

Attitude Coordinated Tracking of Multiple Spacecraft with Control Input Saturation[★]

Qingling Wang^{*,**} Huijun Gao^{*,****} Jiahu Qin^{****,*}
Changbin Yu^{**,†}

^{*} *State Key Laboratory of Robotics and System (HIT), Harbin Institute of Technology, Harbin, China*

^{**} *Australian National University, Canberra, Australia*

^{***} *University of Science and Technology of China*

^{****} *King Abdulaziz University, Jeddah, Saudi Arabia*

[†] *Shandong Computer Science Center, Jinan, China*

Abstract: The attitude coordinated tracking problem of multiple spacecraft with control input saturation is studied in this paper. We first construct a decentralized attitude coordinated tracking algorithm with input saturation, such that spacecraft can track a time-varying desired trajectory and align their attitudes. We then introduce a novel auxiliary dynamical systems to present an attitude coordinated tracking algorithm with input saturation and without angular velocity measurements. Throughout this paper, the information flow between spacecraft is assumed to be undirected. Finally, design examples are given to show the effectiveness of the proposed attitude coordinated tracking algorithms.

1. INTRODUCTION

Attitude coordinated tracking of multiple spacecraft has received significant attention in recent years. In general, spacecraft attitude kinematics and dynamics can be modeled by three-parameter representations (e.g., the Euler angles and modified Rodrigues parameters) and four-parameter representations (e.g., unit quaternion). Several works Chung et al. (2009); Meng et al. (2010); Ren (2010); Zou et al. (2012) have studied the attitude coordinated control problem for spacecraft with three-parameter representations. However, three-parameter representations always exhibit singularity. By using four-parameter representations, the attitude coordinated control problem was investigated in Lawton and Beard (2002); Jin et al. (2008); Chang et al. (2009) with a constant reference attitude. For the time-varying reference attitude, the same problem was presented in Bai et al. (2007); Ren (2007); Wu et al. (2011).

Although many results of multiple spacecraft attitude coordinated control have been given in detail, it is observed that the existing aforementioned literatures have carried out coordinated controller design without control input saturation. In practical attitude control systems, the

control torques offered by actuators are always limited Summers et al. (2009); Akella et al. (2005), and the coordinated controller design without input saturation might not guarantee the stability of the closed-loop system. Recently, by using the Chebyshev neural network, a distributed attitude coordinated control method was proposed for spacecraft with control input saturation when the time-varying reference attitude is available to a subset of spacecraft Zou and Kumar (2012). However, a distributed sliding-mode observer was introduced for attitude coordinated control of spacecraft with transmitting their angular accelerations by using discontinuous signum function. It is known that the implementation issues of this kind of observers will increase the cost and complexity in that more sensors and intensive communication are required. Different from this point, this paper aims to solve the attitude coordinated tracking problem for multiple spacecraft with control input saturation, but without additional cost and complexity.

It is also observed that the attitude coordinated algorithms in the above literatures are mainly based on the assumption that spacecraft know their own and their neighbors' angular velocities. In practise, the angular velocity measurements are not always satisfied due to the failure of mechanical gyroscopes on spacecraft. In Abdessameud and Tayebi (2009), the coordinated attitude control problem for a group of spacecraft was investigated, without considering velocity measurements. However, the coordinated controller design does not consider control input saturation. As an extension, this paper tries to solve the attitude coordinated tracking problem for multiple spacecraft with control input saturation, and remove the requirement for the angular velocity and the relative angular velocities.

Due to the nonlinear dynamics of spacecraft, the problem of attitude coordinated control for a group of spacecraft is more challenging, especially when control input saturation

[★] This work was supported in part by the Australian Research Council through Discovery Project DP-130103610, a Queen Elizabeth II Fellowship under Grant DP110100538, the Program for New Century Excellent Talents in University, China, under Grant NCET-130544, Overseas Expert Program of Shandong Province, a grant from Shandong Academy of Science Development Fund for Science and Technology, and the Pilot Project for Science and Technology in Shandong Academy of Science, the Anhui Provincial Natural Science Foundation (1408085QF105), the Self-Planned Task (NO. SKLRS201308B) of State Key Laboratory of Robotics and System (HIT) and the National Natural Science Foundation of China under Grants (61333012, 61273201, 61203122 and 61375072). Q. Wang was in part supported by the China Scholarship Council for his visit to the Australian National University.

is considered. The main work of this paper includes two aspects:

1) A decentralized attitude coordinated algorithm is proposed with control input magnitude saturation and the requirement of the angular velocity and the relative angular velocities. The topology of the information flow between spacecraft is described by an undirected graph, and the time-varying reference attitude is available to all spacecraft¹. Different from the main work of Zou and Kumar (2012), the implementation of the attitude coordinated controller will not need additional cost and complexity.

2) To remove the measurements of the angular velocity and the relative angular velocities, we use the concept of auxiliary dynamical systems. By introducing one with input saturation, a velocity-free coordinated controller is proposed to align their attitudes and to track a time-varying reference. Here the term *velocity-free* means without angular velocity and the relative angular velocity measurements. The design method is motivated by the work of Abdessameud and Tayebi (2009), and this paper extends the main result of Abdessameud and Tayebi (2009) to the case of considering control input saturation.

