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# Process Identification Using Relay Feedback with a Fractional Order Integrator

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**Abstract:** In this work, a novel relay feedback experiment method is proposed for system identification by connecting a fractional order integrator in front of a basic relay component. With this approach, a linear time invariant process model can be estimated at an arbitrary phase angle in the third or fourth quadrant on the Nyquist curve. Another contribution of this paper is to present a pseudo frequency response concept to generalize the describing function method so as to make it compatible with the frequency response operations. The proposed setup is illustrated through mathematical derivation using the proposed concept. Simulations are provided to verify the effectiveness.

## 1. INTRODUCTION

The relay feedback approach is one of the most commonly used control schemes in the industrial automation. Generally, it has two uses. One is for auto tuning the controller parameters and the other is for model identification. Two detailed surveys on the development of the relay feedbacks can be found in [1, 2]. More recent advances are available in [3]. As this practical-to-implement technique receives so much research attention, more than ten types of its variant have been created in the past two decades. In this paper, inspired by fractional order (FO) modeling and control, a new variant is proposed with unique advantages over other types of variants.

The fractional order modeling and control is an emerging and fast growing topic in recent years [5, 6], whose concept can be briefly stated as to model systems and design controllers with the use of fractional calculus [7]. By taking this effort, the modeling and controller design are equipped with more freedom, which, therefore, can usually provide superior performance than the traditional integer order ones under the same condition. The research in this paper is another exploration in this big trend. In Jeoguk *et, al*'s work [4], a seemingly similar research was carried out in terms of the block diagram connection. However, the big differences lies in the profile of control signals because the FO integrator in [4] is connected behind the relay, which consequently, generates control signals in the "FO integral's" shape instead of the square waveforms directly generated by relays. From this point of view, the setup in the present paper is more loyal to the original relay feedback idea.

For the analysis of relay feedback identification, the describing function (DF) has been the dominant approach. It is the main approach for approximating a linear equivalence of the relay nonlinearities. In this paper, the notion of pseudo frequency response (PFR) is promoted for the input dependant DF so as to demonstrate its operation with the actual frequency response of a linear element. This is feasible because after the approximation with regards to a particular input, the nonlinearity has lost. Hence, the DF is essentially a complex number which can be treated as a gain and phase shift effect on the linear elements. Under this framework, the derivation of different types of the relay feedback variants can be generalized to a universal form. The origin of this idea is inspired by the pseudo transfer function concepts for some special scenarios in model identification, [8, 9].

The rest of the paper is organized as follows. First, the DF method is briefly stated for introducing the PFR concept. Then, five types of relay feedback identification methods are reviewed with this PFR concept. Comments are given on some practical details that are usually ignored in the literature. Afterwards, the proposed relay feedback setup is presented with the highlighted advantages and limitations. Finally, simulations are provided to verify the effectiveness of this setup. A comparison among different types of relay variants for a sample test run is summarized.

## 2. REVIEW OF THE RELAY FEEDBACK IDENTIFICATION AND ITS VARIANTS

### *2.1 The pseudo frequency response*

The describing function of a memoryless nonlinearity, $\psi$ is defined to be the ratio of the first harmonic of its output to that of its input, [10]. An illustrative block diagram is shown in figure 1. Let $v_n(t)$ and $u_n(t)$ denote

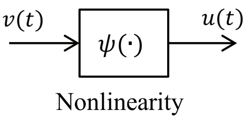



Fig. 1. The nonlinearity block representation.

the Fourier series of the periodic input $v(t)$, and output $u(t)$, respectively,

$$v_n(t) = a_{0_v} + \sum_{k=1}^{\infty}[a_{k_v}\cos(k\omega t) + b_{k_v}\sin(k\omega t)], \tag{1}$$

$$u_n(t) = a_{0_u} + \sum_{k=1}^{\infty}[a_{k_u}\cos(k\omega t) + b_{k_u}\sin(k\omega t)], \tag{2}$$

where $a_k$ and $b_k$ are the triangular form Fourier coefficients, with the footnote $u$ and $v$ for output and input respectively. Then, the describing function of $\psi$ is

$$\Psi = \frac{c_{1_u}}{c_{1_v}}, \tag{3}$$

where $c_{1_u}$ and $c_{1_v}$ are the exponential form Fourier coefficients of the fundamental frequency terms,

$$c_{1_u} = \frac{a_{1_u} - jb_{1_u}}{2}, \quad c_{1_v} = \frac{a_{1_v} - jb_{1_v}}{2}.$$

