Preprints of the 19th World Congress
The International Federation of Automatic Control
Cape Town, South Africa. August 24-29, 2014

Switched model predictive control for networked control systems with time delays and packet disordering ⋆

Andong Liu * Li Yu * Wen-an Zhang *

* *Department of Automation, Zhejiang University of Technology, Zhejiang Provincial United Key Laboratory of Embedded Systems, Hangzhou 310023, P.R. China, (e-mail: lad-andrew@hotmail.com)*

Abstract: The switched model predictive control (MPC) strategy is investigated in this paper for a class of networked control systems (NCSs) with time delays and packet disordering. A new model is proposed to describe the NCS with packet disordering. State feedback controllers are considered and the resulting closed-loop NCS is modeled as a discrete-time switched system. By using the average dwell-time approach and switching between different cost functions, sufficient conditions for exponential stability of the NCS and design procedures for the stabilizing controllers are presented. Finally, an illustrative example is given to demonstrate the effectiveness of the proposed method.

Keywords: Switched model predictive control, Networked control systems, Time delays, Packet disordering.

1. INTRODUCTION

The past few decades have witnessed an ever increasing research interest in networked control systems (NCSs) due to increasing applications of network in mobile sensor networks, remote surgery, and automated highway systems, etc (Hesphanha et al., 2007; Zhang et al., 2013). However, perfect communication is not always possible in many practical systems, especially in a networked environment. Network-induced delay is one of the main problems in NCS, and is usually regarded as a major cause of performance deterioration and potential instability. The control problems of networked systems with network-induced delays have been extensively considered by many researchers, such as Gao and Chen (2007), Wang et al. (2012), Zhang and Yu (2009b) and the references therein.

Since the delay may be larger than one sampling period, more than one control signals may arrive at the actuator during one sampling interval. Moreover, the transmission of data packets does not necessarily follow a "first send first arrive" principle as assumed in most existing works. This means that the newest control data may arrive at the actuator before the older data and thus the older one is discarded during one sampling interval, this is the so-called packet disordering problem (Liu et al., 2011a,b). In Cloosterman et al. (2009) and Cloosterman et al. (2010), modeling and robust stability analysis for NCS with time-varying delay and packet disordering were discussed, and the uncertainties of the delays were transformed into uncertainties of the system models. The guaranteed cost control for multi-input and multi-output NCSs with multi-channel packet disordering was discussed in Li et al. (2011), where the delay is assumed to be time-varying and bounded. It should be noted that in Cloosterman et al. (2009), Cloosterman et al. (2010) and Li et al. (2011) the actuator is event-driven for the packet disordering problem. But an exponential uncertainy was introduced in the NCS model by the event-driven actuator (Hetel et al., 2007). The exponential uncertainty is one of the main difficulties in the analysis and synthesis of NCS with time-varying delays, but it can be avoided by using the time-driven actuator, such as in Liu et al. (2011b) and Zhang and Yu (2009a).

Another important issue that should be considered in the design of NCSs with delays is that the number of the arrived control signals vary over different sampling intervals. Therefore, the closed-loop NCS is naturally a switched system with a group of subsystems describing various system dynamics on the different sampling intervals, such as in Hetel et al. (2007) and Lin and Antsaklis (2005). In Zhang and Yu (2009a), a switched time delay system model was proposed to describe the NCS with time-varying delays and packet disordering. However, in many applications, it is desirable that not only one, but a number of different performance criteria for the controlled system can be taken into account. The different performance criteria to be considered might be associated with different times or stages in a process (Müller and Allgöwer, 2012). This can be easily done in the context of model predictive control (MPC).

In the MPC context, Magni et al. (2008) presented a switched MPC algorithm for nonlinear discrete-time systems with state-dependent switching among different cost functions. A switched MPC algorithm was proposed in Müller and Allgöwer (2012) for nonlinear continuous-time systems by using average dwell-time approach, where switches among different cost functions to be minimized

⋆ This research work was supported by the National Natural Science Foundation of China under the Grants 61104063 and 61273117.

occur at certain sampling instances. For switched systems, a predictive control framework was proposed in Mhaskar et al. (2005) for the constrained stabilization of switched nonlinear systems, where the switching signal has to be known. In Colaneri and Scattolini (2007), a robust MPC algorithm was presented for discrete-time switched linear systems, where the switching signal is a design parameter. In Müller and Allgöwer (2012), a switched MPC algorithm was investigated for switched nonlinear systems by using the average dwell-time approach, where the switching signal not known a priori and the switching times are not a design parameter. For NCS, a novel delay compensation strategy was proposed in Wang et al. (2010) for networked predictive control system by using the average dwell-time approach.

