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A Modeling Approach for HVAC Systems based on the LoLiMoT Algorithm

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Abstract: In heating ventilating and air conditioning (HVAC) systems, typically two variables (air temperature and air humidity) have to be controlled via several (at least two) actuators. Some of the components show nonlinear behaviour. Therefore, HVAC systems belong to the class of nonlinear multi-input-multi-output systems. A well suited approach to control this class of systems is model predictive control, since the time constants of HVAC systems are high (typically in the range of tens or hundreds of seconds) offering enough time to perform the required online optimization. In order to apply *linear* predictive control methods, while taking into account the nonlinearities of the plant, a modeling concept based on a physical plant model and a neuro-fuzzy model is proposed. The neuro-fuzzy model is obtained via the so called local linear model tree (LoLiMoT) algorithm. The generation of a linear state space representation from the neuro-fuzzy model is demonstrated. This linear state space model can then be used in a predicitive control scheme, where the linear model is updated each sampling instant from the neuro-fuzzy model. This technique allows the application of standard linear predictive control while taking into account the nonlinearities of the plant. Simulation and measurement results obtained from an industrial test plant are presented.

1. INTRODUCTION

The operation of heating, ventilating and air conditioning (HVAC) systems requires the compliance with demanding specifications concerning control accuracy of air temperature and air humidity. The applied control technique is a crucial factor influencing the performance of the overall system. Therefore, appropriate control techniques have to be developed. Typical system properties as well as state-of-the-art approaches to control HVAC systems are presented in the following.

A common problem setup is the simultaneous control of air temperature and humidity via the following actuators: Heating and cooling coils are used to control the air temperature. These components are realized as finned tube crossflow heat exchangers. For humidity control, steam humidifiers and cooling coils are used. *All* these actuators influence *both* humidity and temperature, i.e. the given setup describes a multi-input-multi-output (MIMO) control problem. Furthermore, the relative humidity is a nonlinear function of the temperature and some components within HVAC systems show nonlinear behaviour. E.g., the mixing ratio of hot water and return water of the heating coil is a nonlinear function of the valve position.

In the field of HVAC control systems, there exist approaches that take into account the MIMO property of the plant. In Anderson et al. [2008, 2007], an experimental small scale HVAC system is presented. The authors design an $\mathcal{H}_\infty$ controller for the simultaneous control of discharge air temperature and air flow rate. A linearized plant model including dead times is proposed, nonlinearities are regarded as model uncertainties. Humidity is not considered at all. In Semsar-Kazerooni et al. [2008] nonlinear methods are applied to control thermal space and supply air temperatures. The system model is a bilinear system. The proposed concept demonstrates the potential of nonlinear control techniques for HVAC systems in simulation studies. Again, humidity is neglected. Many recent works propose model predictive control (MPC) strategies to control HVAC systems, see Aswani et al. [2012b], Kelman and Borrelli [2011], Aswani et al. [2012a], Naidu and Rieger [2011], Huang and Wang [2008]. In Rehrl and Horn [2011], MPC in combination with exact linearization is applied to the outlet air temperature control of a cooling coil. Actuator saturation cannot be taken into account directly due to the application of the exact linearization method, see e.g. Isidori [1995].

Due to the following reasons, MPC is well suited for the application in HVAC systems:

- The method is directly applicable to MIMO systems.
- Dead times can be handled easily.
- The sampling time of the considered class of systems is in the range of 10 seconds. Therefore, it is possible to solve the online optimization problem in time with standard computer hardware [1].

The use of a nonlinear plant model within the MPC increases the complexity to solve the online optimization problem considerably (e.g. local minima). The non-convex

[1] The presented test plant is equipped with a X20-System with a 650 MHz Celeron PLC from B&R automation (http://www.br-automation.com).

optimization problem is solved via evolutionary programming (EP) in Jalili-Kharaajoo [2005]. However, in EP it is not guaranteed that the found minimum is a global one. In Kelman and Borrelli [2011], Ma et al. [2012] MPC is suggested in order to minimize the energy consumption of HVAC systems. Sequential quadratic programming is applied to solve the optimization problem. It is stated that the computational complexity to solve the low-level, non-convex, MPC problem causes problems with standard HVAC controller hardware.

