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Discrete-time Takagi-Sugeno descriptor models: observer design

Víctor Estrada-Manzo*, Zsófia Lendek, Thierry Marie Guerra***

**University of Valenciennes and Hainaut-Cambrésis, LAMIH UMR CNRS 8201,*
Le Mont Houy, 59313, Valenciennes Cedex 9, France.
(e-mail:{victor.estradamanzo, guerra}@univ-valenciennes.fr)
***Department of Automotion, Technical University of Cluj-Napoca,*
Memorandumului 28, 400114, Cluj-Napoca, Romania.
(e-mail: zsofia.lendek@aut.utcluj.ro)

Abstract: The present paper deals with observer design for discrete-time nonlinear descriptor systems. Through Finsler's Lemma, we cut the link between the observer gains and the Lyapunov function, which relaxes existing conditions and introduces an extra degree of freedom. In addition, the results are expressed as LMI constraints. The effectiveness of the two proposed approaches is illustrated via several examples.

1. INTRODUCTION

Takagi-Sugeno (TS) models are able to exactly represent a large class of nonlinear models via the sector nonlinearity approach (Ohtake et al., 2001). These systems are a collection of linear models blended together with nonlinear convex membership functions (MFs) (Tanaka and Wang, 2001). Therefore, TS models are widely used for analysis and controller synthesis of nonlinear systems through the direct Lyapunov method. The conditions obtained are expressed in terms of linear matrix inequalities (LMIs) which can be solved via convex optimization techniques (Boyd et al., 1994). Several works exist for continuous-time case (Johansson et al., 1999; Tanaka et al., 2003; Rhee and Won, 2006; Sala and Ariño, 2007; Bernal and Guerra, 2010; Guerra et al., 2012) and for discrete-time one (Feng, 2004; Kruszewski et al., 2008; Chadli and Guerra, 2012; Lendek et al., 2012); all of them were presented for the classical TS models. However, one of the drawbacks of using TS models is that the dimension of the resulting LMI problem is exponential in the number of local models, thus increasing the computational cost and possibly making it computationally intractable.

On the other hand, physical systems are naturally described by descriptor models (Luenberger, 1977). Frequently, this type of models appears in control problems (Dai, 1988). After the TS descriptor model was introduced in (Taniguchi et al., 1999), results concerning stability analysis, controller, and observer design have been developed (Guerra et al., 2004, 2007; Tanaka et al., 2007; Guelton et al., 2008; Vermeiren et al., 2012). The TS descriptor model has been used to reduce the number of LMI constraints and to keep the natural form of the nonlinear system.

To apply state feedback control design it is necessary to know all the states of the system; normally, in real applications not all the states are available to be measured, thus the necessity to estimate them arises (Luenberger, 1966). Several works are concerned this subject (Bergsten and Driankov, 2002; Tanaka et al., 1998; Lendek et al., 2010).

The observer design for descriptor models in the continuous-time and the discrete-time has been addressed in (Darouach and Boutayeb, 1995; Zhang et al., 2008; Wang et al., 2012). The results are expressed in terms of equality constraints and/or LMIs. However, all the results consider the output matrix without nonlinear terms, i.e., linear measurements.

This work presents a novel approach for the observer design of discrete-time nonlinear descriptor models. The approach is based on TS representation and Finsler's Lemma. The objective of using Finsler's Lemma is to cut the link between the observer gains and the Lyapunov function (Guerra et al., 2012; Estrada-Manzo et al., 2013), thus it allows finding more general observer structures and therefore more degrees of freedom in terms of decision variables in the LMI constraints, while keeping the same number of LMI conditions. Moreover, the present approaches can be applied even if the output matrix is nonlinear, which outperform the previous works on the subject.

The paper is organized as follows: Section 2 provides some useful notation and introduces the discrete-time TS descriptor model; Section 3 presents the main results on the observer design for discrete-time TS descriptor models; Section 4 illustrates the effectiveness of the proposed approaches via examples.

