

# Optimal Sizing and Allocation of Fixed Reactive Power Compensation

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**Abstract:** This paper proposes an approach to optimize the sizing and allocation of a fixed capacitor in a radial distribution network to compensate reactive power. The optimization problem is formulated as a minimization of the line loss of the network with the load profile within 24 hours. Constraints refer to node voltage quality and power flow. The approach is tested on IEEE 33 nodes radial distribution network and the process of the optimization is analyzed from four aspects which are compensation nodes, power factor, active power load factor and compensation proportion to illustrate its feasibility and affectivity.

*Keywords:* Distribution network; Flow calculation; Reactive power compensation; Fixed capacitor; Static optimization problems.

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## 1. INTRODUCTION

With the scale expansion of distribution power system, the line loss is getting more and more owing to the transmission of reactive power. The compensation of reactive power within the distribution network might reduce the line loss. However, the allocation and sizing of the compensation should be carefully chosen. The proper optimization can reduce line loss of network, and improve the voltage quality Wang, Yang, Wang, Wang, Huang and Zeng (2012). A reactive power optimization problem is a typical nonlinear programming problem with constraints.

At present, there are mainly two kinds of compensation strategies, namely, fixed compensation or adjustable compensation (centralized regulation or distributed regulation) YUAN and HAN (2003). The adjustable compensation, such as Static Var Compensator and Static Compensator, can output varying reactive power according to the supervisor's command or to the operational states of the distribution network to reach good operational performances (line loss and node voltages), which requires lots of information communication and computation. A fixed compensation, such as a fixed capacitor, outputs almost a constant reactive power no matter the system states are. The compensation performance might not be as good as the adjustable compensation. However, such a fixed capacity device requires no communication and computation, the operation is very simple and thus the fixed operational costs are very low. Distribution networks are usually characterized by radial topology. In radial distribution networks, the most widely-used device for reactive power compensation is a shunt capacitor Pires, Antunes and Martins (2012).

Reactive power compensation optimization problems often involve multiple and even conflicting objectives. There exists a global optimum in a single-objective optimization,

while the multi-objective case has a set of solutions from different aspects instead of clear optimal solutions. Many objectives and approaches have been proposed in the scientific literature for reactive power compensation problems.

The optimal objectives are about power loss and/or economical savings in the following literature. In Antunes, Pires, Barrico, Gomes and Martins (2009), the problem of locating and sizing of capacitors for reactive power compensation in radial distribution network is modeled as a multi-objective programming problem, where two (conflicting) objective functions are involved: one is to minimize the line loss of the network and the other is to minimize the installation costs of new reactive power sources. In Pires et al. (2012) and Nojavan, Jalali and Zare (2014), the problem of optimal capacitor placement for the reactive power compensation is formulated to identify the network nodes to install capacitors and the dimension of each capacitor so as to minimize installation costs and power loss. The objective in Haque (1999) is to minimize the power losses and a two stage approach is applied. In the first stage, the objective is to find the nodes where the capacitor singly-installed having the significant effects on the feeder power loss. In the second stage, the capacitor sizes at the selected locations are optimized to overcome any over-compensation.

There are some various approaches applied to solve reactive power compensation optimization problem. For example, a teaching learning based optimization approach, consisting of a teaching phase and a learning phase is applied in Sultana and Roy (2014), while a direct search algorithm implicitly incorporating the power flow calculation is applied in Ramalinga Raju, Ramachandra Murthy and Ravindra (2012). In Kannan, Renuga, Kalyani and Muthukumar (2011), differential evolution and multi agent particle swarm optimization (PSO) are applied to find the sizing and the allocation of the capacitors. A

heuristic search based method and a bacterial foraging based method are applied in Hamouda and Sayah (2013) and Tabatabaei and Vahidi (2011) respectively. A fuzzy adaptive PSO approach is also proposed in Zhang and Liu (2008) to address the multi-objective problem for reactive power compensation. In Antunes et al. (2009), elitist genetic algorithm is applied to characterize the Pareto optimal frontier to obtain the minimized system loss and capacitor installation costs. In Varadarajan and Swarup (2008), the reactive power compensation optimization problem is formulated as a mixed integer power system optimization problem having non-convex, nonlinear objectives and nonlinear constraints and solved with a differential evolution based method.

In summary, the attention in the literature is focused on line power/energy loss and node voltages at a specific time and few efforts are involved in those indices within 24 hours for the radial distribution network, i.e., the power flow is only calculated according to specific load distribution of the network. Actually, the load distribution is varying with respect to the time. Generally, a typical load profile for a node is used to describe the load time-varying, from which the load variation can be seen. When the load profile is considered in the computation of the line loss of the network, reactive power compensation will bring better compensation effects for the actual situations.

This paper proposes an approach to optimize the reactive power compensation of a medium voltage radial distribution network to achieve minimization of the line loss within 24 hours. A typical daily load curve is applied to the network in order to obtain load power values at specific times. The approach is tested on IEEE 33 nodes radial distribution network, which implies the approach is feasible and effective.

