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Market-Based Distributed Task Assignment for Rendezvous Mission over Networks with Limited Connectivity ⋆

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Abstract: This paper extends the market-based task assignment algorithm to handle the networks with limited connectivity. The original algorithm was proposed to allocate tasks which cannot be executed by a single agent, but fully connected network is assumed. After analyzing the existing approaches for task allocation in limited networks, a new communications protocol is proposed which preserve key features of the original algorithm. The data required for task allocation algorithm is relayed via bridge agents. By adopting relaying operation, the proposed strategy allocates tasks regardless of the network topology. Convergence property is also preserved. Numerical simulation is performed to verify the feasibility of the proposed strategy considering the communication load.

Keywords: Task Assignment; UAV; Networks with Limited Connectivity; Rendezvous Mission.

1. INTRODUCTION

During the past few decades, much research has been conducted on task allocation (TA) of multi-agent system (Schumacher et al. (2002); Richards et al. (2002); Jin et al. (2003)). Among many multi-agent system, unmanned aerial vehicles (UAVs) are expected to substitute manned aircrafts for dangerous missions, such as Suppression of Enemy Air Defenses (SEAD). Since more efficient and precise attack by multiple UAVs is possible, unnecessary sacrifice of our/enemy forces can be avoided. To do this, tasks should be allocated properly. However, allocation of the tightly coupled tasks such as rendezvous task to multiple agents is challenging problem. This combinatorial optimization problem is known as non-deterministic polynomial-time hard (NP-hard). Since it is impossible to solve the optimal solution within reasonable time, sub-optimal solver is preferred. Especially, runtime and scalability is important issue for the SEAD mission.

On the other hand, decentralized planner is preferred than centralized planner because it is more robust to the failure of the control center. In recent research on decentralized task allocation, a market-based approach received much attention since it is computationally efficient to implement decentralized planner. Each participant in the virtual market makes decision for its own profit, and this action improves the efficiency of the team. In this virtual market, resources are distributed to participants based on their market mechanism as in auction algorithm by Bertsekas (1989). Dias et al. (2006) provided excellent review and survey of the market-based TA. Farinelli et al. (2006) suggested token-based task assignment algorithm in that the access of the task is controlled by a token. Token passing process activates each agent's turn, thus communication load can be reduced to enable real-time implementation. However, broadcasting process to guarantee the conflict-free solution still requires fully-connected networks. On the other hand, Consensus-Based Bundle Algorithm (CBBA) combined the auction algorithm with the plan consensus scheme guaranteeing *50% performance* of the optimal solution (Choi et al. (2009)). Since agents communicate between nearby agents in CBBA frame, network connectivity is not an critical issue. However, CBBA considers the task that should be allocated only one agent. CBBA was extended to consider complex cooperation using heterogeneous teams by Whitten et al. (2011), but the scale of the messages may be an obstacle in real world implementation.

The market-based TA algorithm developed by authors (Oh et al. (2013)) allocates tasks regarding simultaneous attack

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with polynomial time complexity; however, the limits in communication was not considered. In this study, the previous TA algorithm is revised to deal with the limited network connectivity. To do this, a new global planning strategy is proposed by extending the concept of two local planner-based approaches by Alighanbari and How (2006) and Choi et al. (2009). In the global planner, relaying operation is considered such that the information between indirectly connected agents can be relayed. In fact, multi-hop networking is used for the communication relay, and a new communications protocol is proposed for the relaying operation. To analyze the relationship between a network topology and the amount of the communication data due to the relaying operation, Monte-Carlo simulations are performed.

This paper is organized as follows: Section 2 describes the problem statement and TA algorithm briefly. In Section 3, after analyzing two existing approaches for limited network, global planner algorithm is introduced. The numerical results are illustrated in Section 4 to compare the communication load with different network topology. Finally, conclusions and further works are summarized in Section 5.

