Preprints of the 19th World Congress
The International Federation of Automatic Control
Cape Town, South Africa. August 24-29, 2014

Fault Diagnosis Based on Robust Observer for Descriptor-LPV Systems with Unmeasurable Scheduling Functions ⋆

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Abstract: This paper design a method for fault detection and isolation based on observers for systems modelled as Descriptor-Linear Parameter Varying (D-LPV) with Unmeasurable Scheduling Functions (USF). The first contribution of the paper is deal with the USF problem by transforming the into an uncertain D-LPV system with an estimated scheduling parameter. As a second contribution, a robust LPV observer is designed with H_∞ performance to supply a robust state estimation against uncertainties provided by USF. Sufficient conditions to guarantee the robustness and convergence are obtained via linear matrix inequalities (LMIs). Finally, an observer bank based on robust H_∞ observers is used to generate residuals and perform sensor fault detection and isolation. A numerical example demonstrates the effectiveness of our methods.

1. INTRODUCTION

The majority of model-based FDI schemes rely on linear time-invariant (LTI) models. Unfortunately, in the real world physical systems present a nonlinear behaviour. However, a FDI scheme for nonlinear systems is more complicated and difficult than for linear systems. An attractive alternative to represent nonlinear systems is through Descriptor-Linear Parameter Varying models (D-LPV). D-LPV, also known as multi-models systems, are mathematical models that are able to exactly represent or to approximate to an arbitrary degree of accuracy a large class of nonlinear systems in a compact set of the descriptor linear models (D-LTI). D-LTI systems are obtained by the decomposition of the operating space of a nonlinear system into a finite number of operating zones. The behavior of the system in each zone is represented by a local linear model [Ghorbel et al., 2012]. The linear models are interpolated by scheduling functions which determine the proportion of which model is active. The typical approach considers scheduling functions as depending on a measurable parameter (as the input or the output of the system). Nevertheless, in many applications the scheduling parameter is unmeasurable (as the state of the system). Models which depend on an unmeasurable scheduling parameter (USP) cover a wide class of nonlinear systems compared to models with a measurable scheduling parameter [Bergsten et al., 2002, Yoneyama, 2009, Theilliol and Aberkane, 2011]. On the other hand, descriptor systems, also known as singular systems, are mathematical models with the property of integrate static (algebraic) and dynamical (ordinary differential) equations in the same model. This property improves the capacity of describing a large class of physical systems. Descriptor systems have many important applications, e.g. aircraft modelling [Masubuchi et al., 2004], composition estimation in distillation columns [Aguilera-González et al., 2013], observer design for waste-water treatment plants[Nagy-Kiss et al., 2011], analysis of electrical systems [Duan, 2010], among others. Nevertheless, few results on robust fault diagnosis for D-LPV systems have been reported [Habib et al., 2012a, Boulkroune et al., 2013]. So the study of such problems is of both practical and theoretical importance.

This paper addresses the problem of design a suitable robust LPV observer for fault diagnosis purpose for D-LPV systems with unmeasurable scheduling functions. In order to deal with the unmeasurable parameter, the D-LPV system with USF is transformed into an uncertain system with an estimated scheduling parameter. H_∞ criterion is employed to guarantee flexibility and robustness performance, even in the presence of model-reality differences, disturbances and, the uncertainty provided by the unmeasurable scheduling parameter. Finally, based on the robust LPV observer, a generalized observer scheme bank (GOS) is built to perform robust fault detection and isolation.

The paper is composed as follows: some preliminary results in finding the uncertain state-space error and the problem formulation are presented in Section 2. Section 3 is dedicated to the design of the robust residual generator.

⋆ This work is supported by CONACyT (Consejo Nacional de Ciencia y Tecnología), Mexico, and the Ministére des Affaires Etrangéres, France. The support is gratefully acknowledged.

Section 4 is dedicated to the synthesis of fault diagnosis based on the residual generator. A numerical example is presented in Section 5. Section 6 is dedicated to some conclusions and future works.

