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Event-based PI controller with exponential thresholds ⋆

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Abstract: This paper deals with an event-based PI control strategy for event occurrences reduction. The methodology is based on the send-on-delta technique where the measured value of the error signal is sent to the controller when it crosses predefined quantization levels. We propose to increase exponentially the distance between the thresholds in such a way that the number of events decreases by keeping a satisfactory transient performance and by fixing the maximum steady-state error independently. Simulation results show the effectiveness of the methodology and clarify the physical meaning of the design parameters of the controller.

Keywords: Event-based control, PID control, exponential thresholds, stability.

1. INTRODUCTION

Event-based control has been the subject of a lot of research effort in the last few years (see, for example, (Åström, 2008; Heemels et al., 2008; Heemels and Donkers, 2013)) because it allows the reduction of the information exchanged between the agents that take part in the control loop (sensors, controllers, actuators). Actually, this is a key issue when there are constraints on the communication rate, for instance when data are exchanged in a distributed control system by wired or wireless networks (Otanez et al., 2002; Miskowicz, 2006; Pawlowskyy et al., 2008; Chacón et al., 2013). In these situations, cutting down the traffic load reduces the possibility of lost data and stochastic time delays. Further, the occurrence of large latencies and delay jitter is decresed and the CPU utilization is also reduced. Most of all, when wireless sensors and actuators are employed in process control, reducing the communication rate implies the reduction of the power consumption, which yields an increment of the battery life (Blevins, 2012).

Among the different event-based sampling strategies, one of the most employed is the so called send-on-delta (SOD) sampling (also known as deadband sampling or level crossing sampling (Kofman and Braslavsky, 2006)) where the measured value of the process variable is sent to the controller when the control error (or some function of it) crosses predefined quantization levels (Sánchez et al., 2009). This approach has been also employed in the context of (event-based) Proportional-Integral-Derivative (PID) controllers (Sanchez et al., 2011, 2012). In particular, in (Tiberi et al., 2012), the stability of a first order system without delay is addressed.

A modification of the SOD technique, called symmetric send-on-delta (SSOD) sampling has then been proposed in (Beschi et al., 2012, 2014), where the sampled signal is quantized by a quantity multiple of Δ so that the relationship between the input and the output of the event-generator block is symmetric with respect to the origin and a zero-error threshold surely exists. In this way, the value of the sampled signal has the important property of being independent of the initial conditions and an equilibrium point value can exist (Beschi et al., 2011). Associated with a PI controller, sufficient conditions on system stability and necessary and sufficient conditions on the controller parameters for the existence of equilibrium points without limit cycles have been found for first-order-plus-dead-time (FOPDT) processes.

In any case, a drawback of these techniques is that the reduction of communications is generally obtained at the expense of a decrement of performance. In particular, a steady-state error occurs and there is a trade-off (given by the value of Δ) between the steady-state precision and the number of events (see (Beschi et al., 2014)). Indeed, if a steady-state maximum error is required, the value of Δ has to be fixed at that level and therefore it is not possible to decrease the number of events, especially in the transient response. For this reason, in this paper we propose to modify the event-triggering algorithm where the distance between the thresholds increases exponentially, using an approach similar to the ones presented in (De Persis and Mazenc, 2010; Hayakawa et al., 2009; Liu and Elia, 2004). In this way, a new design parameter, different from Δ, is

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introduced to decrease significantly the number of events while keeping a similar performance. As this parameter (which is the argument of the exponential term) has a clear physical meaning, which will be discussed in the paper by means of an illustrative example, it represents a nice design feature for the user. The presented control algorithm has to be applied when the sensor unit is located in a different entity with respect to the controller and the actuator.

2. CONTROL SCHEME

The architecture of an event-based controller can be divided into three main parts: the sensor unit, which samples the process variable, calculates the error and conveys its value to the control unit, which elaborates the new value of the control action. Finally, the new value of the control action is conveyed from the control unit to the actuator unit, which changes appropriately the process input.

In an event-based control strategy these three tasks can run separately, and in some cases the three parts can be implemented by hardware located in different places of the plant. In order to reduce the communications between the different units, in the event-based controllers the signal values are not sent at constant intervals (note that this operation requires time synchronization of the hardware) but only when the signal changes significantly with respect to the last sent value and without the necessity of synchronization.

