Preprints of the 19th World Congress
The International Federation of Automatic Control
Cape Town, South Africa. August 24-29, 2014

Minimum Realization Tuning Strategy for Dynamic Matrix Control ⋆

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Abstract: In this paper autotuning and self-tuning methods for Dynamic Matrix Control (DMC) are presented with application to single-input single-output (SISO) processes which can be approximated by a first-order-plus-dead-time model. In order to validate the method a nonlinear control valve system described by a Wiener process is simulated. This process was identified by a Hammerstein model and controlled by DMC with output compensator. The proposed tuning methods have its performance compared to another standard method found in literature, showing more conservative results regarding smoothness of the control action while maintaining adequate performance on set-point tracking.

Keywords: Model-based control, nonlinear control, first-order-plus-dead-time, Autotuners, Self-tuning control.

1. INTRODUCTION

The algorithm of model-based predictive control by DMC has kept its essence since its conception in the original works by Cutler and Ramaker (1980). Innovations in this area of predictive control are mainly presented for two different problems: i) the mathematical representation of the model and ii) control parameter tuning of horizons and move suppression factor.

Tuning of DMC is problematic since there are no analytic methods to obtain exact solutions which result in the best desired responses and it is an ongoing topic of research with contributions by Posada and Sanjuan (2008); De Almeida et al. (2009); Matko et al. (2013), to name a few. Qin and Badgwell (2003) presents an extensive review of DMC for industrial applications and Kokate et al. (2010) presents a literature review regarding tuning methods for DMC in SISO processes. Among the autotuning methods Shridhar and Cooper (1997) stands out for ease of usage with a first-order-plus-dead-time (FOPDT) model.

In this paper we present a novel tuning method for DMC with application to SISO processes which can be represented by a FOPDT model. The method acts on two fronts, first with an autotuning approach to ensure reliable operation from the start and second with a self-tuning approach to gain adaptive properties. For the autotuning approach the method is based on a minimum realization of DMC in order to analyze the closed-loop dynamics of the system and tune the move suppression factor. When the autotuning was expanded to better contemplate the process model, considering the settling time and dead-time, it was observed this expanded autotuning approach is an hybrid between our move suppression tuning with (Shridhar and Cooper, 1997) horizons tuning. For the self-tuning approach we use the online minimization of a cost function aiming to reduce set-point tracking error and control ringing.

To validate the method results are presented for the simulation of a nonlinear control valve described by a Wiener process. The valve response is identified by a Hammerstein model and controlled by DMC with output compensator. Results for the proposed tuning are compared to Shridhar and Cooper (1997) showing better characteristics regarding control ringing and set-point tracking.

The rest of this paper is organized as follows. Section 2 makes a brief introduction to DMC, its parameters and output compensation. Section 3 presents the foundation for the proposed tuning method, showing an alternative manual tuning method and the automatic tuning method. Section 4 presents the case study for the nonlinear control valve process. Section 5 presents the results obtained by the proposed method and a comparison with Shridhar and Cooper (1997). Finally, section 6 presents final considerations about the method and future work.

2. DYNAMIC MATRIX CONTROL

Dynamic matrix control was one of the first algorithms of what later became known as model predictive control (MPC) and is widely employed in industrial processes (Qin and Badgwell, 2003). In this type of controller the control actions are calculated with the objective of minimizing a cost function such as:

$$J = \sum_{j=1}^{N_y} \left[\hat{y}(t+j) - y_r(t+j)\right]^2 + \lambda \sum_{j=1}^{N_u} \left[\Delta x(t+j-1)\right]^2 \ , \tag{1}$$

⋆ The first author would like to thank the National Council of Scientific and Technologic Development of Brazil - CNPq for a doctoral scholarship under Grant No. 140592/2012-1.

where $\hat{y}$ is the predicted process output given by a process model, in DMC's case this is $\hat{y} = G\Delta x + f$, free response $f = \Delta G \Delta x + y$ depends only on past control actions, G is the step response dynamic matrix, y_r is the desired set-point, N_y is the prediction horizon, N_u is the control horizon and λ is the move suppression factor. The optimal control action at any instant is given by solving equation 1 as a quadratic optimization problem:

$$\Delta x = (G'G + \lambda I)^{-1} G'(y_r - f) . \tag{2}$$

2.1 Output Compensator

In nonlinear plants the errors between the process and the linear process model severely degrades controller performance creating the necessity of nonlinear models. The Hammerstein model (Narendra and Gallman, 1966) separates plant dynamics in two parts, the first is a static nonlinear (NL) gain and the second represents the linear (L) dynamics:

$$x(t) = f(u(t)) = \sum_{i=1}^{m} \gamma_i u(t)^i$$
$$\frac{Y(z)}{X(z)} = G(z) . \tag{3}$$