Notations: The notations used in this article are standard. For a matrix $A \in \mathbb{R}^{m\times n}$, A^T, A^{-1} and $A^\dagger$ denote its transpose, inverse and pseudoinverse respectively. He$\{A\}$ is a shorthand notation for $A + A^T$. ρ_i and $\hat{\rho}_j$ are shorthand notations of $\rho_i(x(t))$ and $\rho_j(\hat{x}(t))$, respectively. The symbol $\star$ denotes the transposed element in the symmetric positions of a matrix.

2. PRELIMINARIES AND PROBLEM STATEMENT

Consider a continuous D-LPV system affected by unknown inputs such as:

$$\begin{aligned} E\dot{x}(t) &= \sum_{i=1}^{h} \rho_i \left[A_i x(t) + B_i u(t) + B_d d(t)\right] \\ y(t) &= Cx(t), \end{aligned} \tag{1}$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, $d \in \mathbb{R}^q$, and $y \in \mathbb{R}^p$ are the state vector, the control input, the disturbances, and the measured vector respectively. A_i, B_i, B_d, and C are constant matrices of appropriate dimensions. E is a singular matrix with $\mathsf{rank}(E) = r \le n$, h is the number of models. In addition, ρ_i are scheduling functions which depend on the state $x(t)$. The scheduling functions of the h sub-models satisfy the following convex set sum property:

$$\forall i \in [1, 2, ..., h]\,,\ \rho_i(x(t)) \ge 0,\ \sum_{i=1}^{h} \rho_i(x(t)) = 1,\ \forall t. \tag{2}$$

To carry out an appropriate state estimation, the following assumptions are considered:

Assumption 1. [Vermeiren et al., 2012] The D-LPV system(1) is admissible. It means that there exists a Lyapunov function

$$V(x(t)) = x^T(t)E^T P x(t), \tag{3}$$

where $E^T P = P^T E \ge 0$, and whose derivative $\dot{V}(x(t)) = \dot{x}^T(t)E^T P x(t) + x^T E^T P \dot{x}(t)$, is negative.

Assumption 2. [Habib et al., 2012b, Aguilera-González et al., 2013] The D-LPV system (1) is R-observable:

$$\mathsf{rank} \begin{bmatrix} sE - A_i \\ C \end{bmatrix} = n,\ \ \forall i \in [1, 2, ...h]\,. \tag{4}$$

Assumption 3. [Habib et al., 2012b, Aguilera-González et al., 2013] The D-LPV system (1) is I-observable:

$$\mathsf{rank} \begin{bmatrix} E & A_i \\ 0 & E \\ 0 & C \end{bmatrix} = n + \mathsf{rank}\, E,\ \forall i \in [1, 2, ..., h]. \tag{5}$$

The following lemmas will be useful to prove our main results.

Lemma 1. [López-Estrada et al., 2013] The descriptor LPV system (1) is said to be stable with H_∞ performance if there exists a scalar $\gamma > 0$, and a matrix $X = X^T > 0$ such that:

$$\begin{gathered} E^T X = X^T E > 0 \\ \begin{bmatrix} \mathsf{He}\{A_i^T X\} & XB_i & C^T \\ \star & -\gamma^2 I & 0 \\ \star & \star & -I \end{bmatrix} \le 0. \end{gathered}$$

Lemma 2. [Yang et al., 2005] Let M, H and, Φ be real matrices of appropriate dimensions, with matrix $\Gamma(t)$ satisfying $\Gamma^T(t)\Gamma(t) \le I$, then for a given system

$$M + H\Gamma\Phi + \Phi^T\Gamma^T H^T < 0$$

the following property is verified, if and only if there exists a positive scalar $\epsilon > 0$ such that

$$M + \epsilon\Phi^T\Phi + \frac{1}{\epsilon}HH^T < 0$$

or equivalently

$$\begin{bmatrix} M & H & \epsilon\Phi^T \\ H^T & -\epsilon I & 0 \\ \epsilon\Phi & 0 & -\epsilon I \end{bmatrix} < 0.$$

