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Implementation of first order algebraic estimators for numerical filtering and derivation applications

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Abstract:
This paper investigates the use of algebraic estimators for numerical filtering and derivation applications. After giving some explanations for the choice of the estimator order, we focus on the order one. The frequency and time responses are compared with standard filtering and derivation methods, including the Kalman filter. These estimators are finally implemented and tested with real sensor signals. Results show that for quite equivalent performances, Kalman filter is less time consuming, while the first order algebraic filter is easier to implement without *a priori* knowledge on the signal.

1. INTRODUCTION

Numerical estimation of noisy signal is a major challenge in the field of real time applications. Numerical filters are needed to extract the signal from noisy data; this demand is even higher in case of derivation, which tends to amplify the high frequency components of the signal, thus increasing the effect of the noise.

Some solutions are well known in the literature [1]. On the one hand, Finite Impulse Response (FIR) filters (for instance a moving average or the Gaussian filter) are commonly used due to the simplicity of implementation and of their inherent stability [2]. They are only based on the last measured data on a finite time window. However, specific filtering requirements may raise the window length, and thus the computational effort [1]. On the other hand, recursive filters, or Infinite Impulse Response (IIR) filters use the previous values of the estimation. They can reach sharper cut-off characteristics, but on the cost of more tedious settings. Among them, the Kalman filter [3] introduces an evolution model to improve the estimation, thus requiring information on the noise properties.

A new family of signal filters and derivators has been introduced in the last decade: the algebraic estimators [4] [5]. These estimators enable filtering and differentiation by integration methods introduced by Lanczos in [6], they have already been implemented in different applications (for instance [7, 8]). In [9], the authors predict the frequency response from the window length. This estimation method has then been generalized using Jacobi polynomials in [5, 10]. In [11], the noise error contribution of the derivative estimation is investigated.

The purpose of this paper is to investigate the benefits and the limits of the algebraic estimators for real time filtering and derivation of signals. In the following, section 2 introduces the algebraic estimators and gives a geometrical interpretation for the order one, sections 3 and 4 study the effect of the estimator's order for filtering and derivation applications. These estimators are then compared to standard filtering and derivation methods, and implemented on real signals in section 5.

2. STUDY OF THE ALGEBRAIC ESTIMATORS

2.1 Introduction to the algebraic estimators

The algebraic estimators are based on the approximation p of a signal $X(t)$ at time t_0 by its truncated Taylor expansion on a time window $]t_0 - T, t_0]$, where T is the window length. For instance, the Taylor development truncated at the first order is:

$$p(t - t_0) = a_0 + a_1(t - t_0) \tag{1}$$

Per definition, a_0 and a_1 are respectively the estimation of the signal and of its first derivative at time t_0.

The order of the estimator corresponds to the order of the Taylor development. Likewise, the order n estimator can estimate the signal, and its n first derivatives. The algebraic estimators can thus be implemented for both signal filtering and derivation.

The terms of the Taylor development can be identified by means of algebraic methods, based on the Laplace transforms. The computation algorithm of the algebraic estimators is intensively explained by Mboup et al in [4].

For example, we give here the equations for the first order algebraic filter (a_0 in eq. 1):

$$a_0 = \frac{2}{T^2}\int_0^T (2T - 3\tau)X(\tau)\,\mathrm{d}\tau \tag{2}$$

and for the first order algebraic derivator (a_1 in eq. 1):

$$a1 = \frac{6}{T^3}\int_0^T (T - 2\tau)X(\tau)\,\mathrm{d}\tau \tag{3}$$

The definition of the algebraic estimators, based on operational calculus, may be difficult to interpret directly for a non specialist. To easier the implementation of these estimators, we seek a more intuitive understanding of their properties.That is why we looked for a geometrical interpretation for the first order algebraic estimators. The following gives another demonstration for the equations 2 and 3, on the basis of simple geometrical considerations on the signals.

2.2 Interpretation for the order one derivator

Let X be a noisy signal observed on a time window $[t_0 - T, t_0]$. In the following, we are seeking an approximation $\hat{X}$ of the first derivative of X at time $t = t_0 - \frac{T}{2}$.