NLH-DMC (Non Linear Hammerstein - Dynamic Matrix Control) controls the linear portion of the model and the control signal is modified by an output compensator, as shown in figure 1, which calculates the inverse function of the static nonlinearity:

$$u(t) = f^{-1}(x(t)) . \tag{4}$$

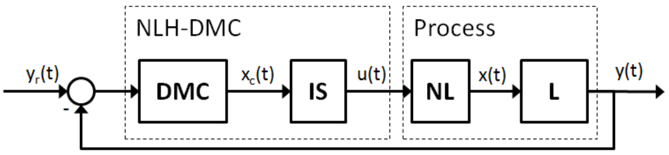

Fig. 1. NLH-DMC block diagram.

In the case where f is a polynomial equation, f^{-1} is given by the iterative search (IS) of the roots of $f(u(t)) - x(t) = 0$. There is no guarantee the inverse function is injective and results only in real values. As such, complex solutions are discarded and in case of multiple solutions $u(t)$ is selected to minimize the criterium $|u(t) - u(t-1)|$.

3. MOVE SUPPRESSION FACTOR TUNING

Considering a process where the linear dynamics can be adequately approximated by a FOPDT model:

$$\frac{Y(s)}{U(s)} = \frac{K_p e^{-\theta_p s}}{\tau_p s + 1} , \tag{5}$$

discretizing the above model by zero-pole mapping $z = e^{sT_s}$, where T_s is the sampling interval:

$$\frac{Y(z)}{U(z)} = \frac{b_0 z^{-d}}{1 + a_1 z^{-1}} , \tag{6}$$

where the discrete delay $d = ceil(\theta_p / T_s) + 1$, $b_0 = K_p \left(1 - e^{-T_s/\tau_p}\right)$ and $a_1 = -exp\left(-T_s/\tau_p\right)$.

Let us consider then the parameter choice $N = N_y = N_u = 1$ for a plant with discrete unit delay, $d = 1$. In practice this choice of parameters is dangerous for a number of reasons, $N = 1$ doesn't appropriately describe the step response dynamics and mis-represents the free response f, $N_y = N_u$ may result in unstable closed-loop dynamics in the presence of non-minimum phase zeros. Later it will be shown that: i) the delay does not affect the tuning of λ in certain parametric considerations; ii) the parameters $N, N_y, N_u = 1$ only purpose is that of an initial analysis of DMC's behavior, after all there are better choices for the horizons.

With these parameters DMC ignores past control actions in the free response, it will use a single one step ahead prediction and seek to minimize a single future control action. This choice is fortuitous for the analysis at hand for it reduces DMC to a minimum realization allowing for a simple algebraical analysis, resulting in $G = G' = b_0$ and $\Delta G = \varnothing$. The free response $f = \Delta G \Delta x + y$ becomes $f = y$ and the control action $\Delta x = (b_0^2 + \lambda)^{-1} b_0 (y_r - y)$.

Manipulating the control law and using a RST structure (Sumar et al., 2009) given by $Ru = Tyr - Sy$, we obtain $R = (G'G + \lambda)\Delta$, $T = G'$ and $S = G'$. The closed-loop transfer function is given by $Y(z)/Y_r(z) = TB/(RA + BS)$ where B and A are, respectively, numerator and denominator of plant model $G(z)$. Finally we achieve a second order system which describes the closed-loop behavior of the controlled system:

$$\frac{b_0^2 z^{-1}}{b_0^2 + \lambda + (a_1 b_0^2 - \lambda + a_1 \lambda) z^{-1} - (a_1 b_0^2 + a_1 \lambda) z^{-2}} . \tag{7}$$

The denominator of equation 7 can be rewritten in the usual form for root locus analysis:

$$\Delta(s) = 1 + \lambda \frac{1 + (a_1 - 1) z^{-1} - a_1 z^{-2}}{b_0^2 + a_1 b_0^2 z^{-1} - a_1 b_0^2 z^{-2}} . \tag{8}$$

Finally, with equations 7 and 8, it is possible to tune the move suppression coefficient to achieve a specific system response, such as finding poles which attend overshoot and settling time specifications.