For an admissible and R/I observable system (1), the following observer is proposed:

$$\begin{aligned} \dot{z}(t) &= \sum_{i=1}^{h} \rho_i(\hat{x}(t)) \left[N_i z(t) + G_i u(t) + L_i y(t)\right] \\ \hat{x}(t) &= z(t) + T_2 y(t) \end{aligned} \tag{6}$$

where $z(t)$ represents the estimated vector and $\hat{x}(t)$ the estimated states. N_i, G_i, L_i, and T_2 are unknown gain matrices of appropriate dimensions to be computed. Additionally, an auxiliary normalized residual vector is defined, to perform fault detection and isolation, as

$$r(t) = \parallel W(y(t) - C\hat{x}(t)) \parallel \tag{7}$$

where W is the residual weighting matrix to determine. As expressed by (7), the residual is the difference between the measured output and an estimated output. In practical applications, the residuals are corrupted by the presence of noise, unknown disturbances, and model uncertainties. Hence, the main objective is to synthesize the gain matrices for the observer (6) to guarantee the convergence of the state estimation error and to generate residual which present robustness against the unmeasurable scheduling parameters ρ_i and disturbances $d(t)$.

3. OBSERVER SYNTHESIS

In this section a methodology to deal with the unmeasurable scheduling problem by transforming the system (1) into an uncertain system (21), is presented. To guarantee the convergence and robust performance, even in the presence of uncertainties, H_∞ criterion is applied to the uncertain system.

Based on (1) and (6), the state-space error e(t) is

$$\begin{aligned} e(t) &= x(t) - \hat{x}(t) \\ e(t) &= (I - T_2 C)x(t) - z(t), \end{aligned}$$

assuming that there exists a $T_1 \in \mathbb{R}^{n\times n}$ matrix as

$$I - T_2 C = T_1 E. \tag{8}$$

By considering (8), the error becomes

$$e(t) = T_1 E x(t) - z(t). \tag{9}$$

The error equation is given by

$$\dot{e}(t) = T_1 E\dot{x}(t) - \dot{z}(t)$$

$$\dot{e}(t) = \sum_{i=1}^{h} \rho_i T_1 \left[A_i x(t) + B_i u(t) + B_d d(t)\right] - \sum_{i=1}^{h} \hat{\rho}_i \left[N_i z(t) + G_i u(t) + L_i y(t)\right] \quad (10)$$

$$\dot{e}(t) = \sum_{i=1}^{h} \rho_i T_1 \left[A_i x(t) + B_i u(t) + B_d d(t)\right] - \sum_{i=1}^{h} \hat{\rho}_i \left[N_i T_1 E x(t) - N_i e(t) + G_i u(t) + L_i y(t)\right] \quad (11)$$

By considering the convex property of the gain scheduling function (2), the term $-\sum_{i=1}^{h} \rho_i(\hat{x}(t)) N_i T_1 E x(t)$ can be handled as

$$\sum_{i=1}^{h} \rho_i \left[\sum_{j=1}^{h} \left[(\rho_j - \hat{\rho}_j)\right] N_r T_1 E\right] x(t) - \sum_{i=1}^{h} \rho_i N_i T_1 E x(t), \quad (12)$$

Similar procedures are done for the remaining terms of (11) and are not presented here due to space limitations. Based on the previous manipulations, the error equation (11) is rewritten as follows:

$$\begin{aligned}\dot{e}(t) = \quad & \sum_{i=1}^{h} \rho_i(x(t)) \{(T_1 A_i - N_i T_1 E - L_i C)x(t) \\ & +(T_1 B_i - G_i)u(t) + N_i e(t) + T_1 B_d d(t) \\ & + \sum_{j=1}^{h} \left[(\rho_j - \hat{\rho}_j)\right] ((N_j T_1 E - L_j C)x(t) \\ & +G_j u(t) + N_j e(t))\}. \quad (13)\end{aligned}$$

In order to guarantee the convergence to zero of the error dynamics, the following constraints are considered

$$T_1 A_i - L_i C - N_i T_1 E = 0 \quad (14)$$
$$G_i - T_1 B_i = 0 \quad (15)$$
$$T_1 B_d = 0. \quad (16)$$

By considering (14), the following equations are equivalent:

$$N_i = T_1 A_i + K_i C \quad (17)$$
$$K_i = N_i T_2 - L_i \quad (18)$$

A particular solution of matrices T_1 and T_2 is computed as

$$[T_1 \; T_2] = [I_n \; 0] \begin{bmatrix} E & B_d \\ C & 0 \end{bmatrix}^{\dagger}. \quad (19)$$

