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A Novel MPC with Chance Constraints for Signal Splits Control in Urban Traffic Network♣

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Abstract: It has been recognized that model predictive control (MPC) approach can be successfully applied to the signal control of the urban traffic systems. In this research, we mainly focus on dealing with the uncertainty of the inflow to the traffic network based on the MPC method. The stochastic process describing the uncertainty and chance constraints are embedded to the mathematic programming problem to prevent the congestion happening on the arteries. A modified MPC algorithm is also developed to solve the novel problem under studies. The simulation results show that the proposed model is closer to the real situation than the deterministic ones. The novel MPC algorithm strictly keeps the traffic flows below the limit of the arteries. Moreover, from computation point of view, the proposed method requires shorter computation time that may meet the real-time control requirement.

Keywords: Urban traffic network, Model predictive control (MPC), Chance constraints

1. INTRODUCTION

It has been a new development philosophy and common goal to create Low Carbon City (LCC). In fact, the urban traffic jams have become one of the major factors to generate urban CO_2 emissions and global warming. Thus the studies on reducing urban traffic jams with advanced control and information technology have been a world-wide attractive research area. Generally, the proper strategy for traffic splits control is considered as one of fundamental and effective measure to improve the efficiency for an existing traffic network. The methods and popular tools in theses researches can be classified to the following categories by their characteristics as noted in (M. Papageorgiou *et al.*, 2003):

1) Isolated Intersection Control (R.B.Allsop, 1971) is one of the earliest and simplest approaches to deal with traffic control problems. Obviously, the optimal of each single intersection does not mean the optimal of the whole traffic network. The vital drawback of these approaches is that the coupling between different intersections is neglected.

2) Fixed-Time Coordinated Control is a popular control strategy. Some well-known traffic control tools such as MAXBAND (Y. F. Li, 2008) and TRANSYT (D. I. Robertson, 1969), (S. C. WONG, 1996) belong to this type of control strategies. The signal splits of each intersection in these strategies are constant and the interactions between intersections are taken into account. Using a reasonable design, these methods are potentially more efficient, but also more costly (M. Papageorgiou *et al.*, 2003), as they require the installation, operation, and maintenance of a real-time control system.

3) Coordinated Traffic-Responsive Strategies are the most complicated control systems, the famous control systems such as SCOOT (P. B. Hunt *et al.*, 1982), OPAC etc. belong to this category. These approaches are modified from the fixed-time coordinated control methods and the distinction is that the new system will adjust the control signal dynamically to adapt to the flow of the traffic network.

Store-and-Forward Based Approach (Gazis and Potts, 1963) is an important modeling method used in coordinated traffic-responsive strategies. The basic idea behind this approach is approximately estimating the outflow of each road in a long period time horizon. Combining it with control theory, the method called TUC (Vaya Dinopoulou *et al.*, 2005) is formulated as a quadratic programming form, and the controller is designed in the classic LQR control law. In later works (Aboudolas *et al.*, 2007; de Oliveira and Camponogara, 2007; K. Aboudolas *et al.*, 2009; Lucas Barcelos de Oliveira *et al.*, 2010; Zhou *et al.*, 2012; Ye *et al.*, 2013), the MPC approach was proposed, and demonstrated that the significant improvements may be induced by replacing the standard LQR control law with the finite moving horizon method such as model predictive control.

It should be noted that the MPC models aforementioned are all deterministic programming ones which can be solved by either classic optimization methods or multi-agents approaches (Lucas Barcelos de Oliveira *et al.*, 2010). As far as we know, the studies that take the uncertainty into consideration on signal control problem are rarely up to now. In (Yafeng Yin, 2008), the authors present three models to determine robust (min-max) optimal signal timings so that the

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[1] **SQP**: Sequential quadratic programming, is an iterative method for

system is less sensitive to fluctuations of traffic flows. In other words, they perform much better against the worst-case scenario. In (Xing Zhang *et al.*, 2010), to improve the performance of the traffic-acturated control systems, a flow-prediction algorithm is embedded into a real-time adaptive signal control model to estimate the future vehicle arrival flow. Different from previous work, in this paper we incorporate the chance constrained programming idea from (Pu Li *et al.*, 2002) to build a stochastic model for traffic signal control. The traditional MPC-based traffic control models intend to minimize the flow through the whole network while the model proposed in this work aims to prevent the arteries in the traffic network over-saturated in stressed load situations. In addition, compared with the work in (Xing Zhang *et al.*, 2010), we focus on the performance of a traffic network rather than that of a single intersection.

