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Robust Predictive PI Controller Tuning

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Abstract: Two decades ago, a predictive PI controller was introduced for regulation of dead time dominated linear time-invariant first-order processes. The introduction involved a simple tuning rule with consideration on easy implementation and good performance. Later, the predictive PI controller has been improved by having an additional low-order filter in the controller structure against high-order modeling uncertainties. Also, several publications have been released with a detailed analysis on performance and robustness in particular with comparison of predictive and conventional PI(D) controllers. However, there have not been tuning rules with a design parameter for fine-tuning performance and dealing with robustness. In this paper, stability and robustness of a predictive PI without an additional filter for any time-invariant system is considered resulting in tuning rules for an optimal performance with a targeted robustness against model mismatch. Consequently, simple controller tuning rules are given for a first-order plus dead time and integrating plus dead time systems.

Keywords: Robust, predictive, PI controller, tuning, dead time.

1. INTRODUCTION

The Smith predictor by O.J.M Smith (1957) served as a starting point for Hägglund's innovation (1992, 1996) on a predictive PI controller (PPI) for processes with long dead times. Later, robustness of the PPI has been improved by Normey-Rico et. al (1997) and studied by Ingimundarson & Hägglund (2001). Recently, some PPI controller variants and an event-based PPI controller for event-triggered control has been proposed by Airikka (2012) and, also, expansion of the PPI to a PPID controller for integrating dead time dominated systems (Airikka, 2013b).

Stability of a PPI controller was not studied by Hägglund (1992, 1996) originally. A detailed Nyquist stability analysis for the PPI controller was given by Airikka (2013a). The stability analysis applies to any linear time-invariant single-input single-output process expressed by its frequency response. The same frequency response based approach has been used as a basis for robust and optimal PPI controller tuning presented in this paper. A cornerstone for the approach lies on the work done for the PI controller by Åström et. al (1998).

The paper is outlined as follow. Section 2 introduces a PPI controller, section 3 gives insight to PPI controller stability, section 4 introduces existing PPI controller tuning rules and section 5 proposes a new optimal and robust frequency response -based tuning method with simplifications to FOPDT (First-Order Plus Dead Time) and IPDT (Integrator Plus Dead Time) systems. Finally, section 6 concludes the paper.

2. PPI CONTROLLER

A closed loop system with a predictive PI controller (PPI) built on a PI controller is illustrated in figure 1. The PPI controller output, control, $u(t)$ is generated by

$$u(t) = k_p e(t) + k_i \int_0^t e(\tau)d\tau - k_{pred} \int_{t-L}^t u(\tau)d\tau \tag{1}$$

where $e(t)$ is a control error at time t. Process dead time estimate L and predictive gain k_{pred} are PPI controller parameters whereas proportional gain k_p and integral gain k_i are PI controller parameters. Optionally, the integral gain can be given using an integral time t_i for $k_i = k_p / t_i$. The PPI controller (1) has four tuning parameters: k_p, k_i, k_{pred} and L.

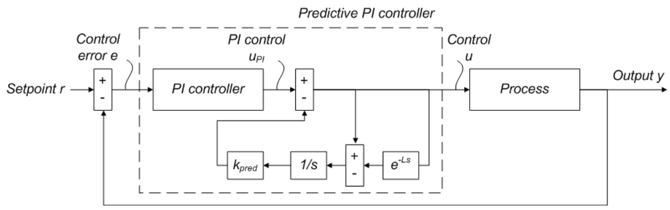

Figure 1. Predictive PI controller (PPI).

In Laplace domain, the PPI controller can be expressed as a series connection of a PI controller $k_{pi}(s) = k_p + k_i / s$ and a prediction filter $f(s)$ (see fig. 2) for allowing prediction with $u(s) = k_{pi}(s) f(s) e(s) = k_{PPI}(s) e(s)$ where

$$f(s) = \frac{s}{s + k_{pred}(1 - e^{-Ls})} \tag{2}$$

The prediction filter includes both predictive gain k_{pred} and the dead time estimate L as design parameters. Figure 3 illustrates the amplitude of the prediction filter for different predictive gains and dead time estimates.