The error equation becomes

$$\dot{e}(t) = \sum_{i=1}^{h} \rho_i \left[N_i e(t) + \sum_{j=1}^{h} \left[(\rho_j - \hat{\rho}_j)\right] \left[(T_1 A_j)\, x(t) + G_j u(t) - N_j e(t)\right]\right]. \quad (20)$$

In order to construct a residual-state-space error system, the states are extended as $x_e(t) = \left[x(t)^T \; e(t)^T\right]^T$. As a result, by considering (20), (1) and (7), the following uncertain residual state-space error system is obtained

$$\bar{E}\dot{x}_e(t) = \sum_{i=1}^{h} \rho_i \left[\left(\bar{A}_i + \Delta\bar{A}_i\right) x_e(t) + \left(\bar{B}_i + \Delta\bar{B}_i\right) u(t)\right] \quad (21)$$
$$r(t) = \bar{C} x_e(t)$$

with

$$\bar{E} = \begin{bmatrix} E & 0 \\ 0 & I \end{bmatrix},\ \bar{A}_i = \begin{bmatrix} A_i & 0 \\ 0 & N_i \end{bmatrix},\ \bar{B}_i = \begin{bmatrix} B_i \\ 0 \end{bmatrix},\ \bar{C} = [0 \; WC].$$

and, matrices $\Delta\bar{A}$ and $\Delta\bar{B}$ defined by

$$\Delta\bar{A}_i = H_A F_A \Phi_A,\ \Delta\bar{B}_i = H_B F_B \Phi_B,$$

with

$$H_A = \begin{bmatrix} 0 & 0 \\ [T_1 A_1 \; ... \; T_1 A_h] & [N_1 \; ... \; N_h] \end{bmatrix},\ F_A = \begin{bmatrix} F & 0 \\ 0 & F \end{bmatrix},$$

$$\Phi_A = \begin{bmatrix} I_A & 0 \\ 0 & -I_A \end{bmatrix}^T,\ I_A = \left[I_{n_1} \; \ldots \; I_{n_h}\right]^T,$$

$$H_B = \begin{bmatrix} 0 \\ [G_1 \; ... \; G_h] \end{bmatrix},\ F_B = [\; F],\ \Phi_B = I_B,$$

$$I_B = \left[I_{m_1} \; \ldots \; I_{m_h}\right]^T,\ F = \begin{bmatrix} (\rho_1 - \hat{\rho}_1) & ... & 0 \\ \vdots & \ddots & \vdots \\ 0 & \ldots & (\rho_h - \hat{\rho}_h) \end{bmatrix}.$$

Note that this transformation is possible due to the convex property (2) which implies $F^T(t)F(t) \leq I$. The problem is reduced to design the observer gains to reject the influence of the control input $u(t)$ and to maximize the robustness against the uncertainty provided by the unmeasurable scheduling function. Sufficient conditions to achieve this objective are given through the following theorem:

Theorem 1. There exists a robust state estimation observer (6) for the D-LPV system (1) with H_∞ attenuation level $\gamma > 0$, if there exist scalars $\epsilon_A > 0$, $\epsilon_B > 0$, matrices $X = \begin{bmatrix} P & 0 \\ 0 & Q \end{bmatrix}$ with $P > 0$, $Q = Q^T > 0$, and gain matrices $K_i = Q^{-1}\Xi_i$, $\forall i \in [1, 2, ..., h]$, such that there exists a solution to the following optimization problem:

$$\begin{gathered}\min_{P,\, Q,\, \Xi_i,\, \epsilon_A,\, \epsilon_B} \gamma \\ \text{s.t.} \\ E^T P = P^T E > 0 \\ \Psi_i \leq 0\end{gathered} \quad (22)$$

where Ψ_i is given in (23) (see next page).