2. NOTATIONS AND PROBLEM STATEMENT

Consider the following discrete-time TS model in the descriptor form:

$$\begin{aligned}\sum_{k=1}^{r_e} v_k\left(z(\kappa)\right)E_k x(\kappa+1) &= \sum_{i=1}^{r} h_i\left(z(\kappa)\right)\left(A_i x(\kappa)+B_i u(\kappa)\right)\\ y(\kappa) &= \sum_{i=1}^{r} h_i\left(z(\kappa)\right)C_i x(\kappa),\end{aligned} \tag{1}$$

where $x \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}^m$ is the control input vector, $y \in \mathbb{R}^o$ is the output vector, and κ is the current sample. The membership functions (MFs) hold the convex sum property in a compact set of the state: $h_i\left(z(\kappa)\right) \geq 0$, $i \in \left\{1,\ldots,2^p\right\}$, $\sum_{i=1}^{r} h_i\left(z(\kappa)\right) = 1$, where p is the number of nonlinearities in the right-hand side. For the p_e nonlinear terms in the left-hand side the MFs are $v_k\left(z(\kappa)\right) \geq 0$, $k \in \left\{1,\ldots,2^{p_e}\right\}$, $\sum_{k=1}^{r_e} v_k\left(z(\kappa)\right) = 1$. Matrices $\left(A_i, B_i, C_i\right)$, $i \in \left\{1,\ldots,r\right\}$ represent the *i*-th linear right-hand side model and E_k, $k \in \left\{1,\ldots,r_e\right\}$ represent the *k*-th linear left-hand side model of the TS descriptor model. The MFs depend on the premise variables grouped in the vector $z(\kappa)$ which are assumed to be known.

In this work, matrix $E_v = \sum_{k=1}^{r_e} v_k\left(z(\kappa)\right) E_k$ is assumed to be regular matrix. This is motivated by mechanical systems, in which E_v contains the inertia matrix and therefore it is regular.

Notation: the following shorthand notation will be used to represent simple convex sums of matrix expressions

$$\Upsilon_h = \sum_{i=1}^{r} h_i\left(z(\kappa)\right)\Upsilon_i, \quad \Upsilon_v = \sum_{k=1}^{r} v_k\left(z(\kappa)\right)\Upsilon_k,$$
$$\Upsilon_{h+} = \sum_{l=1}^{r} h_l\left(z(\kappa+1)\right)\Upsilon_l, \quad \Upsilon_h^{-1} = \left(\sum_{i=1}^{r} h_i\left(z(\kappa)\right)\Upsilon_i\right)^{-1},$$

for multiple convex sums of matrix expressions:

$$\Upsilon_{hh}^{v} = \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{k=1}^{r_e} h_i\left(z(\kappa)\right) h_j\left(z(\kappa)\right) v_k\left(z(\kappa)\right)\Upsilon_{ij}^{k}.$$

An asterisk $(*)$ will be used in matrix expressions to denote the transpose of the symmetric element; for in-line expressions it will denote the transpose of the terms on its left-hand side. In what follows, $x_{\kappa+}$ and x_κ will stand for $x(\kappa+1)$ and $x(\kappa)$ respectively. Arguments will be omitted when their meaning is straightforward.

The following example illustrates why is important to keep the nonlinear descriptor form instead of calculate the "classical" state space with $x_{\kappa+} = A(x)x_\kappa + B(x)u_\kappa$.

