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Fault-Tolerant Control of a Class of Switched Time-Delay Systems with Average Dwelling Time Method

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Abstract: This paper considers fault-tolerant control of a class of uncertain switched linear time-delay systems and its application to water pollution control. Due to the nature of average dwell-time techniques and the representation of actuator faults, this paper has the following features compared with the existing methods in the literature: 1) the proposed method is independent from switching polices provided that switching is on-the-average slow enough; 2) the proposed controller exponentially stabilizes this class of time-delay systems with actuator faults and its nominal systems (i.e., without actuator faults) without necessarily changing any structures and/or parameters of the proposed controllers; 3) the proposed method treats all actuators in a unified way without necessarily classifying all actuators into faulty actuators and healthy ones. Simulation results are provided to illustrate the effectiveness of the proposed method.

Keywords: Switched systems; fault-tolerant control; time-delay; average dwell-time method.

1. INTRODUCTION

Time delay can be seen in various practical applications such as chemical processes, long-distance transmission networks and automotive control. When time delay is in presence of a dynamic system, it may deteriorate transient performance or even leading to instability. When time delay is involved in switched systems, i.e., switched time-delay systems, much attention has been paid on control of switched systems [Wang, et al., 2009; Wang, et al., 2007; Du, et al., 2011; Xiang, et al., 2010] and the references therein. Due to the inherent nature of switched systems [Zhang & Gao, 2010; Zhao & Hill, 2008; Liberzon 2003; Zhao & Dimirovsk, 2004; Sun & Ge, 2004; Xie & Wang, 2003; Hu, et al., 1999; Yurtseven, et al., 2012; El-Farra, et al., 2005] and the complicated features that time delay may bring in, studies on switched time-delay systems are still one of important research topics in control areas.

On the other hand, maintenances or repairs in the highly automated industrial systems cannot be always achieved immediately, for preserving safety and reliability of the systems, the possibility of occurrence and presence of uncertain faults must be taken into account during the system analysis and control design stages to avoid life-threatening prices and heavy economic costs caused by faults [Zhang & Jiang, 2003; Xiao, et al., 2012; Zhang & Jiang, 2008; Han & Yu, 1998; Panagi & Polycarpou, 2011], which makes fault-tolerant control attract more and more attention [Jin, et al., 2007; Veillette, 1995; Wang, et al., 1999; Liang, et al., 2000]. Reference [Jin, et al., 2007] considered decentralized fault-tolerant control for a class of interconnected nonlinear systems consisting of finite subsystems to achieve desired tracking objectives and guarantee stability of the closed-loop systems. However, Jin, et al. (2007) only considered bounded fault functions which satisfy matching conditions and bounded interconnected uncertain structures, but the authors did not explicitly consider faults of the actuators which transmit control signal into the plant. References [Jin, et al., 2007; Veillette, 1995; Wang, et al., 1999; Liang, et al., 2000] designed fault-tolerant controllers for the nonlinear systems with actuator faults to guarantee robust reliable stability of the systems. Their common feature is that actuators are decomposed into two parts, one of which is susceptible to faults, the other part is robust to faults, to compensate for actuator faults effectively. But in order to implement the controller designs, the two-part decomposition has to be known in advance. It is in general difficult to obtain in practice due to uncertain and random feature of faults.

However, there are few results on fault-tolerant control of switched systems and that of switched time-delay systems [Wang, et al., 2009; Wang, et al., 2007; Du, et al., 2011; Jin, et al., 2007; Veillette, 1995; Wang, et al., 2009; Du & Mhaskar, 2010]. References [Wang, et al., 2007; Jin, et al., 2007; Wang, et al., 2009] gave sufficient conditions on robust fault-tolerant control of a class of nonlinear switched systems by decomposing actuators into two parts, in the same way as [Veillette, 1995], i.e., one part is robust to actuator faults, and the other is susceptible to actuator faults. Thus, the method in [Wang, et al., 2007; Jin, et al., 2007; Wang, et al., 2009] has the above-mentioned disadvantages on fault modeling and characterization. In addition, structural uncertainties of input matrices were not considered for this class of nonlinear switched systems in [Wang, et al., 2007; Jin, et al., 2007; Wang, et al., 2009]. In [Wang, et al., 2009], by combining safe-parking method and reconfiguration-based approach the authors proposed two switching strategies to realize fault-tolerant controls of a class of switched nonlinear systems against actuator faults. But when actuator faults occur, the two methods both need to determine reparation time for faulty actuators, which is not easy to acquire in many real-world scenarios. Reference [Du & Mhaskar, 2010] considered an observer-based fault-tolerant control of a class of switched nonlinear system with external disturbances. [Du, et al., 2011] developed an active fault-tolerant control for switched time-delay systems, but it needs to design an observer to detect faults. [Wang, et al., 2009] dealt with the problem of robust fault detection for discrete-time switched time-delay systems without designing controllers.

