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Large-scale system control based on decentralized design. Application to Cuinchy Fontinette Reach ⋆

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Abstract: Large scale engineering systems are typically composed by several subsystems that interact with each other as a result of material, energy and information flows. A high performance control technology such as decentralized control can be employed for control this class of systems.
An optimal decentralized control architecture is presented in this paper in order to ensure the efficient management of an inland navigation network in a global change context. For further logic, we show that if the subsystems present a connectivity from one to another, suitable local feedback action provided by these subsystem (decentralized controllers) may be sufficient if the subsystems are connected. Stabilizability (resp. stability) conditions are given for perfectly decoupled and interconnected coupled (by connectivity parameters) subsystems.

Keywords: Large-scale system; Inland navigation; Decentralized optimal controller; Controller design.

1. INTRODUCTION

Inland navigation networks are large scale systems composed of interconnected rivers and channels (*see* Figure 1). These networks are equipped with locks which are dedicated to the navigation task. To accommodate the navigation, the main objective is to ensure the seaworthiness condition, *i.e.* to maintain the levels close to setpoint designated by Normal Navigation Level (NNL). Achieving this objective is still a challenge due to the complexity of the networks, their dimensions, the impacts of the environment, the effect of extreme events like flood and drought, etc. One of solutions is the design of decentralized controllers [Sawadogo et al., 1998, 2000, Gómez et al., 2002]. It has been long recognized that to control an interconnected system, it is beneficial to decompose it into subsystems, and design control of each subsystem independently on the basis of local subsytem dynamics and the nature of their interconnections [Doan et al., 2009]. Three main reasons due for those [Siljak, 1991]:

- dimension of the system,
- information structure constraints,
- delays for the accuracy of the transmitted information,
- uncertainty.

For the last point, the essential uncertainty arises in the interconnections between different parts of the subsystems, since the local characteristics of each individual subsystem, should be modeled in most practical situations. Decentralized controller design addresses this issue and in other hand can significantly reduce the controller complexity [Jamshidi, 1997], [Rotkowitz and Lall, 2002], [Bakule, 2008]. Two main approaches are proposed to deal with the problem of interconnections [Lunze,

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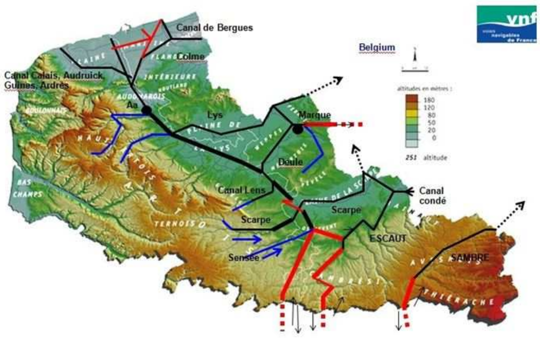

Fig. 1. Inland navigation network

1992]. The first one is the passive approach. A decentralized control is designed for each isolated subsystem for certain desirable performance. The design is independent of the knowledge of interconnections. The control is then applied to the overall system [Siljak, 1991]. The second one is the synthetic approach. In this case, interconnections are explicitly taken into account when designing the controls [Gavel and D.D., 1989]. There is a large number of survey papers [Sandell et al., 1978], [Chae and Bien, 1991], and books [Leondes, 1985], [Gajié and Shen, 1993] which can provide to the reader further information on decentralized control theory and practice.

In the field of water systems, Malaterre and Baume [Malaterre and Baume, 1999] also study the coupling and the possible decouplers when analyzing the coupling of the subsystems. Decentralized control using decouplers are implemented *e.g.* by [Montazar et al., 2005], [Aguilar et al., 2011]. Lemos [Lemos et al., 2007] describes the combination of multiple individual predictive controllers. Begovich [Begovich et al., 2007] used Relative Gain Array technique to check the feasibility of decentralized control of a laboratory irrigation canal. Cantoni [Cantoni et al., 2007] analyses numerically and then gives a theoretical background to the disturbance propagation and explores the possible feedforward combination (decoupler). His results are supported with field experiments. The coupling is further analyzed mathematically, in case of n identical pools in [Li and De Schutter, 2010]. They give guidelines to design decentralized feedback controllers, by having a trade-off between local and global performance, with other words setpoint tracking and disturbance rejection. Another issue concerns the stability of decentralized control. Several researchers have analyzed the stability and the decoupling of decentralized controllers. For example, the possible destabilization of decentralized controllers of water systems is addressed in [Schuurmans, 1997].