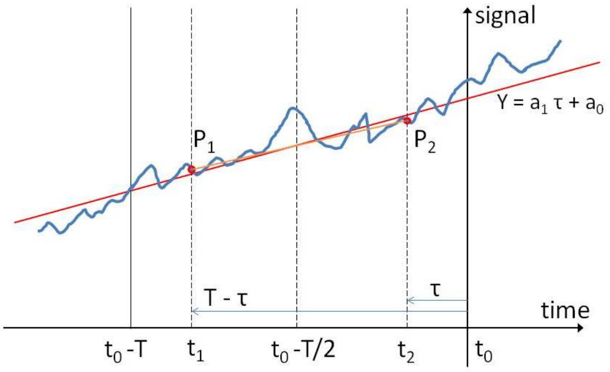

Fig. 1. Principle of the derivative estimation

Let $P_1(t_1)$ and $P_2(t_2)$ be two points of X such as $t_1 \in [t_0 - T, t_0 - \frac{T}{2}[$, $t_2 \in]t_0 - \frac{T}{2}, t_0]$, and $t_1 - (t_0 - T) = t_0 - t_2$ (see fig. 1). Let's call $\tau = t_0 - t_2$. The slope $a(\tau)$ of the segment $[P_1P_2]$ is given by:

$$a(\tau) = \frac{X(\tau) - X(T - \tau)}{T - 2\tau} \tag{4}$$

The first derivative of X at time $t = t_0 - \frac{T}{2}$ can be estimated as the mean value of the slope $a(\tau)$ when τ varies from $t_0 - \frac{T}{2}$ to t_0. The slope is more sensitive to high frequency noise when the points P_1 and P_2 are close to the middle of the window. We should then weight the calculation of the mean in order to promote the furthest points. Different weighting functions have been explored, and the following one allows to retrieve eq. 3:

$$w(\tau) = (\frac{T}{2} - \tau)^2$$

Let a_1 be the weighted average value of the slope:

$$a_1 = \frac{1}{\int_0^{\frac{T}{2}} w(\tau)\,\mathrm{d}\tau}\int_0^{\frac{T}{2}} a(\tau)w(\tau)\,\mathrm{d}\tau \tag{5}$$

Combining equations 4 and 5, we obtain:

$$a_1 = \frac{6}{T^3}\left(\int_0^{\frac{T}{2}} X(\tau)(T - 2\tau)\,\mathrm{d}\tau - \int_0^{\frac{T}{2}} X(T - \tau)(T - 2\tau)\,\mathrm{d}\tau\right)$$

With the variable change $\tau' = T - \tau$, we get:

$$a_1 = \frac{6}{T^3}\left(\int_0^{\frac{T}{2}} X(\tau)(T - 2\tau)\,\mathrm{d}\tau - \int_T^{\frac{T}{2}} X(\tau')(T - 2\tau')\,\mathrm{d}\tau'\right)$$

Which leads to the same formulation as in equation 3. We have then shown that the first order algebraic derivator is an estimation of the derivative of the signal at time $t_0 - \frac{T}{2}$.

2.3 Interpretation for the order one filter

We are now seeking to estimate the signal at time t_0. For this purpose, we firstly estimate the signal with a simple calculation of the mean $\langle X \rangle$ of the signal on the time window. This estimation is delayed by $\frac{T}{2}$.

$$\langle X \rangle = \frac{1}{T}\int_0^T X(\tau)\mathrm{d}\tau$$

The delay is then compensated thanks to the estimated of the derivative a_1 calculated in 2.2:

$$a_0 = \langle X \rangle + \frac{T}{2}a_1 \tag{6}$$

We obtain:

$$a_0 = \frac{1}{T}\int_0^T X(\tau)\mathrm{d}\tau + \frac{3}{T^2}\int_0^T X(\tau)(T - 2\tau)\,\mathrm{d}\tau$$

Here again, we get the same formulation as in eq. 2.

2.4 Conclusion on the geometrical interpretation

We have demonstrated that this filter gives actually the estimation of the signal at time $t_0 - \frac{T}{2}$ via of standard moving average with delay compensation. According to eq. 6, this compensation is based on the estimation a_1 of the first derivative of the signal at time $t_0 - \frac{T}{2}$. If the first derivative changes during the interval $[t_0 - \frac{T}{2}, t_0]$, the delay compensation is faulty. Consequently, this filter tends to lose accuracy and to generate distortion in the signal when the signal second derivative is high.