The rest of this paper is organized as follows: In section 2, we will briefly review the single road model which makes up the whole traffic network. In section 3, a novel network space model with chance constraints is proposed. The simulation and results based on the proposed probabilistically constrained MPC traffic model and strategies are given in section 4. The conclusions and future research directions are addressed in section 5.

2. DETERMINISTIC MODEL

2.1 *The single-way model*

We consider a road z with its downstream intersection j as shown in Fig.1.

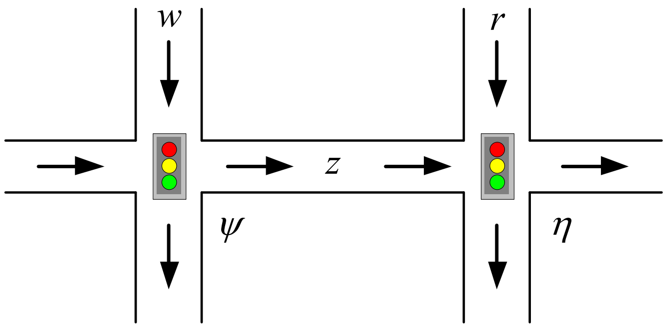

Fig.1. A typical single road model with just one intersection

Here the intersection η has two upstream roads z and r, and without loss of generality, all the roads are assumed to be one-way direction for simplify.

Let the following notations:

$x(t)$: the volume of vehicles in this road at time t.

ΔT: the sample time interval.

C_η: the cycle time of the signal at intersection η.

S_z: the saturation flow at road z.

$I(\eta)$: the set of the upstream link which connecting the junction η.

$O(\eta)$: the set of the downstream link which connecting the junction η.

The decision variable:

$u_{\eta,z}$: the green period of signal assigned to road z at intersection η.

$u_{\eta,r}$: the green period of signal assigned to road r at intersection η.

First, the flow conservation of this road can be represented by the discrete one-dimensional state space model as follows:

$$x(t+1) = x(t) + \Delta T\left(q_{in,z}(t) - q_{out,z}(t) + e(t)\right) \quad (1)$$

Here $e(t)$ equals to the exit flow minus the demand flow generated within the road itself. For simplify, this term will be ignored in the rest of this section.

Here, the outflow term $q_{out,z}(t)$ can be derived as:

$$q_{out,z}(t) = \left(g_z(t)/C_\eta\right)S_z \quad (2)$$

The *Store-and-forward approximate* method aforementioned (C. Diakaki *et al.*, 2002) is used here. $g_z(t) = u_{j,z}(t)$ represents the green light period during a signal control cycle C_j, and S_z is the saturation flow of the road z. This approximate method averages the outflow of road z in a long period view and keeps it constant in a sampling period. The aim is to linearize the traffic network and using a state space model to describe the dynamic of the traffic flow.

Second, we consider the inflow term $q_{in,z}(t)$. Assume the turning rates are given in advance, the traffic flow into link z is expressed as:

$$q_{in,z}(t) = \sum_{w\in I(\psi)} \tau_{w,z} q_{out,w}(t) \quad (3)$$

Where $\tau_{w,z}$ is the turning rate towards link $z \in O(\psi)$ coming from link $w \in I(\psi)$. It means that the roads w and z are connected by the intersection ψ.

Use the Store-and-forward approximating method and let the term u_w represent the green light period for the traffic on road w passing the intersection, then $q_{out,w}(t)$ in (3) can be represented as:

$$q_{out,w}(t) = S_w \cdot \frac{u_{\psi,w}(t)}{C_\psi} \quad (4)$$

Substituted the term $q_{out,z}(k)$ and $q_{in,w}(k)$ in (1) with (2) and (3), then we get:

$$x_z(t+1) = x_z(t) + \Delta T\left[\sum_{w\in I(\psi)} \tau_{w,z}\frac{S_w}{C_\psi}u_{\psi,w}(t) - \frac{S_z}{C_\eta}u_{\eta,z}(t)\right] \quad (5)$$

Additionally, other constraints should be considered:

$$u_{\eta,z}(t) + u_{\eta,r}(t) + l_\eta = C_\eta \quad (6)$$

$$u_{\eta,z}(t), u_{\eta,r}(t) \geq u_{\min} \quad (7)$$

l_η is the lost time per cycle. (6) means that the cycle time of junction j is equal to the summation of the green time of all the stage and lost time. (7) is introduced to guarantee the allocation of sufficient green time to pedestrian phases.