Example. Consider the following system in nonlinear descriptor form with $E(x) = \begin{bmatrix} 1 & -x_1^2 \\ 1 & 1 \end{bmatrix}$, $A(x) = \begin{bmatrix} \sin(x_1)/x_1 & 0.2 \\ 1 & \cos(x_1) \end{bmatrix}$, and $C(x) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}^T$. The representation in the form (1) gives $r_e = 2$ and $r = 4$ due to the number of nonlinearities on the left-side and right-side. To rewrite the original nonlinear system into the "classical" TS representation it is necessary to invert the matrix $E(x)$, resulting in $x_{\kappa+} = \left(E(x)\right)^{-1}\left(A(x)x_\kappa + B(x)u_\kappa\right)$. This means that four different nonlinearities have to be considered, which results in $r = 16$ due to the nonlinearities in $\left(E(x)\right)^{-1}$. Via the quadratic framework, the number of LMI conditions to be verified for the "classical" TS representation is $r^2 + 1 = 257$ while for the TS descriptor model is $r_e r^2 + 1 = 33$. ◊

In order to obtain LMI conditions, the following relaxation scheme will be employed.

Relaxation Lemma (Tuan et al., 2001): Let Υ_{ij}^k be matrices of proper dimensions. Then

$$\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{k=1}^{r_e} h_i\left(z(\kappa)\right) h_j\left(z(\kappa)\right) v_k\left(z(\kappa)\right)\Upsilon_{ij}^{k} < 0,$$

holds if

$$\begin{aligned} &\Upsilon_{ii}^{k} < 0, \quad i \in \left\{1,\ldots,r\right\}, \quad k \in \left\{1,\ldots,r_e\right\} \\ &\frac{2}{r-1}\Upsilon_{ii}^{k} + \Upsilon_{ij}^{k} + \Upsilon_{ij}^{k} < 0, \quad i,j \in \left\{1,\ldots,r\right\},\ k \in \left\{1,\ldots,r_e\right\}, i \neq j \end{aligned} \tag{2}$$

Finsler's Lemma (de Oliveira and Skelton, 2001): Let $x \in \mathbb{R}^n$, $Q = Q^T \in \mathbb{R}^{n\times n}$, and $R \in \mathbb{R}^{m\times n}$ such that $rank(R) < n$; the following expressions are equivalent:

a) $x^T Q x < 0$, $\forall x \in \left\{x \in \mathbb{R}^n : x \neq 0, Rx = 0\right\}$.

b) $\exists M \in \mathbb{R}^{n\times m} : Q + MR + R^T M^T < 0$.

3. MAIN RESULTS

As it was mentioned before, in most applications not all the information is available for control purposes; therefore we propose two approaches for the observer design of discrete-time TS descriptor model. The first approach via a quadratic Lyapunov function; the second one replaces the quadratic Lyapunov function by a more general Lyapunov function.

3.1 Quadratic Case

An observer for the discrete-time TS descriptor model (1) is given by:

$$\begin{aligned} E_v \hat{x}_{\kappa+} &= A_h \hat{x}_\kappa + B_h u + L_{hv}\left(y_\kappa - \hat{y}_\kappa\right) \\ \hat{y}_\kappa &= C_h \hat{x}_\kappa, \end{aligned} \tag{3}$$

where the observer gain is defined as

$$L_{hv} = \sum_{j=1}^{r}\sum_{k=1}^{r_e} h_j\left(z(\kappa)\right) v_k\left(z(\kappa)\right) L_{jk}.$$

The estimation error $e_\kappa = x_\kappa - \hat{x}_\kappa$ and its dynamics are

$$E_v e_{\kappa+} = \left(A_h - L_{hv} C_h\right) e_\kappa. \tag{4}$$

Equality (4) can be written as

$$\begin{bmatrix} A_h - L_{hv} C_h & -E_v \end{bmatrix} \begin{bmatrix} e_\kappa \\ e_{\kappa+} \end{bmatrix} = 0. \tag{5}$$

In order to investigate the stability of the estimation error consider the following Lyapunov function

$$V(e_\kappa) = e_\kappa^T P e_\kappa, \quad P = P^T > 0. \tag{6}$$

The variation of the Lyapunov function (6) is

$$\Delta V(e_\kappa) = e_{\kappa+}^T P e_{\kappa+} - e_\kappa^T P e_\kappa = \begin{bmatrix} e_\kappa \\ e_{\kappa+} \end{bmatrix}^T \begin{bmatrix} -P & 0 \\ 0 & P \end{bmatrix} \begin{bmatrix} e_\kappa \\ e_{\kappa+} \end{bmatrix} < 0. \tag{7}$$