1.1 Paper contribution

In this paper, we discuss on the existence of an optimal state (resp. output) feedback decentralized control of the system under investigation. A complete architecture is studied with two constraitns. The first one is the time-delay for information transmission between two subsystems and the second one is the influence of one controlled subsystem to antoher one. This architecture aims to ensure an efficient management of inland navigation networks by dealing with the interconnections of each subsystems taking into account their interactions with each other (smooth or not). Thus, a local feedback action, provided by these subsystems (decentralized controllers), is tuned to get arround effictively the effect of their interactions. Stability conditions are given to guarantee the feasability and the convergence of decentralized controllers. On the other hand, we show that a perfectly decoupled decentralized control can be deduced via coupled and interconnected subsystems. The above mentioned decouplers can be seen as particular cases of the general design given in this paper.

1.2 Paper outline

This paper is organized as follows. In Section 2 we formulate the problem to describe a suitable design for a decentralized controller by mathematical relationship. In Section 3, we present our main result about stabilizability conditions of decentralized controller form for a large-scale systems. In Section 4, we use the Cuinchy-Fontinette Reach as a demonstrating example to verify the feasability of the approach and at the end, in Section 5, we shall give the conclusion of this work.

2. DECENTRALIZED CONTROLLER DESIGN

2.1 Large scale system modelling

As motivating case study we consider a large-scale system (*LSS*), described by the following equations :

$$LSS : \begin{cases} x(t+1) = f(x,u,t), \;\; x(t_0) = x_0, \;\; x \in \mathbb{R}^n, \\ y(t) = g(x,t), \qquad\qquad t \in \mathbb{N}. \end{cases} \tag{1}$$

where u, x and y are respectively the set of input, state and output terms of the system. t defines the continuous time.

In linear and stationary cases, the large-scale system (*LSS*) can be writting with the following state-space equations:

$$LSS : \begin{cases} x(t+1) = Ax(t) + Bu(t), \quad x(t_0) = x_0 \\ \quad\;\; y(t) = Cx(t) + Du(t), \end{cases} \tag{2}$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, $y \in \mathbb{R}^p$, $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $C \in \mathbb{R}^{p\times n}$, composed of s linear time-delay subsystems LSS_i, described by:

$$LSS_i : \begin{cases} x_i(t+1) = A_{i,i}x_i(t) + B_{i,i}u_i(t) + \displaystyle\sum_{j=1}^{s} \\ \qquad\qquad [A_{i,j}x_j(t-\tau_{i,j}) + B_{i,j}u_j(t-\tau_{i,j})] \\ \qquad\qquad \text{for } i = 1,2,\ldots,s \text{ and } j \neq i, \\ \\ y_i(t) = C_{i,i}x_i(t) + D_{i,i}u_i(t), \end{cases} \tag{3}$$

$\tau \in \mathbb{N}^{n_i}$ is the time delay matrix between the measurement points y_i and y_j. $x_i \in \mathbb{R}^{n_i}$, $u_i \in \mathbb{R}^{m_i}$, are the state and input of subsystem SS_i. $A_{i,i}$, $B_{i,i}$, $A_{i,j}$ and $B_{i,j}$ are coefficients of constant matrices with proper dimensions. The term $\sum_{j=1}^{s}(A_{i,j}x_j(t-\tau_{i,j}) + B_{i,j}u_j(t-\tau_{i,j}))$ is due to the effect of the time-delay input measurement value from one subsystem to each other.

Thus, the objective in this paper is to design a set of optimal output feedback controller whose action would be sufficient to overcome the effect of interactions, and allows efficiently the control of the global system (3) such that:

$$u_i = -K_{i,i}C_{i,i}x_i. \tag{4}$$

In order to do that, stabilizability (resp. stability) conditions are given in the next section for interconnected coupled subsystems using easily the connectivity parameter values.

2.2 Controller tuning

As large engineering systems are typically composed by a number of subsystems, it is quite natural to use a high performance control technology such as decentralized design for control these systems.

In this paper, we discuss about the completly coupled subsystems architecture with an ideal controller feature like is shown in Fig. 2. Herein the block *LSS* illustres each subsystem, the block τ defines the time delay matrix between two measurement points, the block Γ represents the interconnection matrix between each subsystem and K_i for $i = 1,\ldots,s$ represents each optimal control gain.

