p_1}\}, C_1 := 0.2785\delta_1 p_1^* + \frac{\sigma_{p_1}}{2} p_1^{*2}$, the state $x_1(t)$ remains in the constrained set χ_1 and the semiglobal uniform ultimate boundedness of other signals in the closed-loop system are obtained.

Proof: See the Appendix A. ∎

4. CONTROL DESIGN FOR NTH-ORDER SYSTEM

In this section, the results in first-order system are extended to the nth-order system (1) based on backstepping methodology. The intermediate so-called stabilization functions $\alpha_i(t)$ will be designed step by step to render each subsystem the stability. Further, the stabilizing functions $\alpha_i(t)$ require the computation of $\dot{\alpha}_{i-1}(t), \ddot{\alpha}_{i-2}(t), \ldots, \alpha_1^{i-1}(t)$. Accordingly, α_i should be at least $(n-i)$th differentialbe. To this end, consider the following Lyapunov functionals candidate for control design

$$V(t) = \sum_{i=1}^{n} V_{z_i}(t) + \sum_{i=1}^{n} \frac{1}{2}\tilde{p}_i^2(t) \tag{17}$$

where V_{z_i} is defined in (6), $\tilde{W}_i = \hat{W}_i - W_i^*$ and $\tilde{W}_i, \hat{W}_i, W_i^*$ are the NN weight errors, estimates and ideal values respectively, $\Gamma_i = \Gamma_i^T, i = 1, \ldots, n$; $\tilde{p}_i = \hat{p}_i - p_i^*$ and $\tilde{p}_i, \hat{p}_i, p_i^*$ are the estimation errors of unknown bounds, the estimation and ideal values, respectively.

Remark 4.1. Since the boundedness of NN weight estimate can be guaranteed by Lemma (3.1), this paper derives the control input by considering the functional (17) without the inclusion of estimation errors as usual.

The time derivative of $V_{z_i}(t)$ can be obtained as

$$\dot{V}_{z_i}(t) = z_i\Bigg(\frac{k_{c_i}^2}{k_{c_i}^2 - x_i^2}(f_i + g_i z_{i+1} + g_i\alpha_i - \dot{\alpha}_{i-1}) + \dot{\alpha}_{i-1}\Big(\frac{k_{c_i}^2}{k_{c_i}^2 - x_i^2} - \rho_i(z_i, \alpha_{i-1})\Big)\Bigg), \tag{18}$$

where

$$\rho_i(z_i, \alpha_{i-1}) = \frac{k_{c_i}}{2z_i} \log\frac{(k_{c_i} + z_i + \alpha_{i-1})(k_{c_i} - \alpha_{i-1})}{(k_{c_i} - z_i - \alpha_{i-1})(k_{c_1} + \alpha_{i-1}))}.$$

The following lemma show that ρ_i are continuously differentiable up to $n-i$ times:

Lemma 4.1. Tee and Ge (2012). The functions $\rho_i(z_i, \alpha_{i-1})$ are well defined at $z_i = 0$ and $\mathbb{C}^{n-i}$ in the set $\Psi = \{z_i \in \mathbb{R}, \alpha_{i-1} \in \mathbb{R} : |\alpha_{i-1} < k_{c_i}|, |z_i + \alpha_{i-1}| < k_{c_i}\}$.

According to the Lyapunov's direct method, the intermediate stabilizing functions are designed as

$$\alpha_1 = -\kappa_1 z_1 - \hat{W}_1^T S_1(Z_1) - \frac{\hat{p}_1}{g_1} \tanh\left(\frac{\frac{z_1 k_{c_1}^2}{k_{c_1}^2 - x_1^2}}{\delta_1}\right)$$