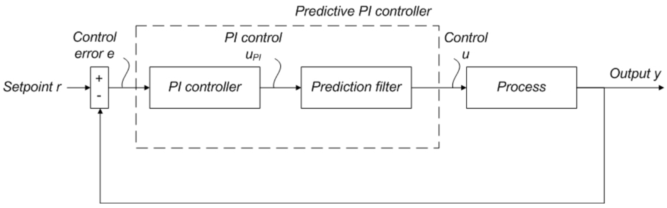

Figure 2. Predictive PI controller (PPI) represented in a series connection of PI controller and prediction filter *f*(*s*).

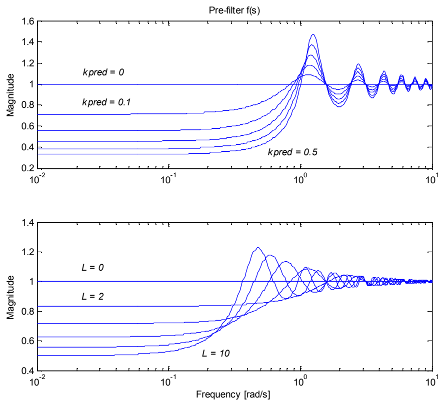

Figure 3. Amplitude responses for a prediction filter *f*(*s*). *Upper*: k_{pred} = 0, 0.1, 0.2, 0.3, 0.4, 0.5 with L = 4. *Lower*: L = 0, 2, 4, 6, 8, 10 with k_{pred} = 0.1.

Compared to the PI controller, the PPI allows larger controller gains resulting in a better setpoint following and load disturbance compensation. Compared to other dead time compensation techniques, the PPI is computationally simple to implement, easier to tune and, most of all, it is capable of regulating integrating processes without requiring any structural modifications like e.g. Smith predictors do.

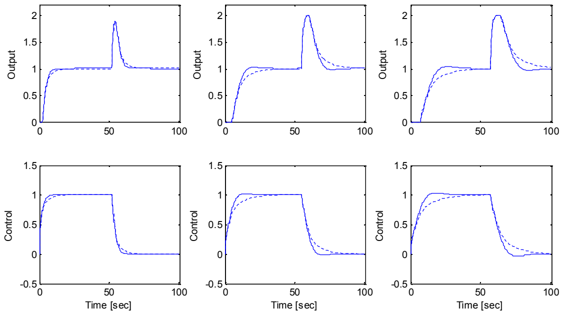

Figure 4. Setpoint and load disturbance responses (upper) with control signals (lower) for PI (dotted) and PPI (solid) controlled FOPDT system when k = 1, T = 1 and L = 2, 5, 7.

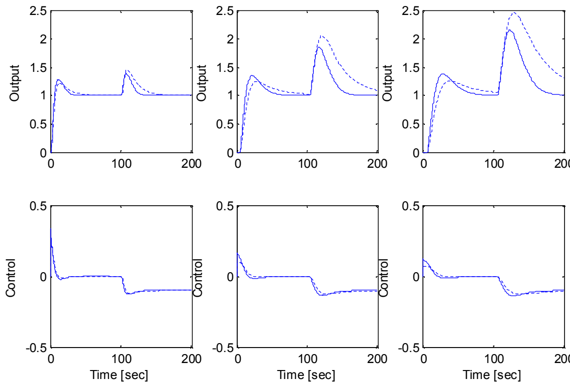

Figure 5. Setpoint and load disturbance responses (upper) with control signals (lower) for PI (dotted) and PPI (solid) controlled IPTD system when k = 1 and L = 2, 5, 7.