2. DISTRIBUTED TASK ASSIGNMENT ALGORITHM FOR RENDEZVOUS

This section begins with the statement of the task assignment problem for rendezvous. To solve this combinatorial optimization problem in a distributed scheme, a market-based approach is adopted. The market mechanism is elaborately designed considering convergence, scalability, and robustness to information inconsistency. The proposed TA algorithm guarantees convergence and polynomial time complexity.

2.1 Problem Statement

Let us consider N agents and M tasks. The objective of a task assignment problem is to find the best match between agents and tasks. In general, a TA problem can be formulated as an integer program with binary decision variables, indicating the status of matching. Most of the existing formulations impose a constraint that a task cannot be assigned to more than one agent. In this study, the integer program should contain constraints for rendezvous. The modified version of general TA formulation (Choi et al. (2009)) can be expressed as follows:

$$
\begin{aligned}
&\underset{x_{ik},p_i}{Minimize} && \sum_{i=1}^{N}\left(\sum_{k=1}^{M} c_{ik}(p_i)x_{ik}\right) \\
&\text{subject to} && \sum_{i=1}^{N} x_{ik} = N_k, \quad \forall k \in \{1,...,M\} \\
& && f(p_1,...,p_N) = 0 \\
& && x_{ik} \in \quad \{0,1\} \quad \forall (i,k) \in \{1,...,N\} \times \{1,...,M\}
\end{aligned} \tag{1}
$$

where decision variable $x_{ik} = 1$, when the agent i is assigned to the task k. The path p_i is a sequence of tasks of the agent i, c_{ik} is the cost that occurs when the agent i performs the task k, and N_k is the number of required agents for the task k. M and N indicate the number of tasks and agents, respectively. If the cost is defined as mission completion time, then it is natural that the cost is a function of path. Function f defines time-dependency constraints that enable rendezvous of agents.

This problem corresponds to a combinatorial optimization problem, which is categorized as NP-hard. Note that the optimal matching of tasks and agents among combinations with repetition should be found since each task should be allocated to the predefined number of agents. Moreover, the order of each path p_i is critical because of the rendezvous requirement. The only known method to find a true optimal solution of this problem is to search the every possible combinations. Since this process demands extremely much computational time, it is not practical in dynamic environments. Rather, suboptimal approach is a promising alternative. In this study, a logical and suboptimal approach appropriate for the rendezvous problem is proposed.

2.2 TA Algorithm for Rendezvous

In this section, the proposed TA algorithm, previously presented by Oh et al. (2013), is briefly introduced. A Recruiting process is the main motivation of this study. The hiring concept is introduced to assign the task that must be performed by more than one agents simultaneously. Building teams for the rendezvous mission resembles with recruiting applicants for project team since each project cannot be performed by one person. All agents should organize a team for each task. Each agent may be a Project Manager (PM) of a certain project(task), or an applicant for the team member of the other projects. Their roles will be decided during negotiation process.

The general process of recruitment is composed of the following six phases: 1) Advertising, 2) Application, 3) Acceptance, 4) Choice, 5) Final approval (FA), and 6) Task assignment. First, PM candidates invite applicants via advertisement, and then all interested applicants apply to the PM candidate. Each PM candidate assesses the suitability of the applicants that applied to his/her project. Since applicants can be accepted from more than one PM candidates, they choose the most attractive PM candidate. After that, some additional process are added to consider rendezvous. The PM candidates must conduct a headcount, because each task should be assigned to the specified number of agents. If the number of applicants choosing the PM candidate is equal to the number of necessary member, then this PM candidate becomes a true PM. PM sends final approval letter to each applicant, and the task is finally assigned. This process is illustrated in Fig. 1.