The rest of the paper is organized as follows. Section 2 formulates the attitude coordinated tracking problem. Section 3 presents the main results on the attitude coordinated tracking problem. Section 4 provides design examples for verifying the theoretical results. Finally, some conclusion remarks are given in Section 5.

Before closing this section, some notations will be stated here. The notation $\text{diag}\{\dots\}$ denotes a block-diagonal matrix. Using $\text{col}\{x_1, x_2, \dots, x_n\}$ to denote a column vector. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations. The notation $P > 0$ (≥ 0) means that P is a real symmetric positive (semi-positive) definite matrix. I and 0 represent, respectively, the identity matrix and zero matrix.

2. PRELIMINARIES AND PROBLEM FORMULATION

2.1 Dynamics of spacecraft attitude

In this paper, we consider N spacecraft. The attitude dynamics of the i -th spacecraft Abdessameud and Tayebi (2009) is

$$\begin{cases} \dot{q}_i = \frac{1}{2}E(q_i)\omega_i \\ J_i\dot{\omega}_i = -s(\omega_i)J_i\omega_i + \tau_i \end{cases}, \quad i = 1, 2, \dots, N, \quad (1)$$

where $\omega_i \in \mathbb{R}^3$ and $J_i \in \mathbb{R}^{3 \times 3}$ are the angular velocity and symmetric positive definite inertia matrix of the i -th spacecraft, respectively, in the body-fixed frame F_b^i . The vector τ_i is the external torque of the i -th spacecraft in F_b^i . The unit-quaternion $q_i = [\sigma_i^T, \eta_i]^T$ is composed of the real part $\eta_i \in \mathbb{R}$ and the vector part $\sigma_i \in \mathbb{R}^3$, and denotes the orientation between the frame F_b^i and the frame F_I . Also, The unit-quaternion q_i is subject to the following condition

$$\eta_i^2 + \sigma_i^T \sigma_i = 1. \quad (2)$$

¹ whether the time-varying reference attitude must be available to a subset of spacecraft or to all spacecraft is still open even without input saturation. (Abdessameud and Tayebi (2009))

Let the vector $v = [v_1, v_2, v_3]^T$, then the notation $s(\cdot)$ denotes a 3×3 skew-symmetric matrix, that is,

$$s(v) = \begin{bmatrix} 0 & -v_3 & v_2 \\ v_3 & 0 & -v_1 \\ -v_2 & v_1 & 0 \end{bmatrix}.$$

For the matrix $E(q_i)$, we have

$$E(q_i) = \begin{bmatrix} \eta_i I_3 + s(\sigma_i) \\ -\sigma_i^T \end{bmatrix}.$$

We define the orthogonal rotation matrix as $R(q_i)$, and it can be obtained as

$$R(q_i) = (\eta_i^2 - \sigma_i^T \sigma_i) I_3 + 2\sigma_i \sigma_i^T - 2\eta_i s(\sigma_i). \quad (3)$$

Assume that the desired attitude for all spacecraft is given by $q_d = [\sigma_d^T, \eta_d]^T$ that denotes the orientation of the desired frame, represented by F_d , and satisfies the following dynamics

$$\dot{q}_d = \frac{1}{2}E(q_d)\omega_d, \quad (4)$$

where $\omega_d \in \mathbb{R}^3$ represents the velocity of the desired spacecraft. We assume that the first and second time-derivatives of ω_d are bounded. The difference between the absolute attitude and the desired attitude of the i -th spacecraft is considered as the attitude tracking error, namely $\bar{q}_i = [\bar{\sigma}_i^T, \bar{\eta}_i]^T$, and is given by

$$\dot{\bar{q}}_i = \frac{1}{2}E(\bar{q}_i)\bar{\omega}_i, \quad (5)$$

where $\bar{\omega}_i = \omega_i - R(\bar{q}_i)\omega_d$ is the angular velocity error vector. Considering the same procedure as in Tayebi (2008), it is known that the error systems for the i -th spacecraft satisfy

$$J_i \dot{\bar{\omega}}_i = -J_i R(\bar{q}_i) \dot{\omega}_d - s(R(\bar{q}_i)\omega_d) J_i R(\bar{q}_i) \omega_d + \tau_i. \quad (6)$$

In what follows, we use the graph $G = (\mathcal{V}, \varepsilon, K)$ to represent the communication topology between spacecraft, where N is a finite nonempty set of nodes $\mathcal{V} = \{1, \dots, N\}$ and edges $\varepsilon = \mathcal{V} \times \mathcal{V}$. The weighted adjacency matrix is $K = [k_{ij}] \in \mathbb{R}^{N \times N}$, where k_{ij} is the coupling strength of the directed edge (j, i) satisfying $k_{ij} \neq 0$ if (j, i) is an edge of G and $k_{ij} = 0$ otherwise. Let $N_i = \{j \in \mathcal{V} : (j, i) \in \varepsilon\}$ be the set of neighbors of node i in G . For any pair of vertices (i, j) , if $k_{ij} = k_{ji}$, the graph is called an undirected graph. Let $d_i = \sum_{j=1}^N k_{ij}$ the in-degree of vertex i , and $D = \text{diag}\{d_1, \dots, d_N\}$ the in-degree matrix of G . The Laplacian matrix $L = [l_{ij}]$ of weighted digraph G is defined by $L = D - E$. The i -th and j -th spacecraft are connected by a graph in this paper, if they have information exchange. We define the relative attitude of the i -th and j -th spacecraft, namely $q_{ij} = [\sigma_{ij}^T, \eta_{ij}]^T$, as