3. PPI CONTROLLER STABILITY

Given any linear time-invariant process using its frequency response $g(j\omega)$ with gain $r(\omega)$ and phase $\varphi(\omega)$

$$g(j\omega) = r(\omega)e^{j\varphi(\omega)} \tag{3}$$

and a PPI controller consisting of a PI controller $k_{pi}(j\omega)$ and a prediction filter $f(j\omega)$ with k_{pred} and L

$$k_{pi}(j\omega) = k_p - j\frac{k_i}{\omega} \tag{4}$$

$$f(j\omega) = \frac{j\omega}{j\omega + k_{pred}(1 - e^{-jL\omega})} = r_f(\omega)e^{\varphi_f(\omega)} \tag{5}$$

where $\omega > 0$ is frequency (rad/sec), the Nyquist stability criterion is expressed as

$$1 + l(j\omega) = 1 + g(j\omega)k_{pi}(j\omega)f(j\omega) = 0 \tag{6}$$

with the loop transfer function being $l = gk_{pi}f$. After solving (6) for proportional gain k_p and integral gain k_i, the following equations in terms of frequency ω are obtained

$$k_p(\omega) = -\frac{\cos(\varphi(\omega) + \varphi_f(\omega))}{r(\omega)r_f(\omega)} \tag{7}$$

$$k_i(\omega) = -\frac{\omega \sin(\varphi(\omega) + \varphi_f(\omega))}{r(\omega)r_f(\omega)} \tag{8}$$

Note that the formulas (7-8) are much simpler to compute than those presented in Airikka (2013a). The parameters $k_p(\omega)$ and $k_i(\omega)$ define a closed stability region on (k_p, k_i) plane for the PPI controller as illustrated in figure 6. Any value taken outside the region results in an unstable closed PPI control loop. By setting either $k_{pred} = 0$ or $L = 0$, the prediction filter reduces to a unit gain $f(j\omega)=1$ with $r_f(\omega)=1$ and $\varphi_f(\omega)=0$ and the formulas (7-8) are simplified to

$$k_p(\omega) = -\cos\varphi(\omega)/r(\omega) \quad (9)$$

$$k_i(\omega) = -\omega\sin\varphi(\omega)/r(\omega) \quad (10)$$

representing the stability region for a PI controller as given in Åström et. al (1998). The PI controller stability regions by (9-10) are plotted in figure 6 for comparison (dotted line).

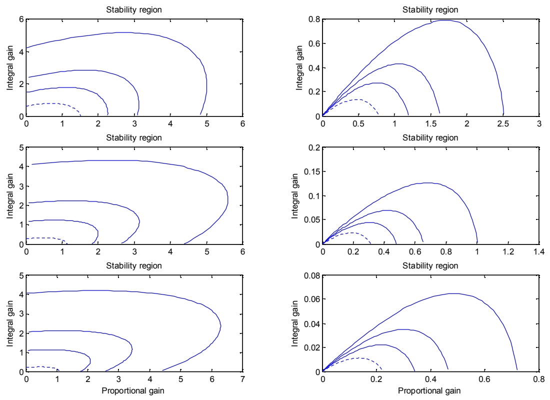

Figure 6. PPI controller stability regions for FOPDT (left) and IPDT (right) systems with predictive gains k_{pred} = 2/*T*, 1/*T* and 0.5/*T* for FOPDT (*k* = 1, *T* = 1, from top to bottom $\tau = L/T$ = 2, 5, 7) and k_{pred} = 2/*L*, 1/*L* and 0.5/*L* for IPDT (*k* = 1, from top to bottom *L* = 2, 5, 7).

A PPI controller stability region (fig. 6) increases with the increased predictive gain k_{pred} and decreases to the stability region of a PI controller (fig. 6, dotted) for $k_{pred}=0$ or $L=0$. All the stability regions shrink as a function of increasing dead time.

4. EXISTING TUNING RULES

In his paper, Hägglund (1992) gave an interesting PPI controller tuning rule for a FOPDT process by suggesting

$$k_p = 1/k,\ t_i = T,\ k_{pred} = 1/t_i \quad (11)$$

The tuning rule is a direct consequence of placing closed loop poles of a PPI controlled FOPDT process to its open loop process poles. The selected criterion makes the practical tuning rather easy but it does not involve a design parameter. Later, Airikka (2013a) proposed a PPI tuning

$$k_p = T/(T_{cl}k),\ t_i = T,\ k_{pred} = 1/T_{cl} \quad (12)$$

where T_{cl} is a design parameter for adjusting the closed loop time constant for setpoint response. The closed loop time constant T_{cl} affects not only the predictive gain k_{pred} but also the proportional gain k_p. Nevertheless, both of the tuning rules apply only to FOPDT systems (18) with a time constant *T*. To the knowledge of the author, there are no other reported PPI controller tuning rules other than above for linear time-invariant processes.