3.1 Root Locus Optimization

For the automatic tuning of the move suppression coefficient we propose the optimization of the root locus of equation 7 on the variable λ, aiming for poles z_{cl} with null imaginary components, to avoid oscillatory behavior, and pole absolute value minimization, for faster settling time. This is a tradeoff between fast response time and small overshoot, being achieved by the following optimization problem:

$$\min_{\lambda} 100 \sum |imag(z_{cl})| + \sum |real(z_{cl})| . \tag{9}$$

We used a quasi-Newton interior-point method (IPM) with Hessian approximation by Broyden Fletcher Goldfarb Shanno (BFGS) algorithm (Nocedal and Wright, 2006) for solving this problem and achieving the optimum λ^*_{at}.

3.2 Online Optimization

Aiming to provide the tuning with adaptive capabilities through self-tuning we propose that for each iteration the following optimization problem must be solved:

$$\min_{\lambda} \, (1-\alpha)\mathrm{E}\left\{|\hat{y}-y_r|\right\} + \alpha\mathrm{E}\left\{|\Delta x|\right\} \ , \tag{10}$$

where α is limited to the range $[0,1]$ and controls the tradeoff between set-point tracking, lower values, and control ringing suppression, higher values. Substituting the definitions from section 2 for $\hat{y}$ and Δx:

$$\begin{aligned} \min_{\lambda} \, (1-\alpha)\mathrm{E}\left\{\left|G(G'G+\lambda I)^{-1}G'(y_r-f)+f-y_r\right|\right\} \\ +\alpha\mathrm{E}\left\{\left|(G'G+\lambda I)^{-1}G'(y_r-f)\right|\right\} \ . \end{aligned} \tag{11}$$

Being a transcendental equation there is no trivial solution and a numeric method must be used, again we chose the same IPM approach. The solution λ^*_{at} found by the auto-tuning approach is used as initial search point for the local optimization at the first control iteration, resulting in a locally optimal λ^*_{st}. In subsequent iterations the last λ^*_{st} is used as initial search point.

The result of this self-tuning approach is a different λ^*_{st} for each iteration, making it necessary to recalculate the control action given by equation 2 in terms of λ^*_{st}.

4. CASE STUDY

In order to assess the proposed methods a nonlinear control valve process commonly found in literature (Wigren, 1990; Al-Duwaish and Naeem, 2001) was simulated. The system is depicted in figure 2, adapted from Cable (2005).

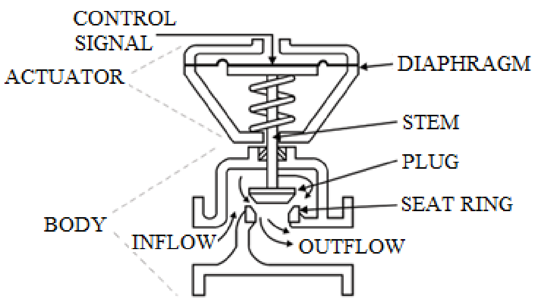

Fig. 2. Control valve process diagram.

This process is described by the following Wiener model:

$$\begin{aligned} x(t) &= \frac{0,616z^{-1}+0,05343z^{-2}}{1-1,5714z^{-1}+0,6873z^{-2}}u(t) \ , \\ y(t) &= \frac{x(k)}{\sqrt{0,1+0,9x^2(t)}} \ . \end{aligned} \tag{12}$$

Its input $u(k)$ is limited to the range [0 0.4] and it has a sampling time of $T_s = 0.1s$. The static curve for this process is presented in figure 3 where the nonlinear output behavior of the process is clearly observable.

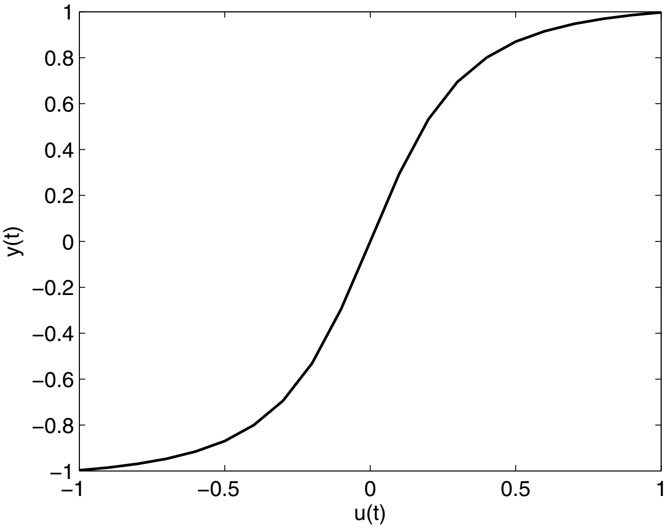

Fig. 3. Static curve, input-output characteristic, for the control valve process.