In this paper, we are motivated to study the switched MPC strategy for a class of NCSs with time delays, and focus on solving the packet disordering problem and the switched dynamic caused by long delay. A logical relation is explicitly established to describe the phenomenon of packet disordering. Then, a switched system model is used to describe the NCS based on the established relation. Considering switching between different cost functions and using the average dwell-time approach, the conditions for exponential stability are derived for the closed-loop NCS, where the switching signal not known a priori and the switching times are not a design parameter. Sufficient conditions for the existence of the state feedback controllers are given in terms of matrix inequalities. Finally, an example is given to demonstrate the effectiveness of the proposed method.

2. MODELING OF THE NCS

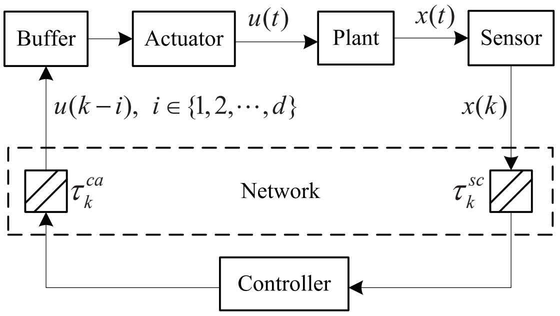

Fig.1. The structure of NCS

The structure of the considered NCS is shown in Fig.1, where the sensor is time-driven and the sampling period is denoted by T, the controller is event-driven, the actuator is time-driven, and has a receiving buffer which contains the most recently received data packet from the controller, τ_k^{sc} and τ_k^{ca} are the sensor-to-controller delay and controller-to-actuator delay, respectively, and $\tau_k = \tau_k^{sc} + \tau_k^{ca}$ is assumed to be bounded by $0 \le \tau_k \le dT$, where d is a known finite integer, $u(k-i)$, $i = 0,\ 1,\ \cdots,\ d$ are the controller outputs. Since the actuator is time-driven, it can be known that $\tau_k \in \mathcal{N}_1 = \{0,\ 1,\ 2,\ \cdots,\ d\}T$. The plant is described by the following continuous-time linear system model

$$\dot{x}(t) = A_p x(t) + B_p u(t) \tag{1}$$

where $x(t) \in R^n$ is the system state, $u(t) \in R^m$ is the control input, A_p and B_p are constant matrices with appropriate dimensions.

Since the network-induced delay may be larger than one sampling period, more than one control signals may arrive at the actuator during one sampling interval, but only one control signal is applied by the actuator. The main problem is how to choose the newest control signal. Moreover, the packet disordering problem will occur since the delay is long delay. A time diagram of the signal transmitting is illustrated in Fig. 2, in which it is assumed that $d = 3$, and at most three control signals may arrive at the actuator during one sampling interval, such as the sampling interval $((k+5)T,\ (k+6)T]$. Furthermore, control signal $u(k+2)$ arrives at the actuator earlier than $u(k+1)$. Thus, $u(k+2)$ is adopted at time $(k+3)T$ while $u(k+1)$ is discarded. It can known that the newest control data arrive at the actuator before the older data and thus the older one is discarded. This phenomenon is called packet disordering.

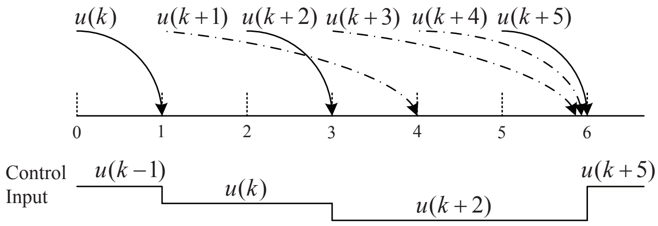

Fig.2. Time diagram of signal transmitting in the NCS

By the aforementioned analysis, it can be seen that the adopted control signal may take values in $\{u(k-d+1),\ \cdots,\ u(k-1),\ u(k)\}$ at each sampling instant, which will result in $d+1$ different system dynamics of the closed-loop system. Moreover, during the sampling interval $(kT,\ (k+1)T]$, the system dynamics are actually determined by $\{\tau_{k-d+1},\ \cdots,\ \tau_{k-1},\ \tau_k\}$. Therefore, we use a vector $\tau(k) = [\tau_{k-d+1},\ \cdots,\ \tau_{k-1},\ \tau_k]$ to represent the control signal which is actually applied at the actuator side, and define a vector-valued function $f:\ \tau(k) \to \sigma(k)$ to map $\tau(k)$ into a scalar $\sigma(k) \in \mathcal{N}_2 = \{0,\ 1,\ \cdots,\ d\}$. The detailed expression of $\sigma(k)$ is given as follows