Basically, the dead time compensation using a PPI controller allows tuning of the proportional and integral part like the process was having no dead time. But apparently this holds only if the process model is accurate with no model mismatch. In practise, this assumption must be relaxed to allow model uncertainties and, therefore, robustness and optimality are considered when proposing a new tuning method.

5. ROBUST AND OPTIMAL TUNING

It was Schei (1994) and, later, Åström et. al (1998) who first proposed an appealing PI control design strategy. The objective was to minimise the integrated control error (*IE*) over infinity for a step load disturbance affecting process input. To guarantee robustness against model uncertainties, the maximum sensitivity M_s (1.2 – 2.0) was used as an optimisation constraint as the inverse of the maximum sensitivity M_s is the shortest distance between the critical point (-1,0) and the loop transfer function on Nyquist plane

$$\frac{1}{M_s} = \min_{\omega}\left|1 + l(j\omega)\right| = \min_{\omega} f(\omega) \quad (13)$$

After solving the necessary condition of (13) for the PPI controller's proportional and integral gain, the following is obtained

$$k_p^*(\omega) = -\frac{\cos(\varphi(\omega)+\varphi_f(\omega))}{r(\omega)r_f(\omega)} \quad (14)$$

$$k_i^*(\omega) = k_i(\omega) - \frac{\omega/M_s}{r(\omega)r_f(\omega)} = -\frac{\omega\left(\sin(\varphi(\omega)+\varphi_f(\omega)) + 1/M_s\right)}{r(\omega)r_f(\omega)} \quad (15)$$

For completed and elegant computation, the sufficient conditions $df(\omega)/d\omega = 0$ and $d^2 f(\omega)/d\omega^2 > 0$ could be solved resulting in an implicit equation for ω similar to that reported for a PI controller by Åström et. al (1998). For practicality, here it is proposed to compute the gains (14-15) for $\omega > 0$ up to the frequency for -270° phase. Then, the maximum integral gain $\max_{\omega} k_i^*(\omega)$ with its frequency and the corresponding proportional gain is selected.

Figure 7 illustrates M_s-curves for a FOPDT system (left) with *k* = 1, *T* = 1 and $\tau = L/T$ = 2, 5, 7 for M_s = 1.2, 1.4, 1.6, 1.8 and 2.0 and the same M_s-curves for an IPDT system (right) with *k* = 1 and *L* = 2, 5, 7. Selection of the maximum integral gain out of each M_s-curve satisfies the optimal criterion of min(IE) with a design constraint M_s. For a PI controller, the formulas (14-15) reduce to

$$k_p^*(\omega) = -\cos\varphi(\omega)/r(\omega) \tag{16}$$

$$k_i^*(\omega) = k_i(\omega) - \omega/(M_s r(\omega)) \tag{17}$$

as shown by Åström et. al (1998).

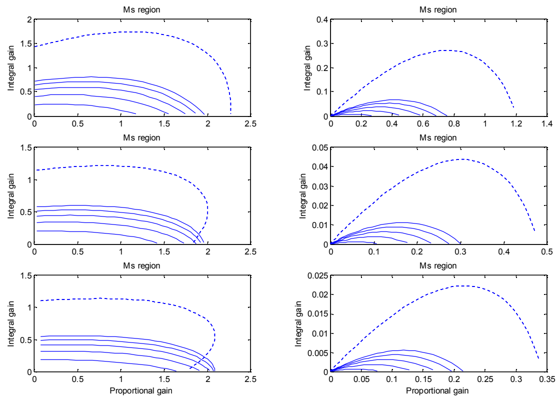

Figure 7. PPI controller's M_s- curves for FOPDT with k_{pred} = 0.5/T (left) and for IPDT with k_{pred} = 0.5/L (right) with the stability boundaries (dotted) when delay L increases from top to bottom.