Remark: Since here we simply assume there are two phases (equal to u_1, u_2) on the intersection, for simplicity, we will not explicitly use the concept of phase which is important in the traffic light control area in the rest of paper.

2.2 The regular MPC model of network

The urban traffic network systems are usually composed of a lot of units described at last section. The network in Fig.2 consists of 13 roads and 6 junctions. All roads are also one-way direction denoted by the arrow lines in the figure. It should be noted that for the roads (e.g. road 1, 2, 3) at the boundary of the system, the inflow equals to the flow coming from the outside. For the internal roads (e.g. road 4, 5), it can be computed using (4).

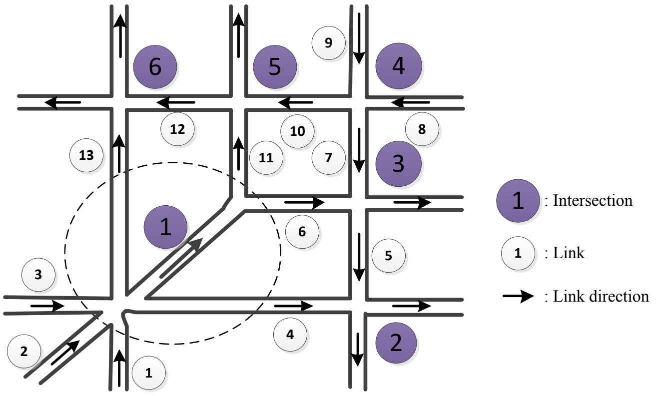

Fig.2. The configuration of the traffic network

Inspired by the moving horizon idea in regular MPC approach, we can generalize (5) to all links in the network, and then we can derive the dynamic equation as follows:

$$\hat{\mathbf{x}}(t+k+1\,|\,t) = \mathbf{A}\hat{\mathbf{x}}(t+k\,|\,t) + \mathbf{B}\hat{\mathbf{u}}(t+k\,|\,t)$$

Here matrix $\hat{\mathbf{x}}(t+k\,|\,t)$ is the vector with each element represents the predicted number of traffic on the corresponding road at time instant $t+k$. $\hat{\mathbf{u}}(t+k\,|\,t)$ is the predicted control actions.

$$\hat{\mathbf{x}}(t+k\,|\,t) = \begin{pmatrix} \hat{x}_1(t+k\,|\,t) \\ \hat{x}_2(t+k\,|\,t) \\ \vdots \\ \hat{x}_n(t+k\,|\,t) \end{pmatrix}_{n\times 1} \quad \hat{\mathbf{u}}(t+k\,|\,t) = \begin{pmatrix} \hat{u}_1(t+k\,|\,t) \\ \hat{u}_2(t+k\,|\,t) \\ \vdots \\ \hat{u}_n(t+k\,|\,t) \end{pmatrix}_{n\times 1}$$

n is the number of links in the network. The state matrix $\mathbf{A}=\boldsymbol{I}$ according to (5), the control input matrix $\mathbf{B}$ is constant by combining the control matrix of each road. (E.g. It is the linear combination of the parameters associated with control $u_{i,w}(t)$, $u_{j,z}(t)$ from $\Delta T(\sum_{w\in I(i)} \tau_{w,z}\left(S_w / C_i\right) u_{i,w}(t) - \left(S_z / C_j\right)$ $u_{j,z}(t))$ in (5)).

And we design the objective $P(t)$ as:

$$\begin{aligned} \min & \sum_{k=1}^{K} \frac{1}{2}\hat{\mathbf{x}}(t+k\,|\,t)'\mathbf{Q}\hat{\mathbf{x}}(t+k\,|\,t) \\ & + \sum_{k=0}^{K-1} \frac{1}{2}\hat{\mathbf{u}}(t+k\,|\,t)'\mathbf{R}\hat{\mathbf{u}}(t+k\,|\,t) \end{aligned} \tag{8}$$

Q weights the states (the number of vehicles in the roads), matrix **R** reflects the penalty imposed on control effort. (This quadric function is identical with the **TUC** method as in (C. Diakaki *et al*., 2002), the detail demonstration on the function can refer to it). Thus, the traditional MPC problem objective $P(t)$ contains two terms, one to minimize the number of traffic in the traffic network and the other to minimize the signal control cost.