In this algorithm, each agent can be both a PM of its own project team and an applicant of other project team. Therefore, the above process should be conducted by all the agents sequentially. One rotation of the whole agents is called one *round*. At each agent's turn, the above six phases should be performed. At each phase, agent conducts designed actions. Since the later process is more closely connected to task allocation, each agent should proceed the process reversely. At each phase, they exchange their

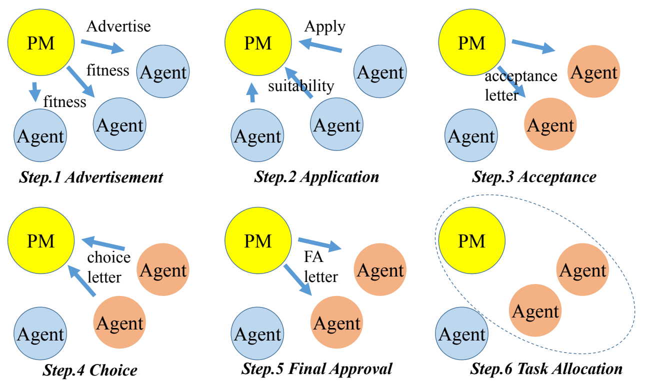

Fig. 1. Conceptual scheme of the proposed TA algorithm.

opinion about task assignment as summarized in Algorithm 1. For more detailed rules about each phase, see Oh et al. (2013).

Algorithm 1 Market-based TA Algorithm

```
procedure TA(i, P, U, T)
    FA Check()              ▷ Get a task if FA is delivered.
    Choice Check()          ▷ Send FA if choice is given.
    Acceptance Check()      ▷ Send choice if accepted.
    Applicants Check()      ▷ Send acceptance if applied.
    Job Seeking()           ▷ Send application.
    Advertisement()         ▷ Advertise.
end procedure
```

To proceed the negotiation process, each agent carries a variable structure $\mathbf{P}$ which include the following personal matrices: a path list $\mathbf{P}_p^{(i)}$, application letters $\mathbf{P}_{app}^{(i)}$, acceptance letters $\mathbf{P}_{acc}^{(i)}$, choice letters $\mathbf{P}_c^{(i)}$, and final approval letters $\mathbf{P}_{fa}^{(i)}$ for the agent i. The matrix A has advertisement information shared by whole agents, and A^i has advertisement information of the agent i. The elements of these matrices are summarized in Table 1.

Table 1. List of matrices of agents

Matrix	Elements
$\mathbf{P}_p^{(i)}$	[1^{st} task, 2^{nd} task,...]
$\mathbf{P}_{app}^{(i)}$	[applicant, task, suitability] (first row)
$\mathbf{P}_{acc}^{(i)}$	[PM, task] (first row)
$\mathbf{P}_c^{(i)}$	[applicant, task] (first row)
$\mathbf{P}_{fa}^{(i)}$	[PM, task] (first row)
A	[PM, task, fitness] (first row)

For example, $\mathbf{P}_{app}^{(i)}$ = [i, k, q] means that the agent i applied for the task k with suitability of q. Moreover, A = [i, k, q; j, l, r] means that the agent i advertised for the task k with fitness q, while the agent j also advertised for the task l with fitness r.

On the other hand, $K = \{1, ..., M\}$ means the set of tasks, and $\mathbf{T}$ is a structured variable having information of the whole tasks. This structured variable is also assumed to be shared by whole agents. For $k \in K$, the states of the task k are three; $\mathbf{T}_{pos}^{(k)}$, $\mathbf{T}_m^{(k)}$, $\mathbf{T}_a^{(k)}$, where $\mathbf{T}_{pos}^{(k)}$ denotes the position of the task k, $\mathbf{T}_m^{(k)}$ denotes the necessary number of the agents for the task k, and $\mathbf{T}_a^{(k)}$ denotes one when the task k is assigned, and zero otherwise. Note that $\mathbf{U}$ is also structured variable of the agent i, including information of position $\mathbf{U}_{pos}^{(i)}$, and velocity $\mathbf{U}_{vel}^{(i)}$.

2.3 Property

The proposed TA algorithm converges to a conflict-free solution within finite time. With polynomial-time complexity, this TA algorithm is scalable to the problem of large size.