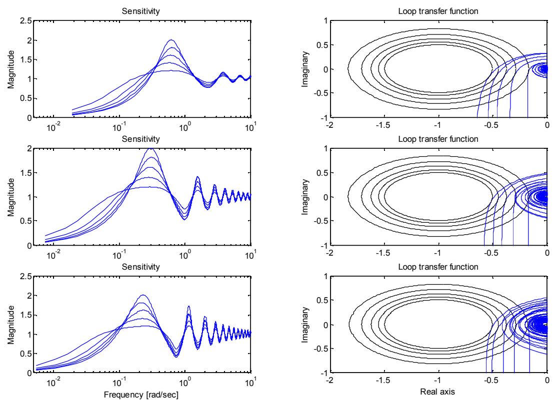

Figure 8. Sensitivity functions (left) and M_s circles with loop transfer function (right) for a PPI controller for FOPDT with k = 1, T = 1 and $\tau = L/T$ = 2, 5, 7 (top to bottom).

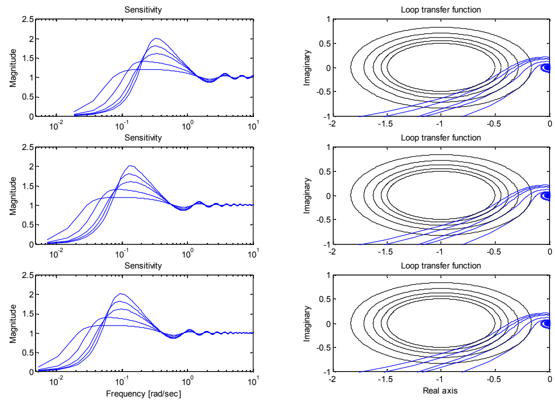

Figure 9. Sensitivity functions (left) and M_s circles with loop transfer function (right) for a PPI controller for IPDT with k = 1 and L = 2, 5, 7 (top to bottom).

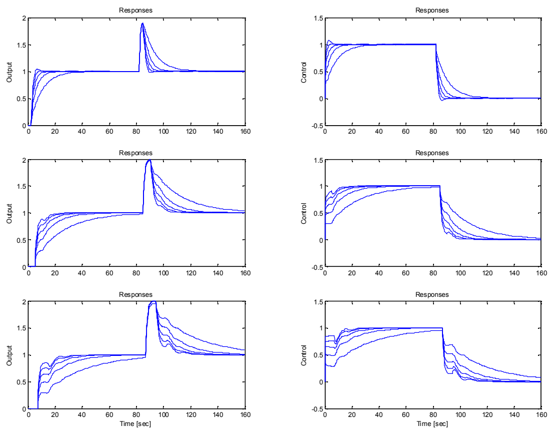

Figure 10. PPI controller setpoint and load disturbance responses (left: output, right: control) for FOPDT with k = 1, T = 1 and $\tau = L/T$ = 2, 5, 7 (top to bottom).

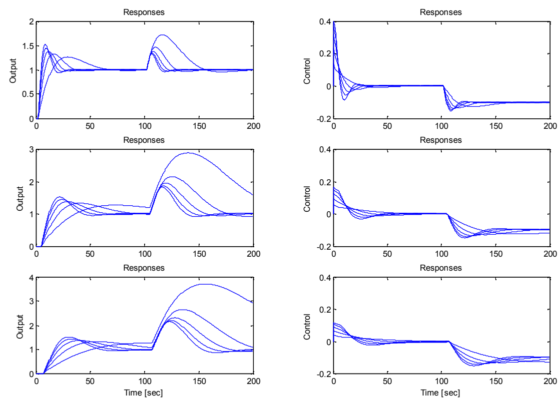

Figure 11. PPI controller setpoint and load disturbance responses (left: output, right: control) for IPDT with k = 1 and L = 2, 5, 7 (top to bottom).

Robust and optimal tuning (14-15) assumes the predictive gain k_{pred} to be given, particularly requiring consideration on how to select the predictive gain. Intuitively, $k_{pred} = 1/(\lambda T)$ is proposed for a FOPDT system and $k_{pred} = 1/(\lambda L)$ for an IPDT system with $\lambda = 0.5 - 5$. However, to come up with easy memorable, yet robust tuning rules, FOPDT and IPDT systems are in particular next considered using (12) with T_{cl} as a design parameter for adjusting both the closed loop speed and robustness to modelling uncertainties.