Consequently, in this paper, a modeling approach for nonlinear MIMO systems is presented and demonstrated in HVAC systems. The proposed model can easily be applied in *linear* MPC, while still taking into account the *nonlinear* plant characteristics. The suggested strategy relies on a linear plant model which is generated from a nonlinear system representation *at each sampling instant*, see Section 4. This linear model is used for the online MPC problem to compute the predicted trajectory of the system state. At the next time step, a new linear model is generated (from the actual system state) which is then again used for prediction. Of course, the linear system is an approximation, however, the accuracy of the approximation is increased by updating the model each time step. A detailed explanation of the two models is given in Section 3.

This paper focuses on the modeling of the plant and the generation of the linear plant model. It is structured as follows: Section 2 gives a description of the test plant which is used to verify the proposed modeling approach. Section 3 deals with the modeling of the plant and the motivation for the application of the proposed approach can be found there. A mathematical model of the considered HVAC system based on physical relations is presented. Furthermore, a neuro-fuzzy model is given to represent the plant dynamics. In Section 4 the computation of a state space model from the neuro-fuzzy model is presented. Section 5 shows the application of the method for modeling a system with two inputs (heating coil power and steam humidifier power) and two outputs (air temperature and air humidity). Section 6 concludes the paper and outlines future work.

2. PLANT

The considered plant is an industrial HVAC system shown in Fig. 1. The white arrows indicate possible air paths: outer air enters the plant from the right, room air enters from the top right hand corner. The conditioned air can be transported into the room or via an air duct to the neighboring factory building. In the problem setup described in the present paper, one heating coil will be used to increase the air temperature, whereas the steam humidifier is used to increase the air humidity.

3. MODELING

In the following, two types of plant models will be presented: a detailed physically motivated model that forms the basis for simulation studies as well as a local linear neuro-fuzzy model created via the so-called local linear model tree (LoLiMoT) algorithm, see e.g. Nelles [2010, 1997]. These models will be referred to as “physical model” and “LoLiMoT model” respectively in the following. The use of these two types of models is motivated by the following reasons:

- The LoLiMoT model offers the opportunity to extract a linear time invariant (LTI) system model in a straightforward way (see Section 4). For the given physical model, the direct computation of linearized models is difficult for the following reason: Due to the segmentation of the heating and cooling coils (see Section 3.1.1), there are numerous state variables that are not measurable. The design of an observer for the nonlinear physical plant model with typically (depending on the number of segments) hundreds of state variables would be a non-trivial task. In order to use the linearized model in a control scheme, the system order should not be that high, i.e. order reduction techniques would be required, whereas the LoLiMoT approach can yield sufficiently good results with local model orders of one, see Schwingshackl et al. [2013]. These arguments justify the use of the LoLiMoT model. The LTI model extracted from the LoLiMoT model will serve as basis for the application in a linear MPC control strategy.
- To identify the LoLiMoT model parameters, *all* inputs (actuating signals and measurable disturbances) have to be excited properly. Some of the disturbances cannot be excited as they are prescribed by, e.g., outdoor air conditions in the real world system. Therefore, the mentioned physical model is used to generate the identification signals for the LoLiMoT model.
- The parameters of the physical model are known from geometry data of the components as well as from material properties. Only few parameters (e.g. heat transfer coefficient, time constants of temperature sensors) are identified from measured data. To identify them, the excitation of the real world system via the actuators is sufficient.

In the following subsections, the physical as well as the LoLiMoT model will be described. A notation describing the variables can be found in the appendix.

3.1 Physical Model

The model of the complete plant is obtained by interconnecting single component models. The core components are the following:

- Heating coils / cooling coils
- Hydraulic system for the heating and cooling coils
- Steam humidifier
- Temperature and humidity sensors

Fig. 2 shows a schematic representation of the relevant plant components. Plant inputs are the actuating signals u_1 and u_2 of heating coil and humidifier, as well as the disturbances d_1 (air inlet temperature) and d_2 (hot water supply temperature). Supply air temperature y_1 and supply air humidity y_2 represent the controlled variables. In the following, descriptions as well as mathematical models of the mentioned components are presented.

Heating/Cooling Coil Both, heating and cooling coil are realized as so-called finned tube, crossflow heat exchanger.