It is important to note that the different units can also be implemented in the same hardware, and clearly in this case the communications between these parts can be done at each interval. In this work, we consider the control and the actuator units located in the same place. For this reason, there are only two signals which require an event-triggered communication: the set-point signal and the error signal. The first one is sent when the operator or a higher hierarchical level changes its value. The error signal is sent by an event-based sampling block, called send-on-exponential-thresholds (for short, SOET) sampling. It receives as input the error signal $e(t)$, and it sends as output signal a signal $e^*(t)$, which represents the event-based sampled error in the presented case.

The event-based sampling block can be described as an automaton, made by an infinite number of states and an internal state variable $i(t) \in \mathbb{Z}$, which identifies the active automaton state.

For each automaton state $i(t)$, the output of the block is set equal to S_i (which could be multiplied by a scaling factor $\beta \in \mathbb{R}$) and two triggering thresholds (respectively $\underline{S}_i$ and $\overline{S}_i$) are defined. When the input signal $e(t)$ crosses the $\underline{S}_i$ threshold i is decremented of a unit, while when $e(t)$ reaches the value $\overline{S}_i$ the state variable $i(t)$ is increases of a unit. In order to avoid the Zeno effect, the presented event-based sampling presents hysteresis, that is $S_i = \underline{S}_{i+1} = \overline{S}_{i-1}$.

As already mentioned, the distance between two adjacent thresholds exponentially increases with the distance of the origin in order to reduce the number of events. In particular, the thresholds are defined as:

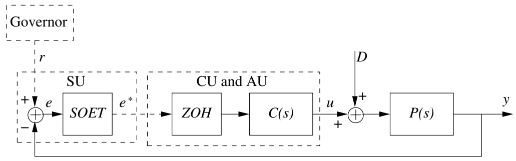

Fig. 1. SOET-PI control system scheme.

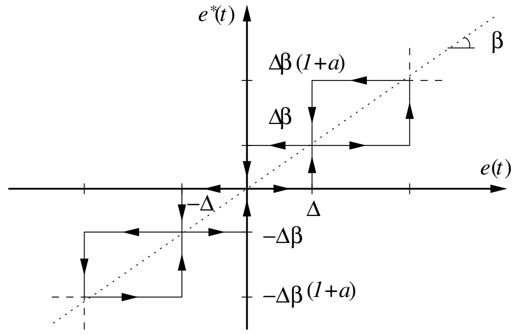

Fig. 2. Relationship between the input $e(t)$ and output $e^*(t)$ of the SOET block.

$$S_i = \begin{cases} \mathrm{sgn}(i(t)) \sum_{k=0}^{|i(t)|-1} a^{|k|}\Delta & \text{if } i(t) \neq 0 \\ 0 & \text{if } i(t) = 0 \end{cases} \tag{1}$$

where $\Delta > 0$ is the amplitude of the thresholds located around the null one and $a \geq 1$ is the exponential increasing factor between two thresholds. The initial condition $i(0)$ is arbitrarily chosen equal to the threshold closest to the initial value of $e(t)$ in order to well-define the system.

The dynamical evolution of the automaton is described as the following discrete system:

$$\begin{aligned} i(t^+) &= \begin{cases} i(t) - 1 & \text{if } e(t) < S_{i-1} \\ i(t) & \text{if } e(t) \in [S_{i-1}, S_{i+1}] \\ i(t) + 1 & \text{if } e(t) > S_{i+1} \end{cases} \\ e^*(t) &= \beta S(i(t)) \end{aligned} \tag{2}$$

Figure 2 shows the graphical representation of the thresholds.

Remark 1. The parameter β can be seen as a cascaded gain which can be included in the controller term. Thus, without loss of generality, we assume $\beta = 1$.

Remark 2. In order to highlight the role of parameter a in the methodology, it can be stated that the minimum number of events which are necessary to perform a step reference transition with amplitude A when there is no overshoot can be calculated by finding the last i which satisfies the relation:

$$\sum_{k=0}^{|i|-1} a^{|k|}\Delta = \frac{a^{|i|} - 1}{a - 1}\Delta \leq A$$

or equally:

$$i = \left\lfloor \log_a \left(1 + (a-1)\frac{A}{\Delta} \right) \right\rfloor$$

when $a \neq 1$ and $\left\lfloor \frac{A}{\Delta} \right\rfloor$ when $a = 1$ (namely, the SSOD sampling case). For this reason, the parameter a allows the designer to reduce the number of events necessary to perform a step. Obviously, it is necessary to take into account that the presence of overshoot can increase the required number of events.