Another good property is that information consensus on the positions of other agents is not necessary. Information consensus is important for cooperation performance, however, in practical situation, consensus algorithm may not make the whole information of agents identical. This process may be slow. If consensus on states of agents and tasks is not converged sufficiently, the results of local planners can be conflicted with each other. On the contrary, the proposed algorithm considers only the superficial information such as $fitness$, not the specific facts such as position. Superficial information does not generate conflict even if specific fact is not consistent for all agents. In this sense, the proposed TA algorithm is robust with respect to the inconsistency of the information. This property is similar to the situation of job interview of recruitment process. Although an interviewer cannot be able to know the whole information of applicants, they select proper applicants on the basis of superficial information. Note that this robustness refers only for TA level, not for rendezvous ability; that is, information to allow rendezvous, such as waiting flag indicating each agent's status, must be agreed to make team members arrive at the common task simultaneously.

3. GLOBAL PLANNING STRATEGY IN LIMITED NETWORK TOPOLOGY

3.1 Analysis of Strategy for Distributed Planner

One of well-known approaches to allocate tasks in limited network topology is employing a local planner for each agent. Using the local planner, each agent makes a decision based on its own information. However, conflicts among the local planning results can take place because of the potential inconsistency in agents' information. Local planner-based algorithm performs additional consensus process to resolve this conflict. Alighanbari and How (2006) suggested local planner-based Robust Decentralized Task Assignment (RDTA) algorithm, which is composed of three phases. In the first phase, information consensus across the fleet is performed. Then, each agent solves several candidate plans of the whole agent by using an MILP (Mixed Integer Linear Programming)-based local planner. In the third phase, each agent exchanges its own candidate plans. Collecting all the candidate plans, each agent performs MILP-based plan consensus algorithm. Finally, the conflict-free solution can be obtained. While this approach can allocate tasks in an almost optimal way, there exists a drawback coming from the way how the local planner works. That is, planning process and consensus process are closely coupled. Since the local planner requires the whole agents' information, information consensus affects on the performance of the local planner. Thus, when information consensus is not sufficiently converged, more candidates of plans are required for good performance, which leads to a heavy burden of communication.

On the other hand, Choi et al. (2009) found a novel approach with the local planner, which is a CBBA consisting of two phases. In the first phase of the CBBA, each agent makes its own task *bundle* in a greedy way. And, in the second phase, each agent updates its own winning bid lists with nearby agent according to the update rule for consensus. If an agent is not the winner of the bidding for a certain task, then this agent should eliminate that task from its bundle. Iterations of these two phases can allocate tasks to the fleet in a decentralized way while guaranteeing 50% optimality. Since the CBBA does not require information consensus, communication load is independent of the degree of the information consensus.

In this study, each task should be allocated to a predefined number of agents, but it is hard to find an update rule for the proposed TA algorithm. It seems that the update rule should define the criteria to determine which team is better than other team. In the CBBA, there is a weak cooperation because the agents do not perform the common task, thus its update rule is relatively simple. However, in this study, the considered problem has many tasks and teams. The rendezvous constraint on resulting path also makes the problem more complicated as mentioned in Eq.(1). Instead, a global planning strategy is presented for the proposed TA algorithm in a sequential way.

3.2 Global Planning Strategy

The proposed algorithm performs a TA in a sequential way. Therefore, it is important to determine the way to sequentially execute turns when the network is disconnected. The information synchronization during the turn is also an important issue. The proposed algorithm requires several values that should be shared: $\mathbf{T}$, A, and L, where the matrix L denotes a letter box for delivery. To maintain the good property of the proposed algorithm, the necessary information is relaying when directed network is disconnected. Information of A, $\mathbf{T}$, and L is propagated in one direction from the precedent agent to the current agent. Using this approach, sequential processing and consensus problem can be solved together. With these background, a new communication protocol for letters is developed for the limited network as summarized in Table 2.

Table 2. Communication protocol for limited network

Matrix	Description
$\mathbf{P}^{(i)}$	Personal information on agent i (not shared)
$\mathbf{U}^{(i)}$	Position, velocity of agent i (not shared)
$\mathbf{T}^{(k)}$	Sharing information on task k
A	Sharing information
L	[Sender, Recipient, Type, Task number, suitability]

The list of shared information is exhibited in Table 2. Note that the third element of L represents a type of letter; 1 for play letter, 2 for FA letter, 3 for choice letter, 4 for acceptance letter, and 5 for application letter. The suitability is only required for the application letter. The concept of TA process for the global planning is illustrated in Fig. 2.