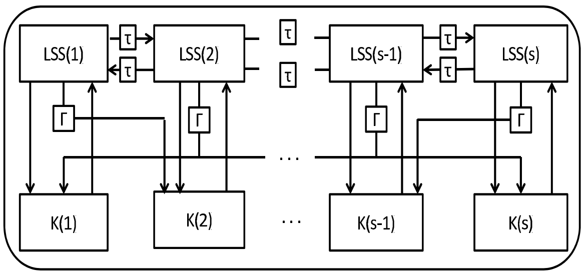

Fig. 2. Interconnected decentralized control

According to this architecture, let us define that:

Definition 1. The global system have a connectivity from and to another subsystem if there exists a matrix Γ definite positive weighting the interconnection of a subsystem i to another subsystem j. In other terms, Γ is the positive weighting interconnection values which are used to model the strength of interconnection. This interconnection matrix can be described by the term:

$$\Gamma_{i,j} = K_{i,j} x_j. \tag{5}$$

While considering that the gains of controllers have strong interaction, the mathematical relationship of input/output pair, describing the control problem is the following:

$$u_i = -(K_{i,i} C_{i,i} x_i + \sum_{j=1}^{s} \Gamma_{i,j}) \text{ with } j \neq i \text{ and for } i = 1, \ldots, s \tag{6}$$

where u_i depends on the locally available state x_i of the i^{th} subsystem

$$LSS_i : x_i(t+1) = A_{i,i} x_i(t) + B_{i,i} u_i(t) \tag{7}$$

For ease the reading, let considerate that the dynamical of the measured state $x(t)$ is given by the simplified relation:

$$LSS : \begin{cases} x(t+1) = A_0 x(t) + B_0 u(t) + A_1 x(t-\tau) \\ \qquad\qquad + B_1 u(t-\tau), \\ y(t) = C_0 x(t) \;\; x(t_0) = x_0 \end{cases} \tag{8}$$

where the matrix C_0 is the identity matrix $I \in \mathbb{R}^{n \times n}$. A_0 and B_0 are respectively the diagonal matrices composed by $A_{i,i}$ and $B_{i,i}$ terms. A_1 and B_1 are respectively the interconnectivity matrices composed by $A_{i,j}$ and $B_{i,j}$ values due to the time delay.

So, the control design problem is described in this case by:

$$u(t) = K x(t), \tag{9}$$

to minimize the quadratic cost function

$$J(u_i) = \sum (x_i^T S x_i + u_i^T R u_i + 2 x_i^T N u_i) dt \tag{10}$$

subjects to the system defined by equation (8).

In equation (10) S, R and N are weighting matrices, or design parameters, where the state-cost matrix, S, weights the state while the performance index matrix, R, weights the control effort. If S is increased while R remains constant, the setting time will be reduced as the states approach zero at a faster rate. The controller in this case maintain the error sufficiently small to guarantee the convergence of controller.

One solution to solve the problem (10) is developed by [Boyd et al., 1994] as:

Theorem 1. For finite-dimensional systems, if there exists matrices $P = P^T > 0$, $S > 0$ and $R > 0$, $\forall i = 1, \ldots, s$ such as

$$\begin{bmatrix} A_0^T P + P A_0 + P & P B_0 + N^T \\ B_0^T P + N & R \end{bmatrix} \geq 0 \tag{11}$$

satisfying the algebraic Riccati equation

$$A_0^T P + P A_0 - (P B_0 + N) R^{-1} (B_0^T P + N^T) + S = 0 \tag{12}$$

then the controller defined by equation (9) with

$$K_{i,i} = R^{-1} (B_0^T P + N^T) \tag{13}$$

stabilize the nominal closed-loop decentralized system described by the relation (8), $\forall i$ values.

Proof 1. The previous results hold, as demonstrated in [Boyd et al., 1994], if the pair (A_0, B_0) are stabilizable, $R > 0$, $S - NR^{-1}N^T \geq 0$ and $S - NR^{-1}N^T$, $A_0 - B_0 R^{-1} N^T$ has no unobservable mode.

One can see easily here that this feature stands up if the following conditions holds:

- The closed-loop system should be stable if each subsystem LSS_i is stable.
- The controller should maintain the error sufficiently small to guarantee its convergence.

The stabilizability proof of this feature is given in Section 3, both to ensure the convergence of the controller and the stability of the overall system.

3. STABILIZATION

In order to state on the stabilizability of the plant using decentralized approach, we focus our study once again on an optimal feedback controller, in interconnected coupled case.