Remark 3. The SSOD sampling (Beschi et al., 2012) can be considered as a particular case of the SOET sampling when $a = 1$.

Similarly to the SSOD-PI controller (Beschi et al., 2012), in the control and actuator unit, the control task is implemented by using a standard (discretized version of) continuous-time PI controller, namely:

$$C(s) = K_p + \frac{K_i}{s} \tag{3}$$

where $K_p \geq 0$ is the proportional gain and $K_i \geq 0$ is the integral gain.

Remark 4. Because the integral action of the PI control is updated each control step there are not equilibrium points outside the error band $\pm\Delta$, therefore the sticking phenomenon is avoided.

The considered process is a (stable) first-order-plus-dead-time (FOPDT) system, which is well-known to model adequately a vast majority of industrial processes and can be described by the following transfer function:

$$P(s) = \frac{K}{\tau s + 1} e^{-Ls} \tag{4}$$

where K is the process gain (which is assumed to be positive without loss of generality), $\tau > 0$ is the time constant and $L \geq 0$ is the apparent dead time. Then, we can write

$$Y(s) = \frac{K}{\tau s + 1} e^{-Ls} \left(U(s) + \frac{D}{s} \right) \tag{5}$$

where $Y(s)$ is the Laplace transform of the process output $y(t)$, $U(s)$ is the Laplace transform of the control action $u(t)$, and D is the amplitude of a constant load disturbance d.

3. STABILITY AND LIMIT CYCLES

In this section, the stability and limit cycles properties of the proposed scheme are analyzed. It is important to note that the event-based systems, as all the nonlinear systems, can be unstable, reach an equilibrium point, or admit a limit cycle. The knowledge of system properties are clearly a powerful tool to adequately tuning the controller.

The SOET-PI controlled system (namely, the feedback loop made by the series between the SOET block (which is applied to the error signal $e(t)$), the controller and the process, as shown in Figure 1 can be described by the following hybrid dynamic equation (Goebel et al., 2009)

$$\begin{cases} e(t) = r - y(t) \\ i(t^+) = \begin{cases} i(t) & \text{if } e(t) \in [S_{i-1}, S_{i+1}] \\ i(t) + 1 & \text{if } e(t) > S_{i+1} \\ i(t) - 1 & \text{if } e(t) < S_{i-1} \end{cases} \\ e^*(t) = \beta S(i(t)) \\ u(t) = K_p v^*(t) + K_i \left(IE(0) + \int_0^t e^*(\xi) d\xi \right) \\ \dot{y}(t) = \frac{K}{\tau} u(t - L) - \frac{1}{\tau} y(t) \end{cases} \tag{6}$$

System (6) can be unstable, can admit limit cycles or can converge to an equilibrium point. Note that, because (2) is symmetrical, it is possible to limit the study only to symmetrical limit cycles.

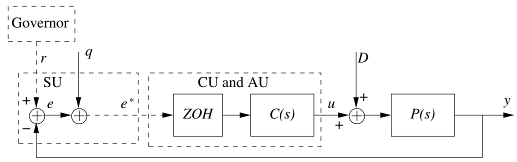

Fig. 3. Equivalent linear scheme.

The stability properties of this system can be studied more easily by considering the equivalent scheme, shown in Figure 3, described by (7), where the SOET block is substituted by an additive disturbance $q(t)$ equal to $q(t) = e^*(t) - e(t)$:

$$\begin{cases} e(t) = r - y(t) \\ w^*(t) = e(t) + q(t) \\ u(t) = K_p e^*(t) + K_i \left(IE(0) + \int_0^t e^*(\xi) d\xi \right) \\ \dot{y}(t) = \frac{K}{\tau} u(t - L) - \frac{1}{\tau} y(t) \end{cases} \tag{7}$$

The key-points of the following propositions are based on the following consideration: for each possible evolution of system (6) starting with arbitrary initial conditions and load disturbance, it is possible to obtain the same output considering the system (7) with the same initial conditions of C and P and the signal $q(t)$ equal to the difference $e^*(t) - e(t)$. Thus, if system (7) is stable and $q(t)$ is bounded also the system (6) is bounded.