$$\dot{q}_{ij} = \frac{1}{2}E(q_{ij})\omega_{ij}, \quad (7)$$

where $\omega_{ij} = \omega_i - R(q_{ij})\omega_j$ is the relative angular velocity and q_{ij} represents the rotation from F_b^j to F_b^i . The following properties are known Abdessameud and Tayebi (2009):

$$R(q_{ij}) = R^T(q_{ji}), \quad \sigma_{ij} = -\sigma_{ji} = -R(q_{ij})\sigma_{ji}. \quad (8)$$

It can be seen from the above definition that attitude tracking control is reached when q_i coincides with q_d for all $i = 1, 2, \dots, N$, that is, $q_i = [0^T, \pm 1]^T$ and $\bar{\omega}_i = 0$ for

all $i = 1, 2, \dots, N$. We also need the following definition of the saturation function.

Definition 1. For a positive scalar ρ , the saturation function $\delta_\rho : \mathbb{R}^m \rightarrow \mathbb{R}^m$ satisfies that $\delta_\rho(x)$ is decentralized, that is, $\delta_\rho(x) = \text{col} \{ \delta_\rho(x_1), \delta_\rho(x_2), \dots, \delta_\rho(x_m) \}$, where it is defined $x = \text{col} \{ x_1, x_2, \dots, x_m \} \in \mathbb{R}^m$ and for each $i = 1, 2, \dots, m$, $\delta_\rho(x_i) = \text{sign}(x_i) \min \{ |x_i|, \rho \}$, where $\text{sign}(\cdot)$ is the signum function.

2.2 Control objectives

In this paper, the main objectives are to design attitude coordinated tracking control algorithms for a group of spacecraft described by system (1).

1) OBJ1: Design an attitude coordinated tracking control algorithm with input saturation, such that each spacecraft tracks the desired trajectory, and the relative attitude and angular velocities between the spacecraft converge to zero, simultaneously, that is, $q_i(t) \rightarrow q_j(t) \rightarrow q_d(t)$ and $\omega_i(t) \rightarrow \omega_j(t) \rightarrow \omega_d(t)$ for all $i, j = 1, 2, \dots, N$.

2) OBJ2: Design a velocity-free attitude coordinated tracking control algorithm with input saturation, such that each spacecraft tracks the desired trajectory, and the relative attitudes and angular velocities between the spacecraft converge to zero, simultaneously, that is, $q_i(t) \rightarrow q_j(t) \rightarrow q_d(t)$ and $\omega_i(t) \rightarrow \omega_j(t) \rightarrow \omega_d(t)$ for all $i, j = 1, 2, \dots, N$.

3. MAIN RESULTS

3.1 Attitude coordinated tracking design for OBJ1

In this section, we consider the first problem (OBJ1) which consists of a simultaneous attitude tracking and the allowance of N spacecraft to align their attitudes.

Consider the following control algorithm for the i -th spacecraft, given by

$$\begin{aligned} \tau_i = & J_i R(\bar{q}_i) \dot{\omega}_d + s(R(\bar{q}_i) \omega_d) J_i R(\bar{q}_i) \omega_d - c_i^p \bar{\sigma}_i - c_i^d \delta_\rho(\bar{\omega}_i) \\ & - \sum_{j=1}^N k_{ij}^p \sigma_{ij} - \sum_{j=1}^N k_{ij}^d [\delta_\rho(\omega_{ij}) - R(q_{ij}) \delta_\rho(\omega_{ji})], \end{aligned} \quad (9)$$

where c_i^p and c_i^d are strictly positive attitude tracking control gains and k_{ij}^p is the edge (i, j) of an weighted undirected graph $G_1 = (\mathcal{V}, \varepsilon, K_p)$ to represent the information flow of the relative attitude q_{ij} while k_{ij}^d is that of $G_2 = (\mathcal{V}, \varepsilon, K_d)$ to represent the information flow of the relative angular velocity ω_{ij} .

In what follows, the main result is obtained according to the above control algorithm (9).

Theorem 1. Consider a group of N spacecraft modeled by (1) under the control algorithm (9). If

$$c_i^p > 2 \sum_{j=1}^N k_{ij}^p \quad (10)$$

holds for all $i = 1, 2, \dots, N$, then, the desired attitude can be tracked and the relative attitude and angular velocities between the spacecraft converge to zero asymptotically, that is, $q_i(t) \rightarrow q_j(t) \rightarrow q_d(t)$ and $\omega_i(t) \rightarrow \omega_j(t) \rightarrow$

$\omega_d(t)$ for all $i, j = 1, 2, \dots, N$. Furthermore, the control algorithm (9) is bounded as follows:

$$\|\tau_i\| \leq \|J_i\| (\omega_1 + \omega_2^2) + c_i^p + c_i^d \rho + \sum_{j=1}^N (k_{ij}^p + 2\rho k_{ij}^d), \quad (11)$$

where ω_1 and ω_2 are the upper bounds of $\dot{\omega}_d(t)$ and $\omega_d(t)$, respectively.