Moreover, we also desire to guarantee the stability of the global closed-loop system under all operation conditions of whole subsystems LSS_i. To do this, set the following expressions which are necessary for the stabilizability criterion:

Definition 2. The equilibrium point $x^* = x_0$ of (8) is stable (in the sense of Lyapunov) at initial time $t = t_0$ if for any $\varepsilon > 0$ there exists a $\delta(t_0, \varepsilon) > 0$ such that

$$\|x(t_0)\| < \delta \implies \|x(t)\| < \varepsilon, \forall t \geq t_0 \tag{14}$$

On the other hand, we know that systems (8) may be infinite dimensions, but the state $x(t)$ evolves on the interval $t \in [0, +\infty)$. Thus,

Definition 3. A large scale system mapping x and u is causal if and only if, for any pair of input signals $u_i(t)$ and $u_j(t)$ such that $u_i(t) = u_j(t)$, $\forall t \leq t_0$, the corresponding states satisfy $x_i(t) = x_j(t)$, $\forall t \leq t_0$.

Theorem 2. The interconnected coupled subsystem (8) is locally stabilizable (resp. stable) by a linear state (output) feedback control law

$$u_i(t) = -K_i x_i(t)$$

such that controller K exits and its diagonal blocks are s separate controllers defined by

$$K = diag(K_i), \;\; \forall i = 1 \, to \, s$$

if

- the large scale system is causal, and
- there exists a symmetric definite positive matrix $P = P^T > 0$ where

$$\mathfrak{L}(P) = \tilde{A_0}^T P \tilde{A_0} - P < 0 \tag{15}$$

for any equilibrium initial state function $x(t_0)$ and with $\tilde{A_0} = (A_0 - B_0 K)$.

The following statement is equivalent with the result above. In fact,

Corrolary 1. If controller K exits and its diagonal blocks are s separate controllers, then a perfectly decoupled decentralized control design is identified. So, the decoupled controlled subsystem is stabilizable with independant local feedback action defined by the linear state (output) feedback control law:

$$u_i(t) = -K_i x_i(t)$$

if there exists a symmetric definite positive matrix $P = P^T > 0$ where

$$\mathfrak{L}(P) = \tilde{A_0}^T P\tilde{A_0} - P < 0$$

for any equilibrium initial state function $x(t_0)$ and with $\tilde{A_0} = (A_0 - B_0K)$.

Proof 2. These statements are straightforward to demonstrate. Indeed, if a perfectly decoupled decentralized control design is identified, the stability property falls within the previous theorem since this is a special case of the previous result.

Theorem 3. The interconnected coupled subsystem (8) is globally stabilizable (resp. stable) by a linear state (output) feedback control law

$$u_i(t) = -(K_{i,i}x_i(t) + \sum_{j=1}^{s}\Gamma_{i,j}) \text{ with } j \neq i \text{ and for } i = 1,...,s$$

if

- locally the system is stable,
- and there exists a symmetric definite positive matrices $Q = Q^T > 0$ and $P = P^T > 0$ which satisfy the following conditions

$$\tilde{A_0}^T Q\tilde{A_0} < 0 \tag{16}$$

$$\tilde{A_i^T}^n \mathfrak{L}(P)\tilde{A_i}^n < 0 \quad \text{with } n \in \mathbb{N} \tag{17}$$

for any equilibrium initial state function $x(t_0)$

Proof 3. A classical way to prove the stability of one system is to use the Lyapunov function concept. Thus, set $V(x)$ the common quadratic Lyapunov-like function defined by the relation

$$V(x) = x(t)^T Px(t) \tag{18}$$

and satisfies the stability condition

$$\Delta V(x) = V(x(t+1)) - V(x(t)) \ < 0, \tag{19}$$

for the system described by (8).

In closed-loop point of view, the dynamical of the measured state $x(t)$ is given by the relation:

$$x(t+1) = (A_0 - B_0K)x(t) - (A_1 - B_1K)x(t-\tau)$$

therefore this last expression is equivalent to the previous one below

$$x(t+1) = \tilde{A_0}x(t) - \tilde{A_1}x(t-\tau)$$

with $\tilde{A_0} = (A_0 - B_0K)$ and $\tilde{A_1} = (A_1 - B_1K)$.

Two cases can be considerate to verify theorems 2 and 3. The first one is to check stability when the system evolves in the interval $t \in [t_0, \tau[$ and the second one when it evolves in the interval $t \in [\tau, \infty)$ for any pair of input signals $u_i(t)$ and $u_j(t)$ such that $u_i(t) = u_j(t) = 0$, with the corresponding states $x_i(t) = x_j(t) = 0$, $\forall t \leq t_0$.

