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A Repetitiveness Index-based Adaptive Two Dimensional Iterative Learning Model Predictive Control

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Abstract: In this paper, we consider about the control strategy design for batch processes with sever non-repetitive disturbances. An index is proposed to measure the repetitive extent of batch processes. An adaptive two dimensional iterative learning model predictive control (ILMPC) method is designed based on this index. The control algorithm is switched between an one-dimensional Model Predictive Control (MPC) and a two time dimensional ILMPC according to this index. Simulation shows the superior effects of the proposed algorithm in handling abrupt changes of plant dynamics.

Keywords: Performance assessment, adaptive model predictive control, iterative learning control(ILC), two-dimensional control, batch processes

1. INTRODUCTION

Different from a continuous process, batch processes have unique characteristics such as nonlinearity, finite time duration and non-stationarity. These characteristics pose challenges for batch process control, especially in the case of model mismatch. It has attracted lots of attention from both academia and industries to improve control performance by exploiting the characteristics of batch processes. Among numerous strategies, ILC (Iterative learning control) is a most widely used one. ILC was initially proposed about two decades ago by Arimoto et al. (1984) for robot manipulation. It is essentially a feedforward control strategy which can improve the performance from batch to batch without the need of knowing precise process dynamics. During the past decade, a lot of work has been done to improve its convergence rate(Longman (2000)). In order to improve the robustness against non-repetitive disturbance, ILC is combined with feedback control. Lee et al. (1999) proposed a method to incorporate MPC (Model Predictive Control (Garcia et al. (1989))) with ILC for a batch reactor. Shi et al. (2006) proposed a robust ILC with both time-wise and batch-wise robust stablity. Further, in 2007, Shi et al. (2007) proposed an integrated design approach to combine ILC with MPC based on a two time dimensional framework. These ILC-based controls can improve performance from batch to batch when disturbances are repetitive. However, when there exists significant non-repetitiveness in the process, ILC may not perform well. At this time, the information from previous batch brought to current batch by ILC can be considered as extra disturbances. As claimed in Chen and Moore (2002), these extra disturbances greatly contribute to the baseline error. The authors also provided two methods to harness two special types of non-repetitiveness respectively. For a general type of non-repetitiveness, currently there is no much work been done. With this understanding, it is important to guage the degree of batch-to-batch repeatability to determine if and how ILC can be used.

In order to do that, it is natural to introduce techniques in process assessment. Performance assessment is quite mature on continuous processes. The motivation is to know 'whether the controller is healthy or not' without perturbing the system. It was initially studied by Desborough and Harris (1992) for a univariate feedback system. It aims to calculate the minimum variance from routine operating data and assess whether the system is under the minimum variance. Later, the same author extended their method to deal with a feedforward and feedback control system (Desborough and Harris (1993)). Recently, a lot of works have been done in extending those methods to multivariate system, eg. Harris et al. (1996). In certain case, it is more important to satisfy other important benchmarks, such as short settling time, small overshoot, than to have the system run under the minimum variance. Huang and Shah (1999) proposed a user defined method to incorporate requirements of different perspectives to be assessed.

In this paper, an adaptive iterative learning model predictive control with tunable weight on previous batch's information is proposed based on an index measuring the repetitiveness of the process.

The paper is organized in the following way. In section 2, an adaptive ILMPC is presented. In section 3, the repetitiveness index is proposed. In section 4, the proposed index is incorporated into the adaptive ILMPC. In Section 5, simulation results are given to show the efficiency of the proposed algorithm. In the last section, conclusions are drawn and further potential work is given.

2. PROBLEM FORMULATION

2.1 Review on one-dimensional MPC

In general, an industrial process can be described as a Controlled Auto-Regression Integrated Moving Average(CARIMA) model as follows.

$$A_o(q^{-1})(1-q^{-1})y(t) = B_o(q^{-1})\Delta_t u(t-1) + \varepsilon(t) \quad (1)$$

where

$$\Delta_t u(t-1) = u(t-1) - u(t-2) \quad (2)$$

$$A_o(q^{-1}) = 1 + a_1 q^{-1} + a_2 q^{-2} + ... + a_n q^{-n} \quad (3)$$

$$B_o(q^{-1}) = 1 + b_1 q^{-1} + b_2 q^{-2} + ... + b_m q^{-m} \quad (4)$$

n and m are the orders of the model, $\varepsilon(t)$ is white noise. We can also denote

$$A_o(q^{-1})(1-q^{-1}) = A(q^{-1}) \quad (5)$$

$$B_o(q^{-1}) = B(q^{-1}) \quad (6)$$

and for simplicity, rewrite the model as

$$A(q^{-1})y(t) = B(q^{-1})\Delta_t u(t-1) + \varepsilon(t) \quad (7)$$

In traditional one-dimensional MPC, firstly define

$$r(t) = \Delta_t u(t) \quad (8)$$

Denote the prediction horizon and controll horizon as N_1 and $N_2 (N_2 \leq N_1)$, we can derive the prediction model at time t as

$$\begin{pmatrix} A_1 & A_2 \end{pmatrix} \begin{pmatrix} y(|_t^{t-n+1}) \\ y_p(|_{t+N_1}^{t+1}) \end{pmatrix} = \begin{pmatrix} B_1 & B_2 \end{pmatrix} \begin{pmatrix} r(|_{t-1}^{t-m+1}) \\ r(|_{t+N_2-1}^{t}) \end{pmatrix} \quad (9)$$

where $y(|_b^a)$ denotes $[y(a)\ y(a+1)\ ...\ y(b-1)\ y(b)]^T$ and

$$(A_1 \mid A_2) = \left(\begin{array}{ccccc|ccccc} a_n & a_{n-1} & a_{n-2} & \dots & a_1 & 1 & 0 & \dots & 0 & 0 \\ 0 & a_n & a_{n-1} & \dots & a_2 & a_1 & 1 & \dots & 0 & 0 \\ 0 & 0 & a_n & \dots & a_3 & a_2 & a_1 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & * & * & * & \dots & a_1 & 1 \end{array}\right) \quad (10)$$

$$(B_1 \mid B_2) = \left(\begin{array}{ccccc|ccccc} b_m & b_{m-1} & b_{m-2} & \dots & b_2 & b_1 & 0 & \dots & 0 & 0 \\ 0 & b_m & b_{m-1} & \dots & b_3 & b_2 & b_1 & \dots & 0 & 0 \\ 0 & 0 & b_m & \dots & b_4 & b_3 & b_2 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & * & * & * & \dots & b_2 & b_1 \end{array}\right) \quad (11)$$

So the predicted output is

$$\begin{aligned} y_p(|_{t+N_1}^{t+1}) =& A_2^{-1}(B_1 r(|_{t-1}^{t-m+1}) + B_2 r(_{t+N_2-1}^{t}) \\ & - A_1 y(|_t^{t-n+1})) \end{aligned} \quad (12)$$

Further, the optimization part of MPC can be formulated as

$$\min_{r(|_{t+N_2-1}^{t})} \|y_r(|_{t+N_1}^{t+1}) - y_p(|_{t+N_1}^{t+1})\|_P^2 + \|r(|_{t+N_2-1}^{t})\|_Q^2 \quad (13)$$

Here P and Q denote the weight for the respective term. By solving this quadratic programming, the value of $r(|_{t+N_2-1}^{t})$ can be obtained. Further, by Eqn (8), the value of the input can be calculated. Based on the idea of receding horizon, only $u(t)$ will be implemented.

2.2 review on two-dimensional ILMPC

When MPC is combined with ILC, a two-dimensional ILMPC can be induced by using information from both time direction and batch direction. Shi et al. (2007) introduced the ILC law by defining

$$u(t,k) = u(t-1,k) + u(t,k-1) - u(t-1,k-1) + r(t,k) \quad (14)$$

Here t is the time index and k is the batch index. The corresponding two dimensional model can be presented as

$$\begin{aligned} A(q^{-1})y(t,k) =& A(q^{-1})y(t,k-1) + B(q^{-1})r(t,k) \\ & + \Delta_k \varepsilon(t,k) \end{aligned} \quad (15)$$

where

$$\Delta_k \varepsilon(t,k) = \varepsilon(t,k) - \varepsilon(t,k-1) \quad (16)$$

Similarly, the prediction model with prediction horizon as N_1 and control horizon as N_2 can be derived as

$$\begin{aligned} y_p(|_{t+N_1}^{t+1}, k) =& A_2^{-1}(B_1 r(|_{t-1}^{t-m+1}, k) + B_2 r(_{t+N_2-1}^{t}, k) \\ & - A_1 y(|_t^{t-n+1}, k) + A_1 y(|_t^{t-n+1}, k-1) \\ & + A_2 y(|_{t+N_1}^{t+1}, k-1)) \end{aligned} \quad (17)$$

And the optimization part is the same as formula (13).

$$\min_{r(|_{t+N_2-1}^{t}, k)} \|y_r(|_{t+N_1}^{t+1}) - y_p(|_{t+N_1}^{t+1}, k)\|_P^2 + \|r(|_{t+N_2-1}^{t}, k)\|_Q^2 \quad (18)$$

2.3 a new adaptive ILMPC

In order to make the algorithm more flexible, a switch variable is introduced to determine whether to turn on or turn off the ILC part by defining

$$\begin{aligned} r(t,k) =& u(t,k) - u(t-1,k) \\ & - \alpha(t,k)(u(t,k-1) - u(t-1,k-1)) \end{aligned} \quad (19)$$

where $\alpha(t,k) \in \{0,1\}$. The corresponding two-dimensional model becomes

$$\begin{aligned} A(q^{-1})y(t,k) =& \alpha(t,k)A(q^{-1})y(t,k-1) + B(q^{-1})r(t,k) \\ & + (\varepsilon(t,k) - \alpha\varepsilon(t,k-1)) \end{aligned} \quad (20)$$

It follows the prediction model

$$\begin{aligned} y_p(|_{t+N_1}^{t+1}, k) =& A_2^{-1}(B_1 r(|_{t-1}^{t-m+1}, k) + B_2 r(_{t+N_2-1}^{t}, k) \\ & - A_1 y(|_t^{t-n+1}, k) + \alpha(t,k)(A_1 \times \\ & y(|_t^{t-n+1}, k-1) + A_2 y(|_{t+N_1}^{t+1}, k-1))) \end{aligned} \quad (21)$$

The rational number α can be considered as a tuning knob to adjust the strength of batch-wise integral function. We focus on two extreme case:

- $\alpha = 0$, the newly proposed method becomes the conventional one-dimensional MPC.
- $\alpha = 1$, it is the same as the two-dimensional ILMPC.

So this tuning knob switch the control strategy between pure one-dimensional MPC and two-dimensional ILMPC. Next we aim to find a method to measure the repetitiveness of the process. When the process is of good repetitiveness, ILC should be integrated by setting $\alpha = 1$ to form the two-dimensional MPC so that we can have batch-wise improvement on the tracking performance. Otherwise, ILC should be turned off by setting $\alpha = 0$ to avoid bringing extra disturbances.

Remark: Here only the SISO system and unconstrained case is considered. The idea presented here can be extended to MIMO constrained cases.

3. DESCRIPTION OF REPETITIVENESS INDEX

In Model Predictive Control, analyzing the error can provide us a lot of useful information. Here repetitiveness is measured from the prediction error. Firstly, define the model-based predicted output $\hat{y}_p(t,k)$ as

$$\hat{y}_p(t,k) = (1 - A_o(q^{-1}))y(t,k) + B_o(q^{-1})u(t-1,k) \quad (22)$$

and the prediction error $\hat{e}_p(t,k)$ as

$$\begin{aligned} \hat{e}_p(t,k) &= y(t,k) - \hat{y}_p(t,k) & (23)\\ &= A_o(q^{-1})y(t,k) - B_o(q^{-1})u(t-1,k) & (24) \end{aligned}$$

$\hat{e}_p(t,k)$ contains the information of model mismatch and unmeasured disturbances. We can further define

$$\Delta_k \hat{e}_p(t,k) = \hat{e}_p(t,k) - \hat{e}_p(t,k-1) \quad (25)$$

This $\Delta_k \hat{e}_p(t,k)$ is the difference between two consecutive batches' model-based prediction error. At the same time, it is the prediction error when previous batch's information is incorporated to current batch. It is reasonable to evaluate the repetitiveness by comparing $\Delta_k \hat{e}_p(t,k)$ with $\hat{e}_p(t,k)$. A normalized repetitiveness index γ is defined as

$$\gamma(t,k) = \sqrt{\frac{\Delta_k \hat{e}_p^2(t,k)}{\Delta_k \hat{e}_p^2(t,k) + \hat{e}_p^2(t,k)}} \quad (26)$$

where $\gamma \in [0,1]$. When $\Delta_k \hat{e}_p(t,k) = \hat{e}_p(t,k)$, $\gamma(t,k) = \frac{\sqrt{2}}{2}$. So when $\Delta_k \hat{e}_p(t,k)$ is quite small compared with $\hat{e}_p(t,k)$, namely $\gamma(t,k) \ll \frac{\sqrt{2}}{2}$ it means the repetitiveness is good and ILC part will be helpful. When $\Delta_k \hat{e}_p(t,k)$ is comparable to $\hat{e}_p(t,k)$, namely $\gamma \in [\frac{\sqrt{2}}{2}, 1]$, the ILC part is not necessary anymore.

At time t, in general digital systems, only the output at time $t-1$ is available. So we can have a prediction of the repetitiveness index at time t by using output of previous time instant as

$$\gamma_p(t,k) = \gamma(t-1,k) \quad (27)$$

Further we can set up the relationship between $\gamma_p(t,k)$ and $\alpha(t,k)$ as

$$\alpha(t,k) = \begin{cases} 1 & \gamma_p(t,k) < \frac{\sqrt{2}}{2} \\ 0 & \gamma_p(t,k) \geq \frac{\sqrt{2}}{2} \end{cases} \quad (28)$$

However, due to the existence of measurement noise in the output, e_p together with γ are random variables. Directly setting α based on a single value of γ may make the system very sensitive to noise, so techniques to make γ more robust will be presented in the following section.

4. STEPS ON REPETITIVENESS ASSESSMENT

As batch processes are instinctively a two-dimensional system, a time and batch integrated method is adopted to do the assessment of repetitiveness index γ. Then prediction can be made based on assessment. The steps are listed in the following.

- Step 1: Initialization.
  From Eqn.(26), it can be known that the assessment can only start from the second batch, so only time-wise information can be utilized. Denote the assessment horizon along time direction as F_n. So for $t > F_n$, we have

$$\gamma_p(t,2) = \sum_{i=1}^{i=F_n} f_i \gamma(t-i,2) \quad (29)$$

  with $f_i > 0$ and $\sum_{i=1}^{i=F_n} f_i = 1$ which is a weighted moving average. For $t \leq F_n$, we can directly set

$$\gamma_p(t,2) = 0 \quad (30)$$

  or a varying horizon filter as

$$\gamma_p(t,2) = \sum_{i=1}^{i=t} f_i \gamma(t-i,2) \quad (31)$$

  where $f_i > 0$ and $\sum_{i=1}^{i=t} f_i = 1$. Since the horizon F_n is a small integer compared with the length of a batch, the initialization value will not have large influence on the overall performance.
- Step 2: Time-wise overall assessment
  After a batch is completed, a repetitiveness assessment can be carried out for the whole batch. The simplest way may be to take the average of $\gamma(t,k)$ as

$$\gamma_c(k) = E(\gamma(:,k)) \quad (32)$$

  Here $E(*)$ denotes to take the average value.
- Step 3: Batch-wise assessment
  Batch-wise assessment is quite similar with time-wise assessment in Step 1. Set the batch-wise assessment horizon as M_n, for $k > M_n$, before a batch is started, assess the similarity based on batch-wise historical data as

$$\gamma_{kp}(t,k-1) = \sum_{i=1}^{i=M_n} m_i \gamma_c(k-i) \quad (33)$$

  where $m_i > 0$ and $\sum_{i=1}^{i=M_n} m_i = 1$, which is a moving average of the time-wise overall assessment γ_c. For $k \leq M_n$, we can have

$$\gamma_{kp}(t,k-1) = \sum_{i=1}^{i=k} m_i \gamma_c(k-i) \quad (34)$$

  and $m_i \geq 0$ and $\sum_{i=1}^{i=k} m_i = 1$.
- Step 4: Prediction based on integrated assessment
  For $k > 2$, when current batch is started, prediction of $\gamma_p(t,k)$ can be made based on time-wise and batch-wise assessment as

$$\gamma_p(t,k) = (1-\omega) \sum_{i=1}^{i=F_n} \gamma(t-i,k) + \omega \gamma_{kp}(t,k-1) \quad (35)$$

  ω is used to adapt the weight of predicion based on current information and batch-wise overall assessment. For $t \in [1, F_n]$, as there is no much time-wise information can be used, ω is taken as 1. The prediction is totally based on batch-wise assessment.
- Step 5: Calculation of $\alpha(t,k)$
  The calculation of $\alpha(t,k)$ can be done as shown in Section 3. A margin can be introduced to the threshold as follows to make this index more robust against noise.

$$\alpha(t,k) = \begin{cases} 1 & \gamma_p(t,k) < \frac{\sqrt{2}}{2} - \delta \\ 0 & \gamma_p(t,k) \geq \frac{\sqrt{2}}{2} - \delta \end{cases} \quad (36)$$

  where δ is a positive small number.

Based on these five steps, both time-wise and batch-wise information on the repetitiveness is integrated in the prediction of the repetitiveness index for the following time spots. Next simulation examples are given to show the effectiveness of this assessment method.

5. SIMULATION

Simulation is conducted on a process as follows

$$y(t,k) = \frac{2.651q^{-1} + 5.298q^{-2} + 0.5805q^{-3}}{1 - 1.454q^{-1} + 0.5285q^{-2} - 0.0473q^{-3}} u(t,k) + w(t,k) \quad (37)$$

$w(t)$ is white noise with variance of 0.01. The control scheme design is based on a reduced order model as

$$y(t,k) = \frac{13.81q^{-2}}{1 - 0.9524q^{-1}} u(t,k) \quad (38)$$

Parameters are taken as: prediction horizon $P_n = 16$, control horizon $C_n = 10$, the weight in the objective function $P = 1$ and $Q = 3000$, the length of time $T = 200$, and the simulation will be conducted for 10 batches. The threshold $\delta = 0.157$, so the criteria used here is

$$\alpha(t,k) = \begin{cases} 1 & \gamma_p(t,k) < 0.55 \\ 0 & \gamma_p(t,k) \geq 0.55 \end{cases} \quad (39)$$

$F_n = 4$ and $f_i = \frac{1}{4}$, each time spot is of a equal weight. $M_n = 2$, $m_1 = 0.8$,$m_2 = 0.2$ and $\omega = 0.2$. Here these parameters are taken in a quite arbitrary way. In the following, it is shown the control performance for the cases with abrupt changes on dynamics. The output of the proposed method is compared with the traditional two dimensional ILMPC which takes $\alpha(t,k) = 1$ for all time spots in each batch.

- Case 1: an abrupt change on process model

We assume in batch 4, the process dynamic changes to

$$y(t,k) = \frac{2.851q^{-1} + 5.298q^{-2} + 0.5805q^{-3}}{1 - 1.454q^{-1} + 0.7285q^{-2} - 0.0473q^{-3}} u(t,k) + w(t,k) \quad (40)$$

then recovers from the 5th batch.
From Fig.1, it is obvious that the proposed adaptive ILMPC can yield a better performance in Batch 5. The poor performance of the 4th batch does not have much influence on Batch 5. The overshoot of Batch 5 is largely decreased compared with the output of traditional method shown in Fig.3. In addition, Fig.4 shows the repetitive index γ has large oscillation, but by proper batch-wise and time-wise filtering strategy, γ_p, as shown in Fig.5, is much smoother. A relatively stable α can be obtained as shown in Fig.6.

- Case 2: a sustained change of process dynamic

Different from Case 1, it is assumed that in Batch 4, the real process dynamics changes to Equ. (40), and keep unchanged for the following batches. By comparing Fig.7 and Fig.8, Fig.9, we can see the outputs do not have too many differences. This example show that the proposed method can guarantee the performance will not be deteriorated when changes are sustained.

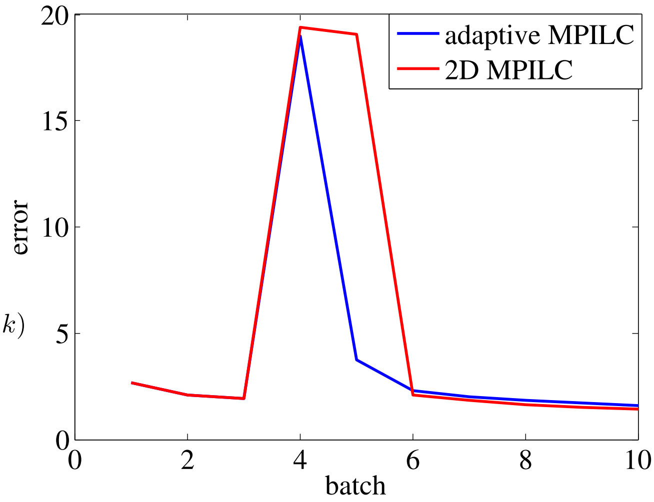

Fig. 1. Case 1: comparison of errors from two methods

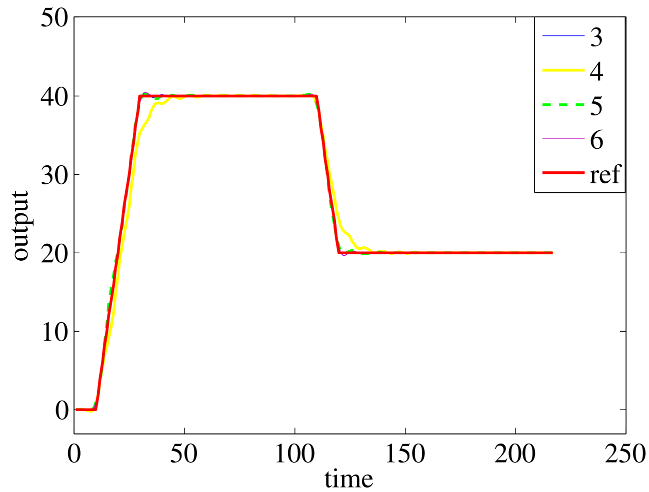

Fig. 2. Case 1: output of the proposed adaptive ILMPC

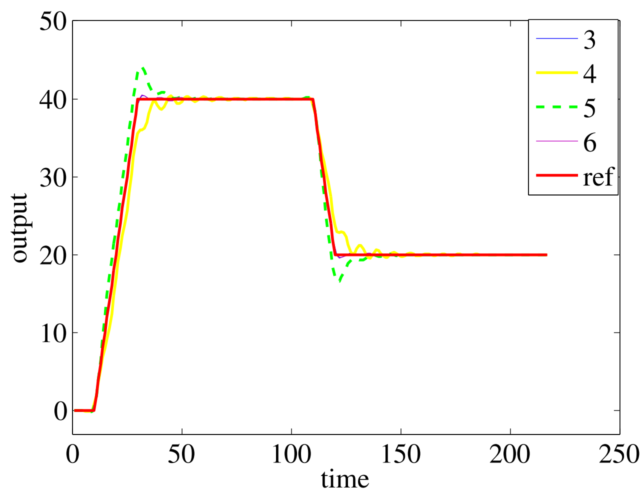

Fig. 3. Case 1: output of the traditional 2D ILMPC

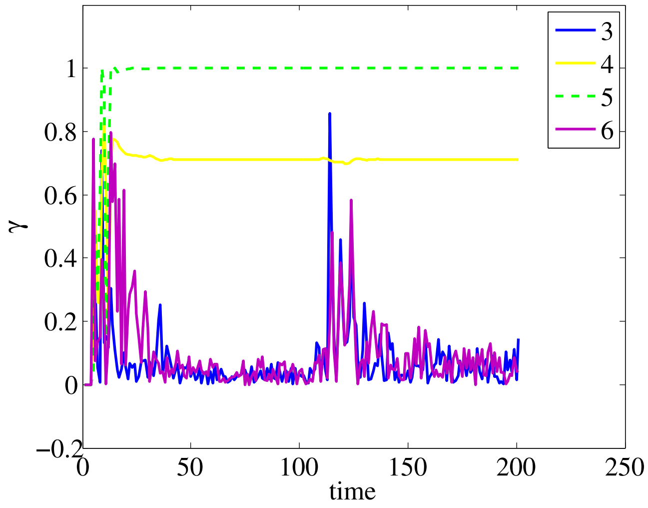

Fig. 4. Case 1: γ of the adaptive ILMPC

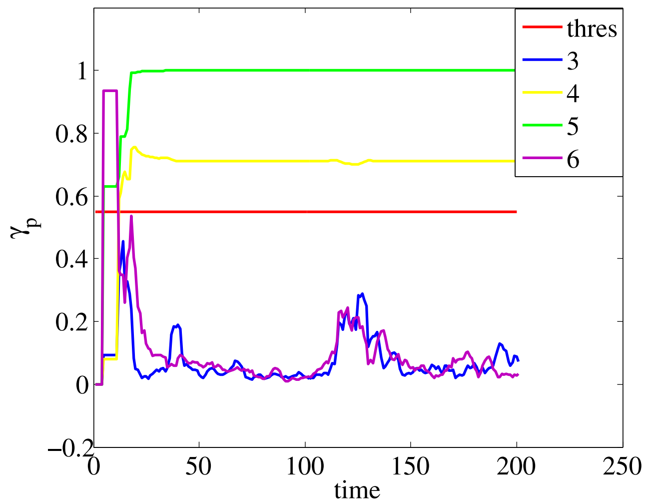

Fig. 5. Case 1: γ_p of the adaptive ILMPC

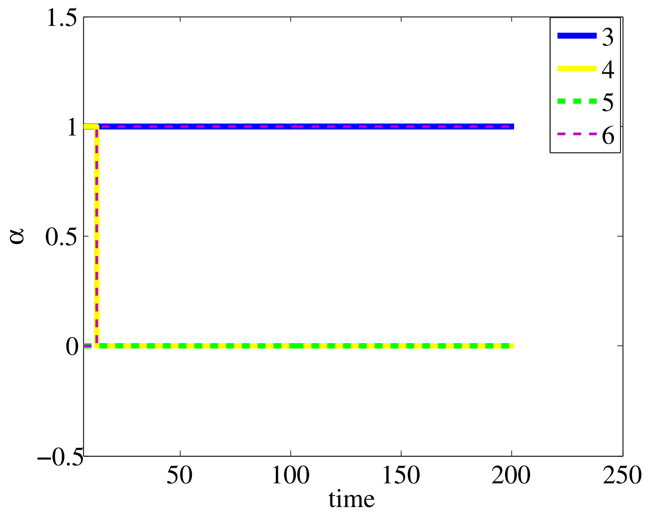

Fig. 6. Case 1: α of the adaptive ILMPC

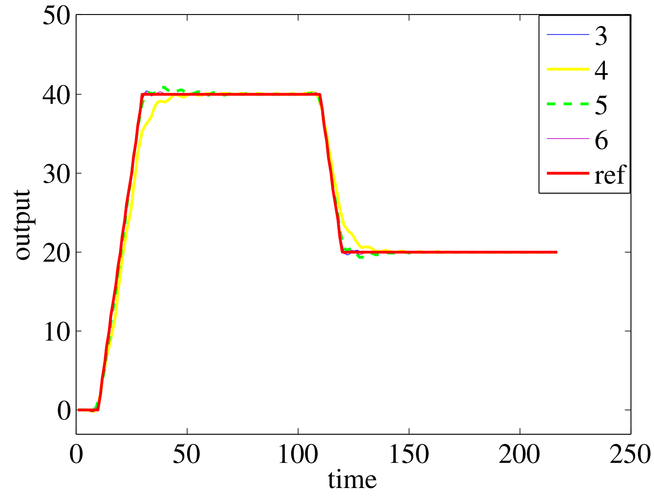

Fig. 7. Case 2: output of the adaptive ILMPC

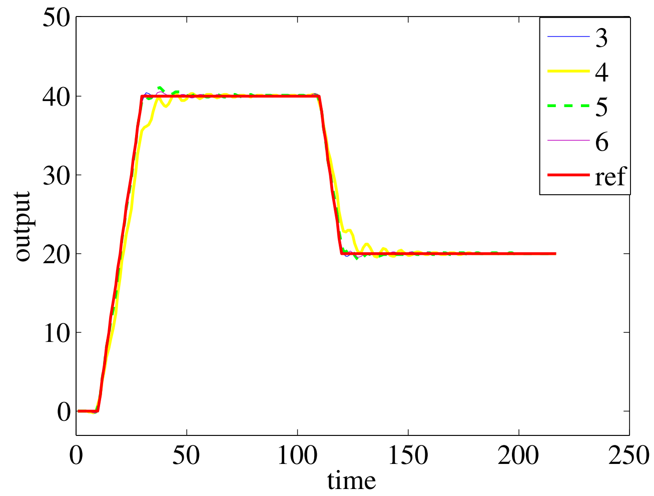

Fig. 8. Case 2: output of the traditional 2D ILMPC

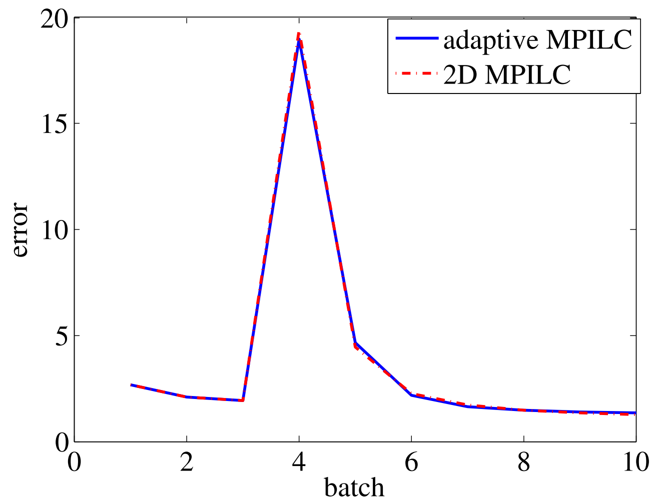

Fig. 9. Case 2: comparison of errors from two methods

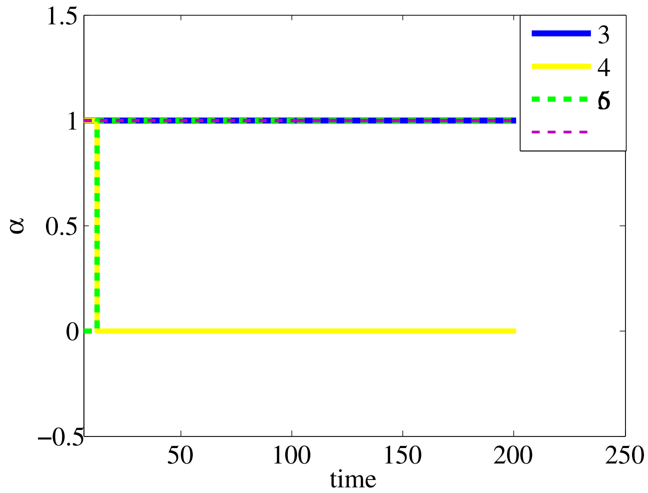

Fig. 10. Case 2: α of the adaptive ILMPC

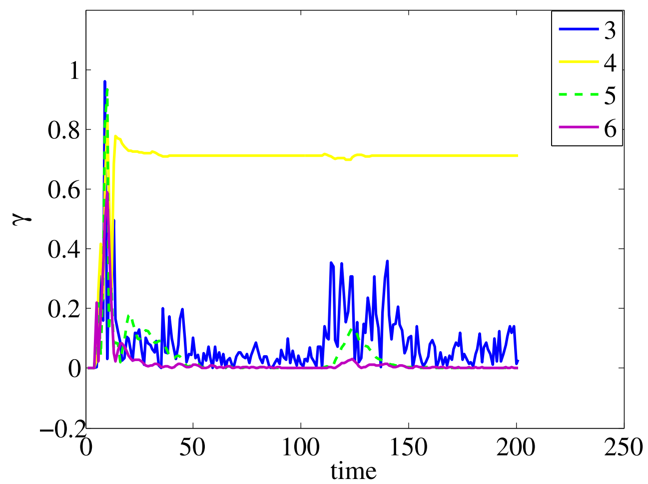

Fig. 11. Case 2: γ of the adaptive ILMPC

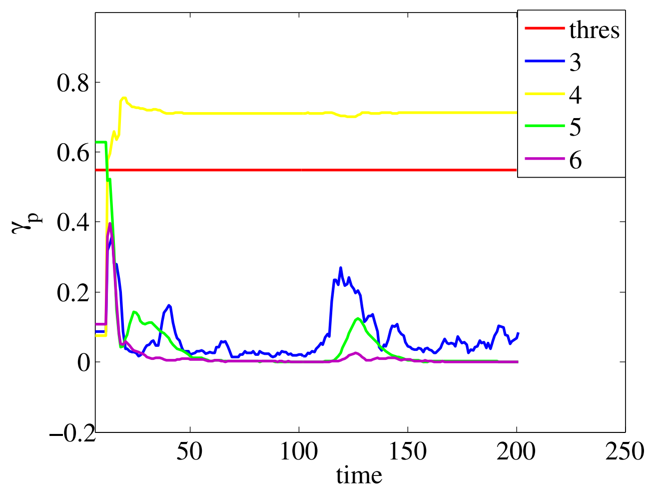

Fig. 12. Case 2: γ_p of the adaptive ILMPC

6. CONCLUSION AND FUTURE WORK

In this paper, we intend to give a clue on the benefits can be obtained by combining repetitiveness assessment with control in batch processes. An index is proposed to measure the repetitiveness. Based on this index, a new adaptive ILMPC is presented to improve the tracking performance when there is non-repetitive disturbances or model change along batch-direction. Simulation show the algorithm is efficient for batch-wise abrupt changes, and can guarantee the performance will not degrade compared with the traditional method when disturbances are repetitive. Further, based on this framework, more detailed work like how to optimize the coefficients of the filters, whether α can be taken as a continuous variable from 0 to 1 and how to prove the stability of this one-dimensional and two-dimensional mixed method can be considered.

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