Furthermore, we rewrite (6), (7) with their combination forms correspondingly as follows.

$$\mathbf{D}\hat{\mathbf{u}}(t+k\,|\,t) = \mathbf{d} \tag{9a}$$

$$\mathbf{C}\hat{\mathbf{u}}(t+k\,|\,t) \geq \mathbf{c} \tag{9b}$$

Combining the objective (8) with constraints (9a), (9b), we can build up a deterministic quadratic programming problem as the one in (Lucas Barcelos de Oliveira *et al*., 2010). Suppose at a time unit k, we derive the K steps predicted optimal control vector $\hat{\mathbf{u}}$ the by solve the above problem. Only one step of $\hat{\mathbf{u}}$ will be taken action. Then move horizon to the next sample time $k+1 \rightarrow k$ and repeat the above step.

The regular deterministic MPC approach has been demonstrated above. At the next subsection, we will take deep investigation into the random disturbance of the inflow variable $q_{in,z}(t)$ and chance constraints will be introduced to the deterministic problem.

3. CHANCE CONSTAINTS

3.1 Random disturbance and chance constraints

In real life, the traffic network is influenced by a lot of uncertain elements, among which the variance of traffic flow through the network is most important. Thus, we assume the inflow roads of network face uncertainty (i.e. the road 1, 2, 3, 8, 9 in Fig.1) and take road 8 in Fig.2 for example. Look back into the inflow term $q_{in,z}(t)$ in the single road space model as (1), it is no doubt that the inflow varies from day to day and hour to hour. However, based on the former information, (normally, the previous traffic information collected over last few months or years is recorded in the database of traffic information systems and can be got easily), we can use pattern recognition and statistic tool to find same or similar pattern matches current situation, and derive the estimation of the variation tendency. Since the gauss distribution can effectively describe the stochastic process in real life and is convenient for the mathematic derivation, without loss of generality, we assume that the inflow $q_{in,z}(t)$ to the road follows a multivariate normal gauss distribution. It should be noted we use ξ whose element ξ_i equals to $q_{in,z}(t+i)$ in the rest of paper for brevity. To simplify the notation, in the rest of this work, $\hat{x}_z(t+i\,|\,t)$ is abbreviated as $\hat{x}_z(t+i)$.

Suppose the initial time is t, the length of the horizon of the control problem is N, using the statistic of the previous data aforementioned, we can give the mean vector $\boldsymbol{\mu}: N\times 1$ where $\boldsymbol{\mu}_i, i=1,\ldots,N$ represents the mean value of the ξ at time instant i, while the matrix $\Sigma: N\times N$ represent the covariance matrix of ξ_i, $i=1,\ldots,N$. These parameters can be computed and given in advance as follows.

$$\boldsymbol{\mu} = \begin{bmatrix} \boldsymbol{\mu}_1 \\ \boldsymbol{\mu}_2 \\ \vdots \\ \boldsymbol{\mu}_N \end{bmatrix} \qquad \Sigma = \begin{bmatrix} \sigma_1^2 & \sigma_1\sigma_2 r_{12} & \cdots & \sigma_1\sigma_N r_{1N} \\ \sigma_1\sigma_2 r_{12} & \sigma_2^2 & \cdots & \sigma_1\sigma_N r_{1N} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_1\sigma_N r_{1N} & \sigma_2\sigma_N r_{2N} & \cdots & \sigma_N^2 \end{bmatrix} \tag{10}$$

Then the probability density function of the inflow sequence is:

$$\varphi(\tilde{\xi}) = \gamma e^{-(1/2)(\tilde{\xi}-\boldsymbol{\mu})^T \Sigma^{-1}(\tilde{\xi}-\boldsymbol{\mu})} \tag{11}$$

Here the parameter γ is constant. Where σ_i is the standard deviation of each individual stochastic variable and $r_{ij} \in [-1,1]$ are the correlation coefficients between the stochastic variables of time point *i* and *j*. For simplicity, the distribution of $\tilde{\xi}$ on any road is assumed to be independent with other roads.

