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H_∞ Fault Tolerant Control of Wind Turbine System with Actuator Faults

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Abstract: This paper focuses on the modeling and $\boldsymbol{H_\infty}$ fault tolerant control of wind turbine system with actuator faults. Stochastic piecewise affine (PWA) modeling tool is adopted to establish the multiple operating points models of wind turbine system with actuator faults under different wind speeds. Based on stochastic PWA model of wind turbine system with actuator faults, the theorem of $\boldsymbol{H_\infty}$ fault tolerant control strategy is presented to solve the PWA state feedback control law of wind turbine system. The performance and the efficiency of the proposed approach are validated via simulations.

1. INTRODUCTION

Wind turbine system is a mechanical electronic hydraulic integrated system which consists of rotor, drive train, gear box, generator and other mechanical equipment. Wind turbine driven by stochastic wind power signal indicates properties of nonlinear switching systems. Control systems perform a vital role in meeting power captured and load alleviation targets in wind turbines. The performance of the designed controller can be easily interrupted by possible faults and failures in different parts of the system. Thus, searching for fault-tolerant controller designs with early fault detection, isolation and successful controller reconfiguration would be very conducive to wind turbine operations.

Fault-tolerant control designs for wind turbine systems are insufficient. In (C. Sloth et al., 2010), the authors presented active and passive fault tolerant control designs for wind turbines. Linear parameter varying control design method was applied which leads to LMI based optimization in the event that active fault tolerant and BLMI in case of passive fault-tolerant problems. The authors in (K. Rothenhagen et al., 2009), presented model-based fault detection and control loop reconfiguration for doubly fed induction generators. In (M. A. Parker et al., 2013), generator-converter fault-tolerant control was investigated for direct drive wind turbines. In (M. Ruba et al., 2009), a fault-tolerant switched reluctance motor was designed for blade pitch control system.

The above mentioned methods are all restricted to the nonlinear characteristics of wind turbines. To overcome this drawback, H_∞ fault tolerant strategy for wind turbine is proposed in this paper based on stochastic PWA model framework.

A number of results have been obtained on analysis and controller design of such piecewise continuous time linear systems during the last few years (M. Johansson et al., 1998; A. Rantzer et al., 2000; A Hassibi, et al., 1998; S Pettersson, et al., 1997) . In the case of discrete time, the authors in (O. Slupphaug, et al., 1999) presented an approach to stabilization of piecewise linear systems based on a global quadratic Lyapunov function. (F. A. Cuzzola et al.,2001; D. Mignone et al., 2000) presented a number of results on stability analysis, controller design, H_∞ analysis, and H_∞ controller design for the piecewise linear systems based on a piecewise Lyapunov function. In (F. A. Cuzzola et al., 2001), the affine term was treated as a disturbance for H_∞ control synthesis. In a recent paper (F. Gang, 2002), a new method was presented to synthesize a H_∞ controller for the piecewise discrete time linear systems. Best to the author's knowledge, there are no work to solve the fault tolerant control problem of the wind turbine within the stochastic PWA framework. In this paper, the stochastic PWA normal and actuator fault models for wind turbine including multiple work regions are established. A reliable piecewise linear quadratic regulator state feedback is designed such that it can make the actuator faults tolerant. A sufficient condition for the existence of the passive fault tolerant controller is derived based on some LMIs.

The paper is organized as follows. In section 2, dynamic model of wind turbine and the control strategy are briefly described. The PWA modeling method of wind turbine is briefly described in section 3. H_∞ Fault-tolerant control is discussed in sections 4. In section 5, simulation results and relevant analysis are given. Finally, conclusions are discussed in section 6.

2. DYNAMIC MODEL OF WIND TURBINE AND THE CONTROL STRATEGY

The inputs of wind turbine (Fig. 1) are wind speed $v(t)$, pitch angle reference $\beta_{ref}(t)$ and generator torque reference $T_{gref}(t)$. The outputs of the system are generator power $P_g(t)$ and high-speed shaft speed $\omega_g(t)$.

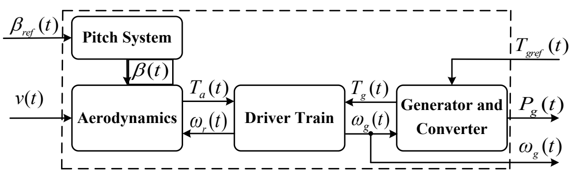

Fig. 1. The structure of wind turbine

2.1 Wind Model

Wind is yielded by superposing two components (I. Munteanu, et al. 2008), as shown in (1).

$$v(t) = v_s(t) + v_m(t) \tag{1}$$

where $v_m(t)$ is the low-frequency component (describing long term, low-frequency variations), i.e. average wind speed and $v_s(t)$ is the turbulence component (corresponding to fast, high-frequency variations).