4.1 Hammerstein Model Identification

The application of DMC in this case study requires the identification of a Hammerstein model from the process described by a Wiener model. For this we considered a nonlinear static gain polynomial function of order $m = 3$, the uneven degree guarantees a real valued root, and a linear function of orders $n_b = 1$ e $n_a = 2$, that is:

$$\begin{aligned} x(t) &= \gamma_1 u(t) + \gamma_2 u(t)^2 + \gamma_3 u(t)^3 \ , \\ \frac{Y(z)}{X(z)} &= \frac{b_0+b_1z^{-1}}{1+a_1z^{-1}+a_2z^{-2}}z^{-1} \ , \end{aligned} \tag{13}$$

and the following root mean squared error (RMSE) optimization problem was elaborated for the identification of model parameters:

$$\min_{b_{0,1},a_{1,2},\gamma_{1,2,3}} \sqrt{\frac{1}{n}\sum_{t=1}^{n}\left(y(t)-\hat{y}(t)\right)^2} \ , \tag{14}$$

where n is the number of samples. Again the IPM approach was used for this optimization task resulting in:

$$\begin{aligned} b_0 &= -1.593896845712190 \ , \\ b_1 &= 0.704732341842690 \ , \\ a_1 &= 2.797526045953196 \ , \\ a_2 &= -0.862005679935081 \ , \\ \gamma_1 &= 0.194536814826656 \ , \\ \gamma_2 &= -0.200460554117093 \ , \\ \gamma_3 &= 0.007938804763741 \ , \end{aligned} \tag{15}$$

resulting in $RMSE = 0.006$. The final model identification results are exhibited in figure 4.

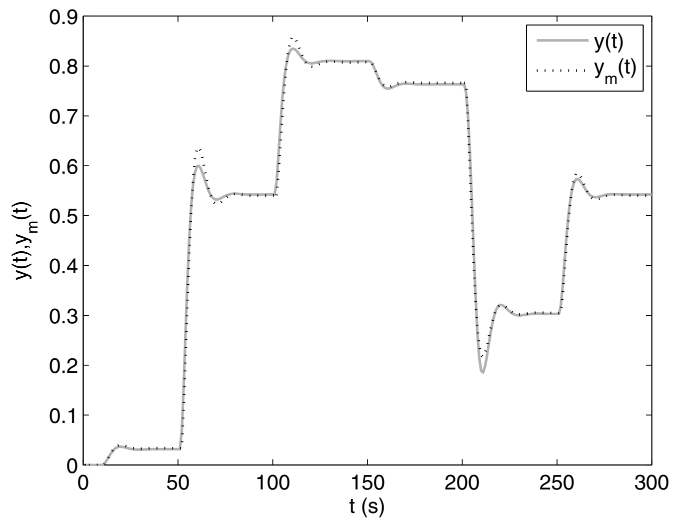

Fig. 4. Control valve process and identified Hammerstein model.

4.2 FOPDT Model Identification

Tuning of horizons requires a FOPDT model, which approximates process' static gain, time constant and dead-time. For this we used the identification method from Smith (1972), applying it to the linear dynamics of equation 13 and resulting in the following parameters:

$$\begin{aligned} K_p &= 17.4658 \ , \\ \tau_p &= 0.2488 \ , \\ \theta_p &= 0.1767 \ . \end{aligned} \tag{16}$$

4.3 Tuning Cases

In order to analyze the proposed method we consider the following 4 tuning cases:

Tuning 1

In this first case the well known DMC tuning proposed by Shridhar and Cooper (1997) is used for comparison.

$$\begin{aligned} N &= \left\lceil 5\frac{\tau}{T_s} + \lceil \theta_p/T_s + 1 \rceil \right\rceil \ , \\ N_y &= N \ , \\ 1 &\leq N_u \geq 6 \ , \\ \lambda &= \frac{N_u K_p^2}{500}\left(3,5\frac{\tau}{T_s} + 2 - \frac{N_u - 1}{2}\right) \ . \end{aligned} \tag{17}$$