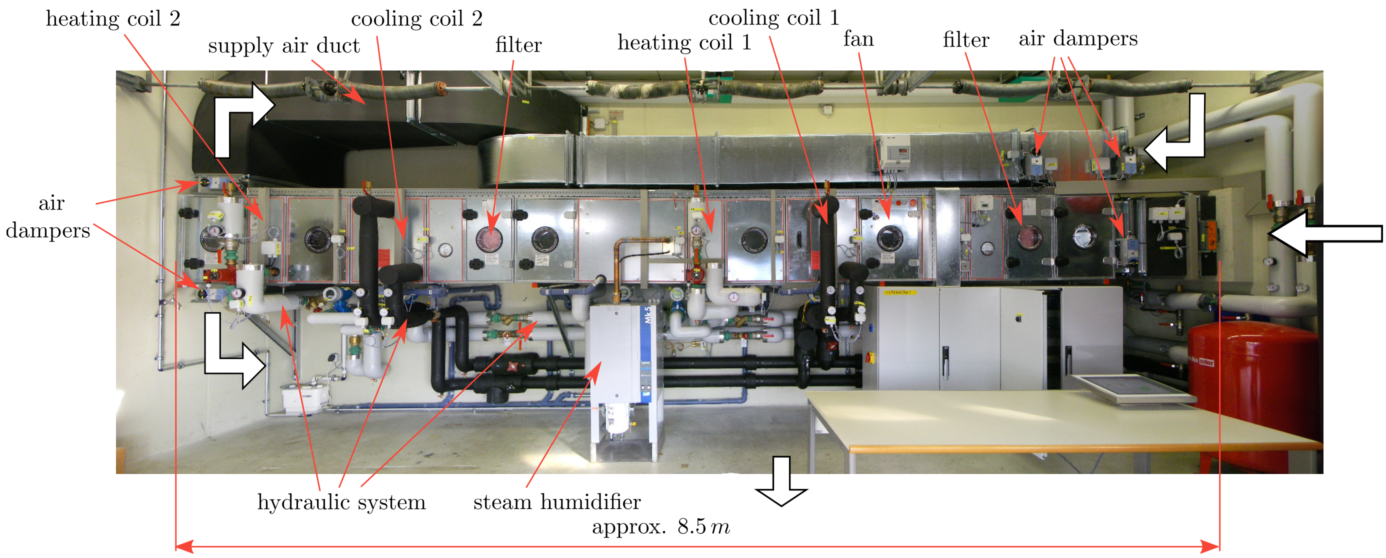

Fig. 1. Picture of the used HVAC test plant.

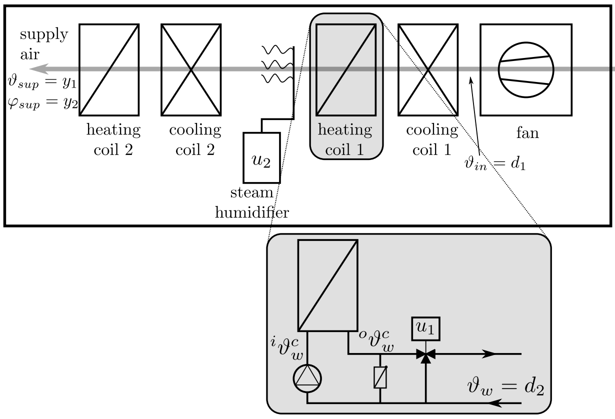

Fig. 2. Schematic representation of the problem setup.

The model inputs are water and air inlet temperatures ${}^{i}\vartheta_w$ and ${}^{i}\vartheta_a$, water and air mass flows $\dot{m}_w$ and $\dot{m}_a$ and the inlet air humidity ${}^{i}x$, expressed as the ratio of watermass to the mass of dry air. The model outputs are outlet temperature of water ${}^{o}\vartheta_w$ and air ${}^{o}\vartheta_a$, as well as the outlet air humidity ${}^{o}x$. For modeling, the tubes of the coil are separated into segments, for each segment, the following model is used, see Wiening [1987], Rehrl et al. [2009]:

$$\frac{d\vartheta^s_t}{dt} = k_1 \left[\frac{1}{2}(\vartheta^s_{w1} + \vartheta^s_{w2}) - \vartheta^s_t\right] + k_2({}^{i}\vartheta^s_a - \vartheta^s_t) + k_3({}^{i}x^s - x^s_t) \quad (1a)$$

$$\frac{d\vartheta^s_{w1}}{dt} = k_4 \left[{}^{i}\vartheta^s_w - \frac{1}{2}(\vartheta^s_{w1} + \vartheta^s_{w2})\right] + k_5(\vartheta^s_t - \vartheta^s_{w1}) \quad (1b)$$

$$\frac{d\vartheta^s_{w2}}{dt} = k_4(\vartheta^s_{w1} - \vartheta^s_{w2}) + k_5(\vartheta^s_t - \vartheta^s_{w2}) \quad (1c)$$