Conventionally, the DF, by default, takes the assumption of a sine wave input, when which is not satisfied a re-derivation of the DF is usually required, [11]. In this work, the notion of pseudo frequency response $\mathbb{G}(u(t))$ is used to replace the input dependant DF in approximating the frequency characteristic of the nonlinear elements, regardless of whether or not the input is a sine wave.

As an example, for the typical relay nonlinearity depicted by the sign function,

$$u = \psi(v) = sgn(v)H = \begin{cases} H, & v \geq 0, \\ -H, & v < 0, \end{cases} \tag{4}$$

its PFR to a cosine wave input is:

$$\mathbb{G}(A\cos(\omega t)) = \frac{\frac{4H}{2\pi} - 0j}{\frac{A}{2} - 0j} = \frac{4H}{\pi A}, \tag{5}$$

which is the same with its PFR to a sine wave input. However, this equality is a coincidence which doesn't always hold true for complex nonlinearities such as non-symmetric ones.

From this generalized point of view, the frequency response of a linear element can be treated as an input *independent* PFR. Next, different kinds of relays are reviewed with this concept.

### *2.2 The ideal relay feedback*

In 1984, Åström and Hägglund introduced the relay feedback technique for automatic tuning the PID controllers by bringing the system to a self-sustained oscillation [12]. This method was then extended by Luyben to identify a transfer function of a distillation process [13].

Assume the process to be identified can be approximated by a linear time invariant (LTI) model $G(s)$; then, a

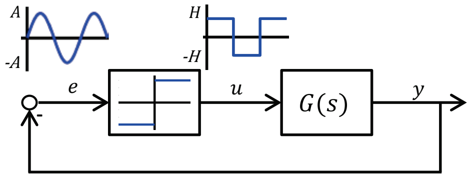



Fig. 2. The block diagram of the ideal relay feedback.

schematic of the relay feedback block diagram is drawn in figure 2. As mentioned in the previous section, the ideal relay with an amplitude of $H$ has a PFR as: $\mathbb{G}_{ideal}(A\sin(\omega t)) = \frac{4H}{\pi A}$. Considering the negative unit feedback, it can push the process to the so called critical oscillation, i.e. the $(-1, 0)$ point on the Nyquist curve shown in figure 3,

$$|\mathbb{G}_{ideal}(A\sin(\omega_u t))G(j\omega_u)| = 1, \tag{6}$$

where $\omega_u$ is the frequency of the process at the oscillation point which is called the ultimate frequency. Thus, the process gain at the phase $\varphi_p = -\pi$ is,

$$|G(j\omega_u)| = \frac{\pi A}{4H}. \tag{7}$$

Depending on the LTI model structures, corresponding parameters can be calculated based on this frequency information. In this paper, first order plus dead time (FOPDT) model is addressed,

$$G(s) = \frac{K}{Ts+1}e^{-Ls}. \tag{8}$$

The parameters $K, L, T$ can be calculated by the following formulae,

$$K = G(0), \tag{9}$$

$$T = \frac{\sqrt{(KK_u)^2 - 1}}{\omega_u}, \tag{10}$$

$$L = \frac{1}{\omega_u}(-\varphi_p - \arctan\sqrt{(KK_u)^2 - 1}), \tag{11}$$

where $K_u = \frac{1}{|G(j\omega_u)|}$ is the ultimate gain, and $K$ is the steady state gain that can be obtained through varieties of methods, such as reading off from a step test or the ratio of the integration of output to input, [16]. The equations for computing other model structures can be referred to [14, 15].