Tuning 2

In this second case the minimum realization tuning of our proposal is used for demonstration purposes.

$$\begin{aligned} N &= 1 \ , \\ N_y &= 1 \ , \\ N_u &= 1 \ , \\ \lambda &= \lambda_{at}^* \ . \end{aligned} \tag{18}$$

Although useful for analysis this tuning is very poor from a practical point of view since $N = 1$ ignores almost the entire step response as well as any delay. Another major problem is the choice of horizons $N_y = N_u = 1$ results in unstable closed-loop poles in the presence of non-minimum phase zeros.

The advantage of this tuning is that in these parametric conditions the move suppression factor tends to be extremely conservative since the control horizon is the smallest possible and it allows an easy algebraical analysis of the closed-loop dynamics, allowing the proposed tuning approaches.

Tuning 3

To improve the former tuning it is necessary to consider both the settling time and delay. With the incorporation of these dynamics the tuning allows DMC to control non-minimum phase processes and high dead-time. With the incorporation of dead-time to the tuning of N and N_y the move suppression coefficient λ becomes independent of delay (Shridhar and Cooper, 1997), justifying disregarding of delay in λ tuning.

By considering the time constant, process delay and future control actions, we achieved a tuning method which is an hybrid between the horizons tuning proposed by Shridhar and Cooper (1997) and the move suppression factor autotuning proposed by us:

$$\begin{aligned} N &= \left\lceil 5\frac{\tau}{T_s} + \lceil \theta_p/T_s + 1 \rceil \right\rceil \ , \\ N_y &= N \ , \\ 1 &\leq N_u \geq 6 \ , \\ \lambda &= \lambda_{at}^* \ . \end{aligned} \tag{19}$$

Tuning 4

This last tuning case expands the former with the inclusion of our self-tuning approach, using λ_{at}^* as initial search point and $\alpha = 0.9$ which prioritizes the suppression of control ringing.

$$\begin{aligned} N &= \left\lceil 5\frac{\tau}{T_s} + \lceil \theta_p/T_s + 1 \rceil \right\rceil \ , \\ N_y &= N \ , \\ 1 &\leq N_u \geq 6 \ , \\ \lambda(t) &= \lambda_{st}^* \ . \end{aligned} \tag{20}$$

The root locus given by equation 8 for this case study is presented in figure 5, indicating the optimal result achieved by the optimization of equation 9 of $\lambda_{at}^* = 850.1$ and the near breakaway point. The parameters for all tuning cases are presented in table 1.

Table 1. Parameters for each tuning case.

Sintonia	N	N_y	N_u	λ
1 (Shridhar and Cooper)	16	16	5	26.46
2 (pure)	1	1	1	850.1
3 (hybrid)	16	16	5	850.1
4 (hybrid self-tune)	16	16	5	λ_{st}^*

5. RESULTS

Results are presented for the four tuning cases in figure 6, with reference 0.1 in 0-5s, 0.7 in 5-10s and 0.3 for the rest of the simulation. A load disturbance is introduced at 15 seconds. These results show Shridhar and Cooper's tuning is extremely aggressive with low settling time but upper and lower control action saturations, higher control ringing and set-point tracking difficulties during saturation. The

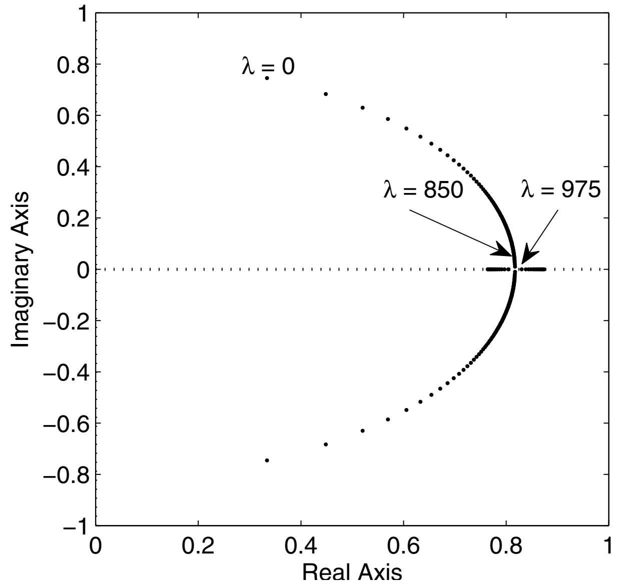

Fig. 5. Root locus for the case study indicating the optimal $\lambda^*_{at} = 850.1$.

pure, hypothetical, tuning is clearly not an effective option. The hybrid and self-tuning approaches present very similar results, with a settling time not much slower from Shridhar and Cooper's, yet with a much more conservative control action and reduced ringing. The self-tuning approach shows slightly, almost unnoticeable, better results for set-point tracking than the autotune approach without sacrificing the conservative control action.