3.2 Model with chance constraints

Utilizing the rolling horizon idea from MPC control, we can get the predictive model about state $\hat{x}$ taking consider the uncertainty as follows:

$$\hat{x}_z(t+i) = \hat{x}_z(t+i-1) + B\hat{u}_{\eta,z}(t+i-1) + \Delta T \cdot \tilde{\xi}(t+i) \tag{12}$$

Here, we define $\hat{x}(0) = x(0)$ is the measured value from the sensors at initial time, and $B = -S_z \cdot \Delta T / C_j$ can be directly derived from (2).

Remark:

1) Because we consider a single road, the state vector and the input vector are both reduced to scalar variables.
2) We assume the state variable *x* can be precisely measured by the sensors installed on the roads. (Using the facilities such as inductive loop detectors)

Assume the limit of the number of traffic on road *z* is N_z. To prevent the road over-saturated, we can add the chance constraints as follows:

$$P(\hat{x}_z(t+i\,|\,t) \le N_z) \ge p \qquad i \in 1,2,...,N \tag{13}$$

Here $P(\bullet)$ is the cumulative distribution function, *p* is the believe level (usually chosen close to 1. Typically, one does not impose *p* = 1 because the system will become overly conservative, actually it can become infeasible because a Gaussian distribution will have a tail towards infinity). It means in the prediction horizon we want to keep the number of traffic on road *z* below the limit N_z with the probability larger than or equal to *p*.

The objective of this problem is similar with the one proposed in (Pu Li et al., 2002).

$$\sum_{i=1}^{N} \left\| \hat{u}_{\eta,z}(t+i\,|\,t) - \hat{u}_{\eta,z}(t+i-1\,|\,t) \right\| \tag{14}$$

This objective is to reduce the oscillation of control variable and prevent the case that $u_{\eta,z}$ grows too large (close to the cycle C_j) which is unrealistic. In the real-life system, the smooth varying systems are friendly to the passengers and drivers.

3.3 Problem transformation

The common method to solve the aforementioned problem is transforming the stochastic constraints to deterministic constraints. Using the similar method in (Pu Li *et al.*, 2002), we can transform the constraints (13) to the following form, in which $\hat{x}_i = \hat{x}(t+i\,|\,t)$ for brevity:

$$P\{\tilde{\xi}_i \le (N_z - B\hat{u}_{\eta,z}(t+i\,|\,t) - \hat{x}_{i-1}) / \Delta T, i = 1,2,...,N\} \ge p \tag{15}$$

According to the probability density function (11), replacing the term $\hat{x}_{i-1}$ with (12), and considering the summation of future *k* steps $\tilde{\xi}_i$, we can get $\sum_{i=1}^{k} \tilde{\xi}_i \le \left(N_z - x_0 - B\sum_{i=1}^{k} \hat{u}_{\eta,z}(t+i\,|\,t)\right) / \Delta T$, define a new vector $\tilde{\xi}'$ whose element $\tilde{\xi}_k' = \sum_{i=1}^{k} \tilde{\xi}_i$, we can derive the new multi-normal distribution $\tilde{\xi}'$ whose dimension is same with $\tilde{\xi}$. The mean $\boldsymbol{\mu}'$ and covariance matrix Σ' of new multi-normal variables $\tilde{\xi}'$ can be derived as follows:

$$\boldsymbol{\mu}' = \mathbf{G}\boldsymbol{\mu} \qquad \Sigma' = \mathbf{G}\Sigma\mathbf{G}^T$$

Then define $\boldsymbol{\alpha}_k = \left(N_z - x_0 - B\sum_{i=1}^{k} \hat{u}_{\eta,z}(t+i\,|\,t)\right) / \Delta T$, we can transform (15) as:

$$\int_{-\infty}^{\alpha_1} \int_{-\infty}^{\alpha_2} \cdots \int_{-\infty}^{\alpha_K} \varphi(\tilde{\xi}') d(\tilde{\xi}') \ge p \tag{16}$$

Fundamentally, (16) is a deterministic constraint. Specially, the right side of the inuality contains the multiple integral term which is difficult to solve.

Above all, it has been shown that the original problem can be transformed to a multiple integral problem which regarded as a special nonlinear programming problem with nonlinear constraints. It can be regarded as a special kind of normal nonlinear programming problem and common tools like **SQP**[1], **BFGS**, may be applied to solve the problem. However, the computation of the joint probability value in (16) demands numerical integration of the multivariate normal distribution function. This is a rather time-consuming task and even prohibitive for real-time control.

