

Machine-cell and Part-family Formation in Cellular Manufacturing Using a Two-phase Clustering Algorithm^{*}

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Abstract: This paper presents a two-phase clustering algorithm for machine-cell and part-family formation in the design of cellular manufacturing systems. The proposed algorithm begins with the determination of initial cluster centers via a linear assignment method using the least similar group representatives in its first phase. A fuzzy C-means clustering method is followed in its second phase for part-family and machine-cell formation using the obtained initial cluster centers. The two-phase algorithm can remedy the problem of clustering inconsistency resulting from the fuzzy C-means method with random initializations. Experimental results on many benchmark data sets based on multiple performance criteria substantiate the effectiveness of the proposed algorithm.

Keywords: Cellular manufacturing, linear assignment, fuzzy C-means clustering

1. INTRODUCTION

Cellular manufacturing is a notion for workplace design which can greatly increase productivity and reduce material handling cost (Singh, 1993). One of the important steps in the design of cellular manufacturing system is machine-cell and part-family formation, which seeks to group parts with similar processing features or requirements into part families and bring dissimilar machines together to form machine cells. The machine-cell and part-family formation problem has been widely studied for decades (e.g., see Selim et al. (1998), Ghosh et al. (2011)) and various approaches have been proposed, such as the K-means algorithm (Unler and Gungor, 2009), the linear assignment algorithm (Wang, 2003), the neural network approach (Pandian and Mahapatra, 2009), the genetic approach (Mahdavi et al., 2009), and the fuzzy C-means algorithm (Chu and Hayya, 1991). Due to the complex nature of machine-part features, there may exist cases where the cluster boundaries are not crisp. Therefore, it is desirable to further develop or apply fuzzy clustering algorithms to deal with the problem efficiently and reliably.

The fuzzy C-means algorithm is a well-known fuzzy clustering algorithm which provides a degree of membership for a datum pattern associated with each cluster (Bezdek, 1981). A distinctive advantage of fuzzy C-means clustering is that it can flexibly and robustly handle natural data with vagueness uncertainty. Fuzzy C-means algorithm has been successfully applied in many areas such as image segmentation, speech enhancement, and edge detection (Miyamoto et al., 2008). One major weakness of fuzzy

C-means clustering is its sensitivity to the initial guess. Different initial setting can potentially result in different local optima or different partitions, and can affect the convergence speed as well.

In this paper, a linear assignment method is applied for providing the initial cluster centers for fuzzy C-means clustering. As the linear assignment itself is a clustering algorithm, the present approach can be regarded as a two-phase algorithm. The algorithm is applied for solving the machine-cell and part-family formation problem in cellular manufacturing. In the first phase, a set of the least similar parts or machines are identified as group representatives by means of comparing dissimilarity coefficients. A linear assignment model is then formulated based on the least similar group representatives, which computes an initial C-partition. In the second phase, fuzzy C-means algorithm iteratively converges each partition to a local optima. The two-phase algorithm is evaluated on many benchmark data sets and is demonstrated to be effective and efficient.

The rest of this paper is organized as follows. Some preliminaries on machine-cell and part-family formation problem and fuzzy C-means clustering are discussed in Section II. The proposed two-phase algorithm is described in Section III. Illustrative examples are provided in Section IV. Experimental results on benchmark data sets are provided in Section V. Finally, Section VI concludes the paper.

2. PRELIMINARIES

2.1 Machine-cell and part-family formation

In machine-cell and part-family formation problems, a manufacturing system is represented as a binary machine-

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part incidence matrix $A = \{a_{ij}\} \in \mathbb{R}^{m \times p}$ where p is the number of parts and m is the number of machines. Element $a_{ki} = 1$ denotes machine k processes part i and $a_{ki} = 0$ denotes otherwise. The machine-cell and part-family formation aims at rearranging rows and columns in A to form a block-diagonal matrix. In this paper, the number of machine cells or part families C is assumed to be known in advance.

2.2 Dissimilarity coefficients

A dissimilarity coefficient quantifies the degree of difference between two datum patterns; i.e., between a pair of machines or parts. Various definitions of dissimilarity coefficients are available in the literature (Sarker and Islam, 1999). Some of commonly used dissimilarity measures are presented below.