$$\begin{aligned} \alpha_i = &-\kappa_i z_i - \frac{k_{c_{i-1}}^2 (k_{c_i}^2 - x_i^2) g_{i-1} z_{i-1}}{k_{c_i}^2 (k_{c_{i-1}}^2 - x_{i-1}^2)} \\ &- \hat{W}_i^T S_i(Z_i) - \frac{\hat{p}_i}{g_i} \tanh\left(\frac{\frac{z_i k_{c_i}^2}{k_{c_i}^2 - x_i^2}}{\delta_i}\right), \\ &i = 2, \ldots, n \end{aligned} \tag{19}$$

where

$$\begin{aligned} Z_i =& [x_1, \ldots, x_i, \alpha_{i-1}, \frac{\partial \alpha_{i-1}}{\partial x_1}, \ldots, \frac{\partial \alpha_{i-1}}{\partial x_{i-1}}, w_{i-1}] \\ &\in \Omega_{z_i} \subset \mathbb{R}^{2i+1} \\ w_{i-1} =& \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{y_d^{(j)}} \dot{y}_d + \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \hat{W}_j} \dot{\hat{W}}_j + \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \hat{p}_j} \dot{\hat{p}}_j \\ \dot{\hat{W}}_i =& \Gamma_i(S_i(Z_i)z_i - \sigma_{w_i}|z_i|\hat{W}_i) \end{aligned} \tag{20}$$

$$\dot{\hat{p}}_i = \frac{z_i k_{c_i}^2}{k_{c_i}^2 - x_i^2} \tanh\left(\frac{\frac{z_i k_{c_i}^2}{k_{c_i}^2 - x_i^2}}{\delta_i}\right) - \sigma_{p_i}\hat{p}_i \tag{21}$$

$$\hat{p}_i(0) \geq 0, \Gamma_i = \Gamma_i^T > 0, \sigma_{w_i} > 0, \sigma_{p_i} > 0. \tag{22}$$

The final control input is specified as

$$u(t) = \alpha_n. \tag{23}$$

Accordingly, the control design yields

$$\dot{V}(t) \leq -\sum_{i=1}^{n} \frac{\kappa_i g_i k_{c_i}^2 z_i^2}{k_{c_i}^2 - x_i^2} - \sum_{i=1}^{n} \frac{\sigma_{p_i}}{2}\tilde{p}_i^2 + 0.2785 \sum_{i=1}^{n} \delta_i p_i^* \tag{24}$$

Using Lemma 1 in Tee and Ge (2012), it is further obtained

$$\dot{V}(t) \leq -\theta V + C, \tag{25}$$

where $\theta := \min\{\kappa_i g_{i,\min}, \min \sigma_{p_i}\} > 0$, $C := 0.2785 \sum_{i=1}^{n} \delta_i p_i^* + \sum_{i=1}^{n} \frac{\sigma_{p_i}}{2} p_i^{*2}$.

In the following, it is proved that the states $x(t) \in \chi := \{x \in \mathbb{R}^n : |x_i| < k_{c_i}, i = 1, \ldots, n\}, \forall t > 0$, and the boundedness of all signals in the closed-loop system.

Theorem 4.1. Consider the closed-loop system consisting of system (1), controller (23) and update laws (20) with (21), then for any bounded initial conditions $x(0) \in \chi := \{x \in \mathbb{R}^n : |x_i| < k_{c_i}, i = 1, \ldots, n\}$ and $\hat{W}_i(0) \in \Omega_{w_i}$. Let

$$A_i = \max_{(\bar{x}_n, \bar{y}_{d_n}, \hat{W}_i) \in \Omega} |\alpha_i(\bar{x}_i, \hat{W}_i, \hat{p}_i, \bar{y}_{d_i})|, i = 1, \ldots, n, \tag{26}$$

where

$$\begin{aligned} \Omega = \{&\bar{x}_n \in \mathbb{R}^n, \bar{y}_{d_n} \in \mathbb{R}^{n+1}, \hat{W}_i \in \mathbb{R}^{l_i} : \\ &|x_i| \leq k_{c_i}, |y_d| \leq A_0, |y_d^{(j)}| \leq Y_j, \\ &||\hat{W}_j|| \leq \frac{s_{n_j}^*}{\sigma_j}, j = 1, \ldots, n\}. \end{aligned} \tag{27}$$