In last decade, average dwell-time switching techniques have been becoming one of popular methods to stabilize switched systems, due that it is more general and flexible than dwell-time techniques [Hespanha & Morse, 1999; Allerhand & Shaked, 2011; Zhang & Shi, 2009; Zhang, et al., 2011; Persis, et al., 2003]. It means that the number of switches in the finite interval is bounded and the average time between successive switchings is greater than or equal to a constant [Zhang & Gao, 2010; Hespanha & Morse, 1999]. Thus, there are several important results on applications of average dwell-time techniques [Wang, et al., 2009; Wang, et al., 2007; Du, et al., 2011; Xiang, et al., 2010; Zhang & Gao, 2010; Zhang & Shi, 2009; Zhang, et al., 2011; Du, et al., 2011; Wang & Shao, 2010; Yang, et al., 2009]. However, to the best of the authors' knowledge, there are few results on applying average dwell-time techniques to fault-tolerant control of switched systems [Wang, et al., 2007; Du, et al., 2011; Xiang, et al., 2010; Wang, et al., 2009, Wang & Shao, 2010; Yang, et al., 2009; Ma & Yang, 2011]. [Wang, et al., 2007, Wang, et al., 2009] need decomposing actuators into faulty actuators and healthy ones. The common feature in [Du, et al., 2011; Xiang, et al., 2010, Wang & Shao, 2010] is that they did not consider structural uncertainties of input matrices. Reference [Yang, et al., 2009] applied average dwell-time method to fault-tolerant control of the switched nonlinear system where the nonlinear item is connected to the system in a parallel way, which can be directly compensated for by control signals. Ma & Yang (2011) proposed an adaptive logic-based switching fault-tolerant control method for a class of nonlinear uncertain systems against actuator faults, and also uses a similar way as [Yang, et al., 2009] to deal with the unmodeled dynamics. However, to our best knowledge, by using average dwell-time techniques the fault-tolerant control of the class of switched linear time-delay systems (1) without necessarily decomposing actuators into faulty actuators and healthy ones has not been investigated yet. This motivates us to study this problem.

This paper deals with the problem of robust fault-tolerant control of a class of switched time-delay linear systems with structural uncertainties existing in both system matrices and input matrices, and proposes a fault-tolerant control method for this class of switched systems by using average dwell-time techniques. The main features and contributions of this paper are highlighted as follows:

(1) The proposed control design works on both the switched time-delay systems with actuator faults and its nominal systems (i.e., without actuator faults) without necessarily changing any structures and/or parameters of the proposed controllers;
(2) The proposed method, unlike [Wang, et al., 2007; Jin, et al., 2007; Veillette, 1995; Wang, et al., 1999; Liang, et al., 2000; Wang, et al., 2009] but in a unified way for easy and practical applications, treats all actuators without necessarily classifying all actuators into faulty actuators and reliable ones;
(3) The proposed method is independent from switching provided that the average switching time is greater than certain dwelling time.

The layout of the paper is as follows. Section II presents the problem statement. The details about designing the controllers of the nonlinear switched systems and its stability analysis are presented in Section III. A numerical example is given in Section IV. Section V concludes the paper.

2. PROBLEM FORMULATION

Consider a class of uncertain switched time-delay systems

$$\begin{aligned} \dot{x}(t) &= (A_i + \Delta A_i)x(t) + E_i x(t-h) + (B_i + \Delta B_i)u_i \\ x(\theta) &= \phi(\theta), \theta \in [-h, 0), \end{aligned} \tag{1}$$

where $x \in R^n$ are system states, $u_i \in R^{q_i}$ is control input, $i:[0,+\infty) \to M = \{1,2,\cdots,m\}$ is a switching signal, A_i , B_i and E_i are known constant matrices, and ΔA_i and ΔB_i are matrix functions representing structural uncertainties. $\phi(\theta)$ is a differentiable vector-valued initial function on $[-h,0]$, and $h > 0$ denotes the state delay.