Expressions (6) and (7) can be written together via Finsler's Lemma

$$\begin{bmatrix} -P & 0 \\ 0 & P \end{bmatrix} + \begin{bmatrix} M \\ N \end{bmatrix} \begin{bmatrix} A_h - L_{hv} C_h & -E_v \end{bmatrix} + (*) < 0. \tag{8}$$

Then, the following result can be stated:

Theorem 1: The estimation error dynamics in (4) are asymptotically stable if there exist matrices $P = P^T > 0$ and F_{jk}, for $j \in \{1, \ldots, r\}$, $k \in \{1, \ldots, r_e\}$ such that the LMI conditions (2) are satisfied with

$$\Upsilon_{ij}^k = \begin{bmatrix} -P & (*) \\ PA_i - F_{jk} C_i & -PE_k - E_k^T P + P \end{bmatrix}. \tag{9}$$

The observer gains are recovered with $L_{hv} = P^{-1} F_{hv}$.

Proof: Recall (8). Choosing the matrices $M = 0$ and $N = P$ yields

$$\Upsilon_{hh}^v = \begin{bmatrix} -P & (*) \\ PA_h - F_{hv} C_h & -PE_v - E_v^T P + P \end{bmatrix} < 0, \tag{10}$$

with $F_{hv} = PL_{hv}$. Finally, applying the relaxation lemma the proof is ended. □

Remark 1: The results for classical discrete-time TS models $x_{\kappa+} = A_h x_\kappa + B_h u_\kappa$ in (Tanaka and Wang, 2001) are included in those of Theorem 1. To see that, for (10) consider $E_v = I$, which results in

$$\begin{bmatrix} -P & (*) \\ PA_h - F_h C_h & -P \end{bmatrix} < 0. \tag{11}$$

Remark 2: The approach presented in Theorem 1 provides an LMI solution to the observer design problem, considering multiples matrices C and avoiding several matrix equalities; for instance, in (Wang et al., 2012) the equality $TE + NC = I$ has to be solved for a common matrix C, therefore the problem becomes almost impossible for different output matrices.

In what follows, a new observer is proposed. This new structure is based on the fact that Finsler's Lemma, in a sense, decouples the observer gains and the Lyapunov function. Moreover, the following approach uses a non-quadratic Lyapunov function thus it allows adding extra slack variables.

3.2 Non-quadratic case

The new observer structure for the discrete-time TS descriptor model (1) is

$$\begin{aligned} E_v \hat{x}_{\kappa+} &= A_h \hat{x}_\kappa + B_h u + H_h^{-1} L_{hv} (y_\kappa - \hat{y}_\kappa) \\ \hat{y}_\kappa &= C_h x_\kappa, \end{aligned} \tag{12}$$

where H_h and L_{hv} are the observer gains to be determined.

The estimation error $e_\kappa = x_\kappa - \hat{x}_\kappa$ and its dynamics are:

$$\begin{bmatrix} A_h - H_h^{-1} L_{hv} C_h & -E_v \end{bmatrix} \begin{bmatrix} e_\kappa \\ e_{\kappa+} \end{bmatrix} = 0. \tag{13}$$

Consider the following non-quadratic Lyapunov function

$$V(e_\kappa) = e_\kappa^T P_h e_\kappa, \quad P_h = \sum_{i=1}^{r} h_i(z(\kappa)) P_i, \quad P_i = P_i^T > 0 \tag{14}$$

The variation of the non-quadratic Lyapunov function (14) along the estimation error is

$$\Delta V(e_\kappa) = e_{\kappa+}^T P_{h+} e_{\kappa+} - e_\kappa^T P_h e_\kappa = \begin{bmatrix} e_\kappa \\ e_{\kappa+} \end{bmatrix}^T \begin{bmatrix} -P_h & 0 \\ 0 & P_{h+} \end{bmatrix} \begin{bmatrix} e_\kappa \\ e_{\kappa+} \end{bmatrix} < 0. \tag{15}$$