If there exist positive constants $\kappa_1, \ldots, \kappa_{n-1}$ that satisfy the following feasibility conditions:

$$k_{c_i} > A_{i-1}(\kappa_1, \ldots, \kappa_{i-1}), \quad i = 1, \ldots, n. \tag{28}$$

where $|y_d(t)| \leq A_0 < k_{c_1}$, then we have the following properties

i) The error signals $z_i(t), i = 1, \ldots, n$ are bounded as

$$|z_i(t)| \leq \sqrt{2(V(0) + \frac{C}{\theta})}. \tag{29}$$

ii) The state $x(t)$ remain in the constrained set χ.
iii) The intermediate stabilization functions $\alpha_i(t), i = 1, \ldots, n-1$ and the control input $u(t)$ are bounded $\forall t > 0$.
Proof: See the Appendix B. ■

Remark 4.2. Compared with previous works in Tee et al. (2009b); Ren et al. (2010), the proposed control scheme in this paper handle the constraints on x_i directly, which is less conservative than considering transformed constraints on tracking errors z_i. With respect to Tee and Ge (2012), the drift system functions in (1) are relaxed to be unknown and not necessary to be assumed as linearly parameterizable. Further, the unknown bounds of neural network approximation errors and neural weight estimation errors as well as radial basis function are also adaptively estimated, which reduces the conservatism in Meng et al. (2012) on requiring these bounds exactly known for control design.

Remark 4.3. The advantage of employing iBLF for control design is the reduce of number of constraints in the optimization problem, i.e., the feasibility conditions $k_{c_i} > A_{i-1}(\xi) + k_{b_i}$ with k_{b_i} are transformed constraints on errors z_i are eliminated, and the initial constrained states x have been expanded to any point in the constrained space χ. Compared with Tee and Ge (2012), the difference is that the bounds $A_i, i = 1, n-1$ on the intermediate stabilizing functions α_i depends on the neural network estimation vector $\hat{W}_i$ and the estimation of unknown bounds $\hat{p}_i$, thus leads to different optimal control gains. Due to the limited space, interested readers can refer to Tee and Ge (2012) on the details of feasibility check.

5. AN APPLICATION EXAMPLE

To demonstrate the validity and performance of proposed method, a 1-link manipulator actuated by a brush dc motor in Tee and Ge (2011) is utilized for application. The dynamics are described as

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -\phi_1 \sin(x_1) - \phi_2 x_2 + \phi_3 x_3 \\ \dot{x}_3 &= -\phi_4 x_2 - \phi_5 x_3 + \phi_6 u \end{aligned} \tag{30}$$

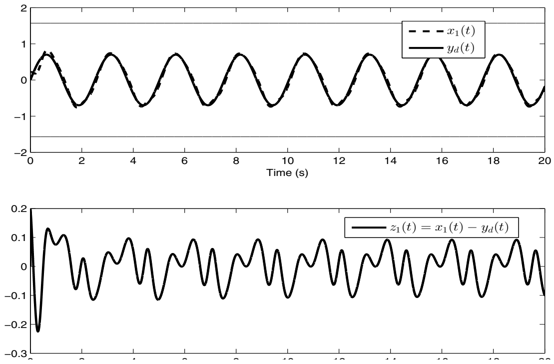
where $x_1 = q, x_2 = \dot{q}, x_3 = I, \phi_1 = mgl/M, \phi_2 = D/M, \phi_3 = k_I/M, \phi_4 = k_{m_3}/k_{m_1}, \phi_5 = k_{m_2}/k_{m_1}, \phi_6 = 1/k_{m_1}$ and control input u represetns the input voltage. . In the simulation, the parameters are chosen as $m = 1, l = 0.15, M = 1, D = 1, k_I = 1, k_{m_1} = 0.05, k_{m_2} = 0.5$ and $k_{m_3} = 10$. The drift nonlinearities in the above model are assumed as unknown in the simulation. The control objective is to guarantee the state constraints $|q(t)| < \pi/2, |\dot{q}(t)| < \pi$ and $|I(t)| < 20$, and let the output $q(t)$ track the desired trajectory $y_d(t) = 0.7\sin(2.5t)$ as closely as possible, and the boundedness of other signals in the closed-loop system.