A typical dissimilarity coefficient between parts i and j in terms of machine processing requirements is defined by the Minkowski metric

$$d_{ij} = \left(\sum_{k=1}^m |a_{ki} - a_{kj}| \right)^{1/r}, \quad (1)$$

where $r > 0$. Specifically, d_{ij} is the Euclidean distance measure when $r = 2$, and d_{ij} is the Manhattan or city block distance measure when $r = 1$. Another measure that can be derived from the Minkowski distance is weighted Minkowski distance measure which can be expressed as

$$d_{ij} = \left(\sum_{k=1}^m w_k |a_{ki} - a_{kj}| \right)^{1/r}, \quad (2)$$

where w_k is a weighting factor on machine k . The weighted Minkowski distance measure can employ a special preference for processing some jobs with special priority on certain machines.

The Bray-Curtis coefficient is another dissimilarity coefficient obtained by entrapping the range d_{ij} of the Manhattan distance measure within $[0 \ 1]$:

$$d_{ij} = \sum_{k=1}^m \frac{|a_{ki} - a_{kj}|}{a_{ki} + a_{kj}}. \quad (3)$$

The Canberra metric coefficient is similar to the Bray-Curtis coefficient in sense that both coefficients lie in the range of $[0 \ 1]$. It is defined as

$$d_{ij} = \frac{1}{m} \sum_{k=1}^m \frac{|a_{ki} - a_{kj}|}{a_{ki} + a_{kj}}. \quad (4)$$

The Hamming distance measure is a binary dissimilarity coefficient which is defined as

$$d_{ij} = \sum_{k=1}^m \delta(a_{ki}, a_{kj}), \quad (5)$$

where

$$\delta = \begin{cases} 1, & a_{ki} = a_{kj}; \\ 0, & a_{ki} \neq a_{kj}. \end{cases}$$

Without loss of generality, the symmetry of dissimilarity coefficients is commonly assumed; i.e., $d_{ij} = d_{ji}$. In addition, a dissimilarity coefficient d_{ij} can be correlated with a normalized similarity coefficient s_{ij} by the relation $d_{ij} = (1 - s_{ij})/s_{ij}$ (Sarker and Islam, 1999).

2.3 Fuzzy C-means clustering

For the convenience of later discussions, we briefly recall fuzzy C-means algorithm (FCM). Let p be the total number of data and m be dimension of data. FCM aims at grouping data into C ($C \in 2, \dots, p - 1$) clusters. It is based on minimization of the following objective function:

$$J_q(U, V) = \sum_{k=1}^p \sum_{i=1}^C u_{ik}^q \|x_k - v_i\|^2, \quad (6)$$

where $q > 1$ is a weighting exponent controlling the amount of clustering fuzzy, u_{ik} is the membership degree of data x_k belonging to cluster i , v_i is the center of the cluster i , U and V are respectively the membership degree matrix and cluster center matrix, and $\|\cdot\|$ is a norm measuring the distance between any data and the center. FCM is carried out through iteratively optimizing the objective function (6) subject to the following constrains

$$\sum_{k=1}^p u_{ik} > 0, \quad \sum_{i=1}^C u_{ik} = 1, \quad 0 \leq u_{ik} \leq 1; \\ i = 1, \dots, C, \quad k = 1, \dots, p. \quad (7)$$

It can be derived (Bezdek, 1981) that the objective function J_q reaches a minimum value for a given cluster center matrix V when u_{ik} is

$$u_{ik} = \sum_{j=1}^C \left(\frac{\|x_k - v_i\|}{\|x_k - v_j\|} \right)^{-\frac{2}{q-1}}, \quad i = 1, \dots, C, k = 1, \dots, p. \quad (8)$$

Accordingly, the cluster center v_i can be updated as follows

$$v_i = \frac{\sum_{k=1}^p u_{ik}^q x_k}{\sum_{k=1}^p u_{ik}^q}, \quad i = 1, \dots, C. \quad (9)$$

In view of (8) and (9), the objective function J_q may converge to a local optima by applying an iterative optimization procedure. The iteration will stop when $\max(|u_{ik}^l - u_{ik}^{l-1}|) \leq \varepsilon$, where $0 < \varepsilon < 1$ is a termination criterion defined by user and l is the iteration step. It is worth pointing out that the solution quality of FCM is highly dependent on the initial cluster center V^0 .

2.4 Performance criteria

The formation quality provided by FCM needs to be evaluated based on certain performance criteria. Specially for the machine-cell and part-family formation problem in cellular manufacturing, there are various performance indexes available in the literature (Agarwal and Sarkis, 1998).

