

Enhancing \mathcal{H}_∞ Norm Estimation using Local LPM/LRM Modeling: Applied to an AVIS^{*}

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Abstract Accurate uncertainty modeling is of key importance in high performance robust control design. The aim of this paper is to develop a new uncertainty modeling procedure that enhances the accuracy of the \mathcal{H}_∞ norm. A frequency response based approach is adopted. The key novelty of this paper is a new method to address the intergrid error using local parametric modeling methods. These local polynomial and rational models enhance the estimates at the discrete frequency grid. Moreover, the presented methods are shown to enhance the intergrid error estimate. This is illustrated using simulations and experiments on an industrial active vibration isolation system. Compared to the local polynomial models, local rational models are able to handle lightly-damped resonances using far fewer data points.

Keywords: Frequency-response; Uncertainty; Norms; Estimation; Polynomials; Signal Processing; System Identification; Robust Control.

1. INTRODUCTION

Robustness is of critical importance in feedback controlled systems, since feedback control can lead to performance degradation or even closed-loop instability. An important example includes an active vibration isolation system (AVIS), where feedback is used to isolate high-precision equipment from external disturbances. The underlying feedback control principle is skyhook damping [Karnopp, 1995]. However, the performance of skyhook damping is limited by high-frequent parasitic resonance phenomena. Such model uncertainties can be taken account explicitly in a robust control design, see, e.g., Zhang et al. [2005] and Chida et al. [2008] for approaches based on \mathcal{H}_∞ optimization. However, the uncertainty estimation in these references is based on rough prior assumptions and hence inaccurate. On the one hand, this can lead to potentially dangerous results, since no stability and performance guarantees can be given if the estimated uncertainty is too small. On the other hand, this can also lead to potentially conservative results, since if the estimated uncertainty is too large, the resulting controller is robust for an overly large class of candidate systems.

Several approaches have been developed in the literature to determine accurate bounds on the model uncertainty. First, model validation techniques have been developed, see Poolla et al. [1994] for time domain results and Smith

and Doyle [1992] for frequency domain results. However, these results are typically overly optimistic, as is shown in Oomen and Bosgra [2009]. Second, model-error-modeling approaches based on parametric system identification with explicit characterization of bias and variance errors has been proposed in Ljung [1999]. An important aspect in these methods is that these require a significant user intervention in the model parameterization step and rely on assumptions that are asymptotic in the data length. Third, non-parametric identification approaches have been adopted, e.g., as in Van de Wal et al. [2002], De Vries and Van den Hof [1994]. In Van de Wal et al. [2002], an identified frequency response function is used directly to evaluate the \mathcal{H}_∞ norm on a discrete frequency grid. In De Vries and Van den Hof [1994], an extended method is presented that bounds the error in between frequency points in a worst-case approach. However, such worst-case methods are well-known to be overly conservative [Vinnicombe, 2001, Section 9.5.2]. Fourth, recently a data-driven \mathcal{H}_∞ norm estimation has been developed in Wahlberg et al. [2010] and Oomen et al. [2014]. This method relies on a sequence of iterative experiments and directly delivers an estimate of the \mathcal{H}_∞ norm, and combines an optimal experiment design while essentially taking intergrid errors into account. Recent application of this method in Oomen et al. [2014] has revealed that these iterative methods lead to much higher \mathcal{H}_∞ estimates compared with traditional frequency response-based methods, thereby underlining the importance of the intergrid error.

Although recently developed data-driven algorithms provide an accurate estimation of the \mathcal{H}_∞ norm, these methods require a sequence of dedicated iterative experiments. In addition, the required number of experiments inflates

^{*} This work is supported by the Fund for Scientific Research (FWO-Vlaanderen), by the Flemish Government (Methusalem), and by the Belgian Government through the Interuniversity Poles of Attraction (IAP VII-Dysco) Program, the Innovational Research Incentives Scheme under the VENI grant "Precision Motion: Beyond the Nanometer" (no. 13073) awarded by NWO (Netherlands Organization for Scientific Research) and STW (Dutch Science Foundation).

in the multivariable case, see Oomen et al. [2014]. The aim in this paper is to develop a method that provides accurate estimates of the \mathcal{H}_∞ norm using data from a single experiment.