Through Finsler's Lemma, inequality (15) under constraint (13) result in

$$\begin{bmatrix} -P_h & 0 \\ 0 & P_{h+} \end{bmatrix} + \begin{bmatrix} M \\ N \end{bmatrix} \begin{bmatrix} A_h - H_h^{-1} L_{hv} C_h & -E_v \end{bmatrix} + (*) < 0. \tag{16}$$

The following result can be stated.

Theorem 2: The estimation error dynamics in (13) are asymptotically stable if there exist matrices $P_j = P_j^T > 0$, H_j, and F_{jk}, for $j \in \{1, \ldots, r\}$, $k \in \{1, \ldots, r_e\}$ such that the LMI conditions (2) are satisfied with

$$\Upsilon_{ijl}^k = \begin{bmatrix} -P_j & (*) \\ H_j A_i - L_{jk} C_i & -H_j E_k - E_k^T H_j^T + P_l \end{bmatrix}, \tag{17}$$

for $l \in \{1, \ldots, r\}$.

Proof: From (16) and assigning $M = 0$ and $N = H_h$ yields

$$\begin{bmatrix} -P_h & (*) \\ H_h A_h - L_{hv} C_h & -H_h E_v - E_v^T H_h^T + P_{h+} \end{bmatrix} < 0, \tag{18}$$

or

$$\Upsilon_{hhh+}^v = \sum_{i=1}^{r} \sum_{j=1}^{r} \sum_{l=1}^{r} \sum_{k=1}^{r_e} h_i(z(\kappa)) h_j(z(\kappa)) h_l(z(\kappa+1)) v_k(z(\kappa)) \cdot \begin{bmatrix} -P_j & (*) \\ H_j A_i - L_{jk} C_i & -H_j E_k - E_k^T H_j^T + P_l \end{bmatrix} < 0.$$

Through the relaxation lemma the proof is concluded. □

Remark 3: Theorem 2 includes Theorem 1. This can be easily shown, by considering $P_h = H_h = P$.

4. EXAMPLES

Some examples are included to illustrate the effectiveness of the proposed method.

4.1 Example 1

Consider a discrete-time TS descriptor model as in (1) with $r = r_e = 2$, $A_1 = \begin{bmatrix} -1.3 & 0.3 \\ -1.6 & 0.5 \end{bmatrix}$, $A_2 = \begin{bmatrix} -0.6 & -0.1 \\ -1 & -1 \end{bmatrix}$, $C_1 = \begin{bmatrix} 0 \\ 1.4 \end{bmatrix}^T$, $C_2 = \begin{bmatrix} 0 \\ 0.6 \end{bmatrix}^T$, $E_1 = \begin{bmatrix} 1 & -0.1 \\ -0.4 & 0.8 \end{bmatrix}$, and $E_2 = \begin{bmatrix} 0.8 & -0.1 \\ -0.4 & 1.6 \end{bmatrix}$. The MFs are defined as follows: $v_{1\kappa} = \dfrac{\sin(x_{2\kappa})/x_{2\kappa} + 0.4278}{0.5722}$, $v_{2\kappa} = 1 - v_{1\kappa}$, $h_{1\kappa} = \dfrac{x_{2\kappa}^2}{4}$, and $h_{2\kappa} = 1 - h_{1\kappa}$. The MFs hold the convex-sum property on the compact set $\Delta = \{|x_{2\kappa}| \le 2\}$.