In order to reduce the distortion, a solution would consist in introducing a time delay δ in the estimator. This reduces the effect of the delay compensation. The purpose is to find an optimal compromise between delay and accuracy. The delay compensation becomes $(\frac{T}{2} - \delta)a_1$. Such an approach has been studied in [5] and [10]. A major advantage for real time applications is to have a steady and known delay. However, a fine tuning of this parameter would require precise *a priori* knowledge on the signal.

2.5 Numerical implementation

The equations 2 and 3 can be written on the form $\hat{X} = \int_0^T \alpha(\tau)X(\tau)\mathrm{d}\tau$. The implementation of the estimators consists in an integration. This can be computed with a classical numerical integration method, for instance using the trapezoid rule.

We obtain the typical expression for a finite impulse response (FIR) filter: $\hat{X} = \sum_{i=0}^{N} \alpha_{N-i} X_i$, where X_i are the measured samples, $\hat{X}$ is the estimate of X (or of its derivative), $N+1$ the number of samples in the time window, and α_i the filter coefficients.

In case of equally spaced samples (constant sampling frequency), these coefficients can be computed *a priori* for a given window length, thus reducing drastically the real-time computational effort. In this case, the implementation consists only in buffering the $N+1$ last samples and multiplying them by the steady filter coefficients α_i.

3. ALGEBRAIC ESTIMATORS FOR FILTERING APPLICATIONS

This section compares the performances of the algebraic filters from order 0 to 3.

3.1 Study on the time domain

In this section, we investigate the algebraic filters in the time domain. Fig. 2 shows the response of the algebraic filters from order 0 to 3 for a sinusoidal signal (50Hz) coupled with Gaussian noise ($\sigma = 0.1$). For this study, the sampling rate of the signals is 10 KHz. The window length of the filters is set to 2ms (i.e. 20 samples).

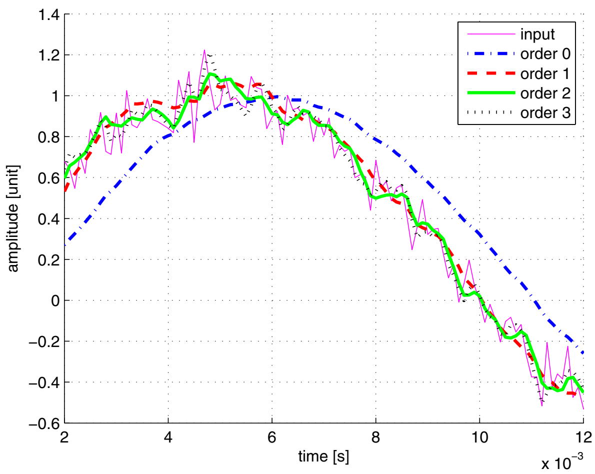

Fig. 2. Response of algebraic filters to a noisy sinusoidal signal

The 0 order filter has a delay of 1ms (the half on the window length), as expected for a symmetrical window FIR filter. For the higher orders, the response seems delay-free.

To compare the noise attenuation, we look at the Signal-to-Noise Ratio (SNR), defined by the ratio between the mean power of the signal (P_{signal}) and the noise variance (P_{noise}). For a FIR filter, the response of a Gaussian white noise is a Gaussian noise, whose variance (or mean power) is given by:

$$P_{noise} = \sigma^2 \sum_{i=0}^{N} \alpha_i^2 \qquad (7)$$

where α_i are the $N+1$ filter coefficients and σ the noise standard deviation [5]. For a noisy sinusoidal signal of amplitude A, the SNR of the filter response is thus given by:

$$\text{SNR} = \frac{P_{signal}}{P_{noise}} = \frac{A^2}{2\sigma^2 \sum_{i=0}^{N} \alpha_i^2} \qquad (8)$$

Table 1 compares the SNR of the response of a noisy sinusoid, with $A = 1$ and $\sigma = 0.1$, for the algebraic filters of orders 0 to 3.

The unfiltered signal SNR is equal to 50 ($= \frac{A^2}{2\sigma^2}$).

Table 1. SNR of filtered noisy sinusoids for algebraic filters

Algebraic filter order	0	1	2	3
response SNR	1025	266	124	74

For the 0 order, the noise is almost totally filtered (compared to the input signal, the SNR is improved by a factor 20). For the order 1, the noise is well filtered (SNR is improved by factor 5), while the remaining noise amplitude increases for the orders 2 and 3.