Remark 5. Note that the amplitude of the signal $q(t)$ is less than $a^{i(t)}\Delta$ when the SOET system is at the state $i(t)$. This consideration plays a key role in determining the following proposition, which states a sufficient condition on the absence of limit cycles which involves the automaton states $\{-N, -N+1, \ldots, 0, \ldots, N-1, N\}$, also denoted as $2N+1$ limit cycles.

Proposition 1. The system (6) does not admit $2N+1$ limit cycles, if

$$m \leq \frac{a^N - 1}{(a-1)a^N}, \quad \text{where} \quad m = \int_0^\infty |h(t)| dt$$

where $h(t)$ is the response of the system (7) when $q(t)$ is an impulse signal.

Proof. By contradiction, suppose that a $2N+1$ limit cycle exists. Thus, the disturbance $q(t)$ is bounded by $a^N\Delta$ (see Remark 4). By using the convolution integral, it is possible to state:

$$|y(t)| = \left| \int_{-\infty}^{\infty} h(\xi) q(t - \xi) d\xi \right| =$$
$$\left| \int_0^{\infty} h(\xi) q(t - \xi) d\xi \right| < a^N \Delta \int_0^\infty |h(\xi)| d\xi = m a^N \Delta.$$

In order for the $2N+1$ limit cycle to exist $y(t)$ has to cross the S_N threshold, namely it has to satisfy the following condition:

$$|y(t)| > S_N,$$

therefore if $m a^N \Delta \leq S(N)$ then a $2N + 1$ limit cycle cannot exist. This condition can be rewritten as:

$$m \leq \frac{a^N - 1}{(a-1)a^N}. \tag{8}$$

Note that, if the inequality is satisfied for a certain N, it is satisfied also for $N+1$ (for all $a \geq 1$). □

Proposition 2. If the equivalent system (7) is asymptotically stable (then $m < \infty$) then system (6) is stable.

Proof. In fact, for any generic initial condition it is not possible for the output to diverge to infinity because the system satisfy (8) for any N sufficiently great. □

The Proposition 2 provide sufficient condition on the system stability (and therefore necessary condition on the instability).

If Proposition 1 holds for $N = 2$, then it is possible to limit the study to the three-state limit cycles, which are equal to the SSOD-case limit cycles analysed in (Beschi et al., 2012), where two propositions to avoid limit cycles are presented and proven. They are reviewed hereafter for the readers convenience. Proposition 3 gives the portion of PI parameters plane (namely, the plane K_p–K_i) where there are no limit cycles when a PI controller is applied, while Proposition 4 gives the set of proportional gains of a P controller which avoids the presence of limit cycles. In order to simplify the propositions, the normalized gains $K_1 = KK_p$ and $K_2 = KK_i\tau$ and the normalized time delay $l = L/\tau$ are used instead of the unnormalized ones.

Proposition 3. In a system described by (6), with $K_1 \geq 0$ and $K_2 > 0$, three states limit cycles cannot occur if the normalized gains K_1 and K_2 are inside the region of the first quadrant delimited by the following parametric curve:

$$K_1(\tilde{t}_1) = \begin{cases} \frac{\tilde{t}_1 - 2\,l + 2\,e^{l} - e^{l+\tilde{t}_1}\left(2\,l - \tilde{t}_1 + 2\right) + 4\,e^{l}\,\sinh(l)}{2\,l\,e^{l} - 2\,l\,e^{l+\tilde{t}_1} + 2\,\tilde{t}_1\,e^{l}\,\sinh(l)} & \text{if } l \leq \tilde{t}_1 < 2l \\ \frac{2}{\tilde{t}_1} - \frac{2\,l - \tilde{t}_1}{\tilde{t}_1\left(e^{l-\tilde{t}_1} - 1\right)} & \text{if } 2l \leq \tilde{t}_1 \leq \bar{t}_1 \end{cases}$$