The main contribution of this paper is a new method for \mathcal{H}_∞ norm estimation by exploiting recently developed Local Polynomial Method (LPM) [Schoukens et al., 2009] and Local Rational Method (LRM) [McKelvey and Guérin, 2012] identification approaches. These frequency-local models constitute a golden mean between simple non-parametric techniques (third approach) and a full parametric model (second approach). In particular, these methods lead to an improved frequency response estimation at the standard discrete Fourier transform (DFT) grid and provide an accurate estimation of the intergrid behavior.

Contents In Section 2, the problem is formulated and illustrated on an example. In Section 3, the local modeling methods – LPM and LRM are introduced, including the relevant notation. The main contribution of this paper, which is a new approach for estimating $\|\Delta\|_\infty$ employing these local methods, is presented in Section 4. The example in Section 2.2 is revisited in Section 5. Afterwards, the technique is illustrated on measurement data of an AVIS in Section 6. Finally, the obtained results and some challenges are discussed in Section 7.

2. PROBLEM FORMULATION

Robust control based on \mathcal{H}_∞ optimization requires a nominal parametric model \hat{P} and a bound on the model error Δ . After the model \hat{P} is determined, either through first principles modeling or system identification, it remains to determine a bound on the model error Δ . The \mathcal{H}_∞ norm of the model error Δ , i.e.

$$\gamma = \|\Delta\|_\infty \triangleq \sup_{\omega} |\Delta(j\omega)| \quad (1)$$

for a single-input single-output (SISO) system, or its weighted form $\|W\Delta V\|_\infty$, is a measure that serves an important purpose in many robust control design methodologies that specifies the ‘size’ of the model class for which a controller is to be designed [Skogestad and Postlethwaite, 2005, Oomen and Bosgra, 2012]. In this paper, the non-weighted \mathcal{H}_∞ norm is considered to facilitate the presentation. The proposed approach can be adapted straightforwardly to incorporate the weighting filters W and V .

In this paper, a frequency response-based approach is pursued. When only a limited amount of data is measured, the frequency resolution of the Frequency Response Function (FRF) is limited and this can cause unreliable estimates of $\|\Delta\|_\infty$ when resonances are present. The goal of this paper is to obtain an accurate estimate of $\|\Delta\|_\infty$ using a finite amount of input-output data of a single (generic) experiment carried out on the set-up explained below.

2.1 Set-up

Consider the linear time-invariant (LTI) discrete-time SISO system $\Delta(q)$ that is shown in Figure 1. The system is excited by input signal $u_\Delta(n)$ and the output $y_\Delta(n) =$

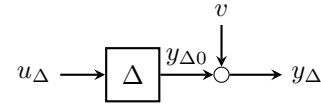


Figure 1. Model-error system Δ in open loop.

$y_{\Delta 0}(n) + v(n)$ where $v(n)$ is colored additive noise (i.e. $v(n) = H(q)e(n)$). We measure N samples of the input u_Δ and output y_Δ and try to determine both the FRF of Δ and $\|\Delta\|_\infty$ from this data.

It is emphasized that the presented results in this paper directly extend to closed-loop systems, as is explained in detail in Section 6.1.

2.2 Motivating Example

Consider the discrete time (model-error) system Δ (as in Figure 1) as unmodeled dynamics:

$$\Delta(z) = \frac{0.412z + 0.405}{z^2 - 1.137z + 0.954}, \quad (2)$$

with sampling time $T_s = 1$ s, which is the zero-order-hold transformed second-order continuous time system. The input u_Δ is white Gaussian noise with unit variance and the disturbing noise variance σ_v^2 is chosen such that a signal-to-noise ratio (SNR)

$$\text{SNR} = \frac{\sqrt{N^{-1} \sum_n y_{\Delta 0}^2(n)}}{\sqrt{N^{-1} \sum_n v^2(n)}} = \frac{\sigma_{y_{\Delta 0}}}{\sigma_v} \approx \frac{\|\Delta\|_2 \sigma_u}{\sigma_v} \quad (3)$$

of 0 dB at the output is obtained.

Using classical Spectral Analysis (SA) techniques – i.e. splitting the signals into N_S segments, applying a window before using the discrete Fourier transform (DFT) and averaging the FRFs over the segments – an FRF of Δ can be obtained. In Figure 2, the ensuing FRF for a Hann window and $N_S = 1$ (no averaging) is shown together with those obtained using the local modeling methods introduced in this paper.