Figure 7 presents the optimal move suppression coefficients for the self-tuning approach. In the beginning it does not deviate from λ^*_{at}, indicating it is a true local optimum. it is interesting how this approach adjusts for more aggressive control during set-point changes to settle the tracking error, yet it presents a much more conservative action in the saturation region, indicating desired adaptive properties.

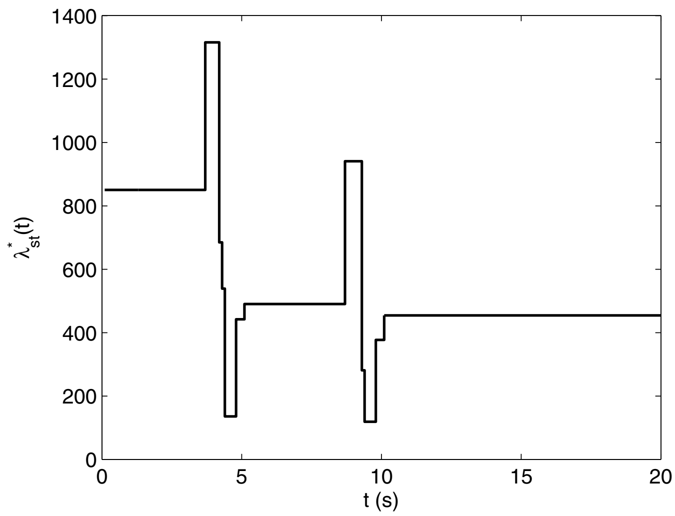

Fig. 7. Move suppression values for self-tuning approach.

6. CONCLUSION

In this paper we presented a novel tuning approach for DMC based on autotuning and self-tuning methods. A minimum realization of DMC was achieved under a particular choice of parameters, allowing for an automatic algebraical analysis of the closed-loop characteristics, resulting in an autotuned move suppression factor. Aiming to provide the method with adaptive properties a minimization problem is solved for each control action, self-tuning the move suppression factor and allowing for more accurate set-point tracking or control ringing attenuation.

This minimum realization purpose is that of an approximation to the move suppression factor effects on the closed-loop response. In practice, the choice of horizons considering FOPDT model parameters such as static gain, time constant and dead-time, resulted in an approach which resembles an hybrid between our move suppression coefficient tuning with the horizons tuning of Shridhar and Cooper (1997).

Our approach was compared to Shridhar and Cooper (1997) showing a much more conservative control action without any lost to set-point tracking performance. In fact, this approach exhibited better set-point tracking performance under control saturation.

It is important to emphasize our method does not inovate the tuning of horizons only the move suppression factor, where a more accurate tuning is attained through automatic calibration. This optimization tuning results in a system response in the performance limit - a moderately aggressive response with low control oscillation. Simulation results for this particular nonlinear case study motivates the application of this method to other classes of processes.

Future work will focus on expanding DMC's minimum realization to a full realization, in order to more accurately assess the closed-loop effects of the move suppression coefficient.