$$K_2(\tilde{t}_1) = \begin{cases} \frac{2 - 2\,e^{2\,l} + 2\,e^{l+\tilde{t}_1} - 2\,e^{l}}{\tilde{t}_1 - \tilde{t}_1\,e^{2\,l} + 2\,l\,e^{l+\tilde{t}_1} - 2\,l\,e^{l}} & \text{if } l \leq \tilde{t}_1 < 2l \\ \frac{2}{\tilde{t}_1} & \text{if } 2l \leq \tilde{t}_1 \leq \bar{t}_1 \end{cases} \tag{9}$$

where $\tilde{t}_1 \in [l, \bar{t}_1]$ with $K_1(\bar{t}_1) = 0$ is the time interval when the state 1 (and the state -1, for symmetry) is assumed.

Proposition 4. In a system described by (6) with $K_i = 0$, if $K_1 < \left(1 - e^{-l}\right)^{-1}$ then three states limit cycles cannot occur.

In order to verify the applicability of the presented results to the standard PI tuning procedure, the relationship between m and the normalized time delay l (which ranges from 0.1 to 3), when the ST tuning rules (Beschi et al., 2014), the AMIGO rules (Åström and Hägglund, 2002) and the SIMC rules (Skogestad, 2003) are calculated, as shown in Figure 4. It is possible to note that $m \leq 1.8$ for any $l \in [0.1, 3]$, therefore Proposition 1 holds with $N \geq 2$ if $a \leq (1 + \sqrt{1+4m})(2m)^{-1} = 1.4371$.

It is also possible to note that all the considered tuning rules satisfy Propositions 1 and 3, therefore we can exclude the presence of limit cycles. In Section 4, an example shows the reduction of the number of the events during the transient due to the presence of the exponential thresholds.

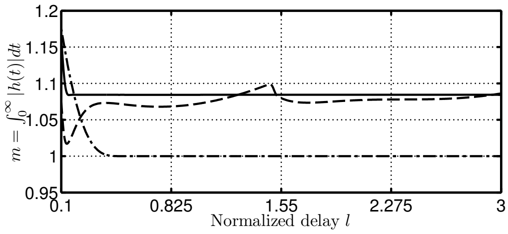

Fig. 4. Relationship between the integral of the absolute value of the impulse response m and the normalized gain l using the ST tuning rule (dashed line), the SIMC tuning rule (solid line) and the AMIGO tuning rule (dash-dot line).

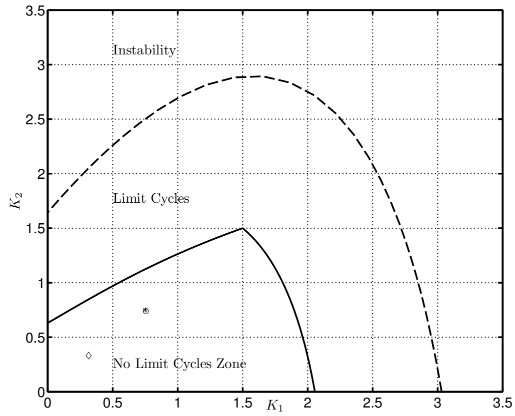

Fig. 5. Plane $K_1 - K_2$ with the sufficient condition on stability (dashed line), and condition on absence of limit cycles (solid line). ◊ AMIGO tuning rule, ∘ ST tuning rule, ★ SIMC tuning rule

Table 1. Values of the parameters K_p and K_i obtained using ST, SIMC, and AMIGO rules.

	K_p	K_i
ST	0.3770	0.0615
SIMC	0.3750	0.0625
AMIGO	0.1575	0.0277

4. SIMULATION RESULTS

In this section, simulation results are provided. In particular, a FOPDT system with $K = 2$, $\tau = 6$, and $L = 4$ is considered. The controller is tuned with the ST, SIMC, and AMIGO tuning rules. The obtained sets of parameters satisfy Proposition 3, as shown in Figure 5 where the no limit cycle area and the instability area, calculated with Proposition 2 and Proposition 3, are plotted. The numerical values of the parameters K_p and K_i for each tuning rule are shown in Table 1.

