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Active Vibration Control for Marine Platform Flatness Using Sliding Mode Scheme

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Abstract: Various industries adopt different techniques for isolating or reducing vibration. This paper investigates flatness control for fitted equipment on marine vessels which are subject to random sea wave. These disturbances significantly affect the operational quality and durability of marine systems, thereby decreasing overall safety. In hostile environments where safety considerations are important, vibration isolators are often used to suppress the effects of vibrations. The system described here is based on a four degree-of-freedom active marine suspension model, which allows for effective disturbance rejection. Sliding Mode Control (SMC) is then used to reduce the disturbance simulated as random sea waves. It has been shown that this method offers the advantages of easy implementation, reduced maintenance requirements, increased system efficiency and reduced vibration.

Keywords: Vibration, marine systems, disturbances rejection, sliding mode control, simulation,

1. INTRODUCTION

In the last two decades, different control system applications have been proposed to reduce vibration inputs to machinery installed on marine structures (see for example, Inman, 1989; Fuller, Elliott, & Nelson, 1996). Currently, the application of these control systems in the marine industry has been relatively slow, despite such systems being applied practically in the structural control of skyscrapers and cabin noise reduction in aeroplanes (S.Daley (2003)).Vibrations in marine vessels occur as structural resonance, and have currently become a major concern to engineers.

Usually, vibration isolators are used to reduce vibrations transmitted from the marine vessel to the fitted equipment (Y.P.Xiong (2005)).This is achieved by employing a conventional approach that constitutes either an active/passive solution that is reliant on spring mounts (W.T. Thomson (1988)), The passive approach is designed to minimise the effects of the excited structural resonance but is limited by its performance at lower frequencies. To solve this problem, active control methods have been applied to limit disturbances to the installed machinery. These methods have been implemented successfully and have resulted in improved performance in relation to the investment costs.
(Darbyshire&Kerry(1997);Winberg,Johansson,&Lago(2000))
In the work presented here, the marine body and fitted equipment is considered as two cuboid shaped structures linked in parallel. The model used in this paper is based on those developed in (Y.P. Xiong (2005)) but is much realistic as it considers more degrees of freedom. Furthermore, previous research report (S.Daley (2003); Y.P. Xiong (2005)) has only focused on sinusoidal excitation of roll, while the present work considers non-sinusoidal periodic excitation of roll and pitch as well as heave. Generally sea waves caused the pitch, heave and roll motions and on the other hand internal and external forces such as rudders, propellers and environment conditions such as wind and sea currents induce surge, sway, and yaw motions.Sliding mode control (SMC) has previously been applied to active vehicle (N. Yagiz (2000);Abbas Chamseddine(2006))and marine system (Tannuri (2010)) but not to the system under consideration. In this paper, a SMC strategy is proposed to reduce the effect of vibrations on a top platform when the marine vessel is subjected to random sea wave. The purpose of control is to use the four actuators to reduce the vibration of the top platform which can be the base for the fitted equipment (e.g.engine). Therefore, a model between the vibration of the top platform and the four actuators need to be established. The paper is organized as follows: formulation of model is presented in section 2; sliding mode control strategy for the system is given in section 3.Section 4 provides the formulation of the wave disturbances while section 5 and 6 presents the simulation and results respectively. Conclusion is discussed in section 7.

2. MODEL FORMULATION

The model of a full marine suspension system is shown in Fig.1.This model is represented as a non-linear and four degree-of-freedom (DOF) system which consists of a top platform (rigid body) and bottom platform (e.g., marine vessel). In this paper, for mathematical convenience the marine body is considered cuboid-shaped as shown Fig.2 (Damitha Sandaruwan (2009)). It is assumed that the two platforms are parallel and linked to each other by a set of non-linear damping actuators, which are located at the four cardinal points of the top platform. The random sea wave acts as a disturbance to the marine vessel so that the top platform vibrates.

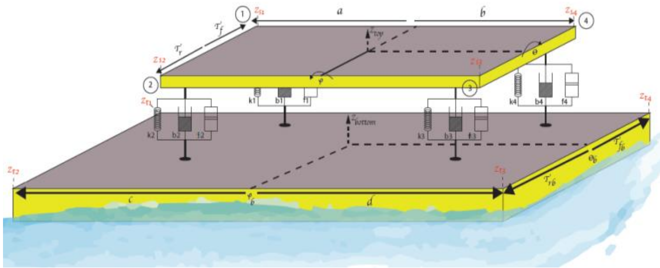

Fig.1. Full marine suspension system model

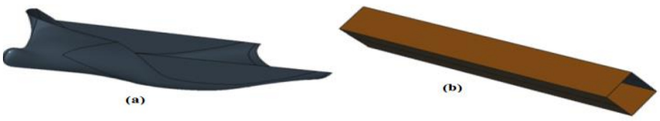

Fig. 2. Real and simplified shapes of marine vessel (Damitha Sandaruwan (2009)).

For this purpose, the well-known Newton's second law can be applied to formulate the mathematical motion expression of the platforms. The static equilibrium position acts at the origin and points at the direction of the angular displacement and the centre of gravity of the model.