3.2 Study in the frequency domain

Fig. 3 shows the transfer function of algebraic filters from order 0 to 3, with a window length $T = 2$ ms.

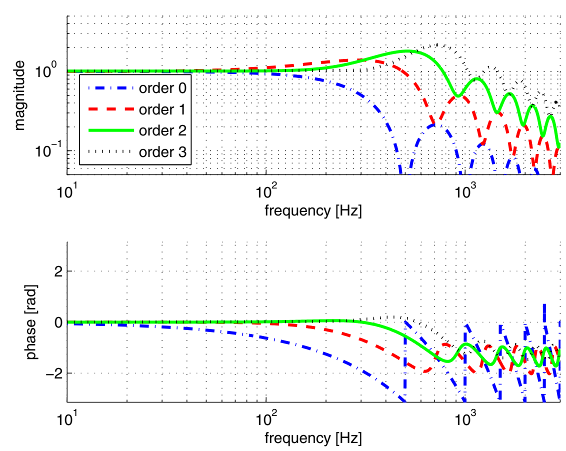

Fig. 3. Frequency response of the algebraic filters

It confirms the observation made in the time domain. These filters are low-pass filters, whose cut-off frequencies f_c increase with the order of the filter. We note that f_c can be approximated according to the filter order n and T the window length T by:

$$f_c \approx \frac{n+1}{2T} \qquad (9)$$

As expected, the 0 order filter (in blue) bode diagram is very close to a standard moving average.

For the orders greater than 0, the phase lift is close to 0 at law frequency. It confirms that the filters do not induce delay in the low frequency components of the signal. The magnitude reaches a peak at a frequency slightly under f_c. The amplitude of this peak increases with the order of the filter (first order: 1.3, second order: 1.9, third order:2.1).

3.3 Conclusion on the algebraic filters

Algebraic filters are a very simple and efficient method for low-pass filtering. The only parameter, the window length, defines the filter cut-off frequency.

While the 0 order filter is a standard moving average, the filter with higher order are delay-free filters. Moreover, these filters amplify a frequency range under the cut-off frequency. The amplitude of this overshoot increases with the order of the filter.

Increment the order of the filter results thus in increasing the cut-off frequency, but also the signal distortion due to the overshot, and increasing the computational effort. This limits the interest for filters of order 2 or more. Consequently, the first order filter is more suitable. The implementation effort only consists in adapting the window length to the desired cut-off frequency, without any other tuning parameter.

4. ALGEBRAIC ESTIMATORS FOR SIGNAL DERIVATION

As well as the signal filtering, the algebraic estimators can be used to estimate the first derivative of a signal. The section compares the algebraic derivators from order 1 to 3. The sampling frequency is still 10 KHz and the window length 2 ms.

4.1 Study of the algebraic derivators in the time domain

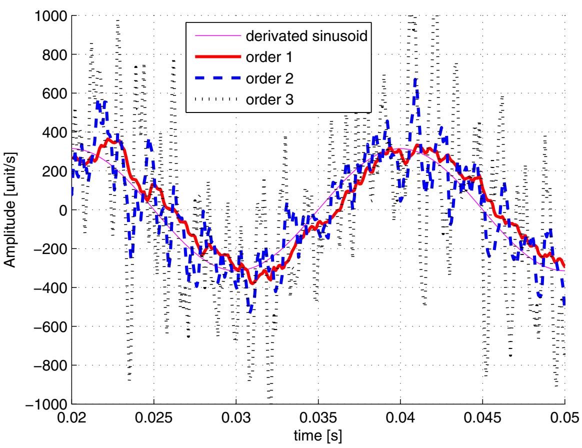

Fig. 4. Response of algebraic derivators to a noisy sinusoid

The response of these derivators to a noisy sinusoidal signal (50 Hz, $\sigma = 0.1$, as in section 3.1) is displayed in Fig. 4. The derivative of the noiseless sinusoid is plotted also (in magenta). While the order 1 derivator (in red) gives an accurate estimation of the derivative, but with a steady delay of 1 ms, the order 2 derivator response (in blue) remains noisy. This is even worst for the order 3.