2.2 Aerodynamic Model

The available power from the wind P_w can be expressed as:

$$P_w = \frac{1}{2}\rho A v^3 \tag{2}$$

where A is the rotor swept area. v is the rotor effective wind speed. ρ is the air density, which is assumed to be constant.

From P_w, the power captured by the rotor P_a is:

$$P_a = P_w C_p(\lambda, \beta) \tag{3}$$

where $C_p(\lambda, \beta)$ is the power coefficient, which depends on the tip-speed ratio λ and the pitch angle β.

The tip-speed ratio is defined as the ratio between the tip speed of the blades and the rotor effective wind speed:

$$\lambda = \frac{R\omega_r}{v} \tag{4}$$

where ω_r is the low-speed shaft speed, and R is the blade length.

2.3 Drive Train Model

The drive train model includes a low-speed shaft, a high-speed shaft, a gear box and flexible device. The drive train dynamics function is given:

$$\begin{cases} \dot{\omega}_r = \frac{T_r}{J_r} - \frac{\omega_r D_s}{J_r} + \frac{\omega_g D_s}{J_r N_g} - \frac{\delta K_s}{J_r} \\ \dot{\omega}_g = \frac{\omega_r D_s}{J_g N_g} - \frac{\omega_g D_s}{J_g {N_g}^2} + \frac{\delta K_s}{J_g} - \frac{T_g}{J_g} \end{cases} \tag{5}$$

where J_r and J_g are the moments of inertia of the low-speed shaft and the high-speed shaft, K_s is the torsion stiffness of the drive train, D_s is the torsion damping coefficient of the drive train, N_g is the gear ratio. δ is the twist of the flexible drive train with $\dot{\delta} = \omega_r - \frac{\omega_g}{N_g}$.

2.4 Pitch System Model

The pitch system can be modeled by a second order transfer function (T. Esbensen et al., 2009). The state equation is:

$$\begin{cases} \dot{\beta} = \dot{\beta} \\ \ddot{\beta} = -\omega_n^2\beta - 2\xi\omega_n\dot{\beta} + \omega_n^2\beta_{ref} \end{cases} \tag{6}$$

2.5 Generator and Converter Model

The generator and converter dynamics can be modeled by a first order transfer function.

$$\dot{T}_g = -\frac{1}{\tau_g}T_g + \frac{1}{\tau_g}T_{gref} \tag{7}$$

where τ_g is the time constant.

The real-time power is described by

$$P_g = \eta_g \omega_g T_g \tag{8}$$

Finally, the dynamics of wind turbine can be achieved:

$$\begin{bmatrix} \dot{\omega}_r \\ \dot{\omega}_g \\ \dot{\beta} \\ \ddot{\beta} \\ \dot{T}_g \end{bmatrix} = \begin{bmatrix} \frac{T_r}{J_r} - \frac{\omega_r D_s}{J_r} + \frac{\omega_g D_s}{J_r N_g} - \frac{\delta K_s}{J_r} \\ \frac{\omega_r D_s}{J_g N_g} - \frac{\omega_g D_s}{J_g {N_g}^2} + \frac{\delta K_s}{J_g} - \frac{T_g}{J_g} \\ \dot{\beta} \\ -\omega_n^2\beta - 2\xi\omega_n\dot{\beta} \\ -\frac{1}{\tau_g}T_g \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \omega_n^2 & 0 \\ 0 & \frac{1}{\tau_g} \end{bmatrix} \begin{bmatrix} \beta_{ref} \\ T_{gref} \end{bmatrix} \tag{9}$$

2.6 Control Strategy

The basic control strategy is described as Fig. 2. The control requirements for the power are different in different wind regions. If $v < v_{cut-in}$, system stops. If $v \in [v_{cut-in}, v_{rated}]$, the system needs to maximize the wind harvested power. If $v > v_{rated}$, the system needs to limit power to the rated and maintain the stability of the system.

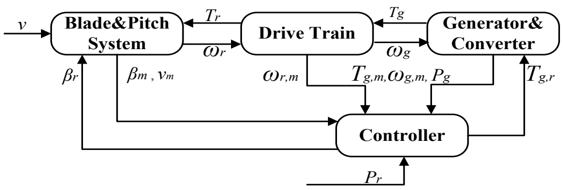

Fig. 2. The control strategy for wind turbine

3. MODELLING WIND TURBINE WITHIN STOCHASTIC PWA MODEL FRAMEWORK

In summary, wind turbine is mainly based on four regions for modeling and control. According to the different wind speeds, models and control strategies need to switch. This section introduces the basic principles of PWA modeling method, and gives modeling ideas of wind turbine.