We now make the assumptions for system (1) as follows:

Assumption 1: Assume that (A_i, B_i) is controllable and that all the states are available for feedback.

Assumption 2: Assume that ΔA_i and ΔB_i are the structural uncertainties with bounded norms, i.e.,

$$\|\Delta A_i\| \le \delta_i \quad \text{and} \quad \|\Delta B_i\| \le \omega_i. \tag{2}$$

Then, one can design state feedback controllers as follows:

$$u_i = K_i x \tag{3}$$

where $K_i \in R^{q_i \times r}$, $i \in M = \{1,2,\cdots,m\}$ are constant matrices.

Given that whether a fault occurs on each actuator or not, a matrix $L_s^{\,i}$ is introduced to represent fault situation of the actuators of the i^{th} subsystem as follows:

$$L_s^i = diag(l_1^i, l_2^i, \cdots, l_q^i) \tag{4}$$

where if $l_j^i = 1$, $j = 1, 2, \cdots, q$, actuator j is normal and if $l_j^i = 0$ actuator j is faulty, and $L_s^i \neq 0$. Therefore, the closed-loop switched nonlinear systems involving uncertain structures and actuator faults are given as follows:

$$\begin{aligned} &\dot{x}(t) = (A_i + \Delta A_i)x(t) + E_i x(t-h) + (B_i + \Delta B_i)L_s^i K_i x(t) \\ &x(\theta) = \phi(\theta), \theta \in [-h, 0), \end{aligned} \tag{5}$$

The control objective is then to design feedback gain matrices K_i $(i \in M)$ such that switched time-delay system (5) under arbitrary switching policies are globally asymptotically stable for all uncertain matrices $\Delta A_i, \Delta B_i$ and the actuator faults.

3. CONTROLLER DESIGN FOR TIME-DELAY SYSTEMS

This section will present the main results on the robust fault-tolerant control of the switched systems (1) and stability analysis of the closed-loop systems (5).

Before presenting the main theorem, we need the following lemma.

Lemma 1: For $\forall x, y \in R^r$ and constant $\varepsilon > 0$ and symmetric positive matrix Π, the inequalities as follows hold:

$$x^T y + y^T x \leq \frac{x^T \Pi x}{\varepsilon} + \varepsilon y^T \Pi^{-1} y \leq \frac{x^T \Pi x}{\varepsilon} + \varepsilon \frac{y^T y}{\lambda_{\min}(\Pi)} \tag{6}$$

Theorem 1: Given positive constants $h, \lambda_0, \varepsilon$. Suppose that Assumptions 1 and 2 are satisfied and that there exist symmetric positive definite matrices P_i, Q_i, H_i and U_i such that the linear matrix inequalities (7) below hold:

$$\begin{bmatrix} \Pi_i & P_i E_i \\ E_i^T P_i & -e^{-2\lambda_0 h} Q_i \end{bmatrix} < 0 \tag{7}$$

where

$$\begin{aligned} \Pi_i = {} & A_i^T P_i + P_i A_i + P_i \left[\frac{1}{\varepsilon_i} H_i + \varepsilon_i \omega_i^2 \mathrm{I}_n + B_i \left(\frac{1}{\varepsilon_i} U_i + \frac{\varepsilon_i}{\lambda_{\min}(U_i)} \mathrm{I}_q + \frac{1}{\varepsilon_i} \mathrm{I}_q \right) B_i^T \right] P_i \\ & + \frac{\varepsilon_i}{\lambda_{\min}(H_i)} \delta_i^2 \mathrm{I}_n + 2\lambda_0 P_i + Q_i \end{aligned} \tag{8}$$

Then,

(1). if the average dwell-time satisfies

$$\tau_a \geq \tau_a^* = \frac{\ln \mu}{2(\lambda^* - \lambda)}, \quad \lambda \in (0, \lambda_0), \lambda^* \in (\lambda, \lambda_0) \tag{9}$$

where positive constant $\mu \geq 1$ satisfying

$$P_i \leq \mu P_j,\ Q_i \leq \mu Q_j,\ \forall i, j \in M \tag{10}$$

Then, the closed-loop systems (5) are globally exponentially stable under arbitrary switching rules with controller gain $K_i = -B_i^T P$, i.e., $u_i = K_i x = -B_i^T P x$ are fault-tolerant feedback controllers which stabilize switched systems (1) globally and exponentially.