- Case 1: the system evolves in the interval $t \in [t_0, \tau[$. Thus,

$$\begin{aligned}\Delta V(x) &= V(x(t+1)) - V(x(t)) \ < 0\\ &= x(t+1)^T Px(t+1) - x(t)^T Px(t) \ < 0\\ &= x(t)^T \tilde{A_0}^T P\tilde{A_0}x(t) - x(t)^T Px(t) \ < 0\\ &= x(t)^T (\tilde{A_0}^T P\tilde{A_0} - P)x(t) \ < 0\end{aligned}$$

therefore, if $\mathfrak{L}(P) = \tilde{A_0}^T P\tilde{A_0} - P < 0$, the inteconnected subsystem (8) is locally stabilizable (resp. stable) by a linear output feedback control law

$$u_i(t) = -K_ix_i(t).$$

- Case 2: the system evolves in the interval $t \in [\tau, \infty)$. Thus, the dynamical of the measured state $x(t)$ is in fact:

$$\begin{aligned}x(1) &= \tilde{A_0}x(0)\\ x(2) &= \tilde{A_0}x(1) = \tilde{A_0}^2 x(0)\\ \vdots &= \vdots\\ x(\tau-1) &= \tilde{A_0}x(\tau-2) = \tilde{A_0}^{\tau-1}x(0)\\ x(\tau) &= \tilde{A_0}x(\tau-1) + \tilde{A_1}x(0)\\ &= (\tilde{A_0}^\tau + \tilde{A_1})x(0)\\ x(\tau+1) &= \tilde{A_0}x(\tau) + \tilde{A_1}x(1)\\ &= (\tilde{A_0}^{\tau+1} + \tilde{A_0}\tilde{A_1} + \tilde{A_1}\tilde{A_0})x(0)\end{aligned}$$

therefore, the derivative of the Lyapunov function has the following schema:

$$\begin{aligned}\Delta V(x) &= V(x(\tau+1)) - V(x(\tau)) \ < 0,\\ &= x(\tau+1)^T Px(\tau+1) - x(\tau)^T Px(\tau) \ < 0\\ &= x(0)^T (\tilde{A_0}^{\tau+1} + \tilde{A_0}\tilde{A_1} + \tilde{A_1}\tilde{A_0})^T P\left(\tilde{A_0}^{\tau+1} +\right.\\ &\left.\tilde{A_0}\tilde{A_1} + \tilde{A_1}\tilde{A_0}\right)x(0) - x(0)^T (\tilde{A_0}^\tau + \tilde{A_1})^T P +\\ &(\tilde{A_0}^\tau + \tilde{A_1})x(0) < 0\\ &= x(0)^T \left[\tilde{A_1}^T \mathfrak{L}(P)\tilde{A_1} + A_0^{T\,\tau}\mathfrak{L}(P)\tilde{A_0}^\tau +\right.\\ &\left.\tilde{A_0}^T Q\tilde{A_0}\right]x(0) < 0\end{aligned}$$

with $Q = \tilde{A_1}^T P\tilde{A_1}$.

Assume that the initial condition $x(0)$ is a stable equilibrium point for the system (8) and this last is locally stable, therefore the coupled subsystem is globally stabilizable (resp. stable) by a linear state (output) feedback control law

$$u_i(t) = -(K_{i,i}x_i(t) + \sum_{j=1}^{s}\Gamma_{i,j})$$

with $j \neq i$ and $i = 1,...,s$, if there exist a Lyapunov function $V(x)$ defined by (18) and a quadratic function $V(x(t))$ such that $V_(x(t_0)) = 0$ and $V_(x(t)) > 0, \forall x > 0$ and $\Delta V(x) < 0, \forall x \neq 0$, then the origin of the state space of the system (8) is globally stable under all operation conditions of overall subsystems LSS_i.

4. ILLUSTRATIVE EXAMPLE

As demonstrating example, we consider a part of the inland navigation network in the north of the France (Fig. 3), the Cuinchy-Fontinette Reach (CFR). Note that the management of this system is ensured by VNF (Voies Navigables de France), and the goal is to maintain the water level of the channel at $NNL = 19.52m$.