3.1 Stochastic PWA Model Form

A linear stochastic discrete-time PWA system is defined by the state-space equation

$$\begin{aligned} x_{k+1} &= A_i x_k + B_i u_k + B_i^w w_k + a_i \\ z_k &= C_i x_k + D_i u_k + D_i^w w_k \end{aligned} \quad \begin{bmatrix} x_k \\ u_k \end{bmatrix} \in \chi_i, x_k \in \bar{\chi}_j \quad (10)$$

where $x_k \in \mathbb{R}^n$ is the state, $u_k \in \mathbb{R}^m$ is the control input and $z_k \in \mathbb{R}^m$ is a performance output. The set $\mathbb{X} \subseteq \mathbb{R}^{n+m}$ of vector $[x_k^T \quad u_k^T]^T$ is $\mathbb{R}^{n+m}$ or a polyhedron containing the origin, $\{\chi_i\}_{i=1}^s$ is a polyhedral partition of $\mathbb{X}$ and $a_i \in \mathbb{R}^n$ are constant vectors. Each χ_i is a cell. For simplicity, assume that cells are polyhedral defined by matrices F_i^x , F_i^u , f_i^x and f_i^u as follows:

$$\chi_i := [x^T \quad u^T]^T \text{ such that } F_i^x x \geq f_i^x \text{ and } F_i^u u \geq f_i^u$$

$$\bar{\chi}_i := x \text{ such that } F_i^x x \geq f_i^x$$

$$S_j := i \text{ such that } \exists x, u \text{ with } x \in \bar{\chi}_j, [x^T \quad u^T]^T \in \chi_i$$

where S_j is the set of all indices i such that χ_i is a cell containing a vector $[x^T \quad u^T]^T$ with $x \in \bar{\chi}_j$ is satisfied. Denote $I = \{1, \ldots, s\}$, which is the set of indices of the cells χ_i; Denote $\mathcal{J} = \{1, \ldots, t\}$, which is the set of indices of the cells $\bar{\chi}_j$. It is important to see that: $\bigcup_{j=1}^t S_j = I$.

3.2 Stochastic PWA Model for Wind Turbine

The linearized drive train dynamics function can be follows:

$$\begin{cases} \dot{\omega}_r = \frac{1}{3J_r}\frac{\partial T_a}{\partial \beta}\beta + \frac{B_{dt}}{N_g J_r}\omega_g + \left(-\frac{B_{dt}+B_r}{J_r} + \frac{1}{J_r}\frac{\partial T_a}{\partial \omega_r}\right)\omega_r + \frac{1}{3J_r}\frac{\partial T_a(t)}{\partial v_r}v(t) \\ \dot{\omega}_g = -\frac{1}{J_g}T_g - \left(\frac{\eta_{dt}B_{dt}}{J_g N_g^2} + \frac{B_g}{J_g}\right)\omega_g + \frac{B_{dt}}{N_g J_g}\omega_r \end{cases} \quad (11)$$

where $\frac{\partial T_a}{\partial \beta}$、 $\frac{\partial T_a}{\partial \omega_r}$ and $\frac{\partial T_a(t)}{\partial v_r}$ are the linearized parameters in different working points; B_g is the viscous friction of the high speed shaft.

Then a linearized overall state space model describing the dynamics of the wind turbine can be given:

$$\begin{bmatrix} \dot{T}_g \\ \dot{\beta} \\ \ddot{\beta} \\ \dot{\omega}_g \\ \dot{\omega}_r \end{bmatrix} = \begin{bmatrix} A_{11} & 0 & 0 & 0 & 0 \\ 0 & 0 & I & 0 & 0 \\ 0 & A_{54} & A_{55} & 0 & 0 \\ a_{71} & 0 & 0 & a_{77} & \frac{B_{dt}}{N_g J_g} \\ 0 & a_{84} & 0 & \frac{B_{dt}}{N_g J_r} & a_{88} \end{bmatrix} \begin{bmatrix} T_g \\ \beta \\ \dot{\beta} \\ \omega_g \\ \omega_r \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ e_{81} \end{bmatrix} v(t) + \begin{bmatrix} B_{11} & 0 \\ 0 & 0 \\ 0 & B_{42} \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} T_{gref} \\ \beta_{ref} \end{bmatrix} (12)$$

where $A_{11} = -\frac{1}{\tau_g}, A_{54} = -\omega_n^2, A_{55} = -2\xi\omega_n, a_{71} = -\frac{1}{J_g},$

$$a_{77} = -\left(\frac{\eta_{dt}B_{dt}}{J_g N_g^2} + \frac{B_g}{J_g}\right), a_{84} = \frac{1}{3J_r}\frac{\partial T_a}{\partial \beta}, e_{81} = \frac{1}{3J_r}\frac{\partial T_a(t)}{\partial v_r}, B_{11} = \frac{1}{\tau_g},$$

$$a_{88} = -\frac{B_{dt} + B_r}{J_r} + \frac{1}{J_r}\frac{\partial T_a}{\partial \omega_r}, B_{42} = \omega_n^2.$$

The effective wind (1) can be thought of as a superposition of the mean wind speed v_m and a stochastic component v_s. Following (J. Højstrup, 1982; S. Thomsen, 2006), the stochastic component v_s can be approximated by a linear second order transfer function driven by a white noise process.

$$\begin{aligned} \dot{\omega}_1 &= \omega_2 \\ \dot{\omega}_2 &= -a_1\omega_1 + a_2\omega_2 + a_3 e \end{aligned} \quad (13)$$

where $\omega_1 = v_s$, $e \in \mathcal{N}(0,1)$ and a_1, a_2, a_3 are parameters depending on the mean wind speed.