4. TRAFFIC NETWORK MODEL AND SOLVING

4.1Model development

To embed our chance constraint into the MPC model as shown (8), (9), we arbitrary classify all the roads into two categories. Define set $\mathcal{A} = \{\text{road } z, z \text{ is an artery}\}$, and set $\mathcal{B} = \{\text{road } z, z \notin \mathcal{A}\}$ is the complement of set $\mathcal{A}$. And the overall problem can be formulated as follows:

For the subset $\mathcal{B}$:

$$P_1(t): \ \min \sum_{k=1}^{K} \frac{1}{2} \hat{\mathbf{x}}_{\mathcal{B}}(t+k\,|\,t)' \mathbf{Q}_{\mathcal{B}} \hat{\mathbf{x}}_{\mathcal{B}}(t+k\,|\,t) + \sum_{k=0}^{K-1} \frac{1}{2} \hat{\mathbf{u}}_{\mathcal{B}}(t+k\,|\,t)' \mathbf{R}_{\mathcal{B}} \hat{\mathbf{u}}_{\mathcal{B}}(t+k\,|\,t)$$

s.t. $\hat{\mathbf{x}}_{\mathcal{B}}(t\,|\,t) = \mathbf{x}_{\mathcal{B}}(t)$

For $k = 0,1,...,K-1$:

[1] **SQP**: Sequential quadratic programming, is an iterative method for nonlinear optimization. **BFGS**: Broyden–Fletcher–Goldfarb–Shanno algorithm, is Quasi-Newton second-derivative line search family method.

$$\begin{aligned}&\hat{\mathbf{x}}_{\mathcal{B}}(t+k+1\,|\,t)=\mathbf{A}_{\mathcal{B}}\hat{\mathbf{x}}_{\mathcal{B}}(t+k\,|\,t)+\mathbf{B}_{\mathcal{B}}\hat{\mathbf{u}}_{\mathcal{B}}(t+k\,|\,t)\\&\mathbf{C}_{\mathcal{B}}\hat{\mathbf{u}}_{\mathcal{B}}(t+k\,|\,t)\ge\mathbf{c}_{\mathcal{B}}\\&\mathbf{D}_{\mathcal{B}}\hat{\mathbf{u}}_{\mathcal{B}}(t+k\,|\,t)=\mathbf{d}_{\mathcal{B}}\end{aligned}\tag{17}$$

For the subset $\mathcal{A}$:

$$P_2(t):\quad \min\sum_{k=1}^{N}\left\|\hat{\mathbf{u}}_{\mathcal{A}}(t+k)-\hat{\mathbf{u}}_{\mathcal{A}}(t+k-1)\right\|$$

$$\begin{aligned}\text{s.t.}\quad&\hat{\mathbf{x}}_{\mathcal{A}}(t\,|\,t)=\mathbf{x}_{\mathcal{A}}(t)\\&\text{For }k=0,1,...,K-1:\\&\quad\hat{\mathbf{x}}_{\mathcal{A}}(t+k+1\,|\,t)=\mathbf{A}_{\mathcal{A}}\hat{\mathbf{x}}_{\mathcal{A}}(t+k\,|\,t)+\mathbf{B}_{\mathcal{A}}\hat{\mathbf{u}}_{\mathcal{A}}(t+k\,|\,t)\\&\quad\mathbf{C}_{\mathcal{A}}\hat{\mathbf{u}}_{\mathcal{A}}(t+k\,|\,t)\ge\mathbf{c}_{\mathcal{A}}\\&\quad\mathbf{D}_{\mathcal{A}}\hat{\mathbf{u}}_{\mathcal{A}}(t+k\,|\,t)=\mathbf{d}_{\mathcal{A}}\end{aligned}\tag{18a}$$

Chance Constraints:

$$P(\hat{\mathbf{x}}_z(t+k\,|\,t)\le N_z)\ge p_z\qquad k\in 1,2,...,N\;\;z\in\mathcal{A}\tag{18b}$$

There are following two objectives:

P_1: The objective which is the same as the original problem to minimize the number of vehicles in the traffic network plus the control costs.

P_2: The objective to keep the traffic flow on these arteries unsaturated combining with the constraints (18).

The reason that we divide all the roads into these two sets is that we want to investigate the impact if we carry the new control method out in the arteries while the other roads are controlled with the original performance index. It should be pointed that we emphasis on the arteries which are given high priority in control action.