Fig. 3. The real system in exploitation

The CFR has a major importance for the management of this inland navigation network because it is located in the center of the network, between the upstream lock of Cuinchy at the East of the town Bethune and the downstream lock of Fontinettes at the Southwest of the town Saint-Omer. So, the head part of the

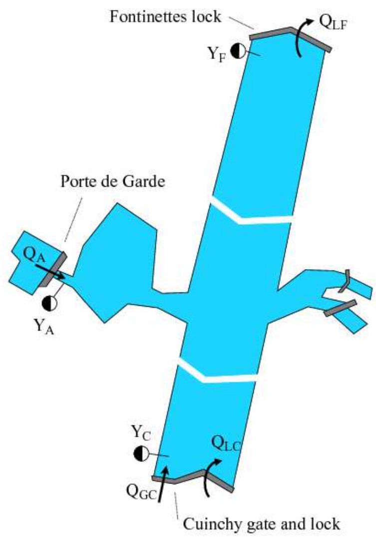

Fig. 4. Scheme of the Cuinchy-Fontinettes navigation reach

CFR system has $28.7Km$ (Fig. 4) from Cuinchy sluice gate to Aire-sur-la-Lys. While, the bottom part has $13.6Km$ from Aire-sur-la-Lys sluice gate to Saint-Omer.

By computing the system's model for control purposes, we consider the NNL operating points. However, in this section we don't discuss about the identification technique of the model, we just use the model whose architecture has been proposed and validated in [Duviella et al., 2013]. Herein, it shown that the CFR system can be modelled with a state-space linear model with fixed delays described by the relation (3). For the CFR system, the state variable $x(t)$ at the instant t represents the water levels $L_i(t)$, with $i = 1,2,3$ representing three subsystems, which are measured in the reach. The variables $u(t)$ are the input/output discharges $Q_l(t)$ of the reach. The variables $y(t)$ are the water levels in the reach. Each vairable is expressed according to the delay matrix τ whose the element is gotten by data-based procedure or by physical knowledge of the system. Theoretically, the time-delay matrix is given by the following relation [Litrico and Fromion, 2004], while considering two points along the canal separated by a distance D:

$$\tau = \int_0^D \frac{dl}{c(l)+v(l)} \tag{20}$$

where $c(l)$ and $v(l)$ represent respectively the celerity and the velocity. This relation gives the following result for the CFR system:

$$\tau = \begin{pmatrix} 0 & 75 & 110 \\ 75 & 0 & 36 \\ 110 & 36 & 0 \end{pmatrix}.$$

As said above, we use for designing the controller a suitable state-space model for the system. The nominal system matrices are:

$$A_0 = \begin{pmatrix} 0.873 & 0 & 0 \\ 0 & 0.943 & 0 \\ 0 & 0 & 0.781 \end{pmatrix}; B_0 = \begin{pmatrix} 0.00114 & 0 & 0 \\ 0 & 0.00023 & 0 \\ 0 & 0 & 0.00175 \end{pmatrix};$$

$$C_0 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix};$$

and the interconnectivity due to the incertainty is given by the following matrices:

$$A_1 = \begin{pmatrix} 0 & 0.116 & 0.011 \\ 0.044 & 0 & 0.014 \\ 0.092 & 0.126 & 0 \end{pmatrix};$$

$$B_1 = \begin{pmatrix} 0 & 0.00078 & -0.0009 \\ -0.00008 & 0 & 0.00005 \\ -0.00014 & 0.00348 & 0 \end{pmatrix};$$