Then, model (12) can be translated into the following form:

$$\begin{bmatrix} \dot{T}_g \\ \dot{\beta} \\ \ddot{\beta} \\ \dot{\omega}_g \\ \dot{\omega}_r \\ \dot{\omega}_1 \\ \dot{\omega}_2 \end{bmatrix} = \begin{bmatrix} A_{11} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & I & 0 & 0 & 0 & 0 \\ 0 & A_{54} & A_{55} & 0 & 0 & 0 & 0 \\ a_{71} & 0 & 0 & a_{77} & \frac{B_{dt}}{N_g J_g} & 0 & 0 \\ 0 & a_{84} & 0 & \frac{B_{dt}}{N_g J_r} & a_{88} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & -a_1 & -a_2 \end{bmatrix} \begin{bmatrix} T_g \\ \beta \\ \dot{\beta} \\ \omega_g \\ \omega_r \\ \omega_1 \\ \omega_2 \end{bmatrix} + \begin{bmatrix} B_{11} & 0 \\ 0 & 0 \\ 0 & B_{42} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} T_{gref} \\ \beta_{ref} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ e_{81} \\ 0 \\ 0 \end{bmatrix} v_m + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ e_{81} \\ 0 \\ a_3 \end{bmatrix} e (14)$$

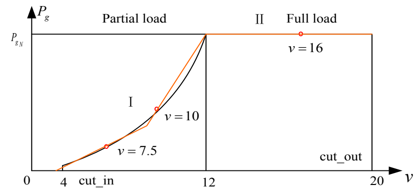

Fig. 3 Wind speed and the corresponding working point

According to Fig. 3, choose 3 working points, the values of $\frac{\partial T_a}{\partial \beta}$,$\frac{\partial T_a}{\partial \omega_r}$ and $\frac{\partial T_a(t)}{\partial v_r}$ in different working points can be obtained according to different wind speeds by an effective wind estimator (K. Østergaard, et al. 2007), where the realization method is described in (R. M. Burkart et al., 2011). The parameters we needed are shown in Tables 1 and 2.

Table 1. Model parameters

J_r	90000	$[\text{kg} \cdot \text{m}^2]$	$P_{g,nom}$	225	[kW]
J_g	10	$[\text{kg} \cdot \text{m}^2]$	$\omega_{r,nom}$	4.29	[rad/s]
K_s	$8 \cdot 10^6$	[Nm/rad]	$\omega_{g,nom}$	105.534	[rad/s]
D_s	$8 \cdot 10^4$	$[\text{kg} \cdot \text{m}^2/(\text{rad} \cdot \text{s})]$	$\omega_{r,min}$	3.5	[rad/s]
N_g	24.6	[-]	$\omega_{g,min}$	86.1	[rad/s]
$\mathcal{R}$	14.5	[m]	θ_{min}	0	[deg]
τ_θ	0.15	[s]	θ_{max}	25	[deg]
τ_r	0.1	[s]	$\lvert\dot{\theta}\rvert_{max}$	10	[deg/s]

Table 2. Parameters of linearized model in different working points

Wind (m/s)	Parameter					
	a_{84}	a_{88}	e_{81}	a_1	a_2	a_3
V=7.5	0.409	0.50	1.90	0.3125	2.92	0.9375
V=10	0.479	0.53	2.31	0.33	3.65	2.3
V=16	0.833	0.53	2.50	0.625	5	5

3.3 Wind Turbine Actuator Fault Model

In this work, we consider actuator faults. Let u_j denote the $j'th$ actuator and u_j^F denote the failed $j'th$ actuator. We model a loss of gain in an actuator as:

$$u_j{}^F = (1-\alpha_j)u_j, 0 \le \alpha_j \le \alpha_{Mj} \quad (15)$$

where α_j is the percentage of failure in the j'th actuator, α_{Mj} is the maximum loss in the j'th actuator. $\alpha_j = 0$ represents the case without faults in the j'th actuator, $0 < \alpha_j < 1$ corresponds to the partial loss of it, and $\alpha_j = 1$ corresponds to complete loss of it. We define α as $\alpha = diag\{\alpha_1, \alpha_2, \dots, \alpha_m\}$. Then $u^F = \Gamma u$, where $\Gamma = (I - \alpha)$. Consider $\boldsymbol{\Gamma_a}$ is the gain factor of actuator fault, which is a diagonal matrix of two elements.