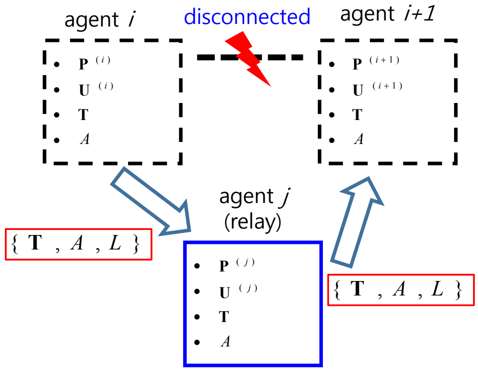

Fig. 2. Conceptual scheme of the global planning strategy.

A play letter, which is newly introduced in this section, includes information about whose turn it is. Until receiving the play letter, each agent does not care about the TA process. Thus, each agent periodically checks whether a play letter is coming or not. When the play letter is delivered to the agent i, the agent i checks if the recipient of the play letter is the agent i. If not, the role of the agent i is to relay information such as A, $\mathbf{T}$, and L. In order to relax communication load, the letters to the agent i are saved in P, and they are deleted from L. If the recipient of the play letter is the agent i, then the proposed TA algorithm is conducted. At the end of the TA, the letters in $\mathbf{P}^{(i)}$ should be augmented to the matrix L. Moreover, if the agent $i+1$ is not directly connected with the agent i, the shortest path to the the agent $i+1$ should be calculated. The agent i sends information variables to the agent j, who is the nearest agent along the shortest path. Note that the recipient in the play letter should be the agent $i+1$, not the agent j. From this strategy, the proposed TA algorithm can be applied even in the environment of limited network topology. Algorithm 2 summarizes the global planning strategy for the proposed algorithm. In Algorithm 2, $|A|$ denotes the number of rows of the inside matrix A, and $L^{(i)}$ denotes the submatrix of L, which consists of the letters to the agent i.

3.3 Property

In the proposed global planning strategy, it is assumed that the network topology should be a connected graph. Only intermittent disconnection can be allowed, and permanent disconnection is not considered. Without any communication, cooperation with other UAV is impossible and therefore it is meaningless. If one agent is isolated, the agent should return to base, while others reallocate tasks similar to the case of agent loss. Another assumption is that the topology of the network is known to all agents. Note that by merging each local topology, the global topology can be achieved. Lastly, the communication topology between agents is bi-directional. If the agent i can transfer data to the agent j, then the agent j also can transfer data to the agent i. Therefore, adjacency matrix of the network topology is symmetric. Since the proposed approach proceeds sequentially, simultaneous communication between the agents is not required.

Based on the above assumptions, two properties can be obtained. First, the TA process is identically proceeded by

Algorithm 2 Global Planning Strategy

procedure GLOBAL TA(i, $\mathbf{P}$, $\mathbf{U}$, $\mathbf{T}$)
  if $|L| > 0$ **then**
    Cut rows of L into $L^{(i)}$, if a recipient is i
    Arrange $L^{(i)}$ into $P(i)$
    if Play letter $\in L^{(i)}$ **then** ▷ TA process
      TA(i, $\mathbf{P}$, $\mathbf{U}$, $\mathbf{T}$)
      Play letter=$[i, i+1, 1, 0, 0]$
      Augment letters into L
      if i is directly connected with $i+1$ **then**
        Send $(\mathbf{T}, A, L)$ to $i+1$
      else
        Find Bridge agent j using Shortest path
        Send $(\mathbf{T}, A, L)$ to j
      end if
    else ▷ Relaying operation
      Next agent = recipient of Play letter
      if i is directly connected with Next **then**
        Send $(\mathbf{T}, A, L)$ to Next
      else
        Find Bridge agent j using Shortest path
        Send $(\mathbf{T}, A, L)$ to j
      end if
    end if
  end if
end procedure