$${}^{o}\vartheta^s_w = 1.5\vartheta^s_{w2} - 0.5\vartheta^s_{w1} \quad (1d)$$

$${}^{o}\vartheta^s_a = e^{\kappa_a}\, {}^{i}\vartheta^s_a + (1 - e^{\kappa_a})\vartheta^s_t \quad (1e)$$

$${}^{o}x^s = e^{\kappa_v}\, {}^{i}x^s + (1 - e^{\kappa_v})x^s_t \quad (1f)$$

The constants k_1 to k_5 and κ_a and κ_v are obtained from material properties, see Wiening [1987]. The segments are interconnected according to the structure of the heating / cooling coil. A coil with 6 rows and 3 segments per row is depicted in Fig. 3. The quantity x^s_t describes the air humidity close to the tube. In case the temperature ϑ^s_t is below the dew point temperature ϑ_{dew}, x^s_t is equal to the saturation air humidity $x_{sat}(\vartheta^s_t)$ (in kg water per kg dry air) at temperature ϑ^s_t. Otherwise, it is equal to ${}^{i}x^s$:

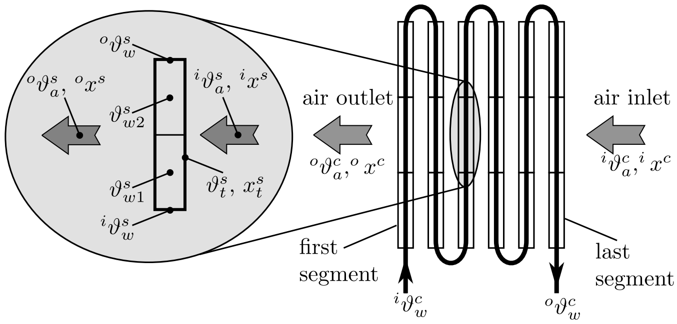

Fig. 3. Structure of the heating/cooling coil.

$$x^s_t = \begin{cases} {}^{i}x^s & \ldots\ \vartheta^s_t > \vartheta_{dew} \\ x_{sat}(\vartheta^s_t) & \ldots\ \vartheta^s_t \leq \vartheta_{dew} \end{cases} \quad (2)$$

The water outlet temperature ${}^{o}\vartheta^c_w$ equals the water outlet temperature of the last segment. The air outlet temperature ${}^{o}\vartheta^c_a$ and air outlet humidity ${}^{o}x^c$ are the arithmetic mean value of the outlet temperatures and humidities of the segments of the last row respectively.

Hydraulic System The hydraulic system for the heating coil is depicted in Fig. 2. For the simulation of the overall systems, fluid dynamics are neglected compared to the temperature and humidity dynamics. Therefore, the hydraulic system is modeled via a static nonlinear curve relating valve position to the mass flows within the circuit.

Steam Humidifier The mass flow of the steam is proportional to the continuously adjustable electric heating power of the steam humidifier [2]. Therefore, the model

$${}^{o}x^{sh} = {}^{i}x^{sh} + ku^{sh} \quad (3)$$

[2] Constant water inlet temperature is assumed and thermal losses are neglected.

is used, where u^{sh} is the heating power in % of the rated power. The parameter k is identified from measured data.

Temperature and Humidity Sensors The temperature and humidity sensors are modeled as first order elements with gain equal to one, i.e. their transfer function is given by $P(s) = \frac{1}{1+sT}$. The time constants T are identified from measured data.

The overall plant model is constructed via interconnection of the component models given above. This overall plant model is used to generate the required input/output data in order to find the coefficients of the LoLiMoT model.

3.2 LoLiMoT Model

The overall model output at time instant k is computed via

$$y_k = \sum_{l=1}^{M} \left(w_{l0} + \sum_{i=1}^{n} \left[w_{li}^{y} y_{k-i} + \sum_{j=1}^{m} w_{li}^{u_j} u_{j,k-i} \right] \right) \Phi_l(\mathbf{u}^*{}_k), \quad (4)$$

where M is the number of local models, m is the number of inputs and n is the order of the local models. The *constant* coefficients w describe the local models, the function Φ_l weights the outputs of the local models depending on $\mathbf{u}^*{}_k$, where

$$\begin{aligned}\mathbf{u}^*{}_k = [&u^*_{1,k-1}\ u^*_{1,k-2} \cdots u^*_{1,k-n}\ u^*_{2,k-1} \cdots u^*_{2,k-n} \\ &\cdots u^*_{m,k-1} \cdots u^*_{m,k-n}\ y_{k-1}\ y_{k-2} \cdots y_{k-n}]^T. \qquad (5)\end{aligned}$$