We consider four values for Δ, equal to 0.1, 0.01, 0.001, and 0, and five values for the parameter a, equal to 1, 1.01, 1.1, 1.2, and 2. Note that the first values of a satisfy Proposition 1, as written in Section 3 (see Figure 4), while using $a = 2$ limit cycles can arise (Proposition 1 is only sufficient condition on the absence of limit cycles).

Remark 6. The notation $\Delta = 0$ corresponds to the standard PI controller (therefore no event-based triggering is used), while the case with $a = 1$ corresponds to the SSOD-PI controller.

The controlled system has to follow a unit step set-point change applied at the time instant $t = 0$, while a unit step load disturbance is applied at the time instant $t = 100$.

It is possible to note that it is sufficient to use a value of a close to 1 to obtain a strong reduction of events without excessively deteriorating the other performance, especially when the value of Δ is small if compared with the set-point variation. On the contrary, a large value of the a parameter can affect the behaviour of the system during the transient, as can be seen in the illustrative examples of Figure 6 (which refers to the ST tuning rule) and 7 (which refers to the SIMC tuning rule), where a remarkable increment of the overshoot causes an increment of the number of events and of the steady-state times (for both the set-point and load disturbance responses), while when the paramenter a is set equal to 1.1 there are no appreciable differences with respect to the standard PI controller and the SSOD-PI controller (note that the different lines are hardly distinguishable). Figure 8 shows the response related to the AMIGO tuning rules. It is possible to see that, in this case, the increment of the overshoot is limited because this rule provides a smooth control. Thus, the number of events when $a = 2$ is strongly reduced.

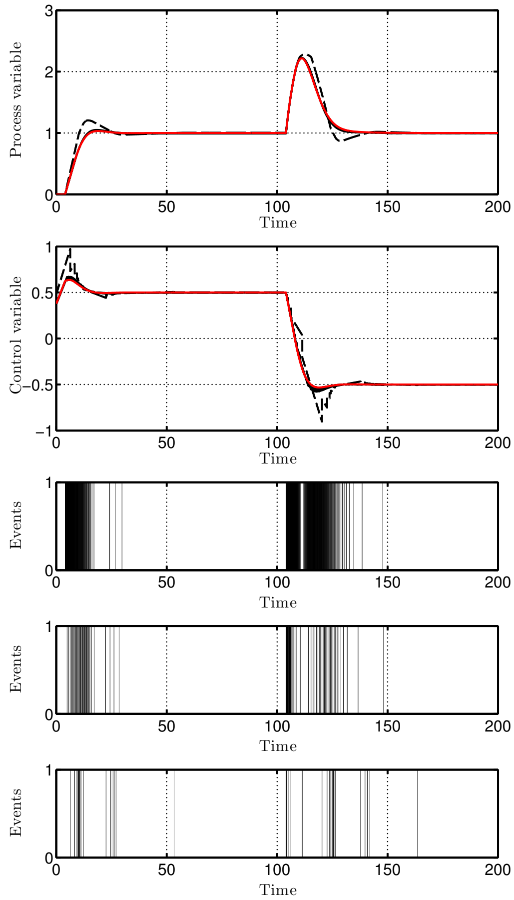

Fig. 6. Response of the SOET-PI controlled system tuned with ST tuning rule with $\Delta = 0.01$ and different values of a. $a = 1$ (SSOD sampling): solid black line; $a = 1.1$: dashed black line; $a = 2$: dash-dot black line; standard PI controller (namely, $\Delta = 0$): red solid line. First plot (from the top): process variable. Second plot: control variable. Third plot: events with $a = 1$. Fourth plot: events with $a = 1.1$. Fifth plot: events with $a = 2$.

5. CONCLUSIONS

In this paper we have proposed a new event-based PI controller based on send-on-delta sampling with exponentially increasing thresholds. This allows the reduction of the number of events without decreasing the transient response performance significantly, both for the set-point following and load disturbance rejection tasks. In particular, the steady-state maximum error is not affected by the choice of the new controller parameter a, which is the argument of the exponential term. The physical meaning of this design parameter has been clarified by means of simulation examples. It turns out that a value of a slightly greater than 1 is sufficient to reduce the number of events significantly without impairing the performance, while a too big value of a causes a larger overshoot in the response and eventually an increment of the number of events because the transient response is longer. A stability analysis has been also performed and it has been shown that the tuning of the PI controller can be done by applying standard tuning rules.