Defining the notation $s_i = \sin i$ and $c_i = \cos i (i = \theta, \varphi, \theta_b, \varphi_b)$ the motion expressions of both platforms are presented below:

The dynamic model of the top platform along the heave motion can be described by the following:

$$\begin{gathered}\ddot{z}_t = \frac{1}{M_c}\{-(k_{s_1}+k_{s_2}+k_{s_3}+k_{s_4})z_t-(b_{s_1}+b_{s_2}+b_{s_3}+b_{s_4})\dot{z}_t-[-T_r \\ (k_{s_2}+k_{s_3})+T_f(k_{s_1}+k_{s_4})]s_\theta-[-T_r(b_{s_2}+b_{s_3})+T_f(b_{s_1}+b_{s_4})]\dot{\theta}c_\theta- \\ [-b(k_{s_4}+k_{s_3})+a(k_{s_1}+k_{s_2})]s_\varphi-[-b(b_{s_4}+k_{s_3})+a(b_{s_1}+b_{s_2})]\dot{\varphi}c_\varphi+ \\ (k_{s_1}+k_{s_2}+k_{s_3}+k_{s_4})z_b+(b_{s_1}+b_{s_2}+b_{s_3}+b_{s_4})\dot{z}_b+[T_{f_b}(k_{s_1}+k_{s_4}) \\ -T_{rb}(k_{s_2}+k_{s_3})]s_{\theta_b}+[-T_{r_b}(b_{s_2}+b_{s_3})+T_{f_b}(b_{s_1}+b_{s_4})]\dot{\theta}_b c_{\theta_b}+[c(k_{s_1} \\ +k_{s2})-d(k_{s_4}+k_{s_3})]s_{\varphi_b}+[c(k_{s_1}+k_{s2})-d(k_{s_4}+k_{s_3})]\dot{\varphi}_b c_{\varphi_b}+u_z\}\end{gathered} \tag{1}$$

where M_c is the mass of the top platform. In addition b_{s_i} and $k_{si} (i = 1,2,3,4)$ are the damping and stiffness coefficients, respectively. The length and width of each platforms are depicted into two sections with 'a' and 'b' representing the length of the top platform an T_r, T_f corresponding to the width. For the bottom platform, 'c' and 'd' corresponds to the length and T_{r_b}, T_{f_b} to its width.

Top platform Pitch motion

$$\begin{gathered}\ddot{\theta} = \frac{c_\theta}{I_\theta}\{-[T_f(k_{s_1}+k_{s_4})-T_r(k_{s_2}+k_{s_3})]z_t-[T_r{}^2(k_{s_2}+k_{s_3})+T_f{}^2 \\ (k_{s_1}+k_{s_4})]s_\theta-[a(T_f k_{s_1}-T_r k_{s_2})-b(T_f k_{s_4}-T_r k_{s_3})]s_\varphi-[T_f(b_{s_1} \\ +b_{s_4})-T_r(b_{s_2}+b_{s_3})]\dot{z}_t-[T_r{}^2(b_{s_2}+b_{s_3})+T_f{}^2(b_{s_1}+b_{s_4})\dot{\theta}c_\theta-[a \\ (T_f b_{s_1}-T_r b_{s_2})-b(T_f b_{s_4}-T_r b_{s_3})]\dot{\varphi}c_\varphi+[T_{f_b}(k_{s_1}+k_{s_4})-T_{r_b}(k_{s_2} \\ +k_{s_3})]z_b+[T_{r_b}{}^2(k_{s_2}+k_{s_3})+T_{f_b}{}^2(k_{s_1}+k_{s_4})]s_{\theta_b}+[c(T_{f_b}k_{s_1}-T_{r_b} \\ k_{s_2})-d(T_{f_b}k_{s_4}-T_{r_b}k_{s_3})]s_{\varphi_b}+[T_f(b_{s_1}+b_{s_4})-T_r(b_{s_2}+b_{s_3})]\dot{z}_b+ \\ [[T_{r_b}{}^2(b_{s_2}+b_{s_3})+T_{f_b}{}^2(b_{s_1}+b_{s_4})]\dot{\theta}_b c_{\theta_b}+[c(T_{f_b}b_{s_1}-T_{r_b}b_{s_2})-d(T_{f_b}b_{s_4}- \\ T_{r_b}b_{s_3})]\dot{\varphi}_b c_{\varphi_b}+u_\theta\end{gathered} \tag{2}$$

where I_θ is the moment of inertia for the top platform corresponding to the direction of 'θ'.

Top platform Roll motion

$$\begin{gathered}\ddot{\varphi} = \frac{c_\varphi}{I_\varphi}\{-[a(k_{s_1}+k_{s_2})-b(k_{s_4}+k_{s_3})]z_t-[a^2(k_{s_2}+k_{s_1})+b^2(k_{s_3}+ \\ k_{s_4})]s_\varphi-[T_f(ak_{s_1}-bk_{s4})-T_r(ak_{s2}-bk_{s_3})]s_\theta-[a(b_{s_1}+b_{s_2})-b \\ (b_{s_4}+b_{s_3})]\dot{z}_t-[T_f(ab_{s_1}-bb_{s4})-T_r(ab_{s2}-bb_{s_3})]\dot{\theta}c_\theta-[a^2(b_{s_2}+ \\ b_{s_1})+b^2(b_{s_3}+b_{s_4})]\dot{\varphi}c_\varphi+[c(k_{s_1}+k_{s_2})-d(k_{s_4}+k_{s_3})]z_b+[c^2(k_{s_2}+ \\ k_{s_1})+d^2(k_{s_3}+k_{s_4})]s_{\varphi_b}+[T_{f_b}(ck_{s_1}-dk_{s4})-T_{r_b}(ck_{s2}-dk_{s_3})]s_{\theta_b}+ \\ [c(b_{s_1}+b_{s_2})-d(b_{s_4}+b_{s_3})]\dot{z}_b+[T_{f_b}(cb_{s_1}-db_{s4})-T_{r_b}(cb_{s2}-db_{s_3})] \\ \dot{\theta}_b c_{\theta_b}+[c^2(b_{s_2}+b_{s_1})+d^2(b_{s_3}+b_{s_4})]\dot{\varphi}_b c_{\varphi_b}+u_\varphi\end{gathered} \tag{3}$$