To model temperature and humidity behaviour, two separate LoLiMoT models of the form (4) were created. In (5), the inputs u_1^* to u_m^* are different for temperature and humidity model. For the temperature model, the following assignment is given (see Fig. 2): $u_1^* = u_1$, $u_2^* = d_1$, $u_3^* = d_2$, and $y = y_1$. In case of the humidity model, the inputs are selected as follows: $u_1^* = u_1$, $u_2^* = u_2$, $u_3^* = d_1$, $u_4^* = d_2$, and $y = y_2$. The coefficients w are obtained using the LoLiMoT-algorithm. An implementation of the algorithm can be found e.g. in Collette [2009], Novak [2012], Mölsä [2007]. In Nelles [2010, 1997], a detailed explanation of the LoLiMoT approach is given.

In the following section, the generation of a state space representation from the LoLiMoT model is presented.

4. LOLIMOT TO STATE SPACE

In Kroll et al. [2000], an approach to compute a fuzzy state space representation from an input/output representation similar to (4) is given. The resulting state space model is *not* a minimal representation. The method proposed in the current paper yields a minimal realization given in Equation (9). It is based on the analytic derivation of a state space representation in observable canonical form from the current delayed inputs and outputs and the LoLiMoT model. This initial model is a local approximation of the LoLiMoT model. In a second step, a regional approximation of the model is computed. Therefore, the parameters obtained from the LoLiMoT model are used as initial values of an optimization procedure. In order to compute the regional approximation around the given operating point, the LoLiMoT model is excited with an identification signal. With the help of this data, the parameters of the state space model are identified. The proposed technique is outlined in the following.

4.1 Analytic computation of the state space model

Equation (4) can be rewritten in the form

$$y_k = \sum_{i=1}^{n} \left(-a_{n-i,k} y_{k-i} + \sum_{j=1}^{m} b_{n-i,k}^{u_j} u_{j,k-i} \right) + w_k^d u_d, \quad (6)$$

where the coefficients $a_{n-i,k}$, $b_{n-i,k}^{u_j}$ and w_k^d are given via

$$w_k^d = \sum_{l=1}^{M} w_{l0} \Phi_l(\mathbf{u}^*{}_k), \quad (7)$$

$$b_{n-i,k}^{u_j} = \sum_{l=1}^{M} w_{li}^{u_j} \Phi_l(\mathbf{u}^*{}_k), \ a_{n-i,k} = -\sum_{l=1}^{M} w_{li}^{y} \Phi_l(\mathbf{u}^*{}_k). \quad (8)$$

The difference equation (6) describes a system with one output y, m inputs u_j and an additional virtual input u_d with the constant value one, corresponding to the coefficient w_k^d.

From (6), a state space representation will be created for further use in the predictive controller shown in Rehrl et al. [Klagenfurt, 2013]. Since there are m inputs and one output, the observable canonical form, see e.g. Chen [1993], will be used. It should be noted that in (6), the coefficients depend on the time instant k. Therefore, in order to obtain the same results as (6), the following state space realization with *time varying* system paramters will be used:

$$\mathbf{x}_{k+1} = \mathbf{A}_k \mathbf{x}_k + \mathbf{B}_k \mathbf{u}_k + \mathbf{b_1}_k u_d \qquad y_k = \mathbf{c}_k^T \mathbf{x}_k \qquad (9)$$

with

$$\mathbf{A}_k = \begin{bmatrix} 0 & \cdots & \cdots & 0 & -a_{0,k+n} \\ 1 & \ddots & & \vdots & -a_{1,k+n-1} \\ 0 & \ddots & \ddots & \vdots & \vdots \\ \vdots & \ddots & \ddots & 0 & -a_{n-2,k+2} \\ 0 & \cdots & 0 & 1 & -a_{n-1,k+1} \end{bmatrix}, \quad \mathbf{b_1}_k = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ w_{k+1}^d \end{bmatrix}, \quad (10a)$$

$$\mathbf{B}_k = \begin{bmatrix} b_{0,k+n}^{u_1} & \cdots & b_{0,k+n}^{u_m} \\ \vdots & & \vdots \\ b_{n-1,k+1}^{u_1} & \cdots & b_{n-1,k+1}^{u_m} \end{bmatrix}, \quad \mathbf{c}_k = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}. \quad (10b)$$

Note that a substitution of Equation (10) into Equation (9) yields Equation (6).