Proof. In view of (6) and (9), the closed-loop angular velocity error dynamics for the i -th spacecraft is

$$\begin{aligned} J_i \dot{\bar{\omega}}_i = & -c_i^p \bar{\sigma}_i - c_i^d \delta_\rho(\bar{\omega}_i) - \sum_{j=1}^N k_{ij}^p \sigma_{ij} \\ & - \sum_{j=1}^N k_{ij}^d [\delta_\rho(\omega_{ij}) - R(q_{ij}) \delta_\rho(\omega_{ji})], \end{aligned} \quad (12)$$

for all $i = 1, 2, \dots, N$. Consider the following Lyapunov function

$$V_1 = \frac{1}{2} \sum_{i=1}^N \bar{\omega}_i^T J_i \bar{\omega}_i + 2 \sum_{i=1}^N c_i^p (1 - \bar{\eta}_i) + \sum_{i=1}^N \sum_{j=1}^N k_{ij}^p (1 - \eta_{ij}).$$

Note that V_1 is positive definite. Then, in view of (5) and (7), the time derivation of V_1 along the trajectories of the closed-loop system (12) is

$$\begin{aligned} \dot{V}_1 = & - \sum_{i=1}^N c_i^d \bar{\omega}_i^T \delta_\rho(\bar{\omega}_i) - \sum_{i=1}^N \sum_{j=1}^N k_{ij}^p \bar{\omega}_i^T \sigma_{ij} \\ & + \frac{1}{2} \sum_{i=1}^N \sum_{j=1}^N k_{ij}^p \sigma_{ij}^T \omega_{ij} \\ & - \sum_{i=1}^N \sum_{j=1}^N k_{ij}^d \bar{\omega}_i^T [\delta_\rho(\omega_{ij}) - R(q_{ij}) \delta_\rho(\omega_{ji})], \end{aligned}$$

where we know that $2(1 - \bar{\eta}_i) = \bar{\sigma}_i^T \bar{\sigma}_i + (1 - \bar{\eta}_i)^2$ for \bar{q}_i and q_{ij} . Using (8) and the facts that $\sigma_{ji}^T R(q_{ji}) = \sigma_{ji}^T$, $\sigma_{ji} = -\sigma_{ij}$ and $k_{ij}^p = k_{ji}^p$ for all $i, j = 1, 2, \dots, N$, it is obtained that

$$\begin{aligned} \frac{1}{2} \sum_{i=1}^N \sum_{j=1}^N k_{ij}^p \sigma_{ij}^T \omega_{ij} = & \frac{1}{2} \sum_{i=1}^N \sum_{j=1}^N k_{ij}^p \sigma_{ij}^T (\bar{\omega}_i - R(q_{ij}) \bar{\omega}_j) \\ = & \sum_{i=1}^N \sum_{j=1}^N k_{ij}^p \bar{\omega}_i^T \sigma_{ij}, \end{aligned}$$

where we have used the facts that $\bar{\omega}_i^T \sigma_{ij}$ and $\sigma_{ij}^T \bar{\omega}_i$ are scalars and $\omega_{ij} = \omega_i - R(q_{ij}) \omega_j$. Furthermore, according to the facts that $R(q_{ji}) = R^T(q_{ij})$, $k_{ij}^d = k_{ji}^d$ and $\omega_{ij} = \bar{\omega}_i - R(q_{ij}) \bar{\omega}_j$, we have

$$\begin{aligned} & \sum_{i=1}^N \sum_{j=1}^N k_{ij}^d \bar{\omega}_i^T [\delta_\rho(\omega_{ij}) - R(q_{ij}) \delta_\rho(\omega_{ji})] \\ = & \sum_{i=1}^N \sum_{j=1}^N k_{ij}^d \bar{\omega}_i^T \delta_\rho(\omega_{ij}) - \sum_{i=1}^N \sum_{j=1}^N k_{ij}^d \bar{\omega}_i^T R(q_{ij}) \delta_\rho(\omega_{ji}) \\ = & \sum_{i=1}^N \sum_{j=1}^N k_{ij}^d \omega_{ij}^T \delta_\rho(\omega_{ij}). \end{aligned}$$

Consequently, \dot{V}_1 can be further simplified as

$$\dot{V}_1 = - \sum_{i=1}^N c_i^d \bar{\omega}_i^T \delta_\rho(\bar{\omega}_i) - \sum_{i=1}^N \sum_{j=1}^N k_{ij}^d \omega_{ij}^T \delta_\rho(\omega_{ij}) \leq 0,$$

where we have used the fact that $\delta_\rho(\cdot)$ is an odd function. Therefore, we have $V_1(t) \leq V_1(0)$. According to the definitions, we know that $\bar{\omega}_i$ and ω_{ij} are bounded, then, it is obtained that $\dot{\bar{\omega}}_i$ and $\dot{\omega}_{ij}$ are bounded. Consequently, it is easy to verify that $\dot{V}_1(t)$ is bounded. According to Barbalat's lemma Khalil and Grizzle (2002), we have $\lim_{t \rightarrow \infty} \dot{V}_1(t) = 0$, that is, $\bar{\omega}_i \rightarrow 0$ and $\omega_{ij} \rightarrow 0$ as $t \rightarrow \infty$.