Proof. By considering Lemma (1), H_∞ performance is guaranteed if the following LMI hold

$$\bar{E}^T X = X^T \bar{E} > 0 \quad (24)$$

$$\begin{bmatrix} \mathsf{He}\left(\left(\bar{A}_i + \Delta\bar{A}_i\right)^T X\right) & X\left(\bar{B}_i + \Delta\bar{B}_i\right) & \bar{C}^T \\ \star & -\gamma^2 I & 0 \\ \star & \star & -I \end{bmatrix} \leq 0. \quad (25)$$

By considering the following

$$M = \begin{bmatrix} \mathsf{He}\left(\bar{A}_i^T X\right) & X\bar{B}_i & \bar{C}^T \\ \star & -\gamma^2 I & 0 \\ \star & \star & -I \end{bmatrix}, \quad (26)$$

The LMI (25) can be rewritten as

$$\left[\begin{matrix}
\mathsf{He}\{A_i^T P\} & 0 & PB_i & 0 & 0 \\
\star & \mathsf{He}\left\{(T_1A_i)^T Q + C\Xi_i\right\} & 0 & [QT_1A_1 \dots QT_1A_h] & [QT_1A_1 + \Xi_1 C \dots QT_1A_h + \Xi_h C] \\
\star & \star & -\gamma^2 I_m & 0 & 0 \\
\star & \star & \star & -\epsilon_A I_{n\times h} & 0 \\
\star & \star & \star & \star & -\epsilon_A I_{n\times h} \\
\star & \star & \star & \star & \star \\
\star & \star & \star & \star & \star \\
\star & \star & \star & \star & \star \\
\star & \star & \star & \star & \star \\
\star & \star & \star & \star & \star
\end{matrix}\right.$$

$$\left.\begin{matrix}
\epsilon_A I_A^T & 0 & 0 & 0 & 0 \\
0 & -\epsilon_A I_A^T & [QG_1 \dots QG_h] & 0 & (WC)^T \\
0 & 0 & 0 & \epsilon_B \Phi_B^T & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
-\epsilon_A I_{n\times h} & 0 & 0 & 0 & 0 \\
\star & -\epsilon_A I_{n\times h} & 0 & 0 & 0 \\
\star & \star & -\epsilon_B I_{m\times h} & 0 & 0 \\
\star & \star & \star & -\epsilon_B I_{m\times h} & 0 \\
\star & \star & \star & \star & -I_p
\end{matrix}\right] \le 0. \qquad (23)$$

$$M + \begin{bmatrix} \mathsf{He}\left(\Delta\bar{A}_i^T X\right) & 0 & 0 \\ \star & 0 & 0 \\ \star & \star & 0 \end{bmatrix} + \begin{bmatrix} 0 & X\Delta\bar{B}_i & 0 \\ \star & 0 & 0 \\ \star & \star & 0 \end{bmatrix} \le 0. \qquad (27)$$

LMI (27) is rewritten in an equivalent form as

$$M + \tilde{H}_A F_A \tilde{\Phi}_A + (\tilde{H}_A F_A \tilde{\Phi}_A)^T + \tilde{H}_B F_B \tilde{\Phi}_B + \left(\tilde{H}_B F_B \tilde{\Phi}_B\right)^T \le 0 \qquad (28)$$

with

$$\tilde{H}_A F_A \tilde{\Phi}_A = \begin{bmatrix} XH_A \\ 0 \\ 0 \end{bmatrix} F_A \,[\Phi_A \;\, 0 \;\, 0]$$

$$\tilde{H}_B F_B \tilde{\Phi}_B = \begin{bmatrix} XH_B \\ 0 \\ 0 \end{bmatrix} F_B \,[0 \;\, \Phi_B \;\, 0]\,.$$

By considering Lemma 2, the previous inequality becomes

$$M + \epsilon_A \hat{\Phi}_A^T \hat{\Phi}_A + \epsilon_B \hat{\Phi}_B^T \hat{\Phi}_B + \frac{1}{\epsilon_A} \hat{H}_A^T \hat{H}_A + \frac{1}{\epsilon_B} \hat{H}_B^T \hat{H}_B \le 0.$$