with respect to the direction of 'φ', the moment of inertia for the top platform is represented as I_φ.

Bottom platform Heave motion

$$\begin{gathered}\ddot{z}_b = \frac{1}{M_{ct}+A_z}\{(k_{s_1}+k_{s_2}+k_{s_3}+k_{s_4})z_t+(b_{s_1}+b_{s_2}+b_{s_3}+b_{s_4})\dot{z}_t+[-T_r \\ (k_{s_2}+k_{s_3})+T_f(k_{s_1}+k_{s_4})]s_\theta+[-T_r(b_{s_2}+b_{s_3})+T_f(b_{s_1}+b_{s_4})]\dot{\theta}c_\theta+[-b \\ (k_{s_4}+k_{s_3})+a(k_{s_1}+k_{s_2})]s_\varphi+[-b(b_{s_4}+k_{s_3})+a(b_{s_1}+b_{s_2})]\dot{\varphi}c_\varphi-(k_{s_1}+k_{s_2} \\ +k_{s_3}+k_{s_4})z_b-(b_{s_1}+b_{s_2}+b_{s_3}+b_{s_4})\dot{z}_b-[T_{f_b}(k_{s_1}+k_{s_4})-T_{rb}(k_{s_2}+k_{s_3}) \\ ]s_{\theta_b}-[-T_{r_b}(b_{s_2}+b_{s_3})+T_{f_b}(b_{s_1}+b_{s_4})]\dot{\theta}_b c_{\theta_b}+[c(k_{s_1}+k_{s2})-d(k_{s_4}+k_{s_3}) \\ ]s_{\varphi_b}+[c(k_{s_1}+k_{s2})-d(k_{s_4}+k_{s_3})]\dot{\varphi}_b c_{\varphi_b}+F_h\}\end{gathered} \tag{4}$$

M_{ct}, A_z represent the mass and added mass of the bottom platform respectively.

Bottom platform Pitch motion

$$\begin{gathered}\ddot{\theta}_b = \frac{c_{\theta_b}}{I_{\theta_b}+A_\theta}\{[T_f(k_{s_1}+k_{s_4})-T_r(k_{s_2}+k_{s_3})]z_t+[T_r{}^2(k_{s_2}+k_{s_3}) \\ +T_f{}^2(k_{s_1}+k_{s_4})]s_\theta+[a(T_f k_{s_1}-T_r k_{s_2})-b(T_f k_{s_4}-T_r k_{s_3})]s_\varphi+ \\ [T_f(b_{s_1}+b_{s_4})-T_r(b_{s_2}+b_{s_3})]\dot{z}_t+[T_r{}^2(b_{s_2}+b_{s_3})+T_f{}^2(b_{s_1}+b_{s_4}) \\ \dot{\theta}c_\theta+[a(T_f b_{s_1}-T_r b_{s_2})-b(T_f b_{s_4}-T_r b_{s_3})]\dot{\varphi}c_\varphi-[T_{f_b}(k_{s_1}+k_{s_4})- \\ T_{r_b}(k_{s_2}+k_{s_3})]z_b-[T_{r_b}{}^2(k_{s_2}+k_{s_3})+T_{f_b}{}^2(k_{s_1}+k_{s_4})]s_{\theta_b}-[c(T_{f_b}k_{s_1} \\ -T_{r_b}k_{s_2})-d(T_{f_b}k_{s_4}-T_{r_b}k_{s_3})]s_{\varphi_b}-[T_f(b_{s_1}+b_{s_4})-T_r(b_{s_2}+b_{s_3})] \\ \dot{z}_b-[T_{r_b}{}^2(b_{s_2}+b_{s_3})+T_{f_b}{}^2(b_{s_1}+b_{s_4})]\dot{\theta}_b c_{\theta_b}-[c(T_{f_b}b_{s_1}-T_{r_b}b_{s_2})-d(T_{f_b} \\ b_{s_4}-T_{r_b}b_{s_3})]\dot{\varphi}_b c_{\varphi_b}+M_p\end{gathered} \tag{5}$$

where I_{θ_b}, A_θ represent the mass and added mass of the bottom platform in the θ_b direction.