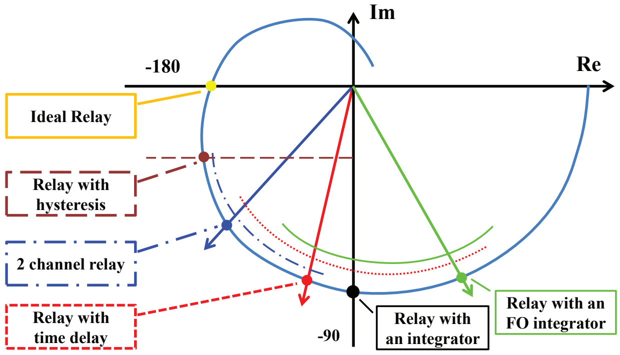



Fig. 3. The frequency response points on the Nyquist curve identified by different types of relay variants.

### *2.3 The relay with hysteresis*

The ideal relay setup is straightforward and simple to implement, but a concern lies in the practical response of the processes with small delay or no delay. In this case, the frequency response has hardly or even no intersection with the negative real axis on the Nyquist plot until reaching very high frequency. Thus, the relay with hysteresis shown in figure 4(a) is used to assure the appearance of the sustained oscillation within reasonable frequency ranges of the industrial processes.

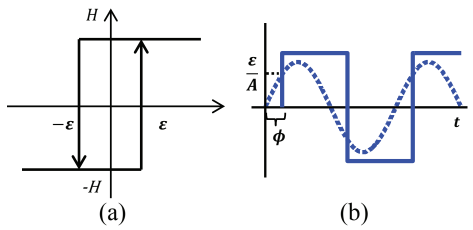



Fig. 4. The schematic of the relay with hysteresis. (a) The nonlinear characteristic; (b) the input and output.

The effect of the hysteresis onto the input signal is like a delay, as shown in figure 4(b). Hence, by evaluating the input and output, the PTF of the relay with hysteresis can be obtained,

$$\mathbb{G}_{hyst}(A\sin(\omega t)) = \frac{4H}{\pi A}e^{-j\phi}, \tag{12}$$

where $\phi = \arcsin(\frac{\varepsilon}{A})$. Again, considering the negative unit feedback, it can be seen, in figure 3, that the frequency response point identified by the relay with hysteresis is $\frac{4H}{\pi A}\angle(-\pi+\phi)$, at which the gain and phase of the process are,

$$|G(j\omega_\phi)| = \frac{\pi A}{4H} \quad \text{and} \quad \varphi_p = -\pi + \phi, \tag{13}$$

respectively. Note that the hysteresis must satisfy $0 < \varepsilon < A$ because otherwise, the relay will output a constant zero. This condition limits the identifiable process phase within the range of $(-\frac{\pi}{2}, -\pi)$.

The same set of equations (9∼11) can be used to compute the parameters by substituting the phase and oscillation frequency if an FOPDT model is expected.

### *2.4 The relay with time delay*

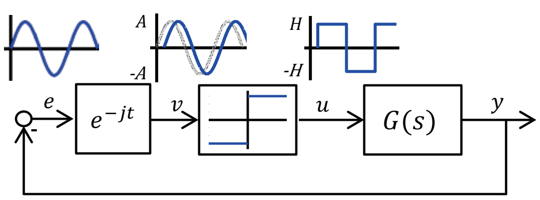



Fig. 5. The block diagram of the relay feedback setup with time delay.

Similar to the relay with hysteresis, the relay with time delay presented in [17] has the same effect of inserting a certain phase shift between the input and output, and it makes no difference whether the delay is placed in front of or behind the relay. The PFR for this type of relay when input is a sine wave is,

$$\mathbb{G}_{delay}(A\sin(\omega t)) = \frac{4H}{\pi A}e^{-jl}, \tag{14}$$

where $l > 0$ is the artificially inserted time delay as shown in figure 5. This PFR is the same with equation (12) except that the delayed time is irrelevant to the amplitude of the input. Without this limitation, the time delay can provide a wider range of phase shift than the hysteresis. Hence, the gain and phase of the process at the identified frequency point are,

$$|G(j\omega_l)| = \frac{\pi A}{4H} \quad \text{and} \quad \varphi_p = -\pi + l. \tag{15}$$

### *2.5 The relay with an integrator*

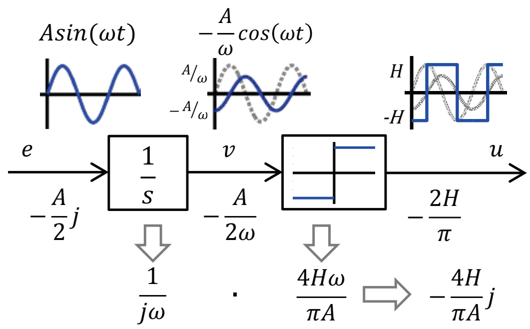



Fig. 6. The block diagram and the signal transform of the relay with an integrator in the front.