$$\sigma(k) = \begin{cases} 0, & \tau_k = 0, \tau_{k-j} \in \mathcal{N}_1,\ j = 1, 2, \cdots, d-1 \\ 1, & \tau_k \ge T, \tau_{k-1} \le T, \tau_{k-j} \in \mathcal{N}_1, \\ & j = 2, \cdots, d-1 \\ 2, & \tau_{k-i} \ge (i+1)T, i = 0, 1,\ \tau_{k-2} \le 2T, \\ & \tau_{k-j} \in \mathcal{N}_1,\ j = 3, \cdots, d-1 \\ \vdots & \vdots \\ d, & \tau_{k-i} \ge (i+1)T,\ i = 0, 1, \cdots d-1 \end{cases} \tag{2}$$

and the adopted control signal is $u(k - \sigma(k))$ at time kT. Denote $A = e^{A_p T}$ and $B = \int_0^T e^{A_P t} dt B_p$, then the NCS under consideration is modeled as the following discrete-time switched system

$$x(k+1) = Ax(k) + Bu(k - \sigma(k)) \tag{3}$$

where $\sigma(k)$ is used as the switching signal of system (3).

Denoting $\Lambda_{i\sigma(k)} = \begin{cases} I, & i = \sigma(k) \\ 0, & i \ne \sigma(k) \end{cases}$, $i \in \mathcal{N}_2$, then system (3) can be represented as follows

$$x(k+1) = Ax(k) + \sum_{i=0}^{d} \bar{B}_{i\sigma(k)} u(k-i) \tag{4}$$

where $\bar{B}_{i\sigma(k)} = \Lambda_{i\sigma(k)} B$. Let $X(k) = \left[\, x^T(k)\ u^T(k-1)\ \cdots\ u^T(k-d+1)\ u^T(k-d) \,\right]^T$, system (4) can be rewritten as the following switched system model

$$X(k+1) = G_{\sigma(k)} X(k) + H_{\sigma(k)} u(k) \tag{5}$$

where

$$G_{\sigma(k)} = \begin{bmatrix} A & \bar{B}_{1\sigma(k)} & \cdots & \bar{B}_{(d-1)\sigma(k)} & \bar{B}_{d\sigma(k)} \\ 0 & 0 & \cdots & 0 & 0 \\ 0 & I & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & I & 0 \end{bmatrix},$$

$$H_{\sigma(k)} = \left[\, \bar{B}^T_{0\sigma(k)}\ I\ 0\ \cdots\ 0 \,\right]^T.$$

The objective of this paper is to design a stabilizing controller $u(k) = K_{\sigma(k)}(k)X(k)$ for the NCS (5) via switched MPC strategy. Then, the closed-loop system of (5) is given as follows

$$X(k+1) = \left(G_{\sigma(k)} + H_{\sigma(k)} K_{\sigma(k)}(k)\right) X(k) \tag{6}$$

For this purpose, let each subsystem of the switched system (5) be associated with a certain performance criterion $J_{\sigma(k)}$ to be minimized. Let $X(k+i|k)$ and $u(k+i|k)$ be the predicted state variables and input variables based on the measurements at time kT, respectively. In order to design a stabilizing MPC controller for the system (6), the following finite horizon optimal control problem is considered

Problem 1. At time kT, solve the optimization problem

$$\begin{aligned} \min\ & J_{\sigma(k)}\left(X(k), U(k), N\right) \\ &= \sum_{j=0}^{N-1} \|X(k+j|k)\|^2_{Q_{\sigma(k)}} + \|u(k+j|k)\|^2_{R_{\sigma(k)}} \\ &\quad + V_{\sigma(k)}\left(X(k+N|k)\right) \end{aligned} \tag{7}$$

subject to

$X(k+i+1|k) = G_{\sigma(k)} X(k+i|k) + H_{\sigma(k)} u(k+i|k)$,
$V_{\sigma(k)}\left(X(k+i|k)\right) = X^T(k+i|k) P_{\sigma(k)}(k) X(k+i|k)$,
$u(k+i|k) = K_{\sigma(k)}(k) X(k+i|k)$,
$X(k|k) = X(k)$,
$u(k|k) = u(k)$,

where $U(k) = [u(k|k),\ u(k+1|k),\ \cdots,\ u(k+N-1|k)]$, N is the size of prediction horizon, $P_{\sigma(k)}$ is positive definite symmetric matrix and $K_{\sigma(k)}(k)$ is the state feedback controller gain to be designed at time kT.

Remark 1. As shown in Müller et al. (2012), the currently active subsystem dynamics and the associated cost function are used in this paper for the whole prediction horizon, though possible switches within the prediction horizon are not known yet.

3. SWITCHED MPC OF THE NCS

In this section, a sufficient exponential stability condition is given for the existence of state feedback controller and a switched MPC algorithm is presented for the NCS with time delays and packet disordering. Before proceeding further, some useful definitions are introduced.

Definition 1. The NCS (6) is said to be exponentially stable, if there exist positive constants c and $\alpha < 1$ such that the solution of system (6) satisfies

$$\|X(k)\| = c\alpha^k \|X_0\|$$

for any initial state $X_0 = X(0) \in R^{(d+1)n}$, where $X(0) = \left[\, x^T(0)\ u^T(-1)\ u^T(-2)\ \cdots\ u^T(-d) \,\right]^T$, $u(i) = 0$, $i = -1,\ -2,\ \cdots,\ -d$.