The SNR of the response can be calculated via equation 8, considering an amplitude $A = 2\pi f_{sinus}$, where f_{sinus} is the input sinusoid frequency. The results are given is table 2. They confirm the major deterioration of the noise attenuation for the orders higher than 1.

Table 2. SNR of noisy sinusoid responses for algebraic derivators

Algebraic derivator order	0	1	2	3
derivative SNR	n.a.	35	2.3	0.4

Enlarging the size of the window would increase the filtering effect (by reducing the cut-off frequency), but it would also affect the accuracy of the derivation (the 50 Hz sinusoid would be filtered too). It is then difficult to build an accurate derivator of order 2 or 3 for this application.

4.2 Study of the algebraic derivators in the frequency domain

The algebraic derivators achieve at the same time a signal derivation and a noise filtering. In order to compare the different derivators, we define the frequency response of the estimator by comparing its frequency to those of a perfect derivator. It means, a perfect derivator would have a magnitude equal to 1 (no signal attenuation) and a phase lift equal to 0 (no delay) on the whole frequency spectrum.

To do so, a step signal (whose derivative is an impulse), is used as input signal. The Fast Fourier Transform (FFT) of the output is divided by the FFT of an impulse signal (considered as perfect derivative), and the result is displayed in a Bode diagram (see Fig. 5). The window length is still $T = 2$ ms.

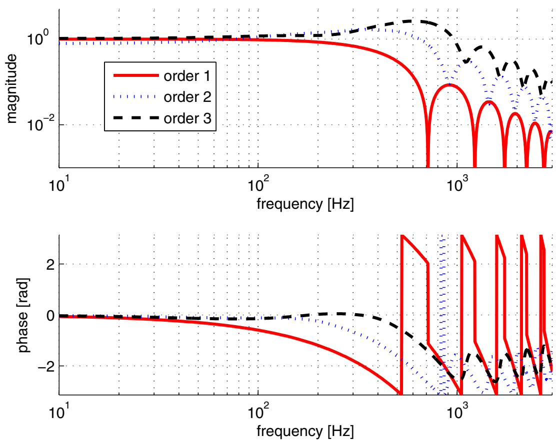

Fig. 5. Frequency response of the algebraic derivator

It confirms that the first order derivator has a steady delay (phase lift proportional to the frequency). The order 2 and 3 derivators are theoretically delay-free (no phase lift at law frequency). However, the attenuation of high frequency noise is not sufficient to compensate the noise amplification due to the derivation.

In the following, we will thus concentrate on the first order algebraic estimators (filter and derivator) to understand their properties and compare their performances with usual estimation methods.

5. COMPARISON A ALGEBRAIC ESTIMATORS WITH USUAL ESTIMATION METHODS

In this section, the first order algebraic estimator is compared to usual FIR and recursive filters. The moving average, the Gaussian filter and the Kalman filter have been selected. The Kalman filter implemented in this comparison is described in the following part.

5.1 Implementation of the Kalman filter

The Kalman filter is well known in the field of numerical signal estimation [3], we will thus not intensively recall its principle. It can be used for denoising and derivation. In this use case, the state vector X, composed of the estimates of the signal $\hat{x}$ and of its first derivative $\hat{\dot{x}}$, can be estimated using the following simple evolution model:

$$\begin{cases} \hat{x}_{i+1} = \hat{x}_i + dt\hat{\dot{x}}_i \\ \hat{\dot{x}}_{i+1} = \hat{\dot{x}}_i \end{cases} \tag{10}$$

The Kalman filter parameters are set while assigning values to the measurement noise matrix R and model noise matrix Q.

R is the estimated value of the variance of the measurement noise.

To define Q, the model error variance is estimated thanks to the first truncated term in 10. The error for $\hat{\dot{x}}_{i+1}$ is of the order of $dt\hat{\ddot{x}}$, and the error for $\hat{x}_{i+1}$ is of the order of $\frac{dt^2}{2}\hat{\ddot{x}}$ (which will be neglected if dt is small). We obtain:

$$Q = \begin{pmatrix} 0 & 0 \\ 0 & \text{Var}(\hat{\ddot{x}})dt \end{pmatrix} \tag{11}$$

To define the variance of $\hat{\ddot{x}}$, we propose the following method: (1) Looking at the physical property, we define the upper limit f_{max} of the signal frequency range, on which the effect of the filtering should be minimal (it means that the desired filter cut-off frequency should be above f_{max}).