From Figure 2 it shows that the SA approach is not advisable to estimate $\|\Delta\|_\infty$, since the frequency resolution is not dense enough to properly capture the actual resonance peak. Increasing N_S will reduce the variability of this estimate, at the cost of a decreased frequency resolution, so it is even more likely to ‘overlook’ a resonance that is detectable from the data.

2.3 Proposed approach

The proposed in this paper is to use local parametric modeling methods, including LPM and LRM to

- enhance the FRF estimate of Δ at the DFT grid; and
- exploit the local parametric models for accurate intergrid error estimation.

A key aspect herein is that the proposed approach does not require a global parametric model of the error system.

3. LOCAL MODELING METHODS

In the following section the LPM and LRM are introduced and some practical consideration are given. We first start

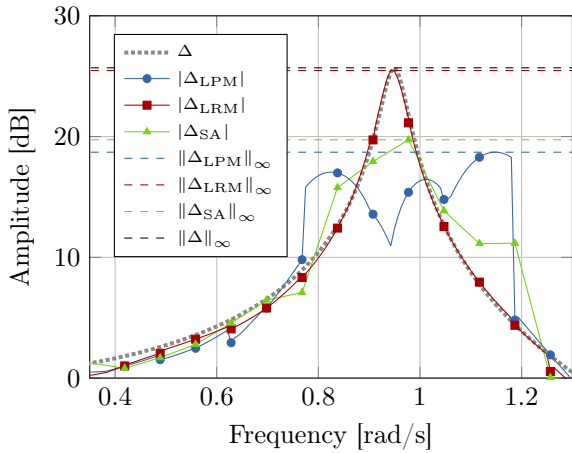


Figure 2. Simulation example revealing that Spectral Analysis (SA) leads to an underestimate $\|\Delta\|_\infty$. The proposed approach using Local Rational Method (LRM) significantly enhances the intergrid estimate as well as the at-grid estimate. For comparison, it is shown that Local Polynomial Method (LPM) does not perform well in this situation. The interpolated LRM and true Δ almost coincide.

from the global system equations and introduce the approximations made by the local modeling methods.

The system in Figure 1 can be described, for an infinitely long data record, by

$$y_\Delta(n) = \Delta(q)u(n) + v(n) = \Delta(q)u(n) + H(q)e(n) \quad (4)$$

where $n = \frac{t}{T_s}$ is the time instant normalized by the sampling time T_s , q^{-1} is the lag operator and both $\Delta(q)$ and $H(q)$ are stable causal real-rational functions. For the practical situation where the data length is limited ($n \in \{0, \dots, N-1\}$), (4) has to include the influence of the system transient $t_{\Delta'}(n)$ and the noise transient $t_H(n)$:

$$y_\Delta(n) = \Delta(q)u(n) + H(q)e(n) + \overbrace{t_{\Delta'}(n) + t_H(n)}^{t_\Delta(n)} \quad (5)$$

where both transient contributions can be combined into a single transient term $t_\Delta(n)$.

By applying the DFT

$$X(k) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} x(n) \exp\left(\frac{-j2\pi kn}{N}\right) \quad (6)$$

to both sides of (5), its frequency domain equivalent is obtained:

$$Y_\Delta(k) = \Delta(\omega_k)U_\Delta(k) + T_\Delta(\omega_k) + V(k) \quad (7)$$

The index k corresponds to the k^{th} frequency bin with frequency $\omega_k = 2\pi k/(NT_s)$.

3.1 The Local Polynomial Method

The LPM revolves around the observation that the transfer function $\Delta(\omega)$ and the leakage term $T_\Delta(\omega)$ are smooth over the frequency, and as such can be approximated well by a Taylor series [Schoukens et al., 2009].