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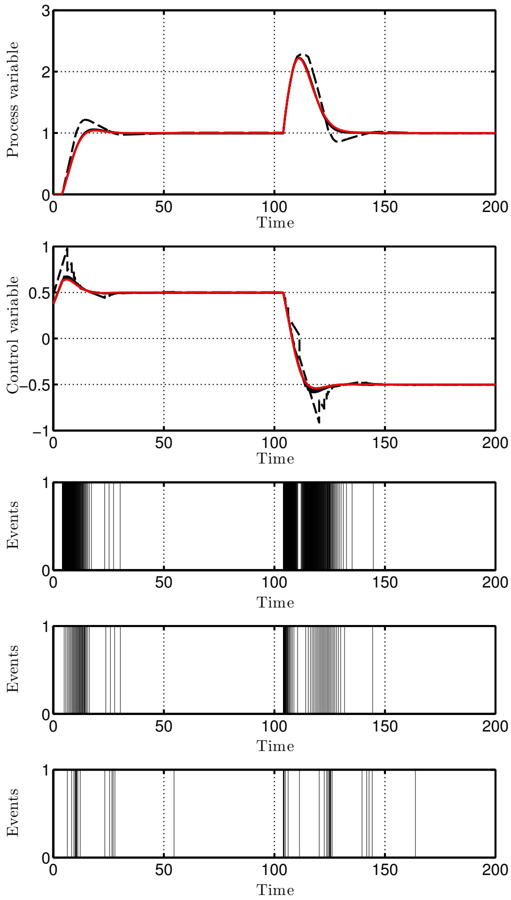

Fig. 7. Response of the SOET-PI controlled system tuned with SIMC tuning rule with $\Delta = 0.01$ and different values of a. $a = 1$ (SSOD sampling): solid black line; $a = 1.1$: dashed black line; $a = 2$: dash-dot black line; standard PI controller (namely, $\Delta = 0$): red solid line. First plot (from the top): process variable. Second plot: control variable. Third plot: events with $a = 1$. Fourth plot: events with $a = 1.1$. Fifth plot: events with $a = 2$.

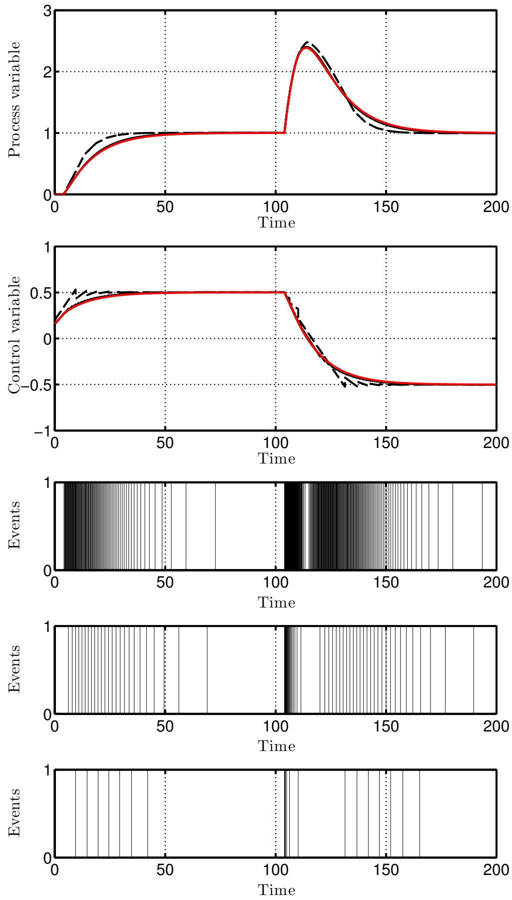

Fig. 8. Response of the SOET-PI controlled system tuned with Amigo tuning rule with $\Delta = 0.01$ and different values of a. $a = 1$ (SSOD sampling): solid black line; $a = 1.1$: dashed black line; $a = 2$: dash-dot black line; standard PI controller (namely, $\Delta = 0$): red solid line. First plot (from the top): process variable. Second plot: control variable. Third plot: events with $a = 1$. Fourth plot: events with $a = 1.1$. Fifth plot: events with $a = 2$.

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