5.1 FOPDT systems

Consider a FOPDT (First-Order Plus Dead Time) system

$$g(s) = \frac{k}{Ts+1} e^{-Ls}, \; L > 0, k \neq 0 \tag{18}$$

First, using a pole placement design with a pole cancellation (Panagopoulos et. al, 1997) with dead time set to $L = 0$, the following with an addition of $k_{pred} = 1/T_{cl}$ is obtained

$$\begin{aligned} k_p &= T/((L+T_{cl})k) \rightarrow T/T_{cl}k \\ t_i &= T \end{aligned} \tag{19}$$

Note that the tuning rule (19) equals to (12). Next, using the pole placement design without pole cancellation (Panagopoulos et. al, 1997) with $L = 0$, the following with an addition of $k_{pred} = 1/T_{cl}$ is resulted

$$\begin{aligned} k_p &= (2T - T_{cl})/(T_{cl}k) \\ t_i &= 2TT_{cl} - T_{cl}^2 \end{aligned} \tag{20}$$

Third, the improved SIMC method (Skogestad, 2012) suggests the following tuning rules for $L = 0$ and, once again, with an addition of $k_{pred} = 1/T_{cl}$

$$\begin{aligned} k_p &= 1/(T_{cl}k) \\ t_i &= \min(T, 4T_{cl}) \end{aligned} \tag{21}$$

Figures 12-14 show results obtained for a FOPDT system with a normalized dead time ratio $\tau = 2, 5, 7$. The results show PPI controller parameters in upper figures (from left to right: proportional, integral and predictive gain) and performance indexes in lower figures (from left to right: maximum sensitivity, IAE for setpoint, IAE for load disturbance). All the figures have a closed loop time ratio $T_{cl}/L = 0.5 - 3$ as horizontal axis and curves with $\tau = 2, 5, 7$ going from top to bottom. The results show that increasing T_{cl} decreases the maximum sensitivity but increases IAE or both setpoint and load disturbance responses. The fastest performance can be found within $T_{cl} = 0.5L...L$ and more conservative performance by setting $T_{cl} > L$. Note that when using tuning (20), the controller parameter get non-positive for $T_{cl} \geq 2T$ limiting the usability of the tuning rules.

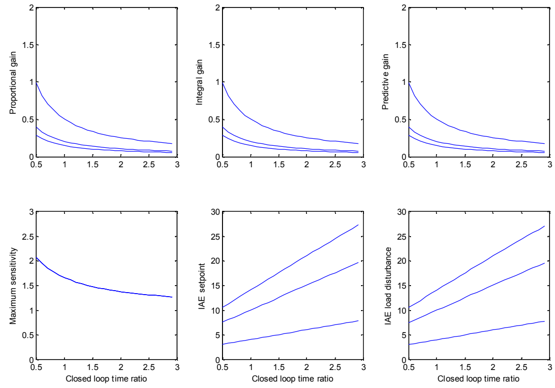

Figure 12. PPI controller performance for FOPDT with pole placement design (19) having pole cancellation for $L = 0$.

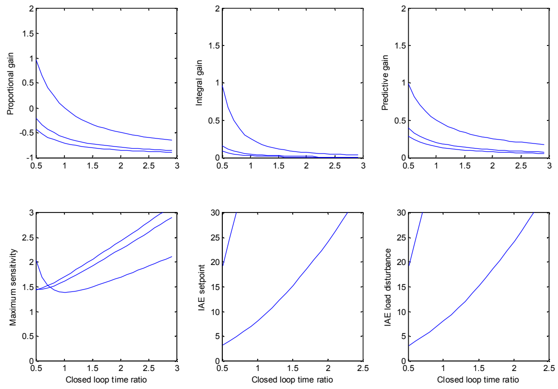

Figure 13. PPI controller performance for FOPDT with pole placement design (20) without pole cancellation for $L = 0$.