Definition 2 (Zhang and Yu, 2009b) **.** For any switching signal $\sigma(k)$ and any $k \geq 1$, let $N_\sigma[0,\ k)$ be the number of switching of $\sigma(k)$ over the interval $[0,\ k)$. If $N_\sigma[0,\ k) \leq N_0 + k/T_\alpha$ holds for $N_0 \geq 0$ and $T_\alpha \geq 0$, then T_α is called the average dwell time and N_0 is the chatter bound.

For simplicity, but without the loss of generality, we choose $N_0 = 0$ in the sequel developments.

Let $U^*(k)$, $P^*_{\sigma(k)}(k)$ be the optimal solution of Problem 1 and $F_{\sigma(k)}(k, N) = J_{\sigma(k)}\left(X(k), U^*(k), N\right)$ be the optimal value of performance index $J_{\sigma(k)}\left(X(k), U(k), N\right)$. Then, we are ready to state the following theorem.

Theorem 1. For given positive scalars $\lambda < 1$ and $\mu > 1$, if there exist matrices $P_i(k) > 0$, $K_i(k)$, $i = 0,\ 1,\ \cdots d$ of appropriate dimensions such that the following inequalities

$$\begin{aligned} & V_i\left(X(k+j+1|k)\right) - V_i\left(X(k+j|k)\right) \\ &\quad \leq -\left(\|X(k+j|k)\|^2_{Q_i} + \|u(k+j|k)\|^2_{R_i}\right) \end{aligned} \tag{8}$$

$$\begin{aligned} & \left(1-\lambda^2\right) V_i\left(X(k+j|k)\right) \\ &\quad \leq \|X(k+j|k)\|^2_{Q_i} + \|u(k+j|k)\|^2_{R_i} \end{aligned} \tag{9}$$

$$P_a(k) \leq \mu P_b(k) \tag{10}$$

$$Q_a \leq \mu Q_b \tag{11}$$

$$R_a \leq \mu R_b, \quad a,\ b \in \mathcal{N}_2 \tag{12}$$

$$T_\alpha > T^*_\alpha = \frac{\mathrm{In}\mu}{2\mathrm{In}\lambda^{-1}} \tag{13}$$

hold, then the NCS (6) is exponentially stable and ensures a decay rate $\alpha = \lambda\mu^{\frac{1}{2T_\alpha}}$.

Proof. If there exists $P_i(k) > 0$, $K_i(k)$, $i = 0,\ 1,\ \cdots d$ satisfying (8), then

$$\begin{aligned} & J_i\left(X(k), \tilde{U}^*(k), N+1\right) \\ &\quad = \sum_{j=0}^{N} \|X(k+j|k)\|^2_{Q_i} + \|u^*(k+j|k)\|^2_{R_i} \\ &\qquad + V_i\left(X(k+N+1|k)\right) \\ &\quad = F_i(k, N) + \|X(k+N|k)\|^2_{Q_i} + \|u^*(k+N|k)\|^2_{R_i} \\ &\qquad + V_i\left(X(k+N+1|k)\right) - V_i\left(X(k+N|k)\right) \\ &\quad \leq F_i(k, N) \end{aligned} \tag{14}$$

where $\tilde{U}(k) = [U(k),\ u(k+N|k)]$. So that

$$F_i\left(k, N+1\right) \leq F_i\left(k, N\right) \tag{15}$$

with $F_i\left(k, 0\right) = V_i(X(k))$. Thus, one has

$$F_i\left(k, N\right) \leq V_i(X(k)) \tag{16}$$

Along the closed-loop trajectory of (6), one has

$$\begin{aligned} &F_i\left(k+1, N\right) - \lambda^2 F_i\left(k, N\right) \\ &= V_i\left(X(k+N+1|k+1)\right) - \lambda^2 V_i\left(X(k+N|k)\right) \\ &\quad - \lambda^2 \sum_{j=0}^{N-1} \left(\|X(k+j|k)\|_{Q_i}^2 + \|u^*(k+j|k)\|_{R_i}^2 \right) \\ &\quad + \sum_{j=0}^{N-1} \Big(\|X(k+j+1|k+1)\|_{Q_i}^2 \\ &\quad + \|u^*(k+j+1|k+1)\|_{R_i}^2 \Big) \end{aligned} \tag{17}$$