(2). Norm estimation of states of systems (7) is measured by

$$\|x(t)\| \leq \sqrt{\frac{b}{a}} e^{-\lambda(t-t_0)} \|x_{t_0}\|, \tag{11}$$

where

$$a = \min_{i \in M} \lambda_{\min}(P_i),$$

and

$$b = \max_{i \in M} \lambda_{\max}(P_i) + h \max_{i \in M} \lambda_{\max}(Q_i).$$

Proof: Define following piecewise Lyapunov candidate function for systems (1):

$$V = V_{i(t)}(x_t) = x^T P_{i(t)} x + \int_{t-h}^{t} e^{2\lambda_0(s-t)} x^T(s) Q_{i(t)} x(s) ds \tag{12}$$

where P_i and Q_i ($i \in M$) are positive definite matrices and satisfying matrix inequalities (7). Then along the trajectory of systems (5), the time derivative of $V(x, z)$ is

$$\begin{aligned} \dot{V}_i = {} & x^T(t)(A_i^T P_i + P_i A_i + \Delta A_i^T P_i + P_i \Delta A_i - 2P_i B_i L_s^i B_i^T P_i - P_i B_i L_s^i \Delta B_i^T P_i \\ & - P_i \Delta B_i L_s^i B_i^T P_i) x(t) + x^T(t-h) E_i^T P_i x(t) + x^T(t) P_i E_i x(t-h) \\ & + x^T(t) Q x(t) - e^{-2\lambda_0 h} x^T(t-h) Q x(t-h) \\ & - 2\lambda_0 \int_{t-h}^{t} e^{2\lambda_0(s-t)} x^T(s) Q x(s) ds \end{aligned} \tag{13}$$

According to the inequality (6) in Lemma 1, for the constant $\varepsilon > 0$ and symmetric positive definite matrices H_i and U_i, also noting Assumption 2, one has

$$\begin{aligned} & x^T \left(\Delta A_i^T P_i + P_i \Delta A_i \right) x \\ & \leq x^T \left(\varepsilon \Delta A_i^T H_i^{-1} \Delta A_i + \frac{1}{\varepsilon} P_i H_i P_i \right) x \\ & \leq x^T \left(\frac{\varepsilon}{\lambda_{\min}(H_i)} \Delta A_i^T \Delta A_i + \frac{1}{\varepsilon} P_i H_i P_i \right) x \\ & \leq x^T \left(\frac{\varepsilon}{\lambda_{\min}(H_i)} \delta^2 \mathrm{I}_n + \frac{1}{\varepsilon} P_i H_i P_i \right) x \end{aligned} \tag{14}$$

$$\begin{aligned} & x^T \left(-2 P_i B_i L_s^i B_i^T P_i \right) x \\ & \leq x^T \left[\varepsilon P_i B_i \left(-L_s^i \right) U_i^{-1} (-L_s^i) B_i^T P_i + \frac{1}{\varepsilon} P_i B_i U_i B_i^T P_i \right] x \\ & \leq x^T \left[\frac{\varepsilon}{\lambda_{\min}(U_i)} P_i B_i \left(-L_s^i \right) \left(-L_s^i \right) B_i^T P_i + \frac{1}{\varepsilon} P_i B_i U_i B_i^T P_i \right] x \end{aligned}$$

$$\leq x^T\left[\frac{\varepsilon\left\|(-L_s^i)(-L_s^i)\right\|}{\lambda_{\min}(U_i)}P_iB_iB_i^TP_i+\frac{1}{\varepsilon}P_iB_iU_iB_i^TP_i\right]x$$
$$=x^T\left[\frac{\varepsilon}{\lambda_{\min}(U_i)}P_iB_iB_i^TP_i+\frac{1}{\varepsilon}P_iB_iU_iB_i^TP_i\right]x \tag{15}$$