The relay feedback with an integrator is another way to identify a process at a frequency response point other than the critical oscillation on the Nyquist curve [18]. When the integrator is connected in front of the relay, the PFR can be obtained by multiplying the frequency response of the integrator with the PFR of the ideal relay, as shown in figure 6,

$$\begin{aligned}\mathbb{G}_{int}(A\sin(\omega t)) &= \frac{1}{j\omega}\mathbb{G}_{ideal}(-\frac{A}{\omega}\cos(\omega t)) \\ &= -\frac{4H}{\pi A}j.\end{aligned} \tag{16}$$

Graphically, it appends an additional $\frac{\pi}{2}$ phase lag to the process output. Hence, the phase of the process at the identified frequency response point is $\varphi_p = -\frac{\pi}{2}$, which is also illustrated in figure 3.

As another variant to this method, the integrator connected behind the relay can be found in [19]. Theoretically, by connecting a differentiator, a point on the positive imaginary axis in figure 3 can be identified, mentioned in [20]. However, this is not widely used in practice.

### *2.6 The two channel relay feedback*

The two channel (TC) relay feedback introduced by Waller *et. al* in 1997 suggests a parallel connection of the ideal relay and the relay with an integrator, [20]. In the similar manner of manipulating transfer functions, the PFR of the TC relay can be obtained by adding up the PFR of the ideal relay in equation (5) and that of the relay with integrator in equation (16),

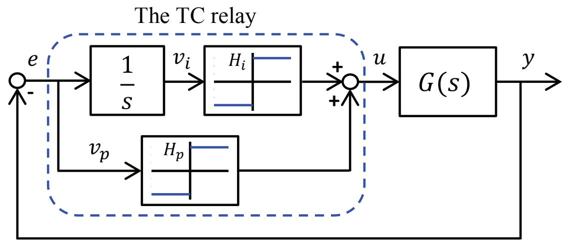



Fig. 7. The block diagram of the TC relay controlled process.

$$\mathbb{G}_{TC}(A\sin(\omega t)) = \mathbb{G}_{ideal}(A\sin(\omega t)) + \mathbb{G}_{int}(A\sin(\omega t))$$
$$= \frac{4H_p}{\pi A} - \frac{4H_i}{\pi A}j. \tag{17}$$

where $H_p$ and $H_i$ are the amplitude of the ideal relay and the integral relay respectively, as shown in figure 7. This can be verified by evaluating the Fourier series of the input and output of the overall setup in the dashed lined box.

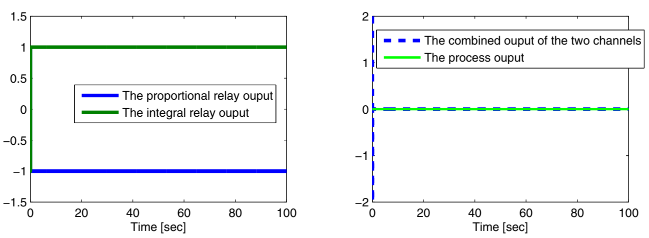
Continued with equation (17), the gain and phase of the process identified by the TC relay feedback are,

$$|G(j\omega_{TC})| = \frac{\pi A}{4\sqrt{H_p^2 + H_i^2}}, \tag{18}$$

$$\varphi_p = -\pi + \arctan(\frac{H_i}{H_p}). \tag{19}$$

It is easy to see that by varying $H_p$ and $H_i$, the frequency response point to be identified can be arbitrarily selected within the third quadrant, yet, only in the third quadrant because $\arctan(\frac{H_i}{H_p}) \in (0, \frac{\pi}{2})$.