If we use $U^*(k)$, $P_i^*(k)$ instead of $U^*(k+1)$, $P_i^*(k+1)$ at time $(k+1)T$, then it follows from optimality that

$$\begin{aligned} &F_i\left(k+1, N\right) - \lambda^2 F_i\left(k, N\right) \\ &\leq \left(1-\lambda^2\right) F_i\left(k, N\right) - \|X(k)\|_{Q_i}^2 - \|u^*(k)\|_{R_i}^2 \\ &\quad + \|X(k+N|k)\|_{Q_i}^2 + \|u^*(k+N|k)\|_{R_i}^2 \\ &\quad + V_i\left(X(k+N+1|k)\right) - V_i\left(X(k+N|k)\right) \end{aligned} \tag{18}$$

One has by inequalities (8), (9) and (16) that

$$\begin{aligned} &F_i\left(k+1, N\right) - \lambda^2 F_i\left(k, N\right) \\ &\leq \left(1-\lambda^2\right) F_i\left(k, N\right) - \|X(k)\|_{Q_i}^2 - \|u^*(k)\|_{R_i}^2 \\ &\leq \left(1-\lambda^2\right) V_i\left(X(k)\right) - \|X(k)\|_{Q_i}^2 - \|u^*(k)\|_{R_i}^2 \\ &\leq 0 \end{aligned} \tag{19}$$

Then, one has

$$F_i\left(k+1, N\right) \leq \lambda^2 F_i\left(k, N\right) \tag{20}$$

On the other hand, if inequalities (10)-(12) are satisfied, one obtains

$$F_a\left(k, N\right) \leq \mu F_b\left(k, N\right), \quad a,\ b \in \mathcal{N}_2 \tag{21}$$

For an arbitrary piecewise constant switching signal $\sigma(k)$, it follows from (16), (20) and (21) that

$$\begin{aligned} F_{\sigma(k)}\left(k, N\right) &\leq \mu F_{\sigma(k-1)}\left(k, N\right) \\ &\leq \lambda^2 \mu F_{\sigma(k-1)}\left(k-1, N\right) \\ &\ \ \vdots \\ &\leq \lambda^{2k} \mu^{N_\sigma(0,k)} F_{\sigma(0)}\left(0, N\right) \\ &\leq \lambda^{2k} \mu^{\frac{k}{T_\alpha}} F_{\sigma(0)}\left(0, N\right) \\ &= \alpha^{2k} F_{\sigma(0)}\left(0, N\right) \\ &\leq \alpha^{2k} V_{\sigma(0)}\left(X(0)\right) \end{aligned} \tag{22}$$

We obtain by (9) and (22) that

$$\begin{aligned} \left(1-\lambda^2\right) V_{\sigma(k)}\left(X(k)\right) &\leq F_{\sigma(k)}\left(k, N\right) \\ &\leq \alpha^{2k} V_{\sigma(0)}\left(X(0)\right) \end{aligned} \tag{23}$$

Let $\varepsilon_1 = \min\limits_{\forall\, i \in N} \{\lambda_{\min}(P_i(l))\}$, $\varepsilon_2 = \max\limits_{\forall\, i \in N} \{\lambda_{\max}(P_i(l))\}$, $l = 0,\ 1,\ \cdots$, $\lambda_{\min}(\Delta)$ and $\lambda_{\max}(\Delta)$ are the minimum and maximum eigenvalues of Δ, respectively. It follows from (23) that

$$\left(1-\lambda^2\right) \varepsilon_1 \|X(k)\|^2 \leq \alpha^{2k} \varepsilon_2 \|X(0)\|^2 \tag{24}$$

By the inequality (24), we further obtain

$$\|X(k)\| \leq c\alpha^k \|X(0)\| \tag{25}$$

where $c = \sqrt{\varepsilon_2 / (1-\lambda^2)\, \varepsilon_1}$. The inequality (13) guarantee that $\alpha < 1$. Therefore, the NCS (6) is exponentially stable and ensures a decay rate α according to Definition 1. The proof is completed.

Based on the stability conditions in Theorem 1, the existence conditions for the state feedback controllers are presented in the following theorem.

Theorem 2. For given positive scalars $\lambda < 1$ and $\mu > 1$, the switched MPC Problem 1 can be solved by the following semi-definite programming

$$\min\ \gamma(k) \tag{26}$$

$$\text{s.t. (11), (12), (13) and}$$

$$\begin{bmatrix} -\bar{P}_i(k) & X(k) \\ * & -I \end{bmatrix} < 0 \tag{27}$$

$$\begin{bmatrix} -\bar{P}_i(k) & \Phi(k) & 0 & 0 \\ * & -\bar{P}_i(k) & \bar{P}_i(k) & \bar{K}_i(k) \\ * & * & -\gamma(k) Q_i^{-1} & 0 \\ * & * & * & -\gamma R_i^{-1} \end{bmatrix} < 0 \tag{28}$$

$$\bar{P}_a(k) \leq \mu \bar{P}_b(k), \quad a,\ b \in \mathcal{N}_2 \tag{29}$$

$$(1-\lambda) P_i(k) - Q_i - K_i^T(k) R_i K_i(k) \leq 0 \tag{30}$$

Moreover, if the optimization problem (26) is feasible, then the NCS (6) is exponentially stable and ensures a decay rate $\alpha = \lambda \mu^{\frac{1}{2T_\alpha}}$ with controller gain $K_i(k) = \bar{K}_i(k) \bar{P}_i^{-1}(k)$, where $\Phi(k) = G_i \bar{P}_i(k) + H_i \bar{K}_i(k)$, $\bar{P}_i(k) = \gamma(k) P_i^{-1}(k)$, $\bar{K}_i(k) = K_i(k) \bar{P}_i(k)$, $i = 0,\ 1,\ \cdots d$.