By applying lemma 2 and the Schur complement the inequality becomes

$$\begin{bmatrix}
\mathsf{He}\left(\bar{A}_i^T X\right) & X\bar{B}_i & \bar{C}^T & XH_A & \epsilon_A \Phi_A^T & XH_B & 0 \\
\star & -\gamma^2 I & 0 & 0 & 0 & 0 & \epsilon_B \Phi_B^T \\
\star & \star & -I & 0 & 0 & 0 & 0 \\
\star & \star & \star & -\epsilon_A I & 0 & 0 & 0 \\
\star & \star & \star & \star & -\epsilon_A I & 0 & 0 \\
\star & \star & \star & \star & \star & -\epsilon_B I & 0 \\
\star & \star & \star & \star & \star & \star & -\epsilon_B I
\end{bmatrix} \le 0. \qquad (29)$$

In order to rearrange column 3 and 7 from LMI (29), the LMI is post and pre-multiplying by

$$\begin{bmatrix}
I & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & I & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & I \\
0 & 0 & I & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & I & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & I & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & I & 0
\end{bmatrix},$$

and its transpose, respectively. The LMI (29) results in

$$\begin{bmatrix}
\mathsf{He}\left(\bar{A}_i^T X\right) & X\bar{B}_i & XH_A & \epsilon_A \Phi_A^T & XH_B & 0 & \bar{C}^T \\
\star & -\gamma^2 I & 0 & 0 & 0 & \epsilon_B \Phi_B^T & 0 \\
\star & \star & -\epsilon_A I & 0 & 0 & 0 & 0 \\
\star & \star & \star & -\epsilon_A I & 0 & 0 & 0 \\
\star & \star & \star & \star & -\epsilon_B I & 0 & 0 \\
\star & \star & \star & \star & \star & -\epsilon_B I & 0 \\
\star & \star & \star & \star & \star & \star & -I
\end{bmatrix} \le 0.$$

Note that by replacing $X = \begin{bmatrix} P & 0 \\ 0 & Q \end{bmatrix} > 0$ and the extended matrices of (21) implies (23). Finally, it is clearly that for given matrices $P > 0$ and $Q = Q^T > 0$, the equality constraint (24) has the equivalent form given in (22). This completes the proof. $\square$

4. SENSOR FAULT DETECTION AND ISOLATION

The fault isolation problem is to determine in which sensor the fault has occurred. For purposes of sensor fault diagnosis, a bank of observers, as in [Frank, 1990], is adopted. Each observer is dedicated to one single fault f_k as proposed in the generalized observer scheme. As result, each residual is sensitive to all but one fault. Hence, for each sensor fault, the descriptor-LPV system corrupted by faults and unknown inputs is described by

$$\begin{aligned} E\dot{x}(t) &= \sum_{i=1}^{h} \rho_i(x(t))\,[A_i x(t) + B_i u(t) + B_d d(t)] \\ y(t) &= C^k x(t) + D_f^k f_k(t) \end{aligned} \qquad (30)$$

Where C^k and D_f^k are the matrix and sensor fault distribution without the k^{th} component. If each derived system (30) satisfies Assumptions (1)-(3), then a bank of k residuals are generated by

$$\dot{z}^k(t) = \sum_{j=1}^{h} \rho_j(\hat{x}(t)) \left[N_j^k z(t) + G_j^k u(t) + L_j^k C^k x(t) \right]$$

$$\hat{x}^k(t) = z^k(t) + T_2 C^k x(t). \tag{31}$$

$$r^k(t) = \| \, W^k(y(t) - C^k \hat{x}^k(t)) \, \| \, . \tag{32}$$

Finally, fault detection and isolation is done by evaluating the r^k residual produced by the k^{th} robust LPV observer. Note that the isolation scheme can only isolate a single fault at the same time. This is based on the fact that the probability for two or more faults to occur at the same time is very small in a real situation [Habib et al., 2012b].