(2) In the next step, we consider the sinusoidal signal $y(t) = \sin(2\pi f_{max}t)$, and tune Q so as to minimize the effect of the filter on y. As $\ddot{y}(t) = -(2\pi f_{max})^2 y(t)$, and $\text{Var}(\sin(t)) = \frac{1}{2}$, the variance of $\ddot{y}$ can be easily calculated.

We obtain: $Q = \begin{pmatrix} 0 & 0 \\ 0 & \frac{(2\pi f_{max})^2 dt}{2} \end{pmatrix}$

Remark : the filter cut-off frequency depends on both Q and R; however, its actual value should be slightly above the frequency f_{max}.

As the matrix R and Q are time-invariant, the Kalman gain matrix K_i and the error variance matrix P_i converge to a steady state. This steady state gain, noted K, can be computed *a priori*, thus reducing drastically the computational effort in case of a real time application. So, the Kalman recursive algorithm is simplified: (1) prediction phase:

$$\begin{cases} \hat{x}_i^- = \hat{x}_{i-1} + \text{dt}\hat{\dot{x}}_{i-1} \\ \hat{\dot{x}}_i^- = \hat{\dot{x}}_{i-1} \end{cases} \tag{12}$$

(2) update phase:

$$\begin{cases} \hat{x}_i = \hat{x}_i^- + k_1(x_i - \hat{x}_i^-) \\ \hat{\dot{x}}_i = \hat{\dot{x}}_i^- + k_2(x_i - \hat{x}_i^-) \end{cases} \tag{13}$$

where k_1 and k_2 are the fixed gains of K, which can be computed *a priori* via a recursive method.

5.2 Computational effort

In case of a real time embedded application, another interesting filter characteristic is the computational effort. The number of elementary operation needed to implement the filter can be evaluated. The effort during the initialization phase (for instance the coefficient or gain computation) is generally not relevant and is hence not taken into account.

The moving average with a window length N is given by:

$$\hat{x}_i = \hat{x}_{i-1} + \frac{1}{N}(x_i - x_{i-N+1}) \tag{14}$$

where $\hat{x}_i$ is the estimate of the signal x.

For FIR filters, like the algebraic or the Gaussian filters, the estimate is given by :

$$\hat{x} = \sum_{i=0}^{N} \alpha_{N-i} x_i \tag{15}$$

where α_i are the filter weighting coefficients.

The equations 12, 13, 14 and 15 lead us to the table 3, which compares the number of elementary operations needed to implement each of these filters.

Table 3. Comparison of the computational effort

	Moving avg	Algebraic filter	Kalman
addition	2	$N-1$	4
multiplication	1	N	3

The computational effort for a fixed-gain Kalman filter is very low (7 operations for both the filtering and the derivation). The effort for the algebraic filter increases proportionally to the number of samples in the window. For a small window, the effort stays comparable to the Kalman filter. However, when the window size increases, the algebraic filter becomes heavier.

5.3 Comparison in the frequency domain

We are here comparing the previously cited filtering methods (see Fig. 6). The parameters have been set to have a similar cut-off frequency (around 500 Hz).

For the Kalman filter, the parameter f_{max} is set to 400 Hz, and the matrix R is set to 0.01 ($\sigma_{noise} = 0.1$). The window length for the algebraic estimators (filter and derivator) is 2.5 ms, for the Gaussian filter: 1.4 ms and for the moving average: 1 ms.

Like the algebraic filter, the Kalman filter presents a magnitude overrun (but slightly less pronounced). The phase lift at low frequency is *zero* for both, which indicates

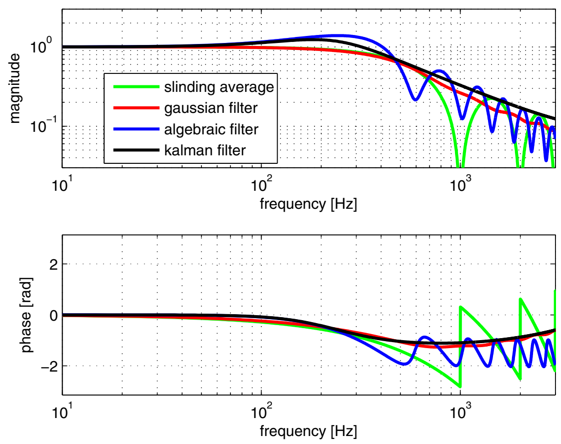

Fig. 6. Frequency response of the different filters

a delay free response. This is not the case for the moving window and the Gaussian filter.