Bottom platform Roll motion

$$\begin{gathered}\ddot{\varphi}_b = \frac{c_{\varphi_b}}{I_{\varphi_b}+A_\varphi}\{[a(k_{s_1}+k_{s_2})-b(k_{s_4}+k_{s_3})]z_t+[a^2(k_{s_2}+k_{s_1})+b^2 \\ (k_{s_3}+k_{s_4})]s_\varphi+[T_f(ak_{s_1}-bk_{s4})-T_r(ak_{s2}-bk_{s_3})]s_\theta+[a(b_{s_1}+ \\ b_{s_2})-b(b_{s_4}+b_{s_3})]\dot{z}_t+[T_f(ab_{s_1}-bb_{s4})-T_r(ab_{s2}-bb_{s_3})]\dot{\theta}c_\theta+[a^2 \\ (b_{s_2}+b_{s_1})+b^2(b_{s_3}+b_{s_4})]\dot{\varphi}c_\varphi-[c(k_{s_1}+k_{s_2})-d(k_{s_4}+k_{s_3})]z_b-[c^2 \\ (k_{s_2}+k_{s_1})+d^2(k_{s_3}+k_{s_4})]s_{\varphi_b}-[T_{f_b}(ck_{s_1}-dk_{s4})-T_{r_b}(ck_{s2}-d \\ k_{s_3})]s_{\theta_b}-[c(b_{s_1}+b_{s_2})-d(b_{s_4}+b_{s_3})]\dot{z}_b-[T_{f_b}(cb_{s_1}-db_{s4})-T_{r_b}(c \\ b_{s2}-db_{s_3})]\dot{\theta}_b c_{\theta_b}-[c^2(b_{s_2}+b_{s_1})+d^2(b_{s_3}+b_{s_4})]\dot{\varphi}_b c_{\varphi_b}+M_r\end{gathered} \tag{6}$$

$I_{\varphi b}$ and A_{φ} are the moments of inertia and added mass for the bottom platforms in the φ_b disrection. The reader is referred to (Damitha Sandaruwan (2009); Jianjun Long (2008)) for an explanation of how $A_i(i = z, \varphi, \theta)$ is calculated.
The dynamics of the non-linear suspension system, which are described by equations (1-6), are written in the following standard state space form:
The dynamics of the non-linear suspension system, which are described by equations (1-6), are written in the following standard state space form:

$$\begin{cases} \dot{x}(t) = f_i(x(t)) + g_i u_i(t) + d(t) \\ y(t) = hx(t)(i = z_t, z_b, \varphi, \theta, \varphi_b, \theta_b) \end{cases} \tag{7}$$

where further notations are used as follows:

$x_1 = z_t$ Heave motion of the top platform.
$x_2 = \dot{z}_t$ Heave velocity of the top platform.
$x_3 = \theta$ Pitch angle of the top platform.
$x_4 = \dot{\theta}$ Pitch angular velocity of the top platform.
$x_5 = \varphi$ Roll angle of the top platform.
$x_6 = \dot{\varphi}$ Roll angular velocity of the top platform.
$x_7 = z_b$ Heave motion of the bottom platform.
$x_8 = \dot{z}_b$ Heave velocity of the bottom platform.
$x_9 = \theta_b$ Pitch angle of the bottom platform.
$x_{10} = \theta_b$ Pitch angular velocity of the bottom platform.
$x_{11} = \varphi_b$ Roll angle of the bottom platform.
$x_{12} = \dot{\varphi}_b$ Roll angular velocity of the bottom platform.

$d(t) = (F_h(t), M_p(t), M_r(t))^T$ represents the vector of external disturbances induced to the active system due to the sea wave irregularities

3. APPLICATION OF SLIDING MODE CONTROL

The designed control strategy is a sliding mode controller that aims to reduce the disturbances. The control input signal applied to the system aims to achieve an asymptotically stable response in the presence of model uncertainty and disturbances. Equations (1-6) present the model of the system. The output of the system is considered as: $y_1 = x_1, y_2 = x_3$ and $y_3 = x_5$ .As such, the error $e(t)$ between the actual and the desired trajectory can be written as:

$$e_i = y_i - y_{ref,i} \qquad i = 1,2,3 \tag{8}$$

where set-points $y_{ref,i}$ is assumed to be zero as the object lies horizontally on the platform. The switching surface s is defined as:

$$s = \dot{e} + \lambda e = \begin{cases} \dot{e}_{z_t} + \lambda_z e_z \\ \dot{e}_\theta + \lambda_\theta e_\theta \\ \dot{e}_\varphi + \lambda_\varphi e_\varphi \end{cases} \tag{9}$$

where λ is a positive constant. The time derivative of $'s'$ is given by:

$$\dot{s} = \ddot{e}_i + \lambda_i \dot{e}_i = f_i + g_i u_i + \lambda_i \dot{e}_i \tag{10}$$

The equivalent control obtained is selected in a way, such that $\dot{s}_i = 0 (i = z_t, \varphi, \theta)$ when $s_i = 0$ as follows:

$$\begin{aligned} u_{z_t,eq} = &-\{-(k_{s_1} + k_{s_2} + k_{s_3} + k_{s_4})z_t - (b_{s_1} + b_{s_2} + b_{s_3} + b_{s_4})\dot{z}_t - [-T_r \\ &(k_{s_2} + k_{s_3}) + T_f(k_{s_1} + k_{s_4})]s_\theta - [-T_r(b_{s_2} + b_{s_3}) + T_f(b_{s_1} + b_{s_4})]\dot{\theta} c_\theta - \\ &[-b(k_{s_4} + k_{s_3}) + a(k_{s_1} + k_{s_2})]s_\varphi - [-b(b_{s_4} + k_{s_3}) + a(b_{s_1} + b_{s_2})]\dot{\varphi} c_\varphi + \\ &(k_{s_1} + k_{s_2} + k_{s_3} + k_{s_4})z_b + (b_{s_1} + b_{s_2} + b_{s_3} + b_{s_4})\dot{z}_b + [T_{f_b}(k_{s_1} + k_{s_4}) \\ &- T_{rb}(k_{s_2} + k_{s_3})]s_{\theta_b} + [-T_{r_b}(b_{s_2} + b_{s_3}) + T_{f_b}(b_{s_1} + b_{s_4})]\dot{\theta}_b c_{\theta_b} + [c(k_{s_1} \\ &+ k_{s2}) - d(k_{s_4} + k_{s_3})]s_{\varphi_b} + [c(k_{s_1} + k_{s2}) - d(k_{s_4} + k_{s_3})]\dot{\varphi}_b c_{\varphi_b}\} - M_c \lambda \dot{e}_z \end{aligned} \tag{11}$$

$$\begin{aligned} u_{\theta,eq} = &-\{-[T_f(k_{s_1} + k_{s_4}) - T_r(k_{s_2} + k_{s_3})]z_t - [T_r{}^2(k_{s_2} + k_{s_3}) + T_f{}^2 \\ &(k_{s_1} + k_{s_4})]s_\theta - [a(T_f k_{s_1} - T_r k_{s_2}) - b(T_f k_{s_4} - T_r k_{s_3})]s_\varphi - [T_f(b_{s_1} \\ &+ b_{s_4}) - T_r(b_{s_2} + b_{s_3})]\dot{z}_t - [T_r{}^2(b_{s_2} + b_{s_3}) + T_f{}^2(b_{s_1} + b_{s_4})\dot{\theta} c_\theta - [a \\ &(T_f b_{s_1} - T_r b_{s_2}) - b(T_f b_{s_4} - T_r b_{s_3})]\dot{\varphi} c_\varphi + [T_{f_b}(k_{s_1} + k_{s_4}) - T_{r_b}(k_{s_2} \\ &+ k_{s_3})]z_b + [T_{r_b}{}^2(k_{s_2} + k_{s_3}) + T_{f_b}{}^2(k_{s_1} + k_{s_4})]s_{\theta_b} + [c(T_{f_b} k_{s_1} - T_{r_b} \\ &k_{s_2}) - d(T_{f_b} k_{s_4} - T_{r_b} k_{s_3})]s_{\varphi_b} + [T_f(b_{s_1} + b_{s_4}) - T_r(b_{s_2} + b_{s_3})]\dot{z}_b + \\ &[[T_{r_b}{}^2(b_{s_2} + b_{s_3}) + T_{f_b}{}^2(b_{s_1} + b_{s_4})]\dot{\theta}_b c_{\theta_b} + [c(T_{f_b} b_{s_1} - T_{r_b} b_{s_2}) - d(\\ &T_{f_b} b_{s_4} - T_{r_b} b_{s_3})]\dot{\varphi}_b c_{\varphi_b}\} - \frac{I_\theta \lambda \dot{e}_\theta}{c_\theta} \end{aligned} \tag{12}$$

$$\begin{aligned} u_{\varphi,eq} = &-\{-[a(k_{s_1} + k_{s_2}) - b(k_{s_4} + k_{s_3})]z_t - [a^2(k_{s_2} + k_{s_1}) + b^2(k_{s_3} \\ &+ k_{s_4})]s_\varphi - [T_f(ak_{s_1} - bk_{s4}) - T_r(ak_{s2} - bk_{s_3})]s_\theta - [a(b_{s_1} + b_{s_2}) - \\ &b(b_{s_4} + b_{s_3})]\dot{z}_t - [T_f(ab_{s_1} - bb_{s4}) - T_r(ab_{s2} - bb_{s_3})]\dot{\theta} c_\theta - [a^2(b_{s_2} \\ &+ b_{s_1}) + b^2(b_{s_3} + b_{s_4})]\dot{\varphi} c_\varphi + [c(k_{s_1} + k_{s_2}) - d(k_{s_4} + k_{s_3})]z_b + [c^2(k_{s_2} \\ &+ k_{s_1}) + d^2(k_{s_3} + k_{s_4})]s_{\varphi_b} + [T_{f_b}(ck_{s_1} - dk_{s4}) - T_{r_b}(ck_{s2} - dk_{s_3})]s_{\theta_b} \\ &+ [c(b_{s_1} + b_{s_2}) - d(b_{s_4} + b_{s_3})]\dot{z}_b + [T_{f_b}(cb_{s_1} - db_{s4}) - T_{r_b}(cb_{s2} - db_{s_3} \\ &)]\dot{\theta}_b c_{\theta_b} + [c^2(b_{s_2} + b_{s_1}) + d^2(b_{s_3} + b_{s_4})]\dot{\varphi}_b c_{\varphi_b}\} - \frac{I_\varphi \lambda \dot{e}_\varphi}{c_\varphi} \end{aligned} \tag{13}$$