According to the result that $\dot{\bar{\omega}}_i$ is bounded, it is easy to demonstrate that $\ddot{\bar{\omega}}_i$ is bounded, which shows that $\bar{\omega}_i \rightarrow 0$ as $t \rightarrow \infty$ by reapplying Barbalat's lemma. In view of the closed-loop system (12) and the facts that $\bar{\omega}_i \rightarrow 0$, $\dot{\bar{\omega}}_i \rightarrow 0$ and $\omega_{ij} \rightarrow 0$ as $t \rightarrow \infty$, we have

$$c_i^p \bar{\sigma}_i + \sum_{j=1}^N k_{ij}^p \sigma_{ij} = 0, \quad i = 1, 2, \dots, N. \quad (13)$$

Following the similar procedure as in Lawton and Beard (2002), and using the fact that $\omega_{ij} = \bar{\omega}_i - R(q_{ij})\bar{\omega}_j$ for all $i, j = 1, 2, \dots, N$, we can conclude that if (10) holds, (13) has a unique solution $[\bar{\sigma}_1^T, \bar{\sigma}_2^T, \dots, \bar{\sigma}_N^T]^T = 0$ as a result of the strictly diagonally dominant matrix. Then we can obtain that $\bar{\sigma}_i \rightarrow \bar{\sigma}_j \rightarrow 0$ as $t \rightarrow \infty$. Finally, we can conclude that $q_i(t) \rightarrow q_j(t) \rightarrow q_d(t)$ for all $i, j = 1, 2, \dots, N$. Moreover, since $\bar{\omega}_i \rightarrow 0$, $\dot{\bar{\omega}}_i \rightarrow 0$ and $\omega_{ij} \rightarrow 0$ as $t \rightarrow \infty$, we conclude that $\omega_i(t) \rightarrow \omega_j(t) \rightarrow \omega_d(t)$ for all $i, j = 1, 2, \dots, N$. Furthermore, it is known from the above results that the control algorithm (9) is bounded by (11). This completes the proof. ■

Remark 1. Note that the control algorithm (9) is given to be bounded as (11). Therefore, it is easy to set the desired bounds on the control torques via an proper choice of the control gains.

3.2 Velocity-free attitude coordinated tracking design for OBJ2

In this section, we consider the second problem (OBJ2) which consists of a simultaneous velocity-free attitude tracking and the allowance of N spacecraft to align their attitudes.

To remove the use of measurements of $\bar{\omega}_i$ and ω_{ij} , which are not always satisfied due to the failure of mechanical gyroscopes on spacecraft, we use the concept of auxiliary systems, which is introduced in Tayebi (2008), and propose the dynamic of i -th auxiliary system as follows:

$$\dot{p}_i = \frac{1}{2} E(p_i) \beta_i, \quad (14)$$

where $\beta_i \in \mathbb{R}^3$ will be designed later. But the difference between the auxiliary system and the attitude tracking error for spacecraft i , namely $\bar{p}_i = [\bar{\theta}_i^T, \bar{\epsilon}_i^T]^T$, is defined as

$$\begin{cases} \dot{\bar{\theta}}_i = \frac{1}{2} \delta_\rho(\bar{\epsilon}_i I_3 + s(\bar{\theta}_i)) \Omega_i \\ \dot{\bar{\epsilon}}_i = -\frac{1}{2} \delta_\rho^T(\bar{\theta}_i) \Omega_i \end{cases}, \quad (15)$$

where $\Omega_i = \bar{\omega}_i - R(\bar{p}_i) \beta_i$ and $R(\bar{p}_i)$ is the rotation matrix respected to \bar{p}_i . Let the unit-quaternion auxiliary system to each pair spacecraft (i, j) be

$$\dot{p}_{ij} = \frac{1}{2} E(p_{ij}) \beta_{ij},$$

where $\beta_{ij} \in \mathbb{R}^3$ will be designed later. Also, the relative attitude error between the i -th and j -th spacecraft, namely $\bar{p}_{ij} = [\bar{\theta}_{ij}^T, \bar{\epsilon}_{ij}^T]^T$, is defined as

$$\begin{cases} \dot{\bar{\theta}}_{ij} = \frac{1}{2} \delta_\rho(\bar{\epsilon}_{ij} I_3 + s(\bar{\theta}_{ij})) \Omega_{ij} \\ \dot{\bar{\epsilon}}_{ij} = -\frac{1}{2} \delta_\rho^T(\bar{\theta}_{ij}) \Omega_{ij} \end{cases}, \quad (16)$$

where $\Omega_{ij} = \omega_{ij} - R(\bar{p}_{ij}) \beta_{ij}$.

Remark 2. The mechanism of auxiliary systems is well explained in Abdessameud and Tayebi (2009). The differences are the dynamics of attitude errors between spacecraft and the auxiliary systems (15) and the dynamics of relative attitude errors between spacecraft (16).