A drawback of the TC relay is that when $H_p = H_i$, it cannot be used to identify processes with little time delay, because the output of the two channels will cancel each other instead of bringing the system to oscillation, as demonstrated in figure 8. That means it cannot identify lots of processes at the phase of $-\frac{3}{4}\pi$ in practice.




(a) The two channels (b) The outputs

Fig. 8. A demonstration of the TC relay output cancelation.

Besides the aforementioned setups, there are other variants of the basic relay feedbacks, such as the biased relay [21] and the parasitic relay [22] which identifies multiple frequency response points in a single relay test. The derivation of PFRs are not enumerated here.

## 3. THE PROPOSED RELAY FEEDBACK

In the proposed relay feedback setup, a fractional order integrator is connected in front of the relay, as shown in figure 9, to provide a competitive adjustable phase shift range to the processes.

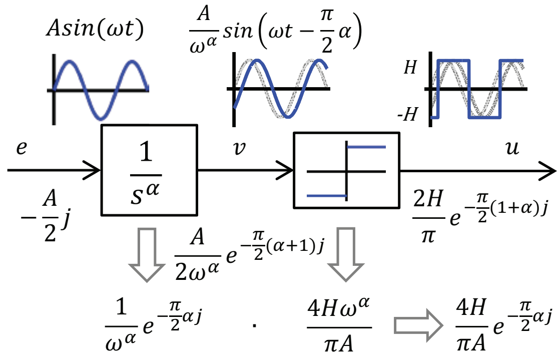



Fig. 9. The block diagram and signal transform of the relay with an FO integrator.

In this context, the fractional order integral follows the Riemann-Liouville (R-L) definition in [7], and the block in figure 9 is the Laplace transform of the R-L FO integral operator,

$$\mathcal{L}[I^\alpha f(t)] = \frac{1}{s^\alpha}F(s), \tag{20}$$

where $\alpha \in \mathbf{R}^+$. For practical reasons such as stability concerns [28], $\alpha$ is set in $(0, 2)$.

### *3.1 The PFR of the relay with an FO integrator*

The FO integrator is a linear element, its PFR is just its frequency response,

$$\frac{1}{(j\omega)^\alpha} = \frac{1}{\omega^\alpha}e^{-\frac{\pi}{2}\alpha j}. \tag{21}$$

Following the previously elaborated PFR concept, the PFR of the entire setup including the relay is,

$$\mathbb{G}_{FOint}(A\sin(j\omega)) = \frac{1}{(j\omega)^\alpha}\mathbb{G}_{ideal}\left(\frac{A}{\omega^\alpha}\sin\left(\omega t - \frac{\pi}{2}\alpha\right)\right)$$
$$= \frac{1}{(j\omega)^\alpha}\frac{4H\omega^\alpha}{\pi A} = \frac{4H}{\pi A}\frac{1}{(j)^\alpha}. \tag{22}$$

Alternatively, this can be derived in the following procedure, as illustrated in figure 9. Assume the input $e(t)$ to this relay setup has a sinusoidal first harmonic. After the FO integrator, the output is shifted in a phase angle corresponding to the fractional order $\alpha$,

$$v(t) = \frac{A}{\omega^\alpha}\sin\left(\omega t - \frac{\pi}{2}\alpha\right), \tag{23}$$

According to the properties of the Fourier series of FO operators, the Fourier series of an FO integrated function equals to the FO integration of the Fourier series of the function, [23],

$$F\left\{I^\alpha x(t)\right\} = I^\alpha F\left\{x(t)\right\}. \tag{24}$$

Thus, the Fourier coefficient of the first harmonic of the signal after the FO integrator, $v(t)$, is

$$c_{1_v} = \frac{1}{(j\omega)^\alpha}\frac{A}{2j} = \frac{A}{2\omega^\alpha}e^{-\frac{\pi}{2}(1+\alpha)j}. \tag{25}$$