To analyse the system uncertainties and to introduce a control law, proportional rate control is imposed by selecting the second control term as:

$$u^* = -kg_i^{-1}\,\mathrm{sgn}(s_i) \qquad i = z_t, \theta, \varphi \tag{14}$$

where $k \succ 0$ is the control gain. Details are given as:

$$u^*_{z_t} = -\frac{k}{M_c}\mathrm{sgn}(s_{z_t}) \tag{15}$$

$$u^*_\theta = -\frac{k\cos\theta}{I_\theta}\mathrm{sgn}(s_\theta) \tag{16}$$

$$u^*_\varphi = -\frac{k\cos\varphi}{I_\varphi}\mathrm{sgn}(s_\varphi) \tag{17}$$

As a result, the control inputs are:

$$u_i = u_i^* + u_{i,eq} = g_i^{-1}(-f_i - \lambda_i \dot{e}_i - k\,\mathrm{sgn}(s_i)) \tag{18}$$

For the controlled system to reach the desired objective u_z^d, u_θ^d and u_φ^d can be interpreted as the desired force in the $'z'$ direction and in the direction of the given torques θ and φ. However, the relationship between these three inputs and the four actuator forces is given as:

$$\begin{bmatrix} u_{z_t} \\ u_\theta \\ u\varphi \end{bmatrix} = \begin{bmatrix} -1 & -1 & -1 & -1 \\ T_f & -T_r & -T_r & T_f \\ a & a & -b & -b \end{bmatrix}^{\dagger} \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{bmatrix} \tag{19}$$

where $[.]^{\dagger}$ is the pseudo-inverse of the matrix[.].If the control law is determined using (18), then the sliding mode is asymptotically stable. Indeed, we can select the Lyapunov function $V = \frac{1}{2} ss^T$, then using equations (10) and (18) it can be seen that $\dot{V} = s^T s = -s^T k \,\mathrm{sgn}(s) \le 0$.

4. FORCE AND MOMENT OF WAVE DISTURBANCE

The disturbance considered for the steady and random system is a type of sub-wave described by (20), which represents the height of water surface along the z axis.

$$\zeta(x_{i,j}, y_{i,j}, t) = \sum_{i=1}^{n} \zeta_{ai} \sin a_i [(x\cos\psi_i + y\sin\psi_i - \omega_i t + \varepsilon_i)] \tag{20}$$

where a_i is the wave number $(a_i = \frac{2\Pi}{\gamma_i})$ and γ_i is the wavelength, the phase ε_i of each component is chosen to be a random variable with uniform distribution between interval [0 2Π] and ω_i is denoted as the circular wave frequency. The angle between X axis and direction of the wave is ψ.

4.1. HEAVE MOTION

For non-zero heights, the rise in the water surface causes a force which pushes the model up or down. The sum, H_a of height fields is given by (21), where $\zeta_{i,j}$ represents the height fields projected vertically along the bottom platform.

$$H_a = \sum_{c}^{d} \sum_{Tf_b}^{T_{rb}} \zeta_{i,j} \tag{21}$$

To obtain the heave motion force, H_a is multiplied by the water density, ρ_w.The net heave motion, F_h is computed as:

$$F_h = H_a \rho_w - R_h \tag{22}$$

where $R_h = K_h M_{ct} |w|^2$ is the resistance force, $K_h, M_{ct}, \dot{w}(= \frac{F_h}{M_{ct}}), w$ are the resistance coefficient for heave motion, the mass of bottom platform, heave acceleration and heave velocity respectively.

4.2 PITCH MOTION

In this work, the pitch is determined by the height difference between the front and the rear of the vessel. The height field deference between the front and rear H_p is given as:

$$H_p = \sum_{c}^{d} \sum_{Tf_b}^{T_{rb}} \zeta_{i,j} \frac{i}{|i|} \tag{23}$$

The net moment of pitch force, M_p is then given as:

$$M_p = K_p{}' H_p \rho_w - R_p \tag{24}$$

where the resistance force against pitch action, $R_p = K_p I_{\varphi b} q$; K_p and $K_p{}'$ are the resistance coefficient of pitch and coefficient for pitch motion respectively.

4.3 ROLL MOTION

The roll is determined using the difference in height field between the port side and starboard side of the vessel. The height field deference between the port and starboard H_r is given as:

$$H_r = \sum_{c}^{d} \sum_{Tf_b}^{T_{rb}} \zeta_{i,j} \frac{j}{|j|} \tag{25}$$

See (Damitha Sandaruwan (2009); YuanhuiWang(2010))for more explanation.

5. SIMULATION

A sea wave was simulated in an indoor pool (assuming zero wind speed) with given dimensions 10m by 10m by 8m, (Fig.3). Two cases were simulated for this pool size using the wave disturbance equations proposed in section 4. The first case has maximum amplitude of 0.8m whilst the second amplitude is set at a maximum of 1.6m. The addition of heave and pitch motions makes the simulation more realistic than just using roll motion as large objects in water experience forces in all three dimensions (Y.P. Xiong (2005)).The aim is to produce a stable wave that would not result in capsizing of ship or overturning of marine bodies. Consequently, the maximum simulated heights were chosen to ensure bodies remain stable.