5. SIMULATION EXAMPLE

A numerical example is presented to illustrate the performance of the proposed method. Consider the D-LPV system (1) under disturbances $d(t)$, described by the following gain matrices

$$E = \mathsf{diag}(1,1,0),\; A_1 = \begin{bmatrix} -10 & 5 & 6.5 \\ 2 & -5.5 & -1.25 \\ -9 & 4 & 8.5 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} -10 & 5 & 6.5 \\ 5 & -4 & -1.25 \\ -2 & 4 & 7 \end{bmatrix},\; A_3 = \begin{bmatrix} -8 & 5 & 6.5 \\ 5 & -4 & -1.25 \\ -5 & 4 & 6 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 0 \\ 1 \\ 0.5 \end{bmatrix},\; B_2 = \begin{bmatrix} 0 \\ 0.7 \\ 1 \end{bmatrix},\; B_3 = \begin{bmatrix} 0 \\ 0.5 \\ 0.6 \end{bmatrix}$$

$$B_d = \begin{bmatrix} 0 \\ 1 \\ 0.5 \end{bmatrix},\; C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}.$$

The gain scheduling functions which depend on the scheduling parameter $x_1(t)$ are defined as

$$\rho_i(x(t)) = \frac{\mu_i(x_1(t))}{\sum_{i=1}^{3} \mu_i(x_1(t))} \tag{33}$$

$$\mu_1(t) = exp\left[\frac{1}{2} \left(\frac{x_1(t) + 0.4}{0.5} \right)^2 \right]$$

$$\mu_2(t) = exp\left[\frac{1}{2} \left(\frac{x_1(t) - 0.4}{0.1} \right)^2 \right]$$

$$\mu_3(t) = exp\left[\frac{1}{2} \left(\frac{x_1(t) - 1}{0.5} \right)^2 \right].$$

In order to evaluate the observer performance, a first simulation is done by considering the system in the fault-free case. The synthesis of the stable D-LPV observer with H_∞ performance (6) has been achieved with Yalmip toolbox [Lofberg, 2004]. The attenuation level obtained by solving (23) is $\gamma = 2.4214 \times 10^{-4}$ which leads to the following state matrices:

$$N_1 = \begin{bmatrix} -5.78 & 3.37 & 2.76 \\ 6.42 & -5.87 & -4.25 \\ -3.21 & 2.93 & 2.12 \end{bmatrix},\; N_2 = \begin{bmatrix} -5.78 & 3.37 & 2.76 \\ 4.42 & -5.12 & -3.50 \\ -2.21 & 2.56 & 1.75 \end{bmatrix}$$

$$N_3 = \begin{bmatrix} -4.78 & 3.37 & 2.76 \\ 5.92 & -5.12 & -3.00 \\ -2.96 & 2.56 & 1.50 \end{bmatrix},\; L_1 = \begin{bmatrix} -2.10 & 1.50 & 1.74 \\ 2.28 & -2.87 & -1.99 \\ -1.14 & 1.43 & 0.99 \end{bmatrix}$$

$$L_2 = \begin{bmatrix} -2.10 & 1.50 & 1.74 \\ 1.28 & -2.3 & -1.81 \\ -0.64 & 1.15 & 0.90 \end{bmatrix},\; L_3 = \begin{bmatrix} -1.60 & 1.50 & 1.74 \\ 2.03 & -2.19 & -1.43 \\ -1.01 & 1.09 & 0.71 \end{bmatrix}.$$

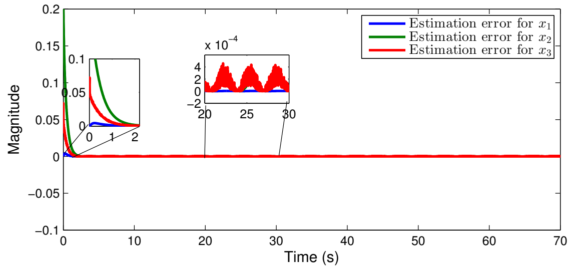

Fig. 1. Square estimation error.

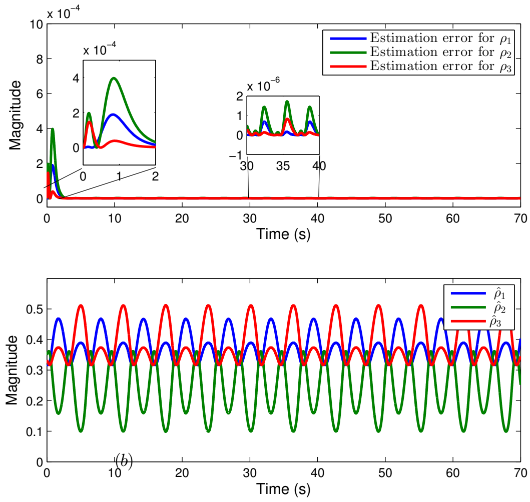

Fig. 2. (a) Scheduling functions estimation error, (b) Evolution of the estimated scheduling functions.