adopting the relay operation. Regardless of the network topology, same TA result can be achieved. Proof of the convergence is still valid in the limited network topology. Second, the maximum number of the letters to deliver has an upper bound as $2N$, which does not depend on the number of tasks. The proof of this is simple; the proposed TA algorithm sends at most two letters at each turn. One letter is one of FA, choice, and acceptance letters. The other is application letter. Since each agent cuts its own letter from the matrix L, the maximum number of rows in L is $2N$.

The number of turns until the convergence will be increased according to the relay operation. However, in the practical situation, the amount of data at each transmit, which determines both a transmission availability and period of communication, usually causes a problem. With sufficiently short period of communication, by virtue of high bandwidth, the increased number of turns due to relay does not make any problem. As in the study of asynchronous version of CBBA (Johnson et al. (2010)), asynchronous implementation of the proposed algorithm will relax the number of turns as well as a burden of the communication.

4. NUMERICAL RESULTS

4.1 Numerical Simulation in SEAD Scenario

The performance of the proposed TA algorithm and the global planning strategy is verified by numerical simulation. Two dimensional military battlefield is considered as simulation environment, and multiple combat UAVs are expected to perform SEAD mission. SAM (Surface-to-Air Missile) sites are distributed in the battlefield. The number of required UAVs can be determined by estimating the degree of risk associated with each SAM site. A UAV is modeled as a point mass, and collision avoidance with other UAVs is assumed to be autonomously performed. The speed of the UAV is set to 200 m/s, and the range of radar is set as 6 km. The simulation was performed by a computer with Intel Core i7 860 @ 2.80GHz and 4GB RAM machine.

Figure 3 shows the result of the numerical simulation in SEAD scenario. Dotted line indicates a resultant path of each UAV. The number inside the parenthesis indicates the number of required UAVs. The dotted circle denotes the range of radar, and the square is the base. The number of required UAVs for each task is randomly set. To simultaneously arrive at each task, path planning algorithm is also considered. The early arrived agents wait other team members while loitering around the task (Oh et al. (2013)).

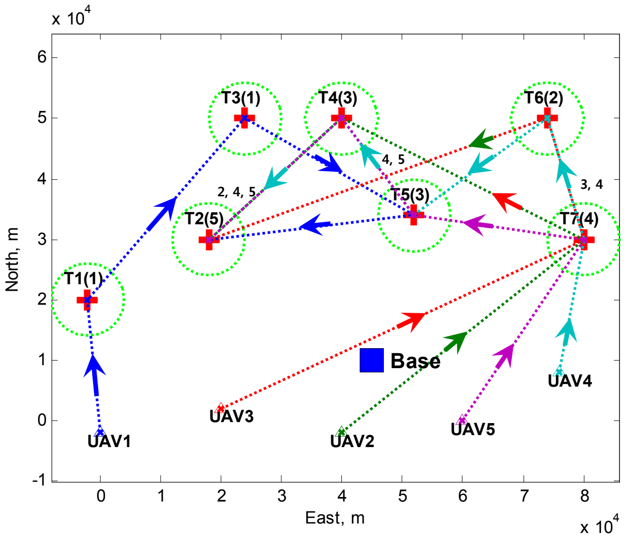

Fig. 3. Simulation results of the TA algorithm.

The number of communication will be increased with the more relaying operations. To compare the number of communication for different network topologies, two limited topologies, line and mesh, are considered as shown in Fig. 4. Diameter of Fig. 4a is four, and diameter of Fig. 4b is two. Diameter of the network is defined as the maximum distance of the two arbitrary vertices of the graph, while distance is the length of the shortest walk between two vertices (Gross and Yellen (2003)).