The model of the system with the loss of gain Γ_a in actuators can be describe by

$$\begin{array}{l} x_{k+1} = A_i x_k + B_i \Gamma_a u_k + B_i^w w_k + a_i \\ z_k = C_i x_k + D_i u_k + D_i^w w_k \end{array} \quad \begin{bmatrix} x_k \\ u_k \end{bmatrix} \in \chi_i, x_k \in \bar{\chi}_j \quad (16)$$

4. H_∞ FAULT TOLERANT CONTROL FOR ACTUATOR FAULTS

Consider the PWA actuator fault system (16), define w_k as follows:

$$\widetilde{w}_k = \begin{bmatrix} w_k \\ a_i \end{bmatrix} \quad (17)$$

Thus, system (16) can be rewritten as:

$$\begin{array}{l} x_{k+1} = A_i x_k + B_i \Gamma_a u_k + \tilde{B}_i^w \widetilde{w}_k \\ z_k = C_i x_k + D_i u_k + \widetilde{D}_i^w \widetilde{w}_k \end{array} \quad \begin{bmatrix} x_k \\ u_k \end{bmatrix} \in \chi_i, x_k \in \bar{\chi}_j \quad (18)$$

where

$$\tilde{B}_i^w = [B_i^w \quad I],\ \widetilde{D}_i^w = [D_i^w \quad I] \quad (19)$$

The H_∞ framework considered here, is based on a finite horizon definition of the l_2 gain and, consequently, the proposed extension of the disturbance input is sensible.

Clearly, it is possible to apply the control approach proposed in (F. A. Cuzzola et al., 2001) directly to the extended system (18). This can be conservative because a_i is not an unknown disturbance but a known term. Unfortunately, in general, a_i is known only when the control signal u_k has already been calculated. Notwithstanding this, under the standard assumption

$$a_i = \bar{a}_j, \forall i \in \mathcal{S}_j, \forall j \in \mathcal{J} \quad (20)$$

an alternative control strategy can be proposed. More precisely, the control is assumed to have the following structure:

$$u_k = \begin{bmatrix} K_j^1 & K_j^2 \end{bmatrix} \begin{bmatrix} x_k \\ \bar{a}_j \end{bmatrix} \quad x_k \in \bar{\chi}_j \quad (21)$$

In this way the controller can also take into account the displacement term $a_i = D\widetilde{w}_k$, where $D = [0 \quad I]$.

By applying the control law (21) to the PWA system (18), we obtain the closed loop PWA actuator fault system:

$$\begin{array}{l} x_{k+1} = \mathcal{A}_{ij} x_k + \tilde{B}_{ij}^w \widetilde{w}_k \\ z_k = \mathcal{C}_{ij} x_k + \widetilde{D}_{ij}^w \widetilde{w}_k \end{array} \quad \begin{bmatrix} x_k \\ u_k \end{bmatrix} \in \chi_i, x_k \in \bar{\chi}_j \quad (22)$$

where

$$\begin{array}{ll} \mathcal{A}_{ij} = A_i + B_i \Gamma_a K_j^1, & \tilde{B}_{ij}^w = \tilde{B}_i^w + B_i K_j^2 D \\ \mathcal{C}_{ij} = C_i + D_i K_j^1, & \widetilde{D}_{ij}^w = \widetilde{D}_i^w + D_i K_j^2 D \end{array} \quad (23)$$

From the above, we can have the following main results:

Lemma 1. Consider system (22) with zero initial condition $x_0 = 0$, if there exists a function $V(x,u) = x^T P_i x$ for $[x^T \ u^T]^T \in \chi_i$ with $P_i = P_i^T > 0$, satisfying the dissipativity inequality

$$\forall w \in \mathbb{R}^r, \forall k \ge 0,\ V(x_{k+1}, u_{k+1}) - V(x_k, u_k) < W(z_k, w_k)$$

with supply rate

$$W_\infty(z,w) = (\gamma^2 \|w\|^2 - \|z\|^2), \gamma > 0,$$

i.e. $\forall k, V(x_{k+1}, u_{k+1}) - V(x_k, u_k) < (\gamma^2 \|w_k\|^2 - \|z_k\|^2)$ (24)

then, the H_∞ performance condition (25) is satisfied.

$$\sum_{k=0}^{N} \|z_k\|^2 < \gamma^2 \sum_{k=0}^{N} \|w_k\|^2 \quad (25)$$

Furthermore, if the following matrix inequalities (26) are satisfied, then condition (24) is fulfilled.

$$\forall j \in \mathcal{J}, \forall i \in \mathcal{S}_j, \forall l \ with\ (l,i) \in \mathcal{S}, M_{l,ij} < 0. \quad (26)$$

where

$$M_{l,ij} = \begin{bmatrix} A_{ij}^T P_i A_{ij} - P_j + C_{ij}^T C_{ij} & C_{ij}^T D_j^w + A_{ij}^T P_i B_j^w \\ D_j^{wT} C_{ij} + B_j^{wT} P_i A_{ij} & B_j^{wT} P_i B_j^w + D_j^{wT} D_j^w - \gamma^2 I \end{bmatrix}$$

In this last case, system (22) is PWQ stable.