Matrices G_i and T_2 are not displayed here due to space limitations but can be computed by solving (15)-(19). Initial conditions are considered as $x(0) = [0, 1, -1]^T$, $z(0) = [0.1, 0, 0]^T$. A sinusoidal input $u(t) = 5sin(t)$ is applied. The disturbance $d(t)$ included in the process is zero-mean noise with a standard deviation of 0.3. As illustrated in Fig. 1, the observer performs well with an square estimation error close to zero. Fig. 2a shows the estimation error of the gain scheduling functions. Clearly, a good estate-estimation implies also good estimation of the scheduling functions. Fig. 1b display the evolution of the scheduling functions which gives information about the contribution of each model to the global behaviour over time. As can be observed, in both Fig. 1 and Fig. 2a-b, due to the uncertain approach and the H_∞ performance, the observer converges fast and asymptotically. System disturbances and model uncertainties are well attenuated. A second simulation is done by considering $D_f = I_3$ in order to consider one fault for each sensor. To provide useful residuals a generalized bank of observers as detailed in section 4 is constructed. Assumptions 1-3 are verified for all derived systems. By solving (23), the attenuation levels obtained for each observer are $\gamma_1 = 3.5683 \times 10^{-4}$, $\gamma_2 = 3.022 \times 10^{-4}$, and $\gamma_3 = 3.087 \times 10^{-4}$. These small values of

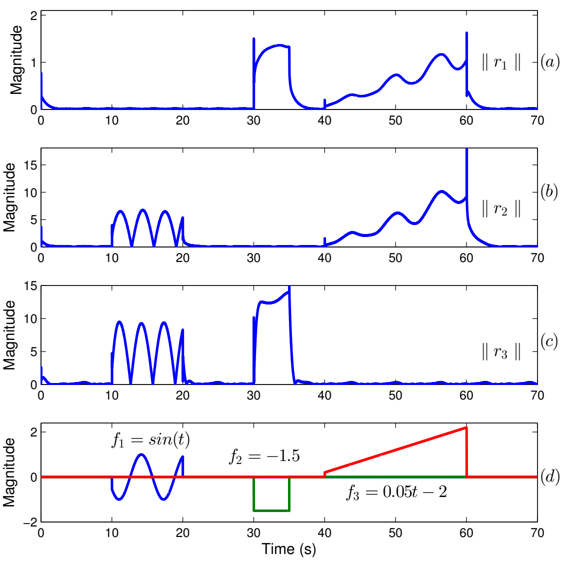

Fig. 3. (a)-(c) Normalized residual vectors $\| r_i \|$, (d) Faults.

the attenuation levels guarantee the objective performance and the robustness against the uncertainty provided by the scheduling functions for all the observers. The faults considered for each sensor are shown in Fig. 3d. The fault which occurred on the first sensor is a sinusoidal fault, the fault on the second sensor is an abrupt fault whereas the fault on the third sensor is an incipient fault. The fault identification is done by comparing the generated residuals with the incidence matrix given in the generalized observer scheme. For example, for the fault occurred in the sensor 1 all the residual have some changes at $t = 10s$ except r_1. Clearly, in this case the fault is identified and isolated in sensor 1. In general, for all cases, the fault detection turns out to be successful.

6. CONCLUSIONS

A robust observer for D-LPV systems with unmeasurable gain scheduling parameter was proposed. To deal with the unmeasurable scheduling problem, the system was transformed into an uncertain system based on the convex property of the scheduling functions. Sufficient conditions to ensure the convergence and H_∞ performance were given in terms of LMIs. A generalized LPV observer scheme was considered to perform the fault detection and isolation problem. In fault-free and faulty cases, the method provides useful residuals despite disturbances and the error given by the unmeasurable scheduling parameter.

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