When $v(t)$ passes through the ideal relay, it becomes a shifted square wave expressed as,

$$u(t) = \begin{cases} -H \ , \ 0 < t < \frac{T}{4}\alpha \text{ and } -\frac{T}{4}\alpha + kT < t < \frac{T}{4}\alpha + kT, \\ H \ , \qquad \frac{T}{4}\alpha + kT < t < \frac{3T}{4}\alpha + kT, \end{cases}$$

where $T$ is the oscillation period and $k = 0, 1, 2, \ldots$ The Fourier coefficient of its first harmonic is,

$$c_{1_u} = \frac{-\frac{4H}{\pi}\sin\left(\frac{\pi}{2}\alpha\right) - j\frac{4H}{\pi}\cos\left(\frac{\pi}{2}\alpha\right)}{2} = \frac{2H}{\pi}e^{-\frac{\pi}{2}(1+\alpha)j}. \qquad (26)$$

To obtain the PFR of the overall setup, $c_{1_u}$ is divided by $c_{1_e}$,

$$\mathbb{G}_{FOint}(A\sin(j\omega)) = \frac{c_{1_u}}{c_{1_e}} = \frac{\frac{2H}{\pi}e^{-\frac{\pi}{2}(1+\alpha)j}}{\frac{A}{2j}} = \frac{4H}{\pi A}e^{-\frac{\pi}{2}\alpha j} = \frac{4H}{\pi A}\frac{1}{(j)^{\alpha}}, \qquad (27)$$

which matches the result in equation (22) perfectly. Thus, the gain and phase of the process identified by the relay with an FO integrator are,

$$|G(j\omega_{FO})| = \frac{\pi A}{4H} \quad \text{and} \quad \varphi_p = -\pi + \frac{\pi}{2}\alpha. \qquad (28)$$

### *3.2 Identifying LTI model parameters*

When such a relay setup is connected to a process in a negative unit feedback loop, the frequency response of the process at the investigated point can be obtained. Then, the FOPDT model parameters $T$ and $L$ can be, again, calculated from the equations ($10 \sim 11$), with the corresponding $\omega$ and $\varphi_p$ substituted.

### *3.3 The advantages and limitations*

The major advantages of the proposed relay feedback setup over other variants are listed below:

(1) The relay with an FO integrator provides a wider selectable phase range for the process to be identified. So, for processes of slow dynamics such as the temperature control in chemical or bio engineering, it is more meaningful and realistic to approximate a model based on the frequency response that is close to a nominal operational point.
(2) Although the relay with time delay provides a even larger range of the identifiable process phase, it worths a notice that the pure delay results in a zero output at the beginning. By contrast, the FO integrator behaves rather a phase shift effect instead of a pure delay, as illustrated in figure 10. Hence, a quarter of the oscillation period can be saved for identifying ultra-slow processes.
(3) The shifted phase of the FO integrator can be predetermined. This is unlike using the relay with hysteresis, where there is no way to do so without priori knowledge of the process output. The reason lies in the dependency of the shifted phase on the amplitude of the process output, as shown by equation (12).

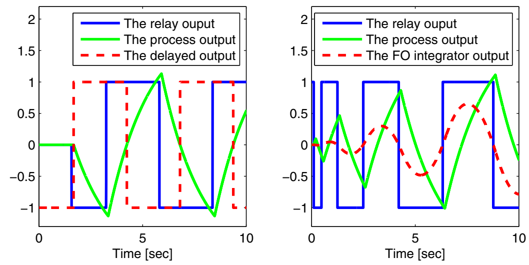



Fig. 10. Comparison of the initial response. Left: with delay behind; Right: with FO integrator.

On the other hand, the relay with an FO integrator cannot get rid of the common limitations suffered by other relay feedback variants in that it is essentially an approach based on the following assumptions:

(1) The output of the process has a first harmonic close to the sine wave;
(2) The higher order harmonics of the process output need to be small enough to be neglected.

Un-satisfaction either of the above conditions will result in big identification error. For example, a first order process usually outputs a triangle shape waveform under the relay test, and an identification error ranging from $-18\% \sim 27\%$ can be expected [25, 26, 27]. Large measurement noise will also affect the identification.

## 4. SIMULATION EXAMPLE

Consider the first element in the transfer function matrix of the Wood-Berry process [24],

$$G(s) = \frac{12.8}{16.7s + 1}e^{-s}. \qquad (29)$$

Six types of relay feedbacks are simulated to obtain the frequency response information of the process individually. A sample plot of a test run is shown in figure 11. The identified model parameters are listed in table I, where $A$ and $T_o$ are the amplitude and period of the oscillation, respectively, and the err(%) $= 1 - \tilde{T}/T$ is the identification error for $T$. (Note: only the error for T is listed because A usually can be determined accurately, and L is computed based on T.) It can be seen that with the specified relay parameters, the TC relay gives the smallest identification error while the relay with an integrator performs worst.