The proof can be found in (F. A. Cuzzola et al., 2001).

Theorem 1. For PWA system (16), there exists a state feedback control law of type (21), which can guarantee PWQ Lyapunov stability and fulfill the dissipativity constraint $\forall w \in \mathbb{R}^r, \forall k \ge 0, V(x_{k+1}, u_{k+1}) - V(x_k, u_k) < W(z_k, w_k)$

with supply rate

$$\widetilde{W}_\infty(z_k, w_k) = (\gamma^2 \| [w_k^T \ a_i^T]^T \|^2 - \|z_k\|^2) = (\gamma^2(\|w_k\|^2 + \|a_i\|^2) - \|z_k\|^2), \gamma > 0, \begin{bmatrix} x_k \\ u_k \end{bmatrix} \in \chi_i \quad (27)$$

If there exist matrices $Q_i = Q_i^T > 0$ with $i \in I$ and matrices G_j, Y_j, K_j^2 with $j \in \mathcal{J}$, such that $\forall j \in \mathcal{J}, \forall i \in \mathcal{S}_j, \forall l$ with $(l,i) \in S_{all}$

$$\begin{bmatrix} Q_i - G_j - G_j^T & (A_i G_j + B_i \Gamma_a Y_j)^T & (C_i G_j + D_i Y_j)^T & 0 \\ A_i G_j + B_i \Gamma_a Y_j & -Q_l & 0 & \tilde{B}_i^W + B_i \Gamma_a K_j^2 D \\ C_i G_j + D_i Y_j & 0 & -I & \widetilde{D}_i^W + D_i K_j^2 D \\ 0 & (\tilde{B}_i^W + B_i \Gamma_a K_j^2 D)^T & (\widetilde{D}_i^W + D_i K_j^2 D)^T & -\gamma^2 I \end{bmatrix} < 0 \quad (28)$$

holds, then the feedback gains K_j^1 with $j \in \mathcal{J}$ are given by:

$$K_j^1 = Y_j G_j^{-1} \quad (29)$$

Proof: Using Schurz's lemma, (25) can be rewrite as follows:

$$\begin{bmatrix} -P_i & (A_i + B_i \Gamma_a K_j^1)^T & (C_i + D_i K_j^1)^T & 0 \\ A_i + B_i \Gamma_a K_j^1 & -Q_l & 0 & \tilde{B}_i^W + B_i \Gamma_a K_j^2 D \\ C_i + D_i K_j^1 & 0 & -I & \widetilde{D}_i^W + D_i K_j^2 D \\ 0 & (\tilde{B}_i^W + B_i \Gamma_a K_j^2 D)^T & (\widetilde{D}_i^W + D_i K_j^2 D)^T & -\gamma^2 I \end{bmatrix} < 0 \quad (30)$$

where $A_{ij} = A_i + B_i \Gamma_a K_j^1, C_{ij} = C_i + D_i K_j^1$.

Now, letting $Q_i = P_i^{-1}$ and $Y_j = K_j^1 G_j$, we can obtain inequality (28) by multiplying (30) from the left by $\mathrm{diag}(G_j^T \; I \; I \; I)$ and the right by $\mathrm{diag}(G_j \; I \; I \; I)$. Obviously, being $Q_i > 0$, $\forall i \in I$, the control matrix K_j^1 can be reconstructed as in (29).

5. SIMULATIONS AND THE RESULT ANALYSIS

According to the modeling method described in part 3, we can obtain the following PWA models of wind turbine:

If $0 < E_1 x_k \le 8$, then

$$\begin{bmatrix} \dot{T}_g \\ \dot{\beta} \\ \ddot{\beta} \\ \dot{\omega}_g \\ \dot{\omega}_r \\ \dot{\omega}_1 \\ \dot{\omega}_2 \end{bmatrix} = \begin{bmatrix} -10 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & -123.4 & -13.332 & 0 & 0 & 0 & 0 \\ -0.1 & 0 & 0 & -13.2 & 325.2 & 0 & 0 \\ 0 & -0.409 & 0 & 0.036 & -0.5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & -0.3125 & -2.92 \end{bmatrix} \begin{bmatrix} T_g \\ \beta \\ \dot{\beta} \\ \omega_g \\ \omega_r \\ \omega_1 \\ \omega_2 \end{bmatrix} + \begin{bmatrix} 10 & 0 \\ 0 & 0 \\ 0 & 123.4 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} T_{gref} \\ \beta_{ref} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1.9 \\ 0 \\ 0 \end{bmatrix} \times 7.5 + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1.9 \\ 0 \\ 0.9325 \end{bmatrix} e$$