$$x^T\left(-P_iB_iL_s^i\Delta B_i^TP_i-P_i\Delta B_iL_s^iB_i^TP_i\right)x$$
$$=x^T\left(P_iB_i(-L_s^i)\Delta B_i^TP_i+P_i\Delta B_i(-L_s^i)B_i^TP_i\right)x$$
$$\leq x^T\left[\varepsilon P_i\Delta B_i\Delta B_i^TP_i+\frac{1}{\varepsilon}P_iB_i(-L_s^i)(-L_s^i)B_i^TP_i\right]x \tag{16}$$
$$\leq x^T\left[\varepsilon\omega^2P_iP_i+\frac{1}{\varepsilon}\left\|(-L_s^i)(-L_s^i)\right\|P_iB_iB_i^TP_i\right]x$$
$$=x^T\left(\varepsilon\omega^2P_iP_i+\frac{1}{\varepsilon}P_iB_iB_i^TP_i\right)x$$

Using (14)-(16) and (7), one has

$$\begin{aligned}\frac{d(x^TP_ix)}{dt}\leq{}& x^T[A_i^TP_i+P_iA_i+P_i(\frac{1}{\varepsilon}H_i+\varepsilon\omega^2L_n\\ &+B_i(\frac{1}{\varepsilon}U_i+\frac{\varepsilon}{\lambda_{\min}(U_i)}\mathrm{I}_q+\frac{1}{\varepsilon}\mathrm{I}_q)B_i^T)P_i\\ &+\frac{\varepsilon}{\lambda_{\min}(H_i)}\delta^2\mathrm{I}_n]x+x^T(t)P_iE_ix(t-h)\\ &+x^T(t-h)E_i^TP_ix(t)\end{aligned} \tag{17}$$

Thus,

$$\begin{aligned}&\dot V_i(t)+2\lambda_0V_i\\ &\leq\begin{pmatrix}x(t)\\x(t-h)\end{pmatrix}^T\begin{bmatrix}\Pi_i & P_iE_i\\ E_i^TP_i & -e^{-2\lambda_0h}Q_i\end{bmatrix}\begin{pmatrix}x(t)\\x(t-h)\end{pmatrix}\leq 0\end{aligned} \tag{18}$$

where the second inequality relation holds from (7).

Using (10) and (12), one obtains

$$V_i(t)\leq\mu V_j(t),\qquad\forall i,j\in M,i<j \tag{19}$$

From (18), one obtains

$$\dot V_i\leq-2\lambda_0V_i \tag{20}$$

For any given $t>0$, let $t_0<t_1<t_2<\cdots<t_k=t_{N_i(t_0,t)}$ denote the time instants at which switching occurs over the operating interval (t_0,t). Thus, taking integration of both sides of (20) and utilizing (19) yield

$$\begin{aligned}V(x_t)=V_i(x_t)&\leq e^{-2\lambda_0(t-t_k)}\mu V_{i(t_k^-)}(x_{t_k})\\ &\leq e^{-2\lambda_0(t-t_{k-1})}\mu V_{i(t_{k-1}^-)}(x_{t_{k-1}})\\ &\leq e^{-2\lambda_0(t-t_{k-2})}\mu^2 V_{i(t_{k-2}^-)}(x_{t_{k-2}})\\ &\leq e^{-2\lambda_0(t-t_{k-3})}\mu^3 V_{i(t_{k-3}^-)}(x_{t_{k-3}})\\ &\leq\cdots\leq e^{-2\lambda_0(t-t_0)+k\ln\mu}V_{i(t_0)}(x_{t_0})\end{aligned} \tag{21}$$

From (19) and (21), one has

$$\begin{aligned}V(x_t)&\leq e^{-2\lambda^*(t-t_0)+\frac{t-t_0}{\tau_a}\ln\mu}V_{i(t_0)}(x_{t_0})\\ &\leq e^{-2(\lambda^*-\frac{1}{2\tau_a}\ln\mu)(t-t_0)}V_{i(t_0)}(x_{t_0})\\ &\leq e^{-2\lambda(t-t_0)}V_{i(t_0)}(x_{t_0})\end{aligned} \tag{22}$$

(10) and (12) then lead to

$$a\left\|x(t)\right\|^2\leq V(x_t)\leq b\left\|x_t\right\|^2 \tag{23}$$

From (22) and (23), one has

$$\left\|x(t)\right\|^2\leq\frac{1}{a}V(x_t)\leq\frac{b}{a}e^{-2\lambda(t-t_0)}\left\|x(t_0)\right\|^2$$

Therefore, we have the conclusion (11).