As Sanner and Slotine (1992) pointed, Gaussian RBFNNs arranged on a regular lattice on R^n can uniformly approximate sufficiently smooth functions on closed bounded subsets. Accordingly, in the simulation studies, the localization of centers and widths are chosen on a regular lattice in the respective compact sets. In particular, we set three nodes for each input dimension of $\hat{W}_1^T S_1(Z_1)$, $\hat{W}_2^T S_2(Z_2)$ and $\hat{W}_3^T S_2(Z_3)$, thus we have 27 nodes (i.e, $l_1 = 27$) with centers $\mu_l = 0.0$ evenly spaced in $[-2, 2] \times [-1, 1] \times [-1, 1]$ and widths $\eta_l = 1.0$ for NN $\hat{W}_1^T S_1(Z_1)$, and 243 (i.e, $l_2 = 243$) nodes with centers $\mu_l = 0.0$ evenly spaced in $[-2, 2] \times [-4, 4] \times [-4, 4] \times [-3, 3] \times [-3, 3]$ and widths $\eta_l = 1.0$ for NN $\hat{W}_2^T S_2(Z_2)$, and 2187 (i.e, $l_2 = 2187$) nodes with centers $\mu_l = 0.0$ evenly spaced in $[-2, 2] \times [-4, 4] \times [-4, 4] \times [-4, 4] \times [-20, 20] \times [-2, 2] \times [-2, 2]$ and widths $\eta_l = 1.0$ for NN $\hat{W}_3^T S_3(Z_3)$.

The initial conditions are selected as $[x_1(0), x_2(0), x_3(0)]^T = [0.2, 0.8, 0]^T$, which lie in the predefined constrained set, and the desired reference signal $y_d(t) = 0.7\sin(2.5t)$. Using the Matlab command *fmincon.m*, we obtain $\kappa_1^* = 10.6275, \kappa_2^* = 8.0006$ and choose $\kappa_3 = 10$. Other parameters are selected as $\delta_1 = \delta_2 = \delta_3 = 0.1, \sigma_{p_1} = \sigma_{p_2} = \sigma_{p_3} = 0.5$. The initial neural network weight estimates and unknown bounds estimates are assumed as $\hat{W}_1 = \hat{W}_2 = \hat{W}_3 = 0$ and $\hat{p}_1 = \hat{p}_2 = \hat{p}_3 = 0$, repsectively. Fig. 1 shows that the tracking objective is achieved without the violation of constraint on x_1, and Fig. 2 shows the evolutions of state trajectories without the violation of constraints on states. The boundedness of NN estimation weights and adapting parameters are also presented in Fig. 3 and Fig. 5, respectively. Fig. 4 shows the boundedness of control input, and the peaks in the initial control input are due to the states approach the constraint boundaries.

6. CONCLUSION

In this paper, a novel neural networks based control design has been proposed for strict-feedback nonlinear systems subject to both full state constraints and unknown system



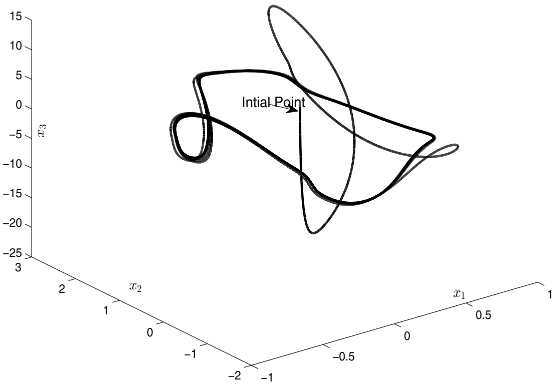


Fig. 2. The state trajectories in constrained space.