If $8 < E_1 x_k \le 12$, then

$$\begin{bmatrix} \dot{T}_g \\ \dot{\beta} \\ \ddot{\beta} \\ \dot{\omega}_g \\ \dot{\omega}_r \\ \dot{w}_1 \\ \dot{w}_2 \end{bmatrix} = \begin{bmatrix} -10 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & -123.4 & -13.332 & 0 & 0 & 0 & 0 \\ -0.1 & 0 & 0 & -13.2 & 325.2 & 0 & 0 \\ 0 & -0.479 & 0 & 0.036 & -0.53 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & -0.33 & -3.65 \end{bmatrix} \begin{bmatrix} T_g \\ \beta \\ \dot{\beta} \\ \omega_g \\ \omega_r \\ \omega_1 \\ \omega_2 \end{bmatrix} + \begin{bmatrix} 10 & 0 \\ 0 & 0 \\ 0 & 123.4 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} T_{gref} \\ \beta_{ref} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 2.31 \\ 0 \\ 0 \end{bmatrix} \times 10 + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 2.31 \\ 0 \\ 2.3 \end{bmatrix} e$$

If $12 < E_1 x_k \le 18$, then

$$\begin{bmatrix} \dot{T}_g \\ \dot{\beta} \\ \ddot{\beta} \\ \dot{\omega}_g \\ \dot{\omega}_r \\ \dot{w}_1 \\ \dot{w}_2 \end{bmatrix} = \begin{bmatrix} -10 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & -123.4 & -13.332 & 0 & 0 & 0 & 0 \\ -0.1 & 0 & 0 & -13.2 & 325.2 & 0 & 0 \\ 0 & -0.833 & 0 & 0.036 & -0.53 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & -0.625 & -5 \end{bmatrix} \begin{bmatrix} T_g \\ \beta \\ \dot{\beta} \\ \omega_g \\ \omega_r \\ \omega_1 \\ \omega_2 \end{bmatrix} + \begin{bmatrix} 10 & 0 \\ 0 & 0 \\ 0 & 123.4 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} T_{gref} \\ \beta_{ref} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 2.5 \\ 0 \\ 0 \end{bmatrix} \times 16 + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 2.5 \\ 0 \\ 5 \end{bmatrix} e$$

where $E_1 = [0\ 0\ 0\ 0\ 0\ 1\ 0]$.

According to Theorem 1, we can successfully obtain the piecewise linear state feedback matrixes of WECS system in normal case and actuator failure with the pitch system actuator gain fault β_f and the generator gain fault T_{gf} (let gain loss factor $\Gamma_a = \begin{bmatrix} 0.6 & 0 \\ 0 & 0.6 \end{bmatrix}$) case. The feedback matrixes calculated in these two cases are shown as follows:

The normal wind turbine:

$$K_1 = \begin{bmatrix} -0.0859 & -0.0149 & -0.0009 & 0.0018 & 0.2721 & 0.0000 & 0.0000 & -0.0066 & -0.0247 \\ -0.0334 & -0.1671 & -0.0121 & 0.0080 & 2.4130 & 0.0000 & 0.0000 & -0.0130 & 0.0000 \end{bmatrix}$$

$$K_2 = \begin{bmatrix} 7.6202 & -0.7650 & -0.0482 & 0.0031 & 0.1483 & 0.0000 & 0.0000 & -0.4561 & -0.8690 \\ 81.4923 & -24.3547 & -1.6292 & 1.4002 & 396.2525 & 0.0000 & 0.0000 & -17.8043 & -0.0099 \end{bmatrix}$$

$$K_3 = \begin{bmatrix} 0.3367 & -0.0142 & -0.0009 & -0.0024 & 0.2854 & 0.0000 & 0.0000 & -0.0055 & -0.0142 \\ 3.9585 & -0.1744 & -0.0186 & -0.0379 & 3.7296 & 0.0000 & 0.0000 & -0.0441 & 0.0000 \end{bmatrix}$$

Actuator within fault tolerant:

$$K_1 = \begin{bmatrix} -0.1431 & -0.0248 & -0.0015 & 0.0030 & 0.4536 & 0.0000 & 0.0000 & -0.0110 & -0.0411 \\ -0.0556 & -0.2784 & -0.0201 & 0.0134 & 4.0216 & 0.0000 & 0.0000 & -0.0217 & 0.0000 \end{bmatrix}$$

$$K_2 = \begin{bmatrix} 12.7003 & -1.2750 & -0.0804 & 0.0052 & 24.4886 & 0.0000 & 0.0000 & -0.7601 & -1.4483 \\ 135.8205 & -40.5912 & -2.7154 & 2.3337 & 660.4208 & 0.0000 & 0.0000 & -29.6738 & -0.0166 \end{bmatrix}$$

$$K_3 = \begin{bmatrix} 0.1833 & -0.0077 & -0.0005 & -0.0013 & 0.1554 & 0.0000 & 0.0000 & -0.0030 & -0.0078 \\ 2.1305 & -0.0967 & -0.0102 & -0.0202 & 2.0492 & 0.0000 & 0.0000 & 0.0222 & 0.0000 \end{bmatrix}$$