In the LPM these Taylor series are estimated by considering a local window ($r = -N_W, \dots, N_W$) of $2N_W + 1$ lines around each frequency bin k :

$$\Delta(\omega_{k+r}) \approx \delta_0(k) + \sum_{i=1}^{N_B} \delta_i(k)r^i \triangleq \tilde{\Delta}_k(r) \quad (8)$$

$$T_\Delta(\omega_{k+r}) \approx t_0(k) + \sum_{i=1}^{N_T} t_i(k)r^i \triangleq \tilde{T}_k(r) \quad (9)$$

where $\delta_i(k)$ and $t_i(k)$ are the (complex-valued) coefficients of the local polynomials $\tilde{\Delta}_k$ and \tilde{T}_k . We denote the LPM as $\text{LPM}(N_W, N_B, N_T)$ to explicitly indicate the degrees of the polynomials and the local bandwidth. Around each frequency bin k , $\delta_i(k)$ and $t_i(k)$ are obtained by fitting (7) in least-squares sense with (8) and (9) substituted. Thereby minimizing the local (linear least-squares) cost function

$$\sum_{r=-N_W}^{N_W} \left| Y_\Delta(k+r) - \tilde{\Delta}_k(r)U_\Delta(k+r) - \tilde{T}_k(r) \right|^2. \quad (10)$$

At the edges of the frequency grid (where $k \leq N_W$ or $k \geq \frac{N}{2} - N_W$), the local window index r becomes degenerate. This can be tackled by using e.g. an asymmetric window around k as is done in [Pintelon et al., 2010a] or by exploiting the periodicity of the DFT over the frequency [McKelvey and Guérin, 2012]. In this paper, we limit the notation to the ‘bulk’ of the frequency grid to enhance the presentation of the formulas, and tacitly use the former approach in the actual computation.

Several remarks are appropriate.

- In typical applications of the LPM, mainly the parameters $\delta_0(k)$ and $t_0(k)$ are of interest, as they are an improved estimate of the FRF $\Delta(\omega_k)$ and the spectrum of the leakage $T_\Delta(\omega_k)$. In contrast, in the proposed approach in this paper, all δ parameters are of interest to interpolate the FRF in-between the frequency grid.
- The LPM(N_W, N_B, N_T) is used such that the local fit retains some degrees of freedoms: i.e. $2N_W + 1 - (N_B + N_T + 2) > 0$.
- The separation between Δ and T_Δ in (7) imposes requirements on the input signal. These requirements essentially imply that this is only possible if the input spectrum $[U(\omega_{k-N_W}) \cdots U(\omega_{k+N_W})]$ is sufficiently ‘rough’ over all frequencies. Importantly, this is the case for random noise and random phase multisines [Schoukens et al., 2009].
- Although the LPM is illustrated for the SISO generalized output error (OE) case, it can be extended to work for multiple-input multiple-output (MIMO) systems, with feedback and within the errors-in-variables (EIV) framework as is done in [Pintelon et al., 2010a,b]. For a more detailed exposition of the LPM, we also refer to [Schoukens et al., 2006, 2009, Gevers et al., 2011].

3.2 The Local Rational Method

The LRM builds upon the same basic idea of the LPM, namely that the transfer function and transient contribution in (7) are ‘structured’ over frequency. Instead of using polynomials to capture the local structure, rational models are used as an approximation [McKelvey and Guérin, 2012].

In equation (7) that is fitted locally, the different factors $\Delta(\omega_k)$ and $T_\Delta(\omega_k)$ are expanded into rational functions instead of polynomials:

$$\Delta(\omega_{k+r}) \approx \frac{\sum_{i=0}^{N_B} b_i(k)r^i}{1 + \sum_{i=1}^{N_A} a_i(k)r^i} = \frac{B_k(r)}{A_k(r)} \triangleq \tilde{\Delta}_k(r) \quad (11)$$

$$T_\Delta(\omega_{k+r}) \approx \frac{\sum_{i=0}^{N_T} t_i(k)r^i}{1 + \sum_{i=1}^{N_A} a_i(k)r^i} = \frac{T_k(r)}{A_k(r)} \triangleq \tilde{T}_k(r). \quad (12)$$

We denote LRM(N_W, N_B, N_A, N_T) to be the LRM with the orders and bandwidth defined above. Similarly to the LPM, these equation can be written in a matrix formulation for every frequency. The resulting linear least-squares problem is closely related to the approach of Levy [1959].

4. LOCAL MODELS FOR \mathcal{H}_∞ NORM ESTIMATION

In the previous section, the LPM and LRM have been shown to offer a way to estimate the error-system transfer function Δ with a reduced influence of the transient term T_Δ . Essentially, this is achieved by approximating Δ and T_Δ around frequency bin k by the low-order local models $\tilde{\Delta}_k$ and \tilde{T}_k , being either polynomial or rational functions. In this section the use of the LPM and LRM is extended towards an approach that exploits these estimated local models to obtain a more fine-grained view of the error-system Δ specifically geared towards determining $\|\Delta\|_\infty$.