In Section 1, the motivation of the study has been provided. The rest of this paper is organized as follows. In Section 2, the problem formulation of optimization of the reactive power compensation with the objective is given. Section 3 introduces optimization approach and solutions. Section 4 presents an example of the application to IEEE 33 nodes radial distribution network with the optimization results discussed. Finally, conclusions are drawn in Section 5.

## 2. PROBLEM FORMULATIONS

There are mainly two reasons for the installation of reactive power compensation devices: one is to regulate the voltages at load nodes to be in a specific range, and the other is to compensate the variable consumption to reduce the line loss. In this paper, the sizing and allocation of a fixed capacitor as a reactive power compensation device for a distribution network is studied. To make full advantages of the fixed capacitor, there are two problems should be answered. One is where the capacitor is installed and the other is that what the size of the capacitor is. The solutions to both problems will have impacts on the voltage regulation and line loss of the network. Here, an approach is proposed to optimize the sizing and allocation of the capacitor.

### 2.1 Line Loss Calculation

Reactive power compensation to a distribution network usually pursues two objectives Wang et al. (2012). One is to minimize the line power loss of the network to save the power energy loss and the other one is to reduce the installation cost of the compensation devices. As we know, a reactive power compensation device can be in operation for a long time (the lifespan of the device) once it is installed. Thus the installation cost of the device might be very small compared with the energy cost saving in the lifespan. Therefore, in this paper, only the line loss of the network is thought of as the objective.

The diagram of the IEEE 33 nodes radial distribution network with 33 nodes is shown in Fig. 1 Khodr, Olsina, Jesus and Yusta (2008).

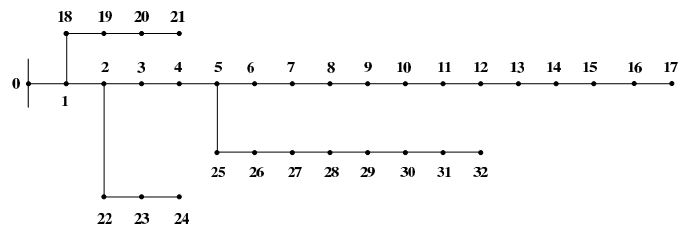


Fig. 1. Single-line diagram of the IEEE 33 nodes distribution network

According to Liao and Zheng (2011), for the given radial distribution network with  $N$  nodes, the line loss of the network within a day can be calculated as:

$$W = \int_{t_0}^{t_f} p_{i,j}(t)dt = 3 \int_{t_0}^{t_f} \sum i_{i,j}^2(t)R_{i,j}dt \times 10^{-3} \quad (1)$$

$$i = 0, 1, 2, \dots, N - 1, j = 0, 1, 2, \dots, N - 1, i \neq j$$

where  $i$  and  $j$  represent the starting and ending nodes in branch  $(i, j)$  respectively,  $p_{i,j}(t)$  is the active power of the  $(i, j)$  branch,  $i_{i,j}(t)$  is the current of the  $(i, j)$  branch,  $R_{i,j}$  is the resistance of the  $(i, j)$  branch and  $t$  is time,  $t_0$  and  $t_f$  are the start time and stop time of the network respectively. To calculate  $W$ , the branch current  $i_{i,j}(t)$  must be obtained firstly. Usually we use power flow calculation to obtain the branch currents.

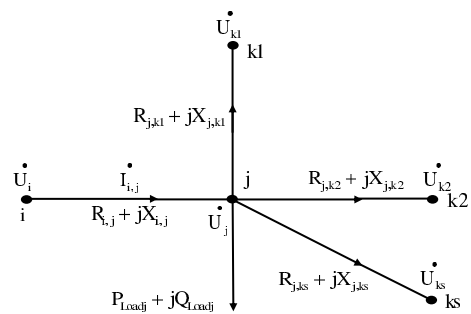


Fig. 2. A typical feeder line

For the distribution network, the voltage of the root node ( $U_0$ ), the load power of the rest nodes ( $P_{Loadi} + jQ_{Loadi}$ ) ( $i = 1, 2, \dots, N - 1$ ), the topology structure of the distribution network and the impedance of the  $(i, j)$  branch ( $Z_{i,j} = R_{i,j} + jX_{i,j}$ ) ( $i = 0, 2, 3, \dots, N - 1, j = 0, 2, 3, \dots, N - 1, i \neq j$ ) are known. The voltage

of each node ( $U_i$ ) ( $i = 1, 2, 3, \dots, N - 1$ ), the current through the ( $i, j$ ) branch ( $i_{i,j}(t)$ ) ( $i = 0, 2, 3, \dots, N - 1, j = 0, 2, 3, \dots, N - 1, i \neq j$ ) and the active power loss of the network are to be calculated Liu, Bi and Dong (2002).

For line loss calculation, branch currents are the focus of attention. An example is illustrated to obtain branch current. A typical branch of a feeder network is shown in Fig. 2.