Employing Theorem 2, the following matrices were obtained:

$$P_1 = \begin{bmatrix} 76.07 & -48.95 \\ -48.95 & 36.33 \end{bmatrix}, \quad P_2 = \begin{bmatrix} 56.18 & -33.37 \\ -33.37 & 26.70 \end{bmatrix},$$

$$H_1 = \begin{bmatrix} 56.66 & -31.01 \\ -24.49 & 32.54 \end{bmatrix}, \quad H_2 = \begin{bmatrix} 53.98 & -19.98 \\ -24.11 & 23.51 \end{bmatrix},$$

$$L_{11} = \begin{bmatrix} -11.85 & -1.26 \end{bmatrix}^T, \quad L_{12} = \begin{bmatrix} -10.63 & -1.87 \end{bmatrix}^T,$$

$$L_{21} = \begin{bmatrix} 15.64 & -40.27 \end{bmatrix}^T, \quad L_{22} = \begin{bmatrix} 12.12 & -38.04 \end{bmatrix}^T.$$

Simulation results with initial conditions $x(0) = \begin{bmatrix} 0.7 & -0.7 \end{bmatrix}^T$ and $\hat{x}(0) = \begin{bmatrix} 0 & 0 \end{bmatrix}^T$ are presented in Figure 1.

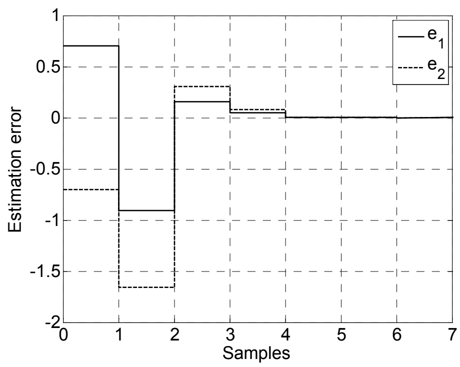

Fig. 1 Estimation error in Example 1.

Note that the conditions in Theorem 1 are infeasible for this system. The next example illustrates Remark 3.

4.2 Example 2

The following example shows the superior performance of the non-quadratic approach, see Remark 3.

Consider a discrete-time TS descriptor model as in (1) with $r = r_e = 2$, $A_1 = \begin{bmatrix} -1 & 1+a \\ -1.5 & 0.5 \end{bmatrix}$, $A_2 = \begin{bmatrix} -1 & 1-a \\ -1.5 & 0.5 \end{bmatrix}$, $C_1 = \begin{bmatrix} 0 \\ 1-b \end{bmatrix}^T$, $C_2 = \begin{bmatrix} 0 \\ 1+b \end{bmatrix}^T$, $E_1 = \begin{bmatrix} 0.9 & 0.1+a \\ -0.4-b & 1.1 \end{bmatrix}$, and $E_2 = \begin{bmatrix} 0.9 & 0.1-a \\ -0.4+b & 1.1 \end{bmatrix}$. The parameters are defined as $a \in [-1,1]$ and $b \in [-1,1]$.

Figure 2 shows feasible regions for the proposed approaches. Results obtained via Theorem 1 are represented by (O) and the ones via Theorem 2 by ($\times$).

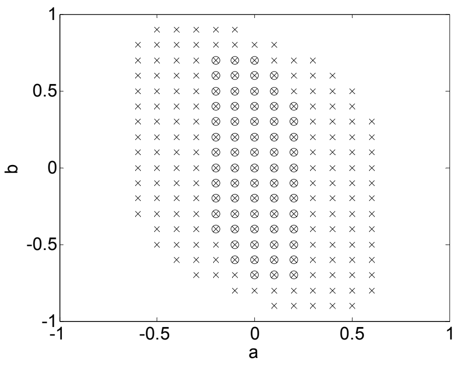

Fig. 2. Feasibility regions in Example 2.

Figure 2 shows that conditions in Theorem 2 outperform the ones in Theorem 1 as it was stated in Remark 3.

It is important to stress that examples here presented are nonlinear descriptor systems with a nonlinear output, which in TS representation relies in different output matrices; from this point of view the approaches on this work outperform the ones presented in previous literature. This work is based on strict LMIs which can be solved via convex optimization techniques.

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