Remember that evaluating the local model $\tilde{\Delta}_k(r)$ corresponds to the value of Δ 's FRF in the frequency ω_{k+r} as captured by the local model around ω_k . To ease the notation, denote

$$\Delta_k(\omega_{k+r}) \triangleq \tilde{\Delta}_k(r). \quad (13)$$

As only a discrete grid of ω_k s is available, the \mathcal{H}_∞ norm (1) based on those measurements is

$$\gamma_{\text{FRF}} \triangleq \max_{\omega_k} |\Delta(\omega_k)| < \|\Delta\|_\infty = \gamma \quad (14)$$

where the discrete grid entails in an underestimate of γ due to the intergrid error, see also Figure 2.

This raises the question what $|\Delta(\omega)|$ should be for any value ω not on the grid. Using the LPM/LRM, we notice that for every ω within the excited frequency grid, two local models that are adjacent to ω can be found. We denote those adjacent local models as $\Delta_{k_L(\omega)}$ and $\Delta_{k_R(\omega)}$. Note that these models are based on the input/output spectra in the vicinity of ω , hence due to their local validity additional information can be extracted regarding the value of $\Delta(\omega)$. Since they are expressed in a parametric formulation ((8) for LPM, (11) for LRM), these can be evaluated locally for a continuous frequency range ω .

With respect to estimating $|\Delta|$ in the \mathcal{H}_∞ norm (1), this means that we have two candidate values that carry information: $|\Delta_{k_L(\omega)}(\omega)|$ and $|\Delta_{k_R(\omega)}(\omega)|$. In view of the worst-case nature of the \mathcal{H}_∞ norm, a suitable solution is to use the largest value: $\max\{|\Delta_{k_L(\omega)}(\omega)|, |\Delta_{k_R(\omega)}(\omega)|\}$.

This leads to the following expression for estimating $\|\Delta\|_\infty$ based on the local models:

$$\|\widehat{\Delta_{\text{LxM}}}\|_\infty \triangleq \max_{\omega} \max\{|\Delta_{k_L(\omega)}(\omega)|, |\Delta_{k_R(\omega)}(\omega)|\}. \quad (15)$$

Instead of formally optimizing (15) over the frequency ω , this expression can be approximated arbitrarily well by evaluating the expression on a sufficiently dense frequency grid. This is the approach we use in the implementation.

5. SIMULATION EXAMPLE REVISITED

In order to get a clear idea of the performance of this method, we illustrate it applied to a simple discrete-time resonant second-order system. This corresponds to the example given in Section 2.2, as depicted graphically in Figure 1.

If we now take a closer look at the results for LRM(5, 2, 2, 2) and LPM(3, 2, 2) for this example as shown in Figure 2, we can see that the LPM is not well-suited to operate in this setting. Since there are very little points near the resonance, and a local polynomial model has a hard time capturing the resonant pole, the obtained FRF estimate is unreliable. Consequently, the estimated \mathcal{H}_∞ norm based on the interpolated local parametric model also is unreliable. These observations are corroborated by the results in [Schoukens et al., 2013], where it is shown that the LPM requires seven points within the 3 dB bandwidth of the resonance for a reliable estimate.

In sharp contrast, it can be observed in Figure 2 that the LRM is able to model the resonance well and yields a great estimate $\|\Delta_{\text{LRM}}\|_\infty = 25.68$ dB in the simulation, where the theoretical value is $\|\Delta\|_\infty = 25.69$ dB. This can be attributed to the fact that the LRM uses a rational model to locally approximate the actual Δ , which means that the (resonant) pole is estimated and used to obtain a local estimate of $\|\Delta\|_\infty$. These results confirm the advantages of the proposed method in uncertainty modeling for robust control design purposes.

6. APPLICATION TO AN AVIS

The proposed method is illustrated on the active vibration isolation system (AVIS) shown in Figure 3. The AVIS consists of a support fixed to the ground and a payload platform, suspended on airmounts, that is free to move in six degrees of freedom. Moreover, the AVIS is equipped with six geophones measuring the velocity and eight linear motors allow to actively compensate for vibration of the platform. To facilitate the presentation only the vertical translation is considered, i.e., a SISO error system is considered.