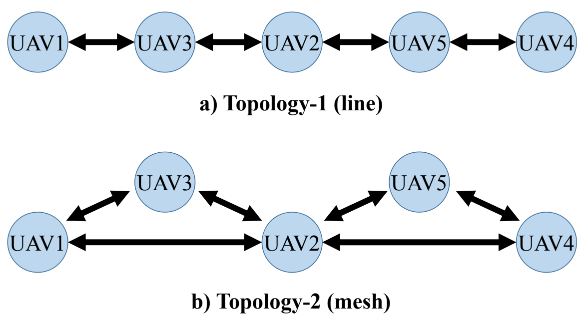

Fig. 4. Two different topology for scenario in Fig. 3.

Table 3 shows the communication loads of the three topologies. N_{turns} denotes the number of total turns,

N_{relay} is the number of relaying, and N_{letter} is the average number of the letters. As expected, increasing diameter leads to the growth of the number of relay. N_{letter} in Topology-1 is less than others, since the letters will be dropped naturally during the relay process.

Table 3. Comparison of communication results

	Diameter	N_{turns}	N_{relay}	N_{letter}
Topology-1(Line)	4	117	58	3.7
Topology-2(Mesh)	2	82	23	4.6
Complete graph	1	59	0	4.8

4.2 Impact of Network Diameter on Communication Load

To check the general tendency of the N_{relay}, 1000 Monte-Carlo simulations are performed. The number of agents and tasks are identical with the above scenario. Initial positions of agents and tasks are randomly generated within 200km by 200km area, and the number of required UAVs for each task is also randomly determined. Random topology is generated using random walk approach; connecting two random vertexes with edge until the graph is connected. To check the connectivity of a graph, second smallest eigenvalue of Laplacian matrix is utilized. If the second smallest eigenvalue of Laplacian matrix is bigger than zero, the given graph is connected (Gross and Yellen (2003)). To check the effect of the additional connections on the connected graph, additional edges are randomly added. Additional edges make the communication network more densely. Figure 5 shows that the results of the Monte-Carlo Simulation, where α is the number of the additional edges. When the randomly generated graph is same as Topology-1 in Fig. 4a, α should be two to change Topology-1 into Topology-2. For the sake of visibility, linear curve fitting algorithm based on least-square estimation is utilized. The tendency of increasing number of the relay with respect to diameter can be easily identified. Moreover, the slope is similar value while the bias is decreased with the growth of α.

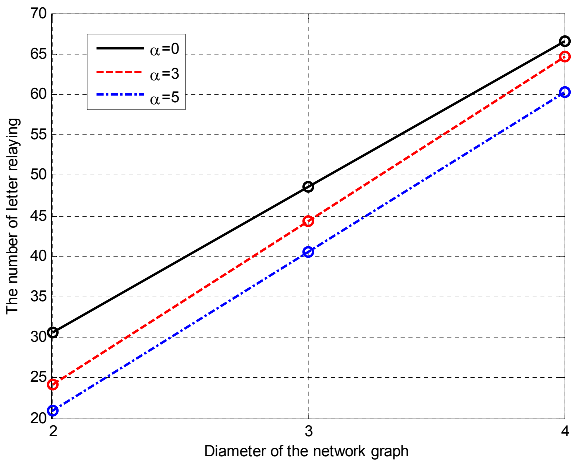

Fig. 5. The number of letter relaying versus diameter of random connected graph.

5. CONCLUSION

This paper proposed the global planning strategy of task assignment algorithm over networks with limited connectivity. Baseline TA algorithm is the market-based distributed task assignment algorithm for the simultaneous arrival of multiple agents. The convergence of the baseline TA algorithm can be proven when the network topology is fully-connected. To preserve the property of convergence of the baseline TA algorithm over the limited network, a new communication protocol is presented. Relay operation is included to the global planning strategy to make the process as in fully-connected network. Numerical simulations showed that the communication load and diameter of the given topology are closely related. Further study will include the asynchronous scheme that can be applied in dynamic environment such as isolation of network or varying topology. Since the proposed scheduling algorithm deals with abstract performance index, this work can be flexibly applied to TA problem with heterogeneous UAVs, and even air traffic management for planning the order of landing on airport.

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