Proof. Summing up both side of inequality (8) from 0 to $N-1$, one has

$$\begin{aligned} &\sum_{j=0}^{N-1} V_i\left(X(k+j+1|k)\right) - V_i\left(X(k+j|k)\right) \\ &\quad \leq -\sum_{j=0}^{N-1} \left(\|X(k+j|k)\|_{Q_i}^2 + \|u(k+j|k)\|_{R_i}^2 \right) \end{aligned} \tag{31}$$

It can be derived from (31) that

$$J_i\left(X(k), U(k), N\right) \leq V_i\left(X(k)\right) \tag{32}$$

this gives an upper bound on the performance objective (7). Then, Problem 1 can be redefined to minimize the upper bound $V_i\left(X(k)\right)$

$$\min\ J_i\left(X(k), U(k), N\right) \leq V_i\left(X(k)\right) \leq \gamma(k) \qquad (33)$$

By applying schur complement, one obtains (33) from (27).

On the other hand, if inequality (8) is true, the following inequality holds

$$\begin{aligned}&\left(G_i + H_iK_i(k)\right)^T P_i(k)\left(G_i + H_iK_i(k)\right) - P_i(k)\\&\quad \leq -Q_i - K_i^T(k)R_iK_i(k)\end{aligned} \qquad (34)$$

It follows from the Schur complement that inequality (34) is equivalent to the following inequality

$$\begin{bmatrix} -P_i^{-1}(k) & G_i + H_iK_i(k) & 0 & 0 \\ * & -P_i(k) & I & K_i^T(k) \\ * & * & -Q_i^{-1} & 0 \\ * & * & * & -R_i^{-1} \end{bmatrix} < 0 \qquad (35)$$

then, pre- and post-multiplying the inequality (35) by $\mathrm{diag}\left\{\gamma^{1/2}(k)I,\ \gamma^{1/2}(k)P^{-1},\ \gamma^{1/2}(k)I,\ \gamma^{1/2}(k)I\right\}$, one obtains (28). Pre- and post-multiplying the inequality (10) by $\gamma(k)I$, we obtain inequality (29). Similarly, we can easily to obtain (30) from inequality (9). Therefore the statements in Theorem 2 are true by Theorem 1. This completes the proof.

With the results of Theorem 2, the state feedback controller can be obtained by solving the optimization problem (26) subject to matrix inequality constraints (11)-(13), (27)-(30). Since (30) is a non-linear matrix inequality, it is difficult to solve the optimization problem (26). Then, the following algorithm is given to solve this problem.

Algorithm 1. Switched MPC for the NCS

0. Initialization: Set $k = 0$ and choose a suitable μ. Determine $i = \sigma(0)$.
1. At time kT, measure the state $x(k)$.
2. Solve the optimization problem (26) subject to (11), (12), (13), (27), (28) and (29) with index $i = \sigma(0)$.
3. Choose a larger enough λ. If (30) is feasible, $K_i(k)$ is the optimal controller gain, otherwise, increase λ by a certain step length, and check the feasibility of the inequality (30).
4. Let $k = k + 1$ and go to Step 2.

Remark 2. If Algorithm 1 is feasible at $kT = 0$, then it is feasible for all time $kT \geq 0$ for an arbitrary switching signal with average dwell-time T_α satisfying (13). To prove the feasibility, we show that the optimal solution of the optimization problem (26) subject to (11)-(13), (27)-(30) at time kT satisfies the inequalities (11)-(13), (27)-(30) at time $(k+1)T$. Since the parameters are independent of the state $X(k)$, inequalities (11)-(13), (28)-(30) are satisfied by using the optimal values computed at time kT instead of the decision variables at time $(k+1)T$. Since inequalities (27)-(29) are satisfied at time kT, one has $F_i(k+1, N) \leq F_i(k, N)$, $i \in \mathcal{N}_2$. From inequalities (27), one obtains that $F_i(k, N) \leq \gamma(k)$ for all $i \in \mathcal{N}_2$, which implies $F_i(k+1, N) \leq \gamma(k)$ at time $(k+1)T$. Then the solution of the optimization problem given in (26) is feasible at time $(k+1)T$. Therefore, Algorithm 1 is feasible at all time $kT \geq 0$.