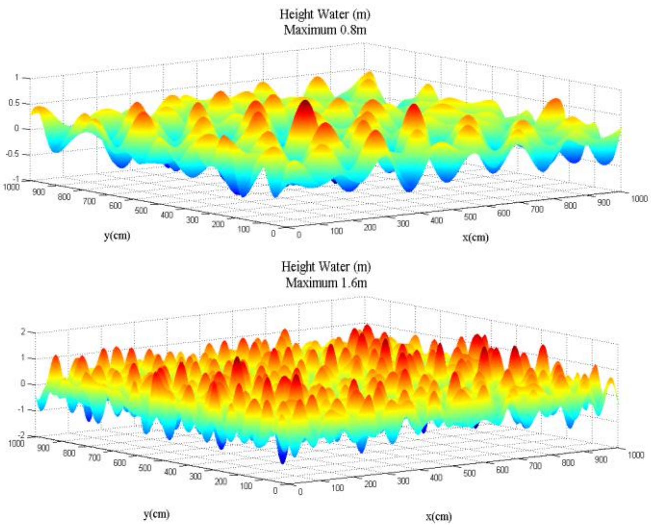

Fig.3. Simulated random sea wave

The simulation parameters for sliding mode control design are shown in Table 1:

Table 1. Design constants

Parameter	Description	value
$\lambda_z, \lambda_\theta, \lambda_\varphi$	Sliding surface slops	1e1
k_z, k_θ, k_φ	Heaven ,Pitch and Roll gains	25e2

Table.2 shows the parameter values used in the proposed model shown in figure 1

Table.2. System parameter values

Parameter	Value	Parameter	Value
M_c	15_{kg}	I_θ	21.6_{kgm}^2
I_φ	$4.6\ _{kgm}^2$	M_{ct}	80_{kg}
$I_{\theta b}+A_\theta$	$145.8\ _{kgm}^2$	$I_{\varphi b}+A_\varphi$	$378.2\ _{kgm}^2$
a	1m	b	1m
c	2m	d	2m
T_f	1m	T_r	1m
T_{fb}	2m	T_{rb}	2m
k_{s_1}, k_{s_4}	$2700\ \frac{N}{m}$	k_{s_2}, k_{s_3}	$2300\ \frac{N}{m}$
b_{s_2}, b_{s_3}	$150\ \frac{N}{m}$	b_{s_2}, b_{s_3}	$120\ \frac{N}{m}$

The parameters representing the marine body(M_{ct}, $I_{\theta b}$, $I_{\varphi b}$) were chosen based on the physical specifications of a 420 (dinghy) sail boat (without sail). These dimensions were used to simulate a bottom platform of the same weight, length and height. The chosen specifications guaranty that the system floats.

6. RESULTS AND DISCUSSION

This section shows the performance of the control design on the two simulated (uncontrolled) cases with maximum heights of 0.8 and 1.6 m respectively.

Fig.4 and Fig.5 show the uncontrolled displacement of the simulated rigid body. It can be seen that the amplitude peaks, at 25cm, are higher than the 6cm limit where vibrations are absorbed for a body with these specifications. The fluctuations are brought by the action of the sea wave on marine body. Similarly, the pitch and roll angles are seen to be very high, much higher than what should be expected for a body that is largely horizontal.

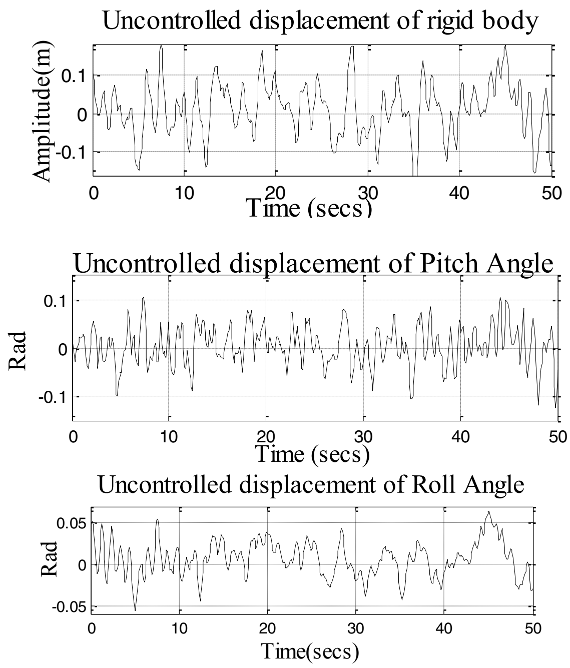

Fig.4. Uncontrolled response of heave, pitch and roll with maximum height water 0.8m

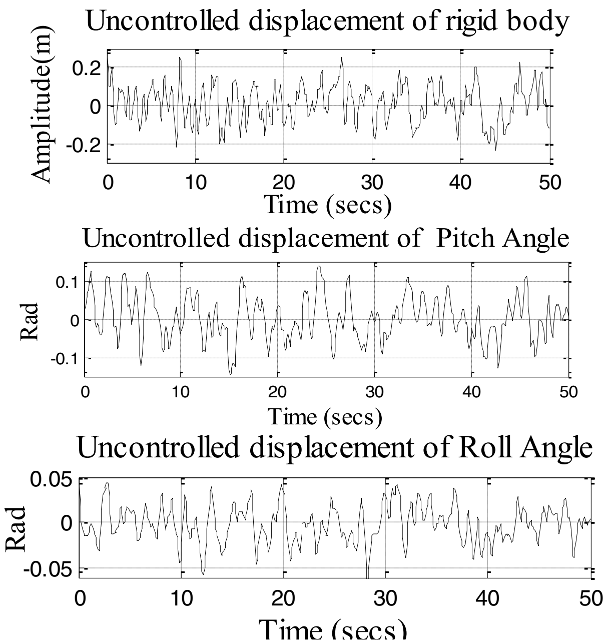

Fig.5. Uncontrolled response of heave, pitch and roll with maximum height water 1.6 m

The controlled action shown in Figs.6-7 is much more improved. The maximum amplitude of the displacement is seen to be within the desirable 15cm limit thus ensuring stability. The pitch and roll angles are also considerably smaller, making the body roughly horizontal. The improved control action on the pitch angle is due to its increased challenge of control over roll angles which is normally measured in a perpendicular plane to the plane of the sea wave. The output response indicates transient stability and suppression of vibration.