In (10), parameters of *future* time instants $k+1, \ldots, k+n$ are required. Since these values are unknown at time instant k, the following approximation is proposed: It is assumed that the system dynamics are slow compared to the sampling rate. Therefore, a "time-shifted" parameter set will be used, i.e.

$$\mathbf{A}_k \approx \begin{bmatrix} 0 & \cdots & \cdots & 0 & -a_{0,k} \\ 1 & \ddots & & \vdots & -a_{1,k-1} \\ 0 & \ddots & \ddots & \vdots & \vdots \\ \vdots & \ddots & \ddots & 0 & -a_{n-2,k-n+2} \\ 0 & \cdots & 0 & 1 & -a_{n-1,k-n+1} \end{bmatrix}, \quad \mathbf{b_1}_k \approx \begin{bmatrix} 0 \\ \vdots \\ 0 \\ w_{k-n+1}^d \end{bmatrix} \quad (11a)$$

$$\mathbf{B}_k \approx \begin{bmatrix} b_{0,k}^{u_1} & \cdots & b_{0,k}^{u_m} \\ \vdots & & \vdots \\ b_{n-1,k-n+1}^{u_1} & \cdots & b_{n-1,k-n+1}^{u_m} \end{bmatrix}, \quad \mathbf{c}_k = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}. \quad (11b)$$

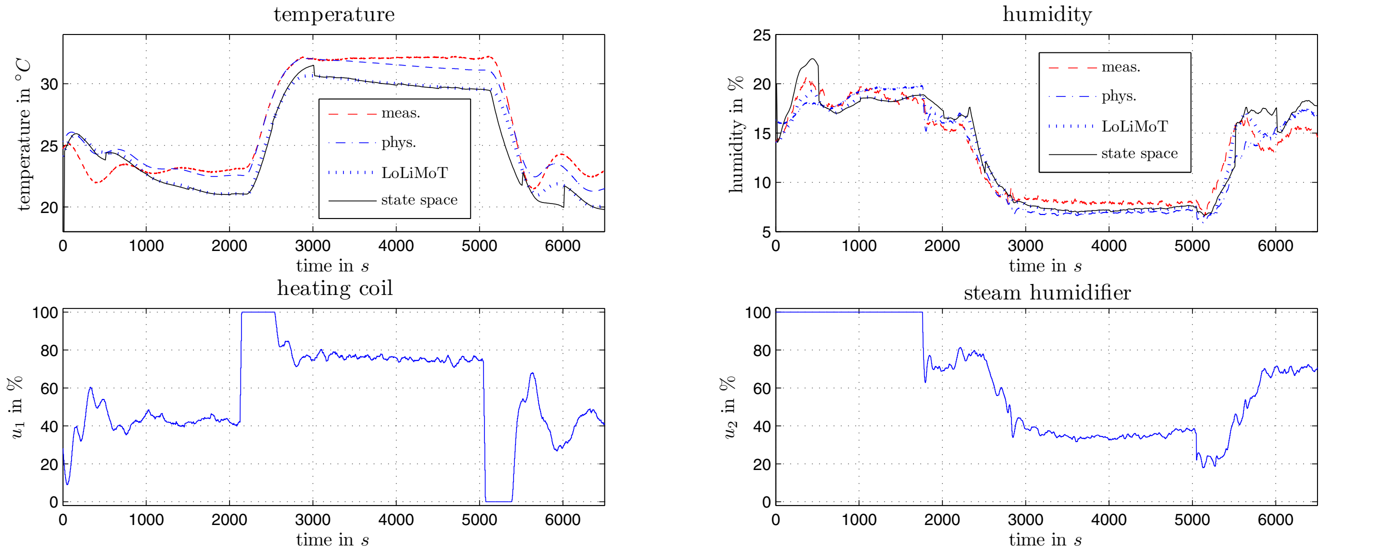

Fig. 4. Plant outputs and actuating signals: measurement, simulation with physical model, LoLiMoT and state space.

4.2 Optimization of the state space model parameters

In the previous section, the analytic computation of the state space parameters (11) was introduced. In contrast to this approach, the parameters of the state space model could be computed via parameter identification as well. However, to find a suitable parameter set, the initial values used for the optimization problem are crucial. Therefore, the coefficients obtained via equation (11) are used as initial values of the optimization problem. The structure of the state space representation (observable canonical form) is fixed. To generate the required data to perform the parameter identification, the LoLiMoT model is excited around the current operating point. Sequentially, each input (i.e. u_1, u_2, d_1 and d_2) is excited with a so called 3-2-1-1 signal, an input signal originally designed for system identification in aircrafts, see e.g. de Visser [2011], Raol et al. [2004]. All other inputs remain constant.