Concerning the derivation (Fig. 7), the Kalman and algebraic derivators (with the same parameters) have a very similar frequency response.

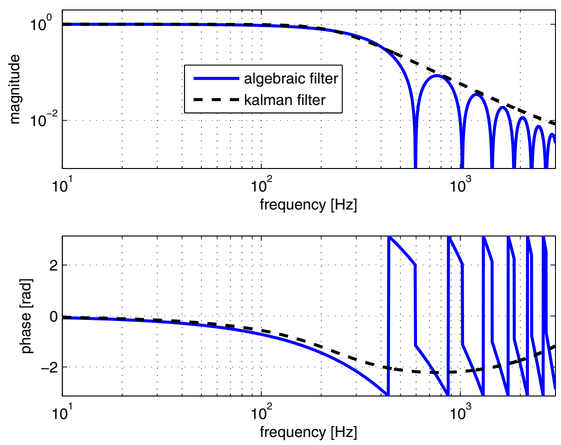

Fig. 7. Frequency response of the algebraic and Kalman derivators

5.4 Comparison in the time domain

Fig. 8 provides the response of the filter and derivator to a noisy sinusoid (50 Hz, $\sigma_{noise} = 0.1$). The moving average and the Gaussian filter have very similar responses. The noise is well filtered, but with a delay of 1 ms. The algebraic and the Kalman filters seem to be also very similar. No delay is observed, but the signal is slightly overestimated (+6% for both filters).

For the derivative estimation (the noiseless sinusoid derivative is plotted in magenta), here again the Kalman and algebraic are very close. The delay is approximately 12 ms.

A quantitative comparison of the noise attenuation is given by the SNR of the response of the noisy sinus. For the FIR filters (moving window, Gaussian and algebraic), the SNR computation is the same than in section 3. For the

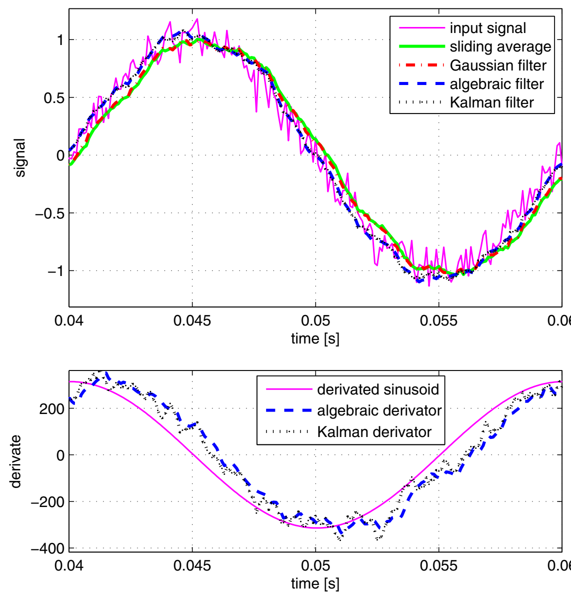

Fig. 8. Response to a noisy sinus

Kalman filter, the estimate of the remaining noise variance is estimated by the diagonal terms of the steady error variance matrix. The resulting SNR are compared is table 4 (Note that the unfiltered input signal SNR is 50).

Table 4. SNR of noisy sinusoid responses for different estimators

Filter type	Moving avg	Gaussian	Algebraic	Kalman
filtering	500	484	315	262
derivation	n.a.	n.a.	60	11

For all filters, the SNR shows a very good noise attenuation. Even better for the moving average and the Gaussian filter, but on the cost of a delay. For the derivators, the noise attenuation is slightly better for the algebraic estimator than for the Kalman filter. This result should however be interpreted carefully due to the different SNR calculation methods for the two estimators.

5.5 Implementation on real signal

The wheel speed belongs to the most relevant signals in the field of vehicle control. This signal and its derivative are important inputs for algorithms of many advance driving assistance systems, such as the ESC (Electronic Stability Control) and the ABS (Anti-lock braking system).