6.1 Control Framework

To implement the approach in this paper, a suitable uncertainty structure is employed that addresses the closed-loop operation of the system. In particular, the so-called dual-Youla framework is adopted [Hansen et al., 1989, Anderson, 1998, Douma and Van den Hof, 2005].

This leads to a model set \mathcal{P} (as shown in Figure 4) that is described by

$$\mathcal{P} \triangleq \left\{ \frac{\hat{N} + D_c \Delta}{\hat{D} - N_c \Delta} \mid \|\Delta\|_\infty \leq \gamma_{\mathcal{P}} \right\}, \quad (16)$$

where $\gamma_{\mathcal{P}}$ is the \mathcal{H}_∞ norm that is to be estimated. The signals u_Δ and y_Δ in Figure 4 are required. It can be

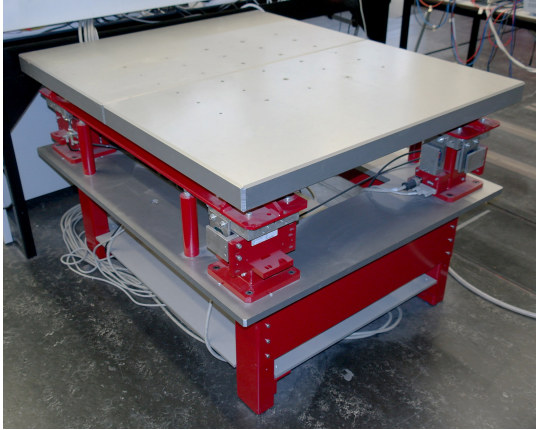


Figure 3. Active vibration isolation system (AVIS).

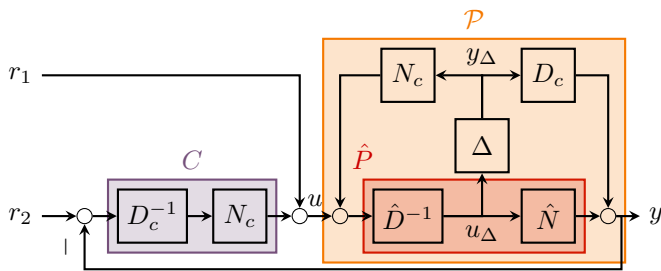


Figure 4. Dual-Youla parametrization of the closed-loop set-up with controller C , plant model set \mathcal{P} and nominal plant model \hat{P} .

shown that u_Δ can be determined noiselessly. Also y_Δ is accessible, but this measurement is disturbed by noise [Anderson, 1998]. This means that Δ in Figure 4 can be estimated in an output error setting equivalent to Figure 1 in the problem formulation.

6.2 Results

We determine an eight-order (global) control-relevant parametric model \hat{P} (Figure 5) directly in the co-prime factorization using the framework of Oomen and Bosgra [2012] that uses an identical \mathcal{H}_∞ criterion for control design and system identification of the form $\|WT(P,C)V\|_\infty$, with weighting filters W and V and $T(P,C)$ the transfer function of r_1 and r_2 to u and y in Figure 4.

We measure the FRF using five periods of a random phase quasi-logarithmic multisine excitation [Geerardyn et al., 2013]. One period consists of 65 536 samples with 1550 excited bins such that $\omega_k \approx 1.001\omega_{k-1}$ and $f_s = 1$ kHz. This signal is applied to the r_1 input of the closed-loop system (keeping the set-point $r_2 = 0$) with an experimental PID controller C^{exp} in the loop. The chosen model provides a reasonable description of the plant, while some unmodeled dynamics end up in Δ . Next, Δ is identified non-parametrically by applying the multisine and calculating the u_Δ and y_Δ given the estimated \hat{P} and known C^{exp} as in Anderson [1998].