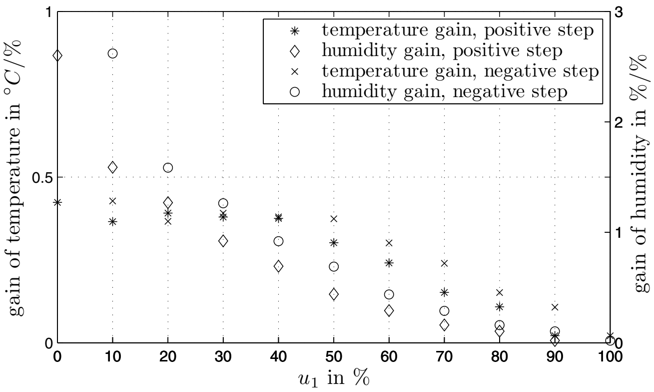

Fig. 5. Gains from input u_1 to temperature and humidity.

5. APPLICATION

The method introduced in Section 4 is demonstrated on the HVAC system given in Sections 2 and 3.1.

5.1 Physical model

The model parameters were obtained from data sheets as well as from material properties. Heat transfer coefficients and sensor time constants were identified from the measured data. Fig. 4 compares the simulation model outputs with the real world system outputs.

The temperature and humidity dynamics are captured quite well by the physical model. Note that the air humidity x in the simulation model is given in kg water per kg dry air, whereas the relative humidity φ in % is measured in the real world system. To compare the results, the relation

$$\varphi = \frac{x}{0.622 + x} \cdot \frac{p}{p_{sat}(\vartheta)} \tag{12}$$

is used to compute the relative humidity φ from x, see e.g. Baehr and Kabelac [2009], Kreith [2000]. In (12), p denotes the air pressure, p_{sat} is the saturation pressure of the water vapor at temperature ϑ. Equation (12) describes an approximately linear relation between x and φ for *constant* temperature ϑ. However, temperature is not constant and due to the nonlinear dependency of the saturation pressure p_{sat} in Pa from temperature ϑ in $°C$,

$$p_{sat} \approx \begin{cases} 611.66\, e^{17.28\left(1-\frac{237.4429}{\vartheta+237.431}\right)} & 0 \le \vartheta \le 60 \\ 611.66\, e^{22.513\left(1-\frac{273.16}{\vartheta+273.16}\right)} & -50 \le \vartheta < 0, \end{cases} \tag{13}$$

φ is as a *nonlinear* function of the temperature ϑ, see e.g. Baehr and Kabelac [2009], Kreith [2000]. Consequently, due to equation (2) and (13) and the nonlinear behaviour of the hydraulics, the system to be modeled is nonlinear. To illustrate the nonlinearity, Fig. 5 shows the gain evaluated for positive and negative steps of heigth 10 % from input u_1 to the outputs ϑ_{sup} and φ_{sup}. In Fig. 4, the physical model outputs are compared against the measured outputs. A compromise between model complexity and model quality was chosen and the depicted curves were regarded as suitable approximation of the real world measurements.

5.2 LoLiMoT

Starting from the physical model, a test sequence for the actuating signals (heating coil u_1, steam humidifier u_2) and for the disturbances (water supply temperature ϑ_w and air inlet temperature ϑ_{in}) is generated. A step sequence with randomized step length ($20\,s$ to $1000\,s$) and randomized amplitudes within the operating range was chosen, see Fig. 6. A comparison of the measured real world outputs, the physical model outputs, the LoLiMoT model outputs and the state space model outputs is given in Fig. 4. The upper left plot in Fig. 4 shows the supply air temperature ϑ_{sup}. The transients are captured quite well by the LoLiMoT model. Please note that the data in Fig. 4 is a different data set than the one used to identify the LoLiMoT parameters (see Fig. 6). The order was set to $n = 2$, the number of local models was selected as $M = 10$, a sampling time of 10 seconds was chosen. Model order and the number of partitions were selected empirically. With $n = 2$ and $M = 10$ the LoLiMoT model captures the behaviour of the physical plant model quite well. The state space model approximates the transients in a satisfactory manner, too. The output of the state space model was computed for 50 steps before its state was reset to match the LoLiMoT output again. For the time between $3000\,s$ and $5000\,s$ where the temperature is almost constant, the LoLiMoT model shows a steady state deviation from the measurements. This steady state error of the model is of minor severity, because the proposed combination of LoLiMoT and state space model will be part of a model predictive control scheme that can typically cope with constant disturbances. In the upper right plot of Fig. 4 it can be seen that the state space model approximates the LoLiMoT model for the humidity even better than for the temperature.