The wheel speed is measured via inductive sensors fixed in front of a rotating impulse wheel (toothed wheel or multipole magnet ring). The lack of precision of these measurements induces noise in the signal. Signal estimation and filtering derivation are consequently generally required upstream of the algorithm.

This section compares the previously studied filtering and derivation methods applied on a wheel speed signal. The signals have been acquired on a test vehicle.

To apply the algebraic filter, the first step is to choose the window length. Fig. 9 compares the result of different window length on the filter response. The best compromise between noise attenuation and signal delay is reached with a window lenght between 200 ms and 300 ms.

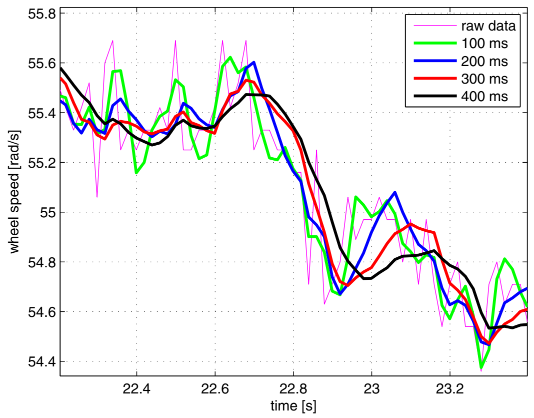

Fig. 9. Effect of the window length on the first order algebraic filter

For the fixed gain Kalman filter, the first step consists in estimating the measurement noise (here, $\sigma_{noise} \simeq 0.2 rad/s$). The optimal gains can be chosen by varying the parameter f_{max} (see Fig. 10). The best compromise is reached with $f_{max} = 0.3$ Hz.

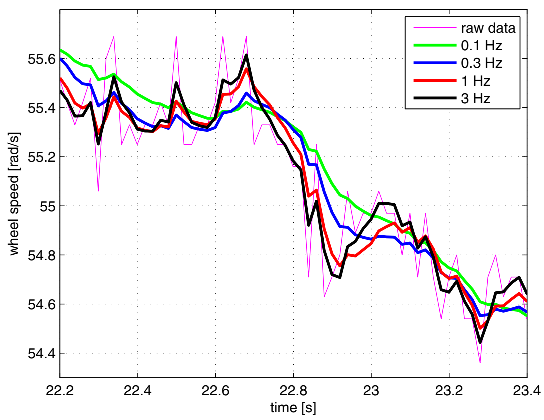

Fig. 10. Effect of the parameter f_{max} on the Kalman filter

We can know compare the 2 estimators on this signal (Fig. 11). The 2 methods give comparable results for the signal filtering, but the algebraic filter seems more reactive for the derivation. This may be improved by tuning the Kalman parameters. However, it exemplifies the difficulty to choose the optimal Kalman gains for real world application, where the noise properties may vary.

6. CONCLUSION

The algebraic estimators are powerful FIR filters, which can be used for low-pass filtering and signal derivation. For both applications, the first order estimator offers the more suitable compromise between noise attenuation and signal distortion. The comparison with a fixed-gain Kalman filter shows very similar performances. Despite a slightly higher computational effort than the Kalman

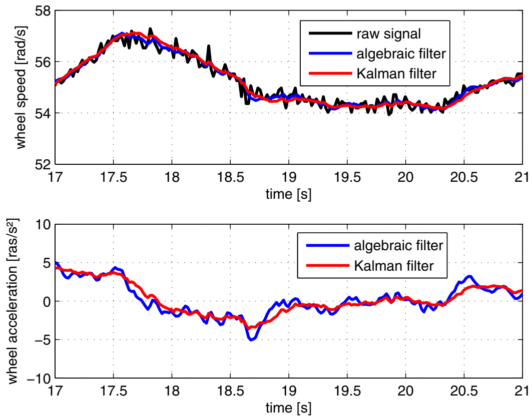

Fig. 11. Estimation and derivation of wheel speed signal

filter, the main advantage of this filter is the simplicity of the implementation: only one parameter defines the filter cut-off frequency, while the Kalman implementation requires *a priori* knowledge of the noise variance and the signal dynamics. Moreover, the filter properties can be adapted in real time by varying the window length (and thus recomputing the weighting parameter), what would be more tedious for adaptating Kalman gain.

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