To test the methods on a low-resolution frequency grid, we use only one out of four excited bins to estimate the

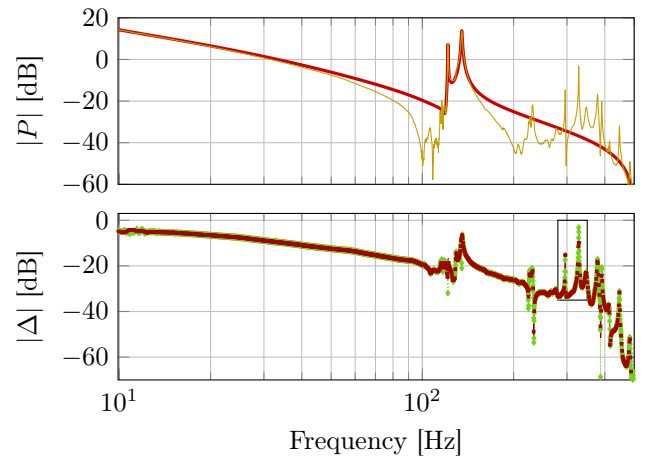


Figure 5. Measured FRF (—) and fitted model \hat{P} (—) of the AVIS on the top. The dual-Youla Δ determined by the proposed method on a coarse (—) and dense (—) frequency grid, as validation are shown on the bottom. Figure 6 shows a zoom of the rectangle.

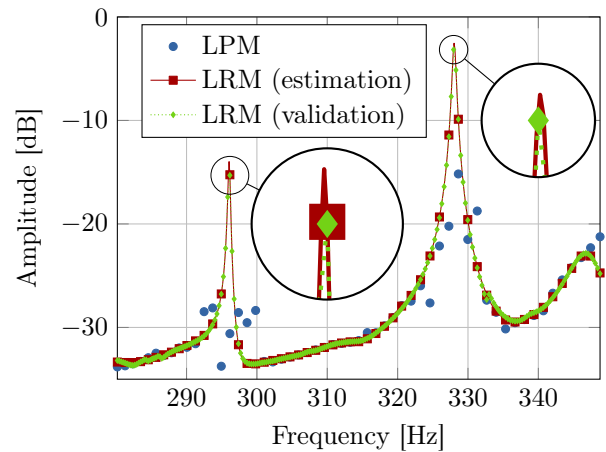


Figure 6. Measured Δ for the AVIS using the LRM and the proposed interpolation. Clearly, the LRM leads to a higher peak value. The validity of the LRM local parametric model is confirmed by the validation measurement at the dense frequency grid, which reveals excellent interpolation properties.

LRM (6, 2, 2, 2) of Δ and apply the proposed interpolation (15). The LRM applied to the full frequency resolution data is used as validation for the method. The results are shown in Figure 5 for the whole frequency band and Figure 6 for a frequency range where two resonances are observed and where the results of LPM (4, 2, 2) are shown for the sparse frequency grid.

In Figure 6 it can be seen that the LPM is not able to represent these sharp resonances well: the resonance is missed almost completely by the blue dots. This can be attributed to an insufficient frequency resolution [Schoukens et al., 2013]. On the other hand, the interpolated LRM on the coarse grid and the validation data agree within about 5%. The peak value near 328 Hz, is shown to be captured by the interpolated LRM in agreement with the validation data. It improves the estimate of $\|\Delta\|_\infty$ at the DFT grid by 7.5 dB using the same data record. This is a significant

improvement that does not rely on an increase in frequency resolution, or equivalently, measurement time.

This suggests that the local rational models with interpolation are a very reasonable approximation of the actual Δ . The LRM is also able to estimate $\|\Delta\|_\infty$, even if the actual peak does not coincide with the discrete frequency grid.

7. CONCLUSION AND FURTHER RESEARCH

High performance, non-conservative robust control design requires an accurate estimate of the \mathcal{H}_∞ norm of the model error Δ . In this paper, a new approach is presented that employs so-called local parametric LRM models that lead to an enhanced estimate of $\|\Delta\|_\infty$ of a SISO error system by interpolating between neighboring local models. In addition, it is illustrated that the LPM is not flexible enough to handle sharp resonances using very few data points. The LRM on the other hand is better suited for that case and is therefore the advisable solution to obtain a better estimate. The technique is illustrated on a simulation example where the LRM-based interpolation was able to retrieve the true \mathcal{H}_∞ norm. Using measurements on an AVIS, it is illustrated that the interpolation-based results can substantially improve $\|\Delta\|_\infty$. Furthermore, the results are validated by a detailed validation measurement. Ongoing research focuses on extending the technique to MIMO systems and the study of its stochastic properties.

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