- Case 1: Existence of a stabilizable decoupled architecture
  Using the relation (13), let verify and compute if controller K exists, and if its diagonal blocks are s separate controllers. The result is:
  $$K = \begin{pmatrix} 1.5316 & 0 & 0 \\ 0 & 8.2000 & 0 \\ 0 & 0 & 0.8926 \end{pmatrix} \times 1.0e^{+3};$$
  If K exists we can compute the associate P matrix which necessary for proving the stabilizability (resp. stability) of the structure. P's value is the following:
  $$P = \begin{pmatrix} 0.1343 & 0 & 0 \\ 0 & 3.5652 & 0 \\ 0 & 0 & 0.0510 \end{pmatrix} \times 1.0e^{+3};$$
  To check if the system is stabilizable with independant local feedback action, we test if the Lyapunov operator is negative for all instant $t > 0$ ($\mathfrak{L}(P) < 0\ \forall t > 0$) according to the theorem 2. The numerical value of this operator is:
  $$\mathfrak{L}(P) = \begin{pmatrix} -0.3196 & 0 & 0 \\ 0 & -3.9485 & 0 \\ 0 & 0 & -0.1989 \end{pmatrix} \times 1.0e^{+6} < 0;$$
  The derivative of the common Lyapunov quadratic function $V(x) = x(t)^T P x(t)$ is negative and therefore the decoupled subsystem is locally stabilizable (resp. stable) by a linear state (output) feedback control law.
- Case 2: Existence of a stabilizable interconnected architecture
  After the computation, the two parts of the controller gain are:
  $$K_0 = \begin{pmatrix} 0.9052 & 0.7149 & -0.1532 \\ 0.1442 & 7.1339 & -0.0291 \\ -0.2352 & -0.2214 & 0.2198 \end{pmatrix} \times 1.0e^{+3};$$
  $$K_1 = \begin{pmatrix} 0 & -2.4637 & -0.0075 \\ 0.1517 & 0 & 0.3322 \\ -0.6833 & 0.9864 & 0 \end{pmatrix} \times 1.0e^{+3};$$
  and with these values, P and Q matrices are:
  $$Q = \begin{pmatrix} 0.4911 & -0.9816 & 0.1056 \\ -0.9816 & 2.4247 & -0.1085 \\ 0.1056 & -0.1085 & 0.1451 \end{pmatrix} \times 1.0e^{+6};$$
  $$P = \begin{pmatrix} 0.0794 & 0.0627 & -0.0134 \\ 0.0627 & 3.1017 & -0.0127 \\ -0.0134 & -0.0127 & 0.0126 \end{pmatrix} \times 1.0e^{+7};$$
  The first criterion of the theorem 3 gives the following result:
  $$\tilde{A_0}^T Q \tilde{A_0} = \begin{pmatrix} 0.1847 & -0.0186 & 0.1831 \\ -0.0186 & 4.0419 & 0.6883 \\ 0.1831 & 0.6883 & 0.4965 \end{pmatrix} \times 1.0e^{+5},$$
  where all eigenvalues have a negative real parts, $\Rightarrow \tilde{A_0}^T Q \tilde{A_0} < 0$.

For the second criterion, we just illustrate the stability test for $n = 2$. Furthermore, for all values of n the condition is always satisfied.

$$\tilde{A_0^T}^2 \mathfrak{L}(P) \tilde{A_0}^2 = \begin{pmatrix} -0.0205 & -0.2636 & 0.0016 \\ -0.2636 & -4.0118 & 0.0591 \\ 0.0016 & 0.0591 & -0.0048 \end{pmatrix} \times 1.0e^{+6},$$

All eigenvalues of this matrix have negative real parts, $\Rightarrow \tilde{A_0^T}^2 \mathfrak{L}(P) \tilde{A_0}^2 < 0$;

$$\tilde{A_1^T}^2 \mathfrak{L}(P) \tilde{A_1}^2 = \begin{pmatrix} -4.1281 & 5.8909 & -2.3450 \\ 5.8909 & -9.3258 & 2.4743 \\ -2.3450 & 2.4743 & -2.1758 \end{pmatrix} \times 1.0e^{+5},$$

This computation gives also a negative real parts for all eigenvalues, $\Rightarrow \tilde{A_1^T}^2 \mathfrak{L}(P) \tilde{A_1}^2 < 0$.

Once again, the derivative of the common Lyapunov quadratic function $V(x) = x(t)^T P x(t)$ is negative and therefore the interconnected architecture is globally stabilizable (resp. stable) by a linear state (output) feedback control law.

5. CONCLUSION

Typically, large engineering systems are composed by a several subsystems that interact with each other. The interconnections between different parts of the systems (subsystems) since the local characteristics of each individual subsystem required in practical situations a performed control technology such as decentralized control. This last is particularly intertesting because it reduces the controller complexity and ease the practical implementation.

We have presented an optimal decentralized control architecture in this paper in order to ensure the efficient management of an inland navigation network whose main objective is to ensure the seaworthiness condition, *i.e.* to maintain the levels close to setpoint designated by Normal Navigation Level (NNL). We have shown that if the subsystems present a connectivity from one to another, suitable local feedback action provided by these subsystem (decentralized controllers) may be sufficient for control purposes. Stabilizability (resp. stability) conditions are given both for perfectly decoupled and interconnected coupled (by connectivity parameters) subsystems controlled by an optimal linear state (output) feedback regulator.

REFERENCES

J. Aguilar, P. Langarita, L. Linares, E. Galvis, K. Horváth, J. Rodellar, and M. Gómez. Control automatico de niveles en un canal experimental dividido en tres tramos. *JIA 2011. II Jornadas del Ingenierá del agua, Modelos numéricos en dinámica fuvial*, 2011.