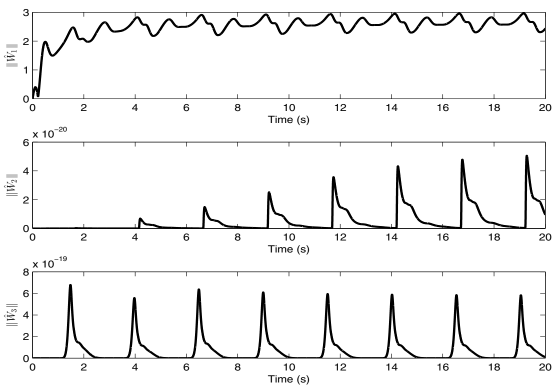


Fig. 3. Boundedness of NN weights

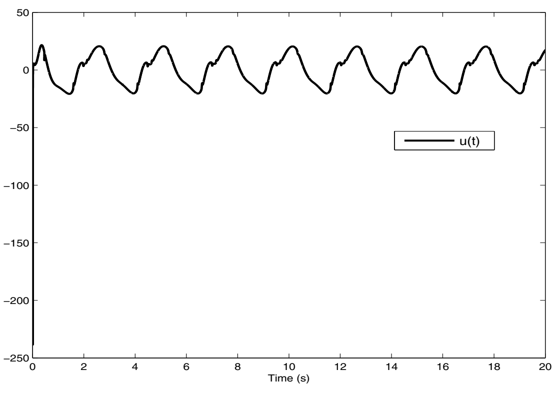


Fig. 4. Control input.

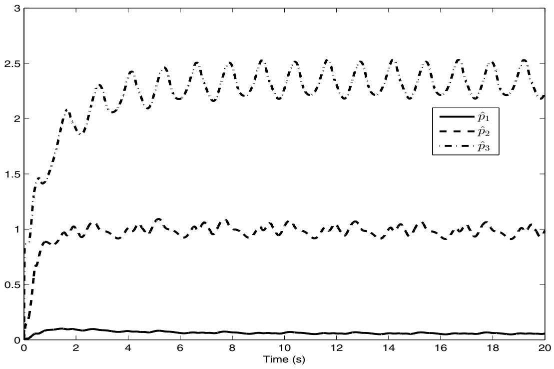


Fig. 5. Boundedness of adapting parameters

drift dynamics. By the incorporation of RBFNN based compensator into iBLF based control design, the proposed control design is valid in the constrained systems with unknown dynamics, and the feasibility conditions are relaxed by avoiding the formulation of transformed constraints on errors. It has been proved that the closed-loop tracking error has been semiglobally uniformly ultimately bounded, all states always remain in the constrained region and other signals in the closed-loop system are also bounded.

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Appendix A. PROOF OF THEOREM 3.1

Consider the following Lyapunov functional candidate

$$V_1(t) = V_{z_1}(t) + \frac{1}{2}\tilde{p}_1^2 \tag{A.1}$$

where $\tilde{p}_1 = \hat{p}_1 - p_1^*$. Based on (12), (13) and (14), the time derivative of V_1 can be obtained as