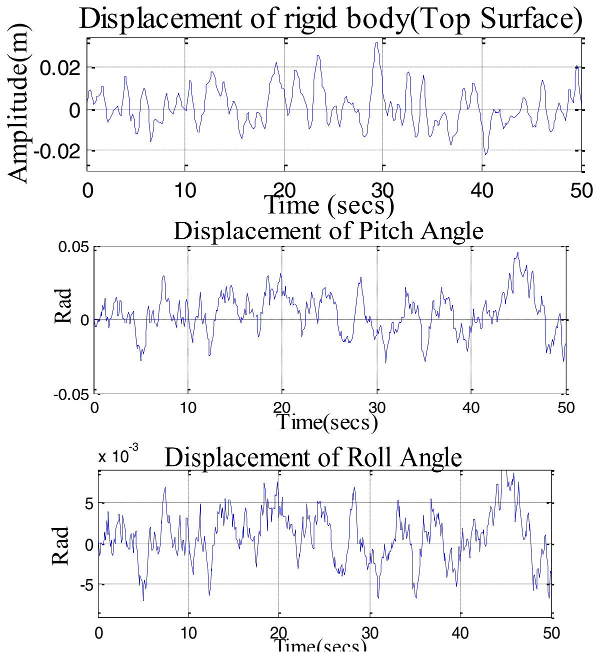

Fig.6. The response of heave, pitch and roll (Output) with maximum height water 0.8m

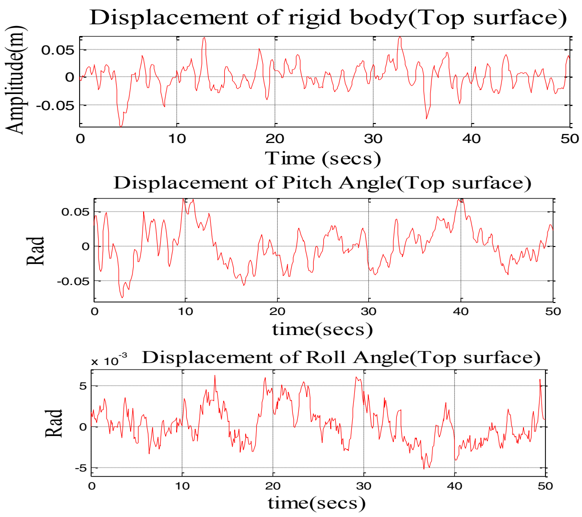

Fig.7. The response of heave, pitch and roll (Output) with maximum height water 1.6m

In Fig.8 and Fig.9 the control actions work to dampen the displacement amplitude. Looking at the peak force values (which coincide with the peak displacements in the uncontrolled case), it can be seen that the actuators generate as much force as required to stabilise the body.

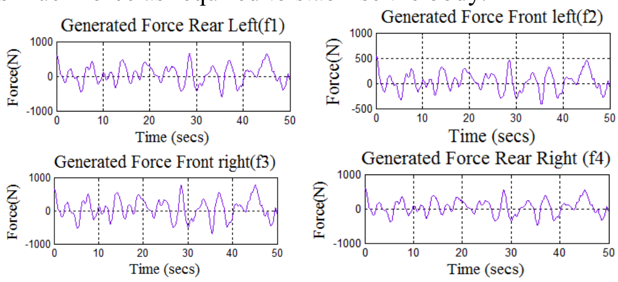

Fig.8. Actuator forces (maximum height water 0.8m)

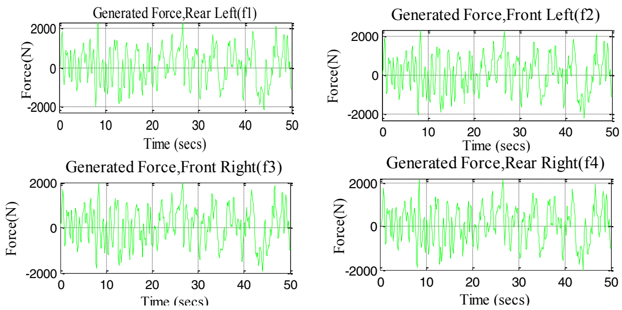

Fig.9. Actuator forces (maximum height water 1.6m)

7. CONCLUSIONS

The work presented in this paper considers a rigid body subjected to a simulated sea wave disturbance. The existence of structural resonance within the marine vessel and the rigid body can lead to instability, therefore a sliding model controller is designed to solve this problem. Simulation results shows that the sea wave disturbance acting on the designed dynamic model indicates unacceptable output responses, therefore a sliding mode controller is designed and used to reduce the vibration caused by the disturbances. Results presented have been carried out to show the effectiveness of the proposed algorithm, where significant improvement on ensuring the flatness have been observed.

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