Remark: Note that (7) is nonlinear matrix inequalities. One cannot solve it with available solver. For solving this issue, pre- and post-multiplying both sides of matrix inequalities (7) by $\Gamma_i=P_i^{-1}$, (7) needs to be carried out and following *LMIs* are generated

$$\begin{bmatrix}\Xi_i & E_iT_i\\ E_iT_i & -e^{-2\lambda_0h}T_iQ_iT_i\end{bmatrix}<0 \tag{24}$$

where

$$\Xi_i=T_iA_i^T+A_iT_i+\left[\frac{1}{\varepsilon_i}H_i+\varepsilon_i\omega_i^2\mathrm{I}_n+B_i\left(\frac{1}{\varepsilon_i}U_i+\frac{\varepsilon_i}{\lambda_{\min}(U_i)}\mathrm{I}_q+\frac{1}{\varepsilon_i}\mathrm{I}_q\right)B_i^T\right]$$
$$+\frac{\varepsilon_i}{\lambda_{\min}(H_i)}\delta_i^2T_iT_i+2\lambda_0T_i+T_iQ_iT_i,$$

Defining $T_iQ_iT_i=Y_i$ and applying Schur Complement Lemma, (3.19) is transformed into

$$\begin{bmatrix}T_iA_i^T+A_iT_i+Y_i++2\lambda_0T_i+\Gamma_i & E_iT_i & T_i\\ * & -e^{-2\lambda_0h}Y_i & 0\\ * & * & -\frac{\varepsilon_i}{\lambda_{\min}(H_i)}\delta_i^2I\end{bmatrix}<0$$

where

$$\Gamma_i=\left[\frac{1}{\varepsilon_i}H_i+\varepsilon_i\omega_i^2\mathrm{I}_n+B_i\left(\frac{1}{\varepsilon_i}U_i+\frac{\varepsilon_i}{\lambda_{\min}(U_i)}\mathrm{I}_q+\frac{1}{\varepsilon_i}\mathrm{I}_q\right)B_i^T\right]$$

4. A NUMERICAL EXAMPLE

In this section, the proposed method will be applied to the following numerical example with model structure as given in Eq. (1) to show the effectiveness of the proposed method.

$$A_1=\begin{bmatrix}-4 & -2 & 0\\ 0 & -4 & 0\\ -0.4 & 1 & -4\end{bmatrix},\qquad B_1=\begin{bmatrix}1 & 0\\ 0.5 & 0.5\\ -0.5 & 1\end{bmatrix},$$

$$A_2=\begin{bmatrix}-2 & -2 & 0\\ 0 & -2.5 & 0\\ -0.4 & 1 & -2\end{bmatrix},\qquad B_2=\begin{bmatrix}1 & 0\\ 0.6 & 0.5\\ -0.6 & 0.9\end{bmatrix} \qquad (25)$$

$$H_1=\begin{bmatrix}0.1 & 0.002 & 0\\ 0.002 & 0.1 & 0\\ 0 & 0 & 0.1\end{bmatrix},\quad U_1=\begin{bmatrix}3 & 0\\ 0 & 3\end{bmatrix},\quad L_s^1=\begin{bmatrix}0 & 0\\ 0 & 1\end{bmatrix}$$

$$H_2=\begin{bmatrix}0.2 & 0.002 & 0\\ 0.002 & 0.2 & 0\\ 0 & 0 & 0.2\end{bmatrix},\quad U_2=\begin{bmatrix}3 & 0\\ 0 & 3\end{bmatrix},\quad L_s^2=\begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix}$$

$$E_1=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix},\qquad E_2=\begin{bmatrix}-0.2 & 0 & 0\\ 0 & -0.2 & 0\\ 0 & 0 & -0.2\end{bmatrix},$$

$x(\theta)=[-100,120,-80]^T, \theta\in[-h,0)$, $h=0.2$, $\varepsilon_1=\varepsilon_2=1.2$, $\delta_1=\delta_2=5$, $\omega_1=\omega_2=0.5$,, $\lambda_0=1.25$ and $\mu=1.1$ 。