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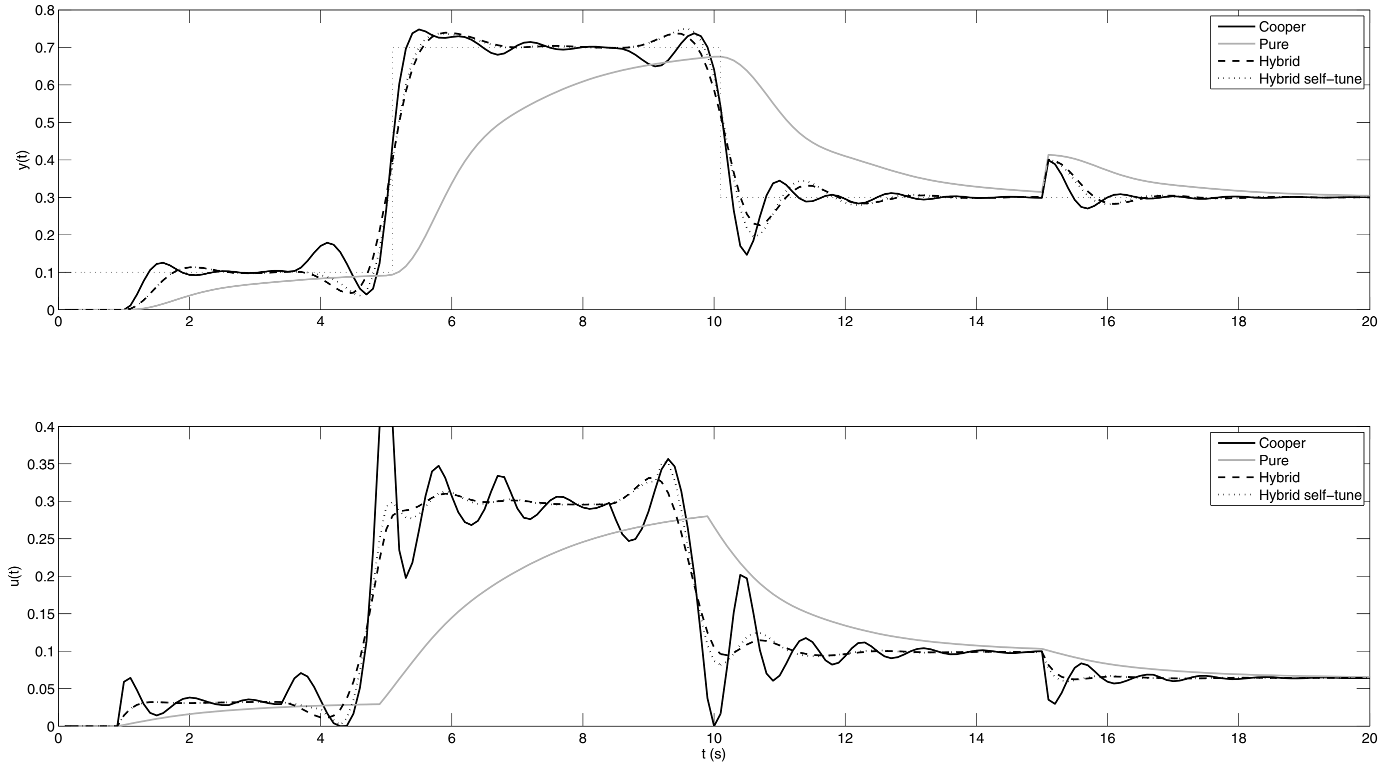

Fig. 6. Step responses for all tuning cases.

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Appendix A. MANUAL TUNING

An alternative to the automatic tuning shown in section 3 is manual tuning. The system response specification for move suppression tuning may be elaborated through classical methods (see Franklin et al. (2002)), using ζ, the damping ratio, and w_n, the undamped natural frequency. For the overshoot specification M_p and for the specification of settling time $t_{s5\%}$: $\zeta = \frac{\sqrt{logM_p{}^2}}{\pi^2+logM_p{}^2}$, $w_n = \frac{3}{\zeta t_{s5\%}}$.

The target poles z_t can be obtained by the classical second order model where the poles are given by $s^2 + 2\zeta w_n s + w_n^2 = 0$, thus $s_{t1,2} = -\zeta w_n \pm j\sqrt{1-\zeta^2}wn$. Using the pole-zero mapping we obtain the discrete poles:

$$z_{t1,2} = e^{-\zeta w_n T_s} e^{\pm j\sqrt{1-\zeta^2}w_n T_s} . \tag{A.1}$$

Finally, tuning is then only a matter of finding the value of λ which places the root locus of the closed-loop system as close as possible to z_t.

A.1 Root Locus Optimization

Considering z_{cl}, the poles of the closed-loop system from equation 7, and z_t, the desired poles obtained from equation A.1, it is necessary to find the value of λ which minimizes the Cartesian distance between these poles:

$$\min_{\lambda} \sqrt{(z_{t1} - z_{cl1})^2 + (z_{t2} - z_{cl2})^2} , \tag{A.2}$$

thus achieving the optimum value λ^*_{at}.

Any optimization method may be employed to solve this nonlinear problem.