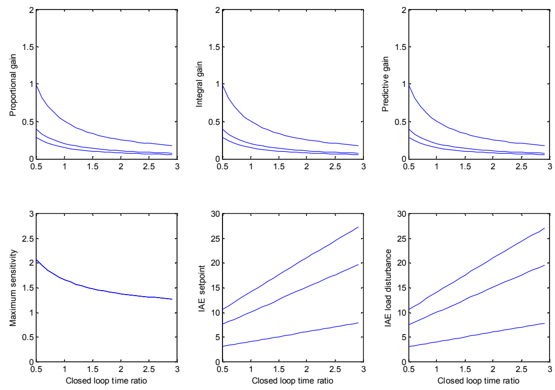

Figure 14. PPI controller performance for FOPDT with improved SIMC method design (21) for $L = 0$.

5.2 IPDT systems

Consider an IPDT system (Integrator Plus Dead Time)

$$g(s) = \frac{k}{s} e^{-Ls}, \; L > 0, k \neq 0 \tag{22}$$

The pole placement design (Panagopoulos et. al, 1997) with an addition of $k_{pred} = 1/T_{cl}$ results in the following tuning rule

$$
\begin{aligned}
k_p &= \frac{L + 2T_{cl}}{k(L + T_{cl})^2} \rightarrow \frac{2}{kT_{cl}} \\
t_i &= L + 2T_{cl} \rightarrow 2T_{cl}
\end{aligned}
\qquad (23)
$$

Second, the improved SIMC method with $L = 0$ suggests

$$
\begin{aligned}
k_p &= 1/(kT_{cl}) \\
t_i &= 4T_{cl}
\end{aligned}
\qquad (24)
$$

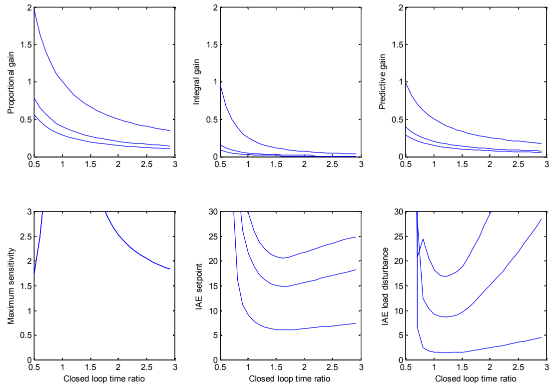

Figure 15. PPI controller performance for IPDT with pole placement design (23) for $L = 0$.

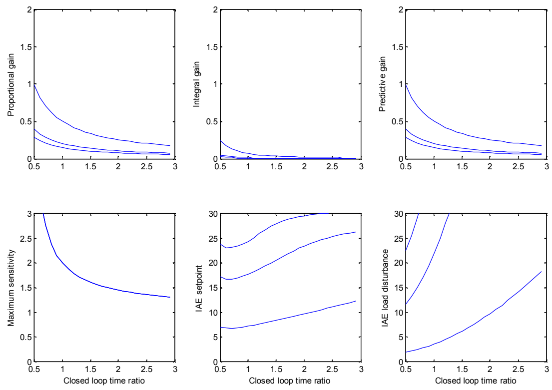

Figure 16. PPI controller performance for IPDT with improved SIMC method design (24) for $L = 0$.

The pole placement based design results in the best IAE performance for $T_{cl} = 1.2L...1.7L$ but at the huge cost of the robustness with the maximum sensitivity being much larger than 3. All in all, the aggressive tuning causes a robustness issue for $T_{cl} \leq 2.5L$. The recommended range is thus only for $T_{cl} > 2.5L$. The SIMC method provides fast but robust tuning for $T_{cl} = L...1.5L$ and more conservative tuning for $T_{cl} > 1.5L$.

6. CONCLUSION

In industrial process control, quite many processes with relatively long dead times can be encountered. Among known dead time compensation methods, a predictive PI (PPI) controller is one of the simplest but yet rather effective. Methods for tuning the PPI controller are still rather few, and, therefore, this paper proposed a new frequency response based PPI controller tuning method where the integrated control error for the step load disturbance is minimised while robustness is secured using a maximum sensitivity as an optimisation constraint. However, as the predictive gain is no involved in optimisation, some guidelines for its selection were also given using a closed loop time constant as a criterion. Finally, simplified tuning rules for FOPDT and IPDT systems were given and compared.

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