6. SUMMARY & CONCLUSION

In the paper, a modeling approach for HVAC systems is presented. In order to apply linear standard methods, in our case linear MPC with a state space representation of the plant, a state space model has to be developed. Since HVAC systems are nonlinear, linearization around one operating point is not appropriate for the whole operating range. Consequently, a strategy that computes a *linear* state space model at *each time instant* that can be used for the application of linear MPC is presented in this paper. Two types of models are involved: a physical model with few parameters to be identified from measurements and a LoLiMoT model that can be identified using the physical plant model. This approach dramatically reduces the time required for measurements compared to directly identifying the LoLiMoT model on the real world system. A method to compute a state space model from the LoLiMoT representation is given. Each time the optimization of the MPC scheme is performed, a new linear state space model is extracted from the LoLiMoT model. With this concept, linear standard techniques can be applied to nonlinear systems. Future investigations will be dedicated to the application of nonlinear optimization techniques to compare the results with the concept proposed in the present paper. Therefore, a MPC using the LoLiMoT model leading to a non-convex optimization problem will be compared to an MPC based on the state space model.

Appendix A. NOTATION

Variables

m	...	number of system inputs
$\dot{m}$	...	mass flow
M	...	number of local models
n	...	system order
$\mathbf{u}_k^*$	...	vector of LoLiMoT inputs
x	...	humidity in kg water per kg dry air
$\mathbf{x}$	...	state vector
ϑ	...	temperature in $°C$
φ	...	relative humidity in %

Indices

a	...	air	s	...	segment
c	...	coil	sh	...	steam humidifier
i	...	inlet	t	...	tube
o	...	outlet	w	...	water

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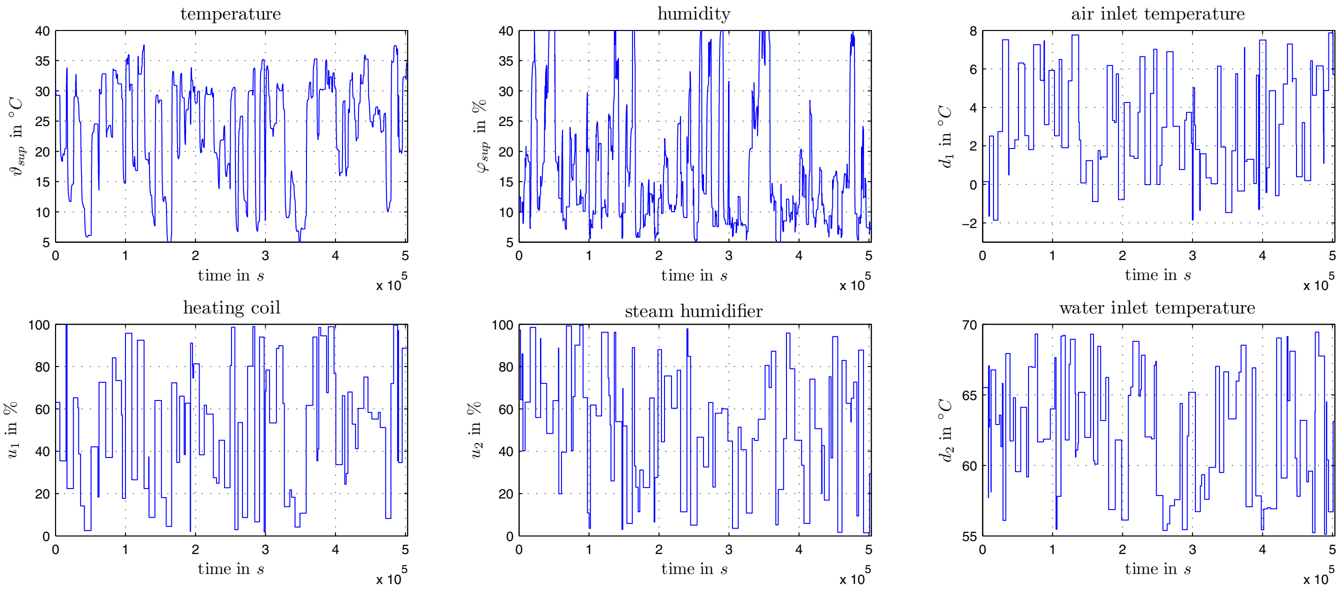

Fig. 6. Identification sequence for generating the LoLiMoT model.

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