Remark 3. The parameters λ and μ should be chosen as small as possible to yield a smaller α and T_α^* in Algorithm 1, which results in a better performance and makes the condition (13) easier to be satisfied. The rules to choose λ and μ can be seen from Zhang and Yu (2009b).

Remark 4. The switched MPC solution to the NCS with delays can also be extended to the case with input/output constraints. By using the similar method in Kothare et al. (1996), we can obtain some similar results for NCS with input/output constraints.

4. NUMERICAL EXAMPLE

In this section, a simulation example is given to demonstrate the effectiveness of the developed method.

Consider an inverted pendulum system with delayed control input (Gao and Chen, 2007), its state-space model is given by

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ \dfrac{3(M+m)g}{l(4M+m)} & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ -\dfrac{3}{l(4M+m)} \end{bmatrix} u(t) \qquad (36)$$

Here the parameters are selected as $M = 1.32kg$, $m = 0.11kg$, $l = 0.25$, $g = 9.8m/s^2$. Choose the sampling period as $T = 0.02s$, then the discretized model for the pendulum system is given by

$$x(k+1) = \begin{bmatrix} 1.0062 & 0.02 \\ 0.6253 & 1.0062 \end{bmatrix} x(k) + \begin{bmatrix} -0.0004 \\ -0.0446 \end{bmatrix} u(k) \qquad (37)$$

It is assumed that the network-induced delay τ_k is bounded by $\tau_k \leq 2T$. Since the upper bound of the delays is $d = 2$, at most three control signals can be involved during one sampling period and they are $u(k-2)$, $u(k-1)$ and $u(k)$. Then, it can be seen from the distribution of the delays that totally three subsystems are involved. Fig. 3 depicts the distribution of the delays and the values of these τ_k during the interval [0, 200) are shown as follows

$$\begin{aligned}&[0002221111\underbrace{0\cdots0}_{10}\underbrace{1\cdots1}_{25}\underbrace{2\cdots2}_{10}\underbrace{1\cdots1}_{25}\underbrace{0\cdots0}_{15}\underbrace{2\cdots2}_{10}\\&\quad\underbrace{0\cdots0}_{15}\underbrace{1\cdots1}_{30}\underbrace{2\cdots2}_{10}\underbrace{0\cdots0}_{10}\underbrace{1\cdots1}_{30}]T\end{aligned} \qquad (38)$$

According to the discussion in Section 2, packet disordering is inevitable in this case. By using the map $f : \tau(k) \to \sigma(k)$, the subsystems are activated in the following sequence

$$\begin{aligned}&[0001222111\underbrace{0\cdots0}_{10}\underbrace{1\cdots1}_{26}\underbrace{2\cdots2}_{10}\underbrace{1\cdots1}_{24}\underbrace{0\cdots0}_{15}1\underbrace{2\cdots2}_{9}\\&\quad\underbrace{0\cdots0}_{15}\underbrace{1\cdots1}_{31}\underbrace{2\cdots2}_{9}\underbrace{0\cdots0}_{10}\underbrace{1\cdots1}_{30}]\end{aligned} \qquad (39)$$

It can be seen from (38) and (39) that the number of switching are 13 and 15, respectively. Moreover, the switching times are also different. These phenomenon are caused by packet disordering.

Choosing the initial condition $x(0) = [\,0.1\ 0\,]^T$, the input constraints $\|u(k)\| \leq 2.8$, and $Q_0 = R_0 = 1.2$, $Q_1 = R_1 = 1.1$, $Q_2 = R_2 = 1$, we obtain $\mu = 1.2$, $\lambda = 0.9849$ by applying Algorithm 1. By the switching sequence of the subsystems and the definition of the average dwell time, we have $T_\alpha = 200/15 = 13.3333$, and therefore the condition $T_\alpha > T_\alpha^* = \mathrm{In}\mu/2\mathrm{In}(1/\lambda) = 11.9582$ is also satisfied. Thus, by Theorem 2, the NCS controlled by the designed controller via network is exponentially stable and has a decay rate $\alpha = 0.9918$. The simulation results are shown in Fig. 4, which depicts the state trajectories and control input trajectory of the closed-loop NCS.