Formation results often contain exceptional elements that indicate discrepancies. Element a_{ij} is an exceptional element if $a_{ij} = 1$ and $\exists k x_{jk} \neq y_{ik}$, where x_{jk} is the binary decision variable defined as $x_{jk} = 1$ if part j is assigned to family k or $x_{jk} = 0$ otherwise, y_{ik} is the binary decision variable defined as $y_{ik} = 1$ if machine i is assigned to machine-cell k or $y_{ik} = 0$ otherwise. The number of exceptional element (EE) can be quantitatively determined by

$$EE = \frac{1}{2} \sum_{k=1}^C \sum_{i=1}^m \sum_{j=1}^p a_{ij} |x_{ik} - y_{jk}|, \quad (10)$$

where C is the number of machine-cells/part-families, m is the number of machines, and p is the number of parts.

The percentage of exceptional elements (PE) is defined as the ratio of EE to the number of unity elements in the incidence matrix (UE):

$$PE = \frac{EE}{UE}, \quad (11)$$

where $UE = \sum_{i=1}^m \sum_{j=1}^p a_{ij}$.

The bond energy (BE) measures the compactness of a permuted matrix:

$$BE = \sum_{i=1}^{m-1} \sum_{j=1}^p a_{ij}a_{i+1,j} + \sum_{i=1}^m \sum_{j=1}^{p-1} a_{ij}a_{i,j+1}. \quad (12)$$

The machine utilization (MU) is defined as the frequency of visits to machines within cells:

$$MU = \frac{UE - EE}{\sum_{k=1}^C m_k p_k}, \quad (13)$$

where m_k and p_k denote the number of machines in cell k and number of parts in family k , respectively.

Group efficiency (GE) is another widely used performance criteria which is defined as

$$GE = \rho \frac{UE - EE}{\sum_{k=1}^C m_k p_k} + (1-\rho) \left(1 - \frac{EE}{mp - \sum_{k=1}^C m_k p_k}\right), \quad (14)$$

where $\rho \in [0, 1]$ is a weighting parameter. When $\rho = 1$, $GE = MU$. Normally $\rho = 0.5$.

3. ALGORITHM DESCRIPTION

3.1 Linear assignment initialization

The essence of the present algorithm lies in the determination of initial cluster center. Several methods have been developed for solving the initialization problem (Kim et al., 2004; Li et al., 2007). As clustering aims at grouping similar data into the same cluster as well separating dissimilar data into different clusters, it is reasonable to conclude that C least similar data belongs to C different partitions. Hence the C least similar data can be identified as C cluster representatives, and each data represents a cluster. A recursive approach to determining cluster centers using dissimilar coefficients can be expressed as

$$\{v_1, v_2\} = \arg \max_{i,j} d_{ij}, \quad (15)$$

$$v_k = \arg \max_{i=1, \dots, p-k+1} \min \{d_{iv_1}, \dots, d_{iv_{C-1}}\}, \quad (16)$$

$$k = 3, \dots, C,$$

where d_{iv_j} denotes the dissimilarity between data x_i and the cluster center v_j . Cheng et al. (2012) showed that the determination of cluster representatives by using (15) and (16) takes a polynomial time of p .

A linear assignment model can be formulated as follows based on the determined cluster representatives:

$$\begin{aligned} \min \quad & \sum_{i=1}^C \sum_{k=1}^p d_{ik} u_{ik} \\ \text{s.t.} \quad & \sum_{k=1}^p u_{ik} > 0, \quad \sum_{i=1}^C u_{ik} = 1, \quad 0 \leq u_{ik} \leq 1; \\ & i = 1, \dots, C, \quad k = 1, \dots, p. \end{aligned} \quad (17)$$

The advantages of the linear assignment model (17) is twofold. First, it is a linear program instead of a linear integer program entailed in some optimization-based clustering approaches (e.g., p -median models (Wang and Roze, 1997)). Second, the number of decision variables is reduced compared with some existing linear program models (pC versus p^2) (Wang, 1999).

3.2 Two-phase clustering algorithm

By applying the linear assignment initialization, an initial cluster center matrix $V^0 = [v_1, v_2, \dots, v_C]^T$ is obtained for the followed fuzzy C-means clustering. The overall clustering algorithm for formation of machine cells and part families in cellular manufacturing can be described as follows:

- (1) Input the clustering number C , the fuzzy exponent q , the maximal iteration number $Loop$, and stopping criteria ε .
- (2) Import the machine-part incidence matrix data $A_{m \times p}$ where $A(i, j) = a_{ij}$.
- (3) Load or compute the dissimilarity coefficients between each pair of data d_{ij} .
- (4) Compute the initial cluster centers using the linear assignment model in Eq. (15) and (16).
- (5) Apply FCM for iteratively minimizing the objective function (6) using the following updating rules

$$u_{ik}^l = \sum_{j=1}^C \left(\frac{\|x_k - v_i^l\|}{\|x_k - v_j^l\|} \right)^{-\frac{2}{q-1}}, \quad i = 1, \dots, C, k = 1, \dots, p, \quad (18)$$

$$v_i^{l+1} = \frac{x_i \sum_{k=1}^p (u_{ik}^l)^q}{\sum_{k=1}^p (u_{ik}^l)^q}, \quad i = 1, \dots, C. \quad (19)$$

where l the current iteration step. The FCM iteration steps are illustrated in Fig. 1.