According to Kirchhoff's current law, the following is true.

$$i_{i,j}(t) = \frac{U_i - U_j}{R_{i,j} + jX_{i,j}} \quad (2)$$

$$i_{i,j}(t) = \sum_{k \in d} \frac{U_j - U_k}{R_{j,k} + jX_{j,k}} + \frac{P_{Loadj} - jQ_{Loadj}}{U_j^*} \quad (3)$$

Then one has

$$\frac{U_i - U_j}{R_{i,j} + jX_{i,j}} = \sum_{k \in d} \frac{U_j - U_k}{R_{j,k} + jX_{j,k}} + \frac{P_{Loadj} - jQ_{Loadj}}{U_j^*} \quad (4)$$

where  $d$  is a set of the branches whose parent node is the node  $j$ .

The above equations are applicable to all branches. So for the  $N$  nodes radial distribution network, there are  $N - 1$  equations and  $N - 1$  node voltages to be calculated. And with the node voltages obtained, the branch currents can be calculated. The equations in the form of Equation (4) are nonlinear obviously, and there are no analytic solutions to the equations. Generally, numerical solutions can be obtained by iterative computation which will be introduced in Section 3.

With branch currents obtained, the line power loss of the network can be calculated. Generally, the theoretical calculation for line energy loss refers to the line energy loss in a day is done in an interval of one hour, which means

$$\begin{aligned} i_{i,j}(t) &= \text{constant} = I_{i,j} \\ 0 &\leq k < t \leq k + 1 \leq 24 \end{aligned} \quad (5)$$

where  $k$  is positive integer.

So,  $W(t)$  can be further expressed as

$$W(t) = \sum_{i=0}^{N-1} \sum_{j=1, j \neq i}^{N-1} 3I_{i,j}^2 R_{i,j} \times 10^{-3} \quad (6)$$

within the  $t$ -th hour.

For the network, the theoretical line loss within a day can be expressed as

$$W = \sum_{t=1}^{24} W(t) \quad (7)$$

When the reactive power compensation capacity  $Q_C$  is compensated to a node  $j$  in the distribution network, the power flow in the network can be changed with the branch current changed. Then, the equation (3) changes into (8)

$$i_{i,j}(t) = \sum_{k \in d} \frac{U_j - U_k}{R_{j,k} + jX_{j,k}} + \frac{P_{Loadj} - j(Q_{Loadj} - Q_C)}{U_j^*} \quad (8)$$

And the equation (4) changes into the equation (9)

$$\frac{U_i - U_j}{R_{i,j} + jX_{i,j}} = \sum_{k \in d} \frac{U_j - U_k}{R_{j,k} + jX_{j,k}} + \frac{P_{Loadj} - j(Q_{Loadj} - Q_C)}{U_j^*} \quad (9)$$

The line loss of the network  $W$  changes with the branch current changed. Thus, appropriate compensation capacity of the reactive power  $Q_C$  may minimize the line loss  $W$ .

## 2.2 Operational Constraints

Two operational constraints for the distribution network must be taken into consideration. One is the node voltage constraint. For each node, the voltage must satisfy

$$(1 - 5\%)U_{rated} \leq U_i \leq (1 + 5\%)U_{rated} \quad (10)$$

where  $U_{rated}$  is the rated voltage.

The other is the security limitation on the branch current. For each branch, the branch current must satisfy

$$0 \leq I_{i,j} \leq I_{i,j}^s \quad (11)$$

where  $I_{i,j}^s$  is the current carrying capacity of the ( $i, j$ ) branch cable.

## 2.3 Optimization Problem

According to the power flow distribution, compensating reactive power to the node in the network will change the power flow distribution of the network. The optimization of the capacitor is to find the node and corresponding capacity of the capacitor in the network such that the line loss of the network within a day is minimized with the operational constraints satisfied. In this paper, the objective is  $f$  expressed as

$$\begin{aligned} f &= \min W \\ \text{s. t. } &\begin{cases} (1 - 5\%)U_{rated} \leq U_i \leq (1 + 5\%)U_{rated} \\ 0 \leq I_{i,j} \leq I_{i,j}^s \end{cases} \end{aligned} \quad (12)$$

## 3. APPROACH AND SOLUTIONS

The power flow equations are nonlinear. It is difficult to obtain analytic solutions to those equations. In engineering, such a problem is generally solved by numerical computation approach specifically. Here we use a numerical computation approach based on the forward and backward substitution method CONG and WANG (2008) which is usually used in engineering.

### 3.1 Forward and Backward Substitution Method

On the basis of analyzing the techniques for power flow calculations of distribution network, the topology structure of distribution network is defined by the tree structure which consisted of special circuit branch structure and node structure CHEN, CHEN, GU and LIU (2010). Branch currents are corrected by postorder-traversing the tree structure and node voltages are corrected by preorder-traversing the tree structure until constringency. With such a method, there is no need to number the distribution network and form