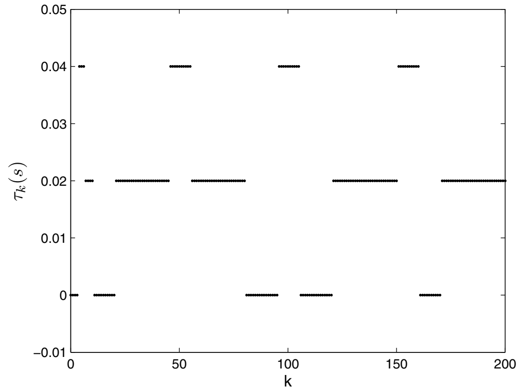

Fig.3. Distribution of the network-induced delays

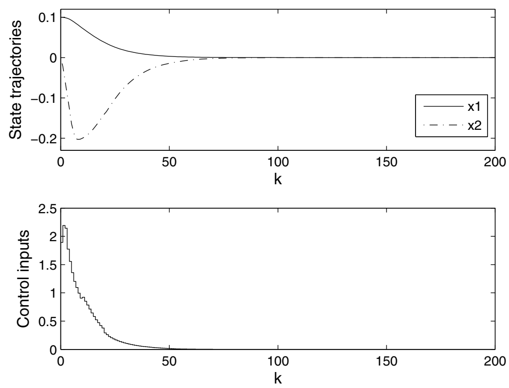

Fig.4. State trajectories and control input of the NCS

5. CONCLUSION

In this paper, the switched MPC strategy was presented for a class of NCSs with time delays and packet disordering. The NCS was modeled as a switched system, which can fully describing the phenomenon of packet disordering. Switching between different cost functions and the average dwell-time approach were used to derive the stability condition and controller design procedure. The effectiveness of the proposed method was illustrated by numerical examples.

REFERENCES

Cloosterman, M.B.G., van de Wouw, N., Heemels, W.P.M.H., et al. (2009). Stability of networked control systems with uncertain time-varying delays. *IEEE Trans. Autom. Control*, 54(7), 1575-1580.

Cloosterman, M.B.G., Hetel, L., van de Wouw, N., et al. (2010). Controller synthesis for networked control systems. *Automatica*, 46(10), 1584-1594.

Colaneri, P., and Scattolini, R.(2007). Robust model predictive control of discrete-time switched systems. in *Proceedings of the 3rd IFAC Workshop on Periodic Control Systems*, 208-212.

Gao, H., and Chen, T. (2007). New results on stability of discrete-time systems with time-varying state delay. *IEEE Trans. Autom. Control*, 52(2), 328-334.

Hesphanha, J.P., Naghshtabrizi, P., and Xu, Y. (2007). A survey of recent results in networked control system. *Proc. IEEE*, 95(1), 138-162.

Hetel, L., Daafouz, J., and Lung, C. (2008). Analysis and control of LTI and switched systems in the digital loops via an event-based modeling. *Int. J. Control*, 81(7), 1125-1138.

Kothare, M.V., Balakrishnan, V., and Morari, M. (1996). Robust constrained model predictive control using linear matrix inequalities. *Automatica*, 32(10), 1361-1379.

Li, J., Zhang, Q., Yu, H., et al. (2011). Real-time guaranteed cost control of MIMO networked control systems with packet disordering. *J. Process Control*, 21(6), 967-975.

Lin, H., and Antsaklis, P.J. (2005). Stability and persistent disturbance attenuation properties for a class of networked control systems: Switched system approach. *Int. J. Control*, 78(18), 1447-1458.

Liu, A., Yu, L., and Zhang, W.A. (2011a). One-step receding horizon H_∞ control for networked control systems with random delay and packet disordering. *ISA Trans.*, 50(1), 44-52.

Liu, A., Yu, L., and Zhang, W.A. (2011b). H_∞ control for network-based systems with time-varying delay and packet disordering. *J. Franklin Inst.*, 348(5), 917-932.

Magni, L., Scattolini, R., and Tanelli, M. (2008). Switched model predictive control for performance enhancement. *Int. J. Control*, 81(12), 1859-1869.

Mhaskar, P., El-Farra, N.H., and Christofides, P.D. (2005). Predictive control of switched nonlinear systems with scheduled mode transitions. *IEEE Trans. Autom. Control*, 50(11), 1670-1680.

Müller, M.A., and Allgöwer, F. (2012). Improving performance in model predictive control: Switching cost functionals under average dwell-time. *Automatica*, 48(2), 402-409.

Müller, M.A., Martius, P., and Allgöwer, F. (2012). Model predictive control of switched nonlinear systems under average dwell-time. *J. Process Control*, 22(9), 1702-1710.

Wang, R., Wang, B., Liu, G.P., et al. (2010). H_∞ controller design for networked predictive control systems based on the average dwell-time approach. *IEEE Trans. Circuits Syst. Express Briefs*, 57(4), 310-314.

Wang, Z., Shen, B., Shu, H., et al. (2012). Quantized H_∞ control for nonlinear stochastic time-delay systems with missing measurements. *IEEE Trans. Autom. Control*, 57(6), 1431-1444.

Zhang, L., Gao, H., and Kaynak, O. (2013). Network-induced constraints in networked control systems–A survey. *IEEE Trans. Ind. Inf.*, 9(1), 403-416.

Zhang, W.A., and Yu, L. (2009a). A robust control approach to stabilization of networked control systems with time-varying delays. *Automatica*, 45(10), 2440-2445.

Zhang, W.A., and Yu, L. (2009b). Stability analysis for discrete-time switched time-delay systems. *Automatica*, 45(10), 2265-2271.