Due to the assumption of multivariate normal distribution of ξ_j, this sub-problem with chance constraints is convex (Kall & Wallace, 1994), while the other sub-problem is a standard quadratic programming problem.

Intuitively, the extreme and most robust way to satisfy the constraint (18) is keeping the green time period for the artery as long as possible. But it is too conservative and unreasonable in practical control situations. Thus we introduce the objective P_2 in order to make the signal varying smoothly. And to keep the traffic network safe and prevent the congestion happening, here we should deal with the objective P_2 first.

4.2 Algorithm

The basic idea to solve the mathematic programming problem has been shown at the previous section. We decompose the optimal problem into two parts, and solve them in series. In the optimization process, the safety objective P_2 is in prior to the original one P_1. The algorithm can be described as follows. Here L is the simulation length.

Initial: k=0,
Step 1: If $k<L$, stop; otherwise, go to step 2.
Step 2: Solve the sub-problem P_2 which assures that the safety of the arteries.
Step 3: Solve the sub-problem P_1 aims to minimize the traffic flow on the rest of the network.
Step 4: k=k+1 and return to Step 1, until the simulation horizon is complete.

In fact, the sub-problem P_1 is a special NLP that is difficult to solve, because the nonlinear constraints make the search sophisticated. Moreover, the problem with multi-integrate-terms could be hardly solved with analytic methods. The key issue to solve the problem is to approximately derive the values of integrate terms in the chance constraints. The work (Pu Li *et al*., 2002) successfully utilizes the method proposed in (T. SZÁNTAI, 1986) and Hammersley sequence sampling (HSS) method. A step by step approach was used by computing the probability of the combinations of the single and bivariate events accurately and to estimate that of the rest combinations through sampling. Instead, for brevity, we use the transformation approach proposed in (ALAN GENZ, 1992) which embedded with a Monte-Carlo sample to solve the problem directly.

Above all, the essential idea of our method is that we divide the whole traffic network into two sub-problems and solve them one by one. It should be pointed out that optimization is carried out by a centralized tool. Thus the control actions and other information on subset $\mathcal{A}$ and subset $\mathcal{B}$ can communicate from one problem to the other without any difficult.

4.3 Model generalization

The stochastic process is not only able to describe the uncertain inflow into the road, but also used to represent the uncertainty of the term $e(t)$ in (1). It represents the flow which generated within the road itself. In other words, according to the data recorded previously, we can approximately derive the mean and covariance matrix of $e(t)$. Then the constraints similar with (18b) could be added to the programming problem. Based on our knowledge, it is significant to consider the term $e(t)$ at morning or evening peak hours due to more traffic flow generated in the urban region during those rush hours.

However, if the interactions exist between two arteries, the same problem with chance constraints may be hard to solve due to the complexity of it grows rapidly.

5. SIMULATION STUDIES

5.1 Simulation Parameters

The simulation studies are based on the traffic network as shown in Fig.2. The parameters of the traffic network are listed as follows:

a) Cycle time C=120s for all intersection; the saturation flow of each road is 3600 vel/h.
b) The turning rates, the weights matrix Q and R are all the same with those defined in (Lucas Barcelos de Oliveira *et al.,* 2010).
c) Without loss of generality and for the sake of simplicity, it is assumed here that only x_8 has uncertain inflow whose initial value equals to 400vel.
d) The upper limit of the volume of vehicles on road 8 is

kept 500vel during the whole simulation.

e) The mean of the input flow is randomly generated in [2000, 2400], and the covariance matrix is built by assigning $\sigma_i = 90 \ \forall i,\ r_{ij} = 0.1\ \forall i, j$ in (3).

f) The confidence level $p = 0.8$.

The simulation has 20 steps and runs on the computer with CPU: Intel® T6500 and 2GB memory. All the programs are coded in Matlab® language.

5.2 Simulation Results

First, we randomly run a lot of cases and pay attention to the number of traffic on road 8, and three randomly selected cases are shown in Fig.3.

In Fig.3 we can see that the value of x_8 during the entire simulation process is all along below (sometimes slightly surpass) the upper limit line (dotted line). It is believed that our chance constraints can perform well to prevent the congestion.

Second, the comparison tests were performed. On one hand, we use a centralized MPC tool to solve the original deterministic problem without considering the chance constraints (i.e. we have no concern on whether the volume of traffic is saturated or not). In this case, other parameters such as the structure of network, the vehicle inflow into the network, the turning rate of each intersection, etc. are same with the first simulation. On the other hand, we use our novel approach to solve the uncertain problem aforementioned. The comparison results on objective P_1 (including x_8) are shown in table.1.