$$\begin{aligned}\dot{V}_1 =& -\kappa_1 g_1 \frac{z_1^2 k_{c_1}^2}{k_{c_1}^2 - x_1^2} \\ &+ g_1 \frac{z_1 k_{c_1}^2}{k_{c_1}^2 - x_1^2}(-\hat{W}_1 S_1(Z_1) - \hat{p}_1 \tanh\left(\frac{\frac{z_1 k_{c_1}^2}{k_{c_1}^2 - x_1^2}}{\delta_1}\right) \\ &+ W_1^* S_1(Z_1) + \epsilon_1) + \tilde{p}_1 \dot{\hat{p}}_1 \\ \leq& -\kappa_1 g_1 \frac{z_1^2 k_{c_1}^2}{k_{c_1}^2 - x_1^2} - \frac{\sigma_{p_1}}{2}\tilde{p}_1^2 + p_1^* \left|\frac{z_1 k_{c_1}^2}{k_{c_1}^2 - x_1^2}\right| \\ &- p_1^* \frac{z_1 k_{c_1}^2}{k_{c_1}^2 - x_1^2} \tanh\left(\frac{\frac{z_1 k_{c_1}^2}{k_{c_1}^2 - x_1^2}}{\delta_1}\right) + \frac{\sigma_{p_1}}{2} p_1^{*2}\end{aligned} \tag{A.2}$$

According to the claim in Plycarpou and Ioannou (1996) and Lemma 1 in Tee and Ge (2012), it is further obtained

$$\dot{V}_1 \leq -\theta_1 V_1 + C_1, \tag{A.3}$$

where $\theta_1 := \min\{\kappa_1 g_{1,\min}, \sigma_{p_1}\}, C_1 := 0.2785\delta_1 p_1^* + \frac{\sigma_{p_1}}{2} p_1^{*2}$. According to the Lemma 1 in Ren et al. (2010), we conclude that $|x_1(t)|$ remains in the constrained set χ_1 provide $|x_1(0)| \in \chi_1$. Further, as $\frac{z_1^2}{2} \leq V_{z_1}(t)$, it is obtained that $|z_1(t)| \leq \sqrt{2(V_1(0) + \frac{C_1}{\theta_1}})$. In terms of the control design (13) and Lemma (16), the control input u is also bounded.

Appendix B. PROOF OF THEOREM 4.1

i) Multiplying (25) by $e^{\theta t}$yields

$$\frac{d}{dt}(V e^{\theta t}) \leq C e^{\theta t}. \tag{B.1}$$

Integrating the above inequality, it yields

$$V \leq (V_1(0) - \frac{C}{\theta}) + \frac{C}{\theta} \leq V(0) + \frac{C}{\theta}, \tag{B.2}$$

Using the fact $\frac{1}{2}\sum_{i=1}^{n} z_i^2(t) \leq V(t)$, it has

$$|z_i(t)| \leq \sqrt{2(V(0) + \frac{C}{\theta})} \quad \forall t > 0, \tag{B.3}$$

which leads to the conclusion.

ii) According to the Lemma 1 in Ren et al. (2010) and inequality (25), it is concluded that $x(t)$ remain in the constrained set $\chi, \forall t > 0$.

iii) As the feasibility condition (28) is satisfied, this paper has $|\alpha_{i-1}(t)| < k_{c_i}, \forall t > 0$, with the result in item (ii), i.e., $|x_i(t)| < k_{c_i}, \forall t > 0$, it is obtained $z_i(t), \alpha_{i-1}(t) \in \Psi, \forall t > 0, i = 1, \ldots, n$, where Ψ is defined in Lemma (4.1). Thus, it is concluded that $\rho_i(z_i(t), \alpha_{i-1}(t))$ and its partial derivatives are bounded $\forall t > 0$. Further, the NN weight estimation vectors $\hat{W}_i, i = 1, \ldots, n$ are bounded according to Lemma (3.1). Similar to z_i, it also has $|\tilde{p}_i(t)| \leq \sqrt{2(V(0) + \frac{C}{\theta}})$, and then $\hat{p}(t)$ is also bounded. Hence, the stabilizing functions α_i and control input u are bounded as well under the design form (19) with the boundedness of y_d and its derivatives, ρ_i and its derivatives, and the constraint satisfaction $|x_i(t)| < k_{c_i}, \forall t > 0, \forall i = 1, \ldots, n$.