- (6) Evaluate the clustering results using one or more performance criteria.

4. ILLUSTRATIVE EXAMPLES

Consider an incidence matrix A as presented in Chandrasekharan and Rajagopalan (1987):

$$A = \{a_{ij}\} = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \end{matrix} & \begin{bmatrix} & & & & & & & & & \\ & & 1 & & & & 1 & & & \\ 1 & & & 1 & & 1 & & 1 & & \\ & 1 & & & 1 & & & & 1 & 1 \\ 1 & & & 1 & & 1 & & 1 & & \\ & & 1 & & & & 1 & & & \\ 1 & & & 1 & & & & 1 & & 1 \\ & 1 & & & 1 & & & & 1 & 1 \end{bmatrix} \end{matrix}, \quad (20)$$

where 0 elements are left blank. In this case, there are 8 machines and 10 parts (i.e., $m = 8$ and $p = 10$).

no.	$m \times p$	C	PE	PE*	BE	BE*	MU	MU*	GE	GE*	Reference
1	5×7	2	0.0000	0.0000	14	14	0.8235	0.8235	0.9118	0.9118	King and Nakornchai (1982)
2	5×7	2	0.1250	0.1250	15	15	0.8235	0.8235	0.8256	0.8256	Waghodekar and Sahu (1984)
3	5×7	2	0.1250	0.1250	15	16	0.8235	0.8235	0.8256	0.8256	Waghodekar and Sahu (1984)
4	10×10	3	0.0000	0.0000	21	21	0.7059	0.7059	0.8029	0.8029	Mosier and Taube (1985)
5	7×11	2	0.1304	0.1304	18	18	0.5263	0.5263	0.7247	0.7247	Kusiak and Chow (1987)
6	8×20	3	0.1475	0.1475	81	84	1.0000	1.0000	0.9583	0.9583	Chandrasekharan and Rajagopalan (1986)
7	10×20	4	0.0000	0.0000	68	68	1.0000	1.0000	1.0000	1.0000	Srinivasan et al. (1990)
8	23×20	2	0.1140	0.1140	78	78	0.4280	0.4000	0.6850	0.6667	Kumar et al. (1986)
9	14×24	4	0.0328	0.0328	67	68	0.6860	0.6860	0.8390	0.8390	King (1980)
10	24×40	7	0.0000	0.0000	198	198	1.0000	1.0000	1.0000	1.0000	Chandrasekharan and Rajagopalan (1989)
11	24×40	7	0.0769	0.0769	163	163	0.9160	0.9160	0.9520	0.9520	Chandrasekharan and Rajagopalan (1989)
12	24×40	7	0.1450	0.1527	140	143	0.8550	0.9116	0.9160	0.9409	Chandrasekharan and Rajagopalan (1989)
13	24×40	7	0.1450	0.1527	144	142	0.8550	0.8473	0.9160	0.9116	Chandrasekharan and Rajagopalan (1989)
14	24×40	7	0.3664	0.3664	95	91	0.6434	0.5971	0.7928	0.7693	Chandrasekharan and Rajagopalan (1989)
15	24×40	7	0.4046	0.4046	76	79	0.5909	0.5909	0.7635	0.7635	Chandrasekharan and Rajagopalan (1989)
16	24×40	7	0.4351	0.4427	68	68	0.5248	0.5290	0.7276	0.7292	Chandrasekharan and Rajagopalan (1989)
17	30×41	2	0.0234	0.0234	67	64	0.1836	0.1836	0.5990	0.5990	Kumar and Vannelli (1987)
18	40×100	10	0.0857	0.0857	577	577	0.9121	0.9121	0.9510	0.9510	Chandrasekharan and Rajagopalan (1987)

Table 1. Summary of clustering results in comparison with the best known results on benchmark problems

in the design of cellular manufacturing systems. The approach utilizes the combination of a linear assignment program and a fuzzy C-means algorithm. One distinctive advantage of the two-phase clustering algorithm is that it is deterministic with constant clustering results. The present algorithm is demonstrated to be effective and efficient according to the experimental results of an extensive comparative study.

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