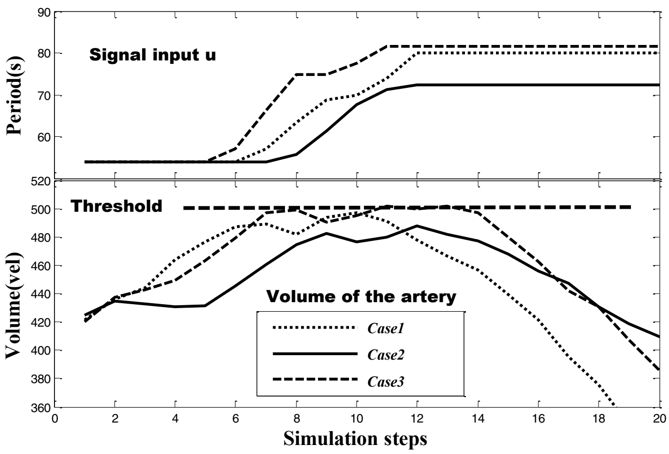

Fig.3. Three randomly selected cases that the state x_8 varies during the simulation

The value 200 in the first column means the initial states of the roads (the number of traffic on the road) except road 8 are randomly generated in [0,200], and we repeat 10 times randomly for one case. Table.1 indicates that the proposed approach performs about 5% worse than the original approach as the cost to guarantee the safety of arties, the reason for that is the solution generated by the new approach concerns on chance constraints and leads to the performance loss on P_1. There is no free lunch.

Remark:

Object (10^6)	200		300		400	
	Origin	New	Origin	New	Origin	New
1	6.24	6.72	8.92	9.18	8.80	9.94
2	5.40	5.98	6.73	7.07	13.4	14.4
3	5.27	5.71	9.60	10.00	10.5	11.4
4	7.45	7.70	5.77	6.14	10.1	10.4
5	6.24	6.55	8.21	8.53	9.23	10.0
6	5.29	5.85	7.99	8.80	11.8	12.6
7	6.66	6.93	7.32	7.92	9.04	9.31
8	7.26	7.69	8.51	8.84	7.89	8.56
9	5.56	6.06	7.09	7.30	12.1	12.9
10	5.49	5.73	8.20	8.41	11.2	11.8
Worse	6.92%		4.95%		6.67%	

[2]Table 1. The performance comparison on objective P_1 with the same initial parameters.

1) During the simulation, we find that the optimization computation spends most of the time on those cases when the number of traffic gets close to upper limit of the road 8. Because the constraints are tight and considerable instances will be exhausted on the integrate term at these cases.
2) The believe level can be changed according to the situation of the road. More tolerable on over-saturation, less value of p can be assigned.

6. CONCLUSION

In our research, we pay attention to the arteries in traffic network which usually undertake stressed load in the traffic peak time. To prevent the congestion happening at these roads, we build a chance constrains programming model and relevant random MPC strategy. Based on the previous knowledge to estimate the multi-variable distribution of the inflow to these arteries, we compared with the classic MPC algorithm, the new algorithm has two differences as follows:
1) Our approach focus far more on the safety of the arteries. We successfully guarantee the security of the arteries with a smooth varying control signal and it is quite significant for the traffic participants.
2) We have to sacrifice the performance of objective P_1 to make our solution robust. The comparison in table.1 indicates that the original algorithm is a little outperformed than ours on the objective P_1. It means that our approach is "lucrative" for the traffic system since limited capacity of non-arteries is scarified.

Thus, the advantage of our work is that by considering the inflow uncertainty, we introduce the chance constraints term into original deterministic model to prevent over-saturated happening on the arteries.

In our opinion, an update and more realistic algorithm can be built up as a switch control system integrating our algorithm with other regular MPC algorithms. When the traffic load is light, the original MPC approach can be used. While the traffic load becomes stressed, our approach should be used.

On the other hand, Poisson distribution should be investigated to make the model more close to the nature of traffic. Also, a lot of other stochastic factors such as accidents, the personal driving habits should be taken into account in future works.

[2] The column "Origin" shows the performance of deterministic MPC while the column "New" shows that of our chance constraints MPC. The unit of all the data is *vel*.

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