The velocity-free attitude coordinated tracking controller is proposed as

$$\tau_i = J_i R(\bar{q}_i) \dot{\omega}_d + s(R(\bar{q}_i) \omega_d) J_i R(\bar{q}_i) \omega_d - c_i^p \bar{\sigma}_i - c_i^d \delta_\rho(\bar{\theta}_i) - \sum_{j=1}^N k_{ij}^p \sigma_{ij} - \sum_{j=1}^N k_{ij}^d [\delta_\rho(\bar{\theta}_{ij}) - R(q_{ij}) \delta_\rho(\bar{\theta}_{ji})], \quad (17)$$

where c_i^p, c_i^d, k_{ij}^p and k_{ij}^d are defined as in (9). The result is stated in the following theorem.

Theorem 2. Consider a group of N spacecraft modeled by (1) under the control algorithm (17). Let the auxiliary systems (15) and (16) be, respectively,

$$\beta_i = \Gamma_i \bar{\theta}_i, \quad \beta_{ij} = \Gamma_{ij} \bar{\theta}_{ij}, \quad (18)$$

where we have $\Gamma_i = \Gamma_i^T > 0$ and $\Gamma_{ij} = \Gamma_{ij}^T > 0$. If the control gains satisfy

$$c_i^p > 2 \sum_{j=1}^N k_{ij}^p \quad (19)$$

holds for all $i = 1, 2, \dots, N$, then, the desired attitude can be tracked and the relative attitude and angular velocities between the spacecraft converge to zero asymptotically, that is, $q_i(t) \rightarrow q_j(t) \rightarrow q_d(t)$ and $\omega_i(t) \rightarrow \omega_j(t) \rightarrow \omega_d(t)$ for all $i, j = 1, 2, \dots, N$. Furthermore, the control algorithm (17) is bounded as (11)

Proof. In view of (6) and (17), the closed-loop angular velocity-free error dynamics of the i -th spacecraft is

$$J_i \dot{\bar{\omega}}_i = -c_i^p \bar{\sigma}_i - c_i^d \delta_\rho(\bar{\theta}_i) - \sum_{j=1}^N k_{ij}^p \sigma_{ij} - \sum_{j=1}^N k_{ij}^d [\delta_\rho(\bar{\theta}_{ij}) - R(q_{ij}) \delta_\rho(\bar{\theta}_{ji})]. \quad (20)$$

for all $i = 1, 2, \dots, N$. Consider the following Lyapunov function

$$V_2 = \sum_{i=1}^N \left[\frac{1}{2} \bar{\omega}_i^T J_i \bar{\omega}_i + 2c_i^p (1 - \bar{\eta}_i) + 2c_i^d (1 - \bar{\epsilon}_i) \right] + \sum_{i=1}^N \sum_{j=1}^N [k_{ij}^p (1 - \eta_{ij}) + 2k_{ij}^d (1 - \bar{\epsilon}_{ij})].$$

Note that V_2 is positive definite. Then, the time derivation of V_2 along the trajectories of the closed-loop system (20) is

$$\begin{aligned} \dot{V}_2 &= \sum_{i=1}^N \bar{\omega}_i^T [-c_i^p \bar{\sigma}_i - c_i^d \delta_\rho(\bar{\theta}_i)] - \sum_{i=1}^N \sum_{j=1}^N k_{ij}^p \bar{\omega}_i^T \sigma_{ij} \\ &\quad - \sum_{i=1}^N \sum_{j=1}^N k_{ij}^d \bar{\omega}_i^T [\delta_\rho(\bar{\theta}_{ij}) - R(q_{ij}) \delta_\rho(\bar{\theta}_{ji})] \\ &\quad + \sum_{i=1}^N c_i^p \bar{\omega}_i^T \bar{\sigma}_i + \sum_{i=1}^N c_i^d \delta_\rho^T(\bar{\theta}_i) \Omega_i \\ &\quad + \frac{1}{2} \sum_{i=1}^N \sum_{j=1}^N k_{ij}^p \sigma_{ij}^T \omega_{ij} + \sum_{i=1}^N \sum_{j=1}^N k_{ij}^d \delta_\rho^T(\bar{\theta}_{ij}) \Omega_{ij} \\ &= - \sum_{i=1}^N c_i^d \delta_\rho^T(\bar{\theta}_i) R(\bar{p}_i) \beta_i + \sum_{i=1}^N \sum_{j=1}^N k_{ij}^d \delta_\rho^T(\bar{\theta}_{ij}) \Omega_{ij} \\ &\quad - \sum_{i=1}^N \sum_{j=1}^N k_{ij}^d \bar{\omega}_i^T [\delta_\rho(\bar{\theta}_{ij}) - R(q_{ij}) \delta_\rho(\bar{\theta}_{ji})] \\ &= - \sum_{i=1}^N c_i^d \delta_\rho^T(\bar{\theta}_i) R(\bar{p}_i) \beta_i - \sum_{i=1}^N \sum_{j=1}^N k_{ij}^d \delta_\rho^T(\bar{\theta}_{ij}) R(\bar{p}_{ij}) \beta_{ij}, \end{aligned}$$

where we have used the fact that $\frac{1}{2} \sum_{i=1}^N \sum_{j=1}^N k_{ij}^p \sigma_{ij}^T \omega_{ij} = \sum_{i=1}^N \sum_{j=1}^N k_{ij}^p \bar{\omega}_i^T \sigma_{ij}$. Then, according to (18) and using the facts that $\bar{p}_i^T R(\bar{p}_i) = \bar{p}_i^T$ and $\bar{p}_{ij}^T R(\bar{p}_{ij}) = \bar{p}_{ij}^T$, the time derivation of V_2 can be further simplified as