Table I. The frequency response information

| | $\varphi_p$ | $A$ | $T_o$ | $\tilde{T}$ | err(%) | $L$ |
|---|---|---|---|---|---|---|
| Ideal | $-180°$ | 0.743 | 3.900 | 13.604 | 18.54 | 1.00 |
| Hyst | $-104.4°$ | 1.121 | 6.360 | 13.561 | 18.80 | 1.23 |
| Delay | $-122.7°$ | 1.444 | 7.580 | 13.561 | 18.79 | 0.79 |
| Int | $-90°$ | 4.909 | 26.51 | 13.358 | 20.01 | 1.29 |
| TC | $-116.5°$ | 3.995 | 11.40 | 16.451 | 1.49 | 1.04 |
| FO int | $-108°$ | 2.929 | 15.55 | 13.548 | 18.88 | 1.22 |

By adjusting the fractional order $\alpha$ from 0.1 to 1.9 with a set size of 0.2, a comprehensive sweep of the process frequency response in the third and fourth quadrant can be performed. The detailed values are listed in table II. Unfortunately, due to cumulated numerical errors, the parameter estimation when $\alpha > 1.3$ fails to work

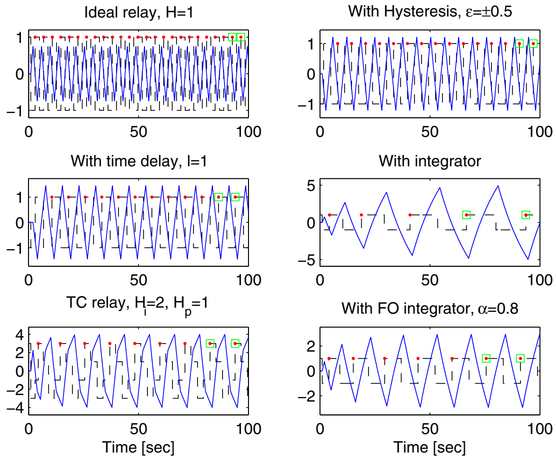



Fig. 11. A sample plot of the test run with 6 types of relay variants.

because the FO integrator is implemented by discretized approximation in the bandwidth of 0.01 and 100. In this case, a better FO computational tool is to be used.

Table II. The frequency information with $\alpha$ changing

| $\alpha$ | $\varphi_p$ | $A$ | $T_o$ | $\tilde{T}$ | err(%) | $L$ |
|---|---|---|---|---|---|---|
| 0.1 | $-171°$ | 0.857 | 4.440 | 13.42 | 19.66 | 1.04 |
| 0.3 | $-153°$ | 1.120 | 5.860 | 13.54 | 18.92 | 1.09 |
| 0.5 | $-135°$ | 1.541 | 8.080 | 13.54 | 18.91 | 1.13 |
| 0.7 | $-117°$ | 2.245 | 11.84 | 13.55 | 18.87 | 1.15 |
| 0.9 | $-99°$ | 3.735 | 20.08 | 13.57 | 18.73 | 1.24 |
| 1.1 | $-81°$ | 6.726 | 39.00 | 14.70 | 17.96 | 1.67 |
| 1.3 | $-63°$ | 10.22 | 73.04 | 14.44 | 13.48 | 2.39 |
| 1.5 | $-45°$ | 12.16 | 122.2 | 17.36 | 3.98 | 1.09 |
| 1.7 | $-27°$ | 12.77 | 228.9 | 28.87 | -72.85 | -7.24 |
| 1.9 | $-18°$ | 12.80 | 530.4 | 66.53 | -298.4 | -43.08 |

## 5. CONCLUSION

In this paper, different types of relay feedback identification methods are reviewed with the pseudo frequency response concept. Comments are given on some usually ignored aspects when implementing these methods. A relay with an FO integrator is presented through theoretical derivation and simulation, with the advantages of providing wider phase range for the identified processes, etc.

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