5.1 Validation of H_∞ Control for the Normal Wind Turbine

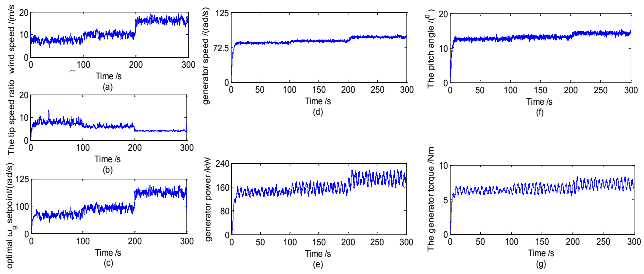

Fig. 4. H_∞ reliable control for normal wind turbine based on stochastic PWA model

Fig. 4 demonstrates the simulation results of wind turbine fault free dynamic response which is regulated by the robust H_∞ controller. Fig. 4(a) is the test wind speed signal which is consisted of mean wind speed v_m and wind speed turbulent v_s ($v = v_m + v_s$). $v_m = 7.5$, 10, 16 (m/s), which is switched at time of 100s, 200s, 300s, respectively; The tip speed ratio and the optimal generator power set point calculated by Max Power Point Tracking (MPPT) algorithm are given by Fig. 4(b),(c), respectively, which are under the control of reliable H_∞ controller designed based on the stochastic PWA model of wind turbine. The generator speed and generator power responses are shown in Fig. 4(d),(e), from which we can conclude the real generator power tracks the optimal set point quite well and the generator speed is controlled with good performance. Fig. 4(f),(g) give the H_∞ controller output of pitch angle β and generator torque T_g. Fig. 4 shows that the H_∞ controller of wind turbine designed based on stochastic PWA model, combining with MPPT works quite well.

5.2 Validation of H_∞ Control for Wind Turbine with Actuators Fault

Fig. 5(a)-(c) show that the MPPT module works quite well. Comparing with Fig. 4(a)-(c), there are little changes. While, comparing with Fig. 4(d),(e), the performance of generator power P_{gf} and generator speed ω_{gf} have deteriorated, and begin to show the trend of losing stability. The control variables display wide range oscillations in Fig. 5(f),(g).It is seen that the normal H_∞ controller for wind turbine cannot deal with the actuator faults with gain factor loss Γ_a. Comparing with Fig. 5(d),(e), Fig. 6(d),(e) demonstrate the response of generator speed and generator power under the control of H_∞ fault tolerant controller designed according to Theorem1 based on the stochastic PWA actuator fault model of wind turbine, Fig. 6(d),(e) show that the performance with fault tolerant design significantly is better than the normal controller.

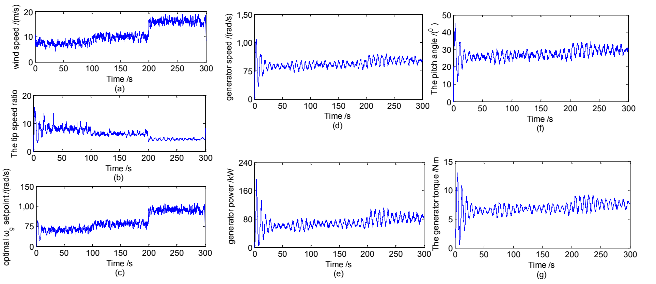

Fig. 5. Actuator faults of wind turbine without fault tolerant control

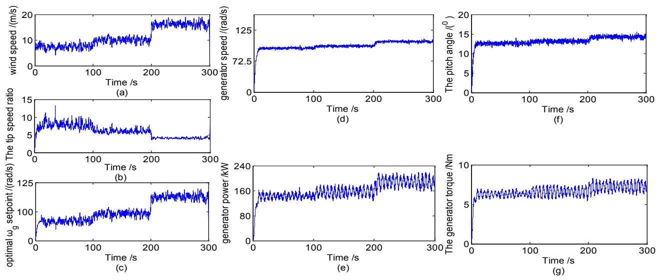

Fig. 6. H_∞ fault tolerant control for actuator faults of wind turbine

6. CONCLUSIONS

This paper proposed a stochastic PWA modeling method to solve wind turbine modeling problem under normal and fault conditions; It provides the H_∞ fault tolerant controller design method for wind turbine under the conditions of actuator faults as theorems form by using the stability theory and H_∞ fault tolerant control method of nonlinear dissipative systems. The simulation results show that stochastic PWA modeling and H_∞ fault tolerant control method presented in this paper can well solve wind turbine modeling and fault tolerant control problems under stochastic wind loads.

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