It is easy to verify that Assumptions 1-2 are satisfied by (25). Let $x(\theta)=[-100,120,-80]^T, \theta\in[-h,0)$. Solving Riccati equation (9) with the parameters given above gives positive definite matrix solutions as shown below:

$$P_1=\begin{bmatrix}0.0138 & -0.0372 & -0.0051\\ -0.0372 & 0.1319 & 0.0171\\ -0.0051 & 0.0171 & 0.0083\end{bmatrix}, P_2=\begin{bmatrix}0.0164 & -0.0367 & -0.0035\\ -0.0367 & 0.1346 & 0.0121\\ -0.0035 & 0.0121 & 0.0095\end{bmatrix}$$

$$Q_1=\begin{bmatrix}0.0063 & -0.0192 & -0.0021\\ -0.0192 & 0.0777 & 0.0083\\ -0.0021 & 0.0083 & 0.0035\end{bmatrix},\ Q_2=\begin{bmatrix}0.0095 & -0.0167 & -0.0005\\ -0.0167 & 0.0942 & -0.0007\\ -0.0005 & -0.0007 & 0.0050\end{bmatrix}.$$

Then, according to $u_i=K_i x=-B_i^T P x$ $(i=1,2)$ the controller can be designed. Taking $\lambda^*=1$ and $\lambda=0.97$, one has $\tau_a^*=\dfrac{\ln\mu}{2(\lambda^*-\lambda)}=1.59$. Take $\tau_a^*\le\tau_a=2$ and choose the following switching rule:

$$i=\begin{cases}1, & t_k=0,3.8,8.8,15\\ 2, & t_k=2,6,11,18\end{cases}$$

to carry out the simulation studies for system (25).

As Fig. 1 shows, the state feedback control law guarantees that systems (25) under arbitrary switching rules are still asymptotically stable when the second actuator of the first subsystem and the first actuator of the second subsystem have faults, as indicated by L_s^1 and L_s^2 . Fig. 1 is the time history of the states, where the stars represent switching points. Fig. 2 and Fig. 3 represent switching sequences of controller gains and switching signal, respectively. From the figures, the effectiveness of the proposed control method is verified.

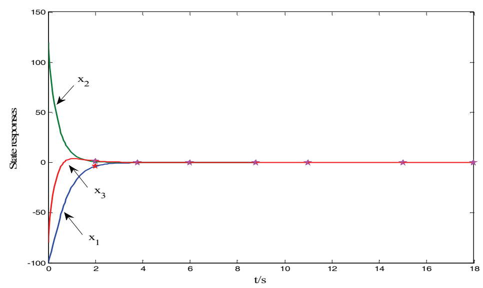

Fig.1 System state responses

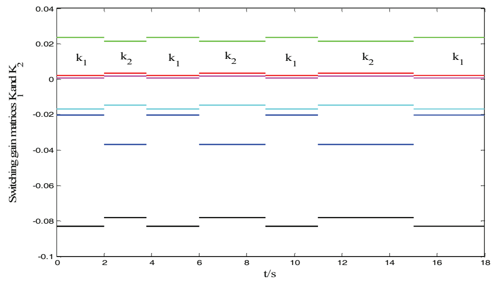

Fig.2 Switching sequences of gain matrices

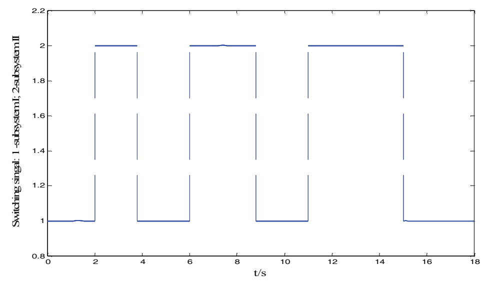

Fig.3 Switching signal

5. CONCLUSIONS

This paper studies robust fault-tolerant control of a class of uncertain switched linear time-delay systems. By designing feedback control law and using the average dwelling time techniques, a sufficient condition is given on globally asymptotical stabilization of the switched nonlinear systems against actuator faults under arbitrarily switching signals provided switching is on-the-average slow enough.

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