$$\begin{aligned} \dot{V}_2 &= - \sum_{i=1}^N c_i^d \delta_\rho^T(\bar{\theta}_i) R(\bar{p}_i) \beta_i - \sum_{i=1}^N \sum_{j=1}^N k_{ij}^d \delta_\rho^T(\bar{\theta}_{ij}) R(\bar{p}_{ij}) \beta_{ij} \\ &= - \sum_{i=1}^N c_i^d \delta_\rho^T(\bar{\theta}_i) \Gamma_i \bar{\theta}_i - \sum_{i=1}^N \sum_{j=1}^N k_{ij}^d \delta_\rho^T(\bar{\theta}_{ij}) \Gamma_{ij} \bar{\theta}_{ij} \\ &\leq 0, \end{aligned}$$

where we have used the fact that $\delta_\rho(\cdot)$ is an odd function. In view of the similar analysis of Theorem 1, it is easy to demonstrate that the conclusion of Theorem 2 still holds. ■

4. DESIGN EXAMPLES

In this section, design examples are given to verify the effectiveness of the proposed controllers. In this simulation, we consider a scenario where there are four spacecraft. We consider an undirected graph whose set of edges is given by $\varepsilon = \{(1, 2), (1, 3), (1, 4), (2, 3)\}$, where the undirected graph $G_1 = (\mathcal{V}, \varepsilon, K_p)$ is assumed to be the same as $G_2 = (\mathcal{V}, \varepsilon, K_d)$. The spacecraft are modeled as rigid bodies, where the values of spacecraft inertia matrices are given as $J_i = \text{diag}\{20, 20, 30\}$. The desired angular

velocity and the initial conditions of the desired attitude are given as

$$\omega_d = [0.1 \sin(0.1\pi t) \ 0.1 \sin(0.1\pi t) \ 0.1 \sin(0.1\pi t)]^T, \\ q_d(0) = [0 \ 0 \ 0 \ 1]^T.$$

We consider two different cases discussed in Section 3.

1) Under the controller (9), let the parameters be $\rho = 2, c_i^d = 60, c_i^p = 60, k_{ij}^p = 5, k_{ij}^d = 5$. The initial conditions of attitude and angular velocity of spacecraft are assigned as follows.

$$\begin{aligned} q(0) &= \text{col}\{q_1(0), q_2(0), q_3(0), q_4(0)\} \\ &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \\ \omega(0) &= \text{col}\{\omega_1(0), \omega_2(0), \omega_3(0), \omega_4(0)\} \\ &= \begin{bmatrix} -0.5 & 0.5 & 0.1 & 0.4 \\ 0.5 & -0.3 & 0.6 & 0.4 \\ -0.45 & 0.1 & -0.1 & -0.5 \end{bmatrix}. \end{aligned} \quad (21)$$

The trajectories of vector parts of spacecraft attitudes are shown in Fig. 1 - Fig. 3. It is clearly seen that the four spacecraft converge to the desired attitude.

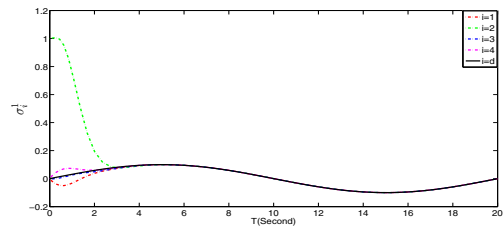


Fig. 1. The vector parts of spacecraft attitudes under Theorem 1, $\sigma_i^1, i = 1, 2, 3, 4, d$.

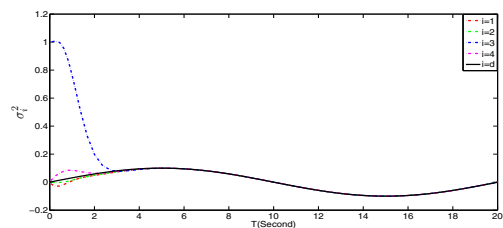


Fig. 2. The vector parts of spacecraft attitudes under Theorem 1, $\sigma_i^2, i = 1, 2, 3, 4, d$.

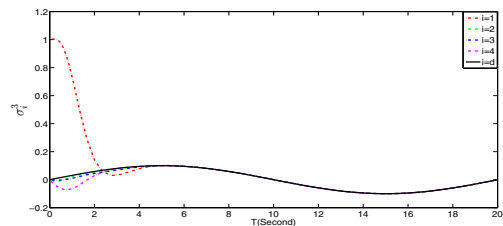


Fig. 3. The vector parts of spacecraft attitudes under Theorem 1, $\sigma_i^3, i = 1, 2, 3, 4, d$.

2) Under the controller (17), let the parameters be $\rho = 0.2, c_i^d = 60, c_i^p = 60, k_{ij}^p = 5, k_{ij}^d = 5, \Gamma_{ij} = \Gamma_i = 5I_3$. The initial conditions of attitude and angular velocity of spacecraft are assigned as (21). In Fig. 4 - Fig. 6, we can see that the four spacecraft reach an agreement and converge to the same desired attitude.

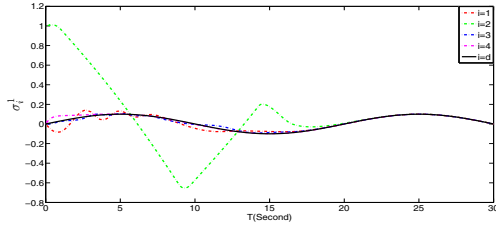


Fig. 4. The vector parts of spacecraft attitudes under Theorem 2, $\sigma_i^1, i = 1, 2, 3, 4, d$.

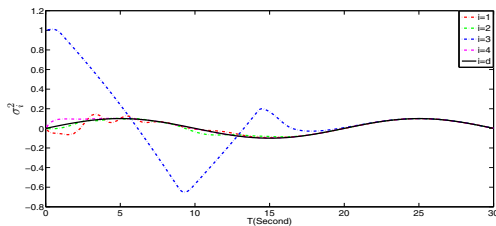


Fig. 5. The vector parts of spacecraft attitudes under Theorem 2, $\sigma_i^2, i = 1, 2, 3, 4, d$.

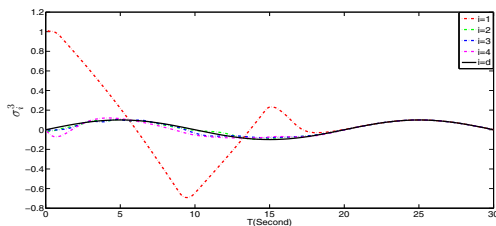


Fig. 6. The vector parts of spacecraft attitudes under Theorem 2, $\sigma_i^3, i = 1, 2, 3, 4, d$.

5. CONCLUSIONS

In this paper, we studied the attitude coordinated tracking problem of multiple spacecraft with control input saturation. Two decentralized attitude coordinated control algorithms with input saturation were proposed with and without angular velocity measurements. Both of the attitude coordinated controllers were proved by the Lyapunov based method. One important feature of the velocity-free attitude coordinated tracking algorithm is the introduction of auxiliary dynamical systems with input saturation. Design examples were provided to demonstrate the effectiveness of the proposed control algorithms. Future work includes extending the results to cases when spacecraft are subject to parametric uncertainty and external disturbances.

REFERENCES

A. Abdessameud and A. Tayebi. Attitude synchronization of a group of spacecraft without velocity measurements.

IEEE Trans. Automat. Control, 54(11):2642 – 2648, 2009.

M.R. Akella, A. Valdivia, and G.R. Kotamraju. Velocity-free attitude controllers subject to actuator magnitude and rate saturations. *Journal of Guidance, Control, and Dynamics*, 28(4):659 – 666, 2005.

H. Bai, M. Arcak, and J. Wen. A decentralized design for group alignment and synchronous rotation without inertial frame information. In *Proc. the 2007 IEEE Conference on Decision and Control*, pages 2552 – 2557, New Orleans, USA, 2007.

I. Chang, S. Park, and K. Choi. Decentralized coordinated attitude control for satellite formation flying via the state-dependent riccati equation technique. *Int. J. of Non-Linear Mechanics*, 44(8):891 – 904, 2009.

S. Chung, U. Ahsun, and J.E. Slotine. Application of synchronization to formation flying spacecraft: Lagrangian approach. *J. of Guidance Control & Dynamics*, 32(2): 512 – 426, 2009.

E. Jin, X. Jiang, and Z. Sun. Robust decentralized attitude coordination control of spacecraft formation. *Systems & Control Letters*, 57(7):567 – 577, 2008.

H.K. Khalil and J.W. Grizzle. *Nonlinear systems*. Prentice-Hall International, Inc., UK, 2002.

J.R. Lawton and R.W. Beard. Synchronized multiple spacecraft rotations. *Automatica*, 38(8):1359 – 1364, 2002.

Z. Meng, W. Ren, and Z. You. Distributed finite-time attitude containment control for multiple rigid bodies. *Automatica*, 46(12):2092 – 2099, 2010.

W. Ren. Distributed attitude alignment in spacecraft formation flying. *Int. J. Adaptive Control and Signal Processing*, 21(2-3):95 – 113, 2007.

W. Ren. Distributed cooperative attitude synchronization and tracking for multiple rigid bodies. *IEEE Trans. Control Systems Technology*, 18(2):383 – 392, 2010.

T.H. Summers, M.R. Akella, and M.J. Mears. Coordinated standoff tracking of moving targets: control laws and information architectures. *J. of Guidance Control & Dynamics*, 32(1):56 – 69, 2009.

A. Tayebi. Unit quaternion-based output feedback for the attitude tracking problem. *IEEE Trans. Automat. Control*, 53(6):1516 – 1520, 2008.

B. Wu, D. Wang, and E.K. Poh. Decentralized robust adaptive control for attitude synchronization under directed communication topology. *J. of Guidance, Control and Dynamics*, 34(4):1276 – 1282, 2011.

A. Zou and K.D. Kumar. Neural network-based distributed attitude coordination control for spacecraft formation flying with input saturation. *IEEE Trans. Neural Networks and Learning Systems*, 23(7):1155 – 1162, 2012.

A. Zou, K.D. Kumar, and Z. Hou. Attitude coordination control for a group of spacecraft without velocity measurements. *IEEE Trans. Control Systems Technology*, 20(5):1160 – 1174, 2012.