L. Bakule. Decentralized control : an overview. *Annual reviews in control*, 32:87–98, 2008.

O. Begovich, J.C. Felipe, and V.M. Ruiz. Real-time implementation of a decentralized control for an open irrigation canal prototype. *in Asian Journal of Control*, 9, 2007.

S. Boyd, L. El Ghaoui, E. Feron, and V. Balakrishnan. *Linear Matrix Inequalities in System and Control Theory*. SIAM, Philadelphia, PA, 1994.

M. Cantoni, E. Weyer, Y. Li, S.K. Ooi, I. Mareels, and M. Ryan. Control of large-scale irrigation networks. *IEEE Special Issue on Technology of Networked Control Systems*, 95:75–91, 2007.

S. Chae and Z. Bien. Techniques for decentralized control for interconnected systems. *in Control and Dynamic Systems*, 41:273–315, 1991.

D. Doan, T. Keviczky, I. Necoara, M. Diehl, and B. De Schutter. A distributed version of han's method for dmpc of dynamically coupled systems with coupled constraints. *in: Proceedings of the 1st IFAC Workshop on Estimation and Control of Networked Systems*, pages 240–245, 2009.

E. Duviella, L. Rajaoarisoa, J. Blesa, and K. Chuquet. Adaptive and predictive control architecture of inland navigation networks in a global change context: application to the cuinchy-fontinettes reach. *IFAC Conference on Manufacturing Modelling, Management, and Control*, 2013.

Z. Gajié and X. Shen. Parallel algorithms for optimal control of large-scale linear systems. *in Springer-Verlag*, 1993.

D.T. Gavel and Siljak D.D. Decentralized adaptive control : Structural conditions for stability. *in IEEE Transactions on Automatic Control*, 34:413–426, 1989.

M. Gómez, J. Rodellar, and J. Mantecón. Predictive control method for decentralized operation of irrigation canals. *in Applied Mathematical Modelling*, 26:1039–1056, 2002.

M. Jamshidi. Large-scale systems : Modeling, control and fuzzy logic. *in Prentice-Hall*, 1997.

J.M. Lemos, L.M. Rato, F. Machado, N. Nogueira, P. Salgueiro, and R. Neves-Silva. Predictive adaptive control of water level in canal pools. *in: Proceedings of the 16th International Conference on Systems Science*, 2007.

C.T. Leondes. Control and dynamic systems. *Academic Press, Orlando, Fl*, pages 22–24, 1985.

Y. Li and B. De Schutter. Control of a string of identical pools using non-identical feedback controllers. *in: Proceedings of the 49th IEEE Conference on Decision and Control*, 2010.

X. Litrico and V. Fromion. Simplified modeling of irrigation canals for controller design. *Journal of irrigation and drainage engineering*, 130(5):373–383, 2004.

J. Lunze. Feedback control of large-scale systems. *in Prentice-Hall*, 1992.

P.O. Malaterre and J-P. Baume. Optimal choice of control action variables and linked algorithms: Comparaison of different alternatives. *in Proceedings of USCID Workshop on modernization of irrigation water delivery systems*, pages 387–405, 1999.

A. Montazar, P.J. van Overloop, and R. Brouwer. Centralized controller for the narmada main canal. *in Irrigation and drainage*, 54:79–89, 2005.

M. Rotkowitz and S. Lall. Decentralized control information structures preserved under feedback. *Proceedings of the IEEE Conference on Decision and Control*, pages 569–575, 2002.

N.R. Sandell, P. Varaiya, M. Athans, and Safonov M.G. Survey of decentralized control methods for large-scale systems. *in IEEE Transactions on Automatic Cotnrol*, 23:108–128, 1978.

S. Sawadogo, R.M. Faye, P.O. Malaterre, and F. Mora-Camino. Decentralized predictive cotnroller for delivery canals. *IEEE International Conference on Systems, Man and Cybernetics*, pages 3880–3884, 1998.

S. Sawadogo, R. Faye, A. Benhammou, and K. Akouz. Decentralized adaptive control of multi-reach irrigation canal. *in Proceedings of IEEE International Conference on Systems, Man and Cybernetics*, 1-5:3437–3442, 2000.

J. Schuurmans. Control of water levels in open channels. *PhD Thesis*, 1997.

D.D. Siljak. Decentralized control of complex systems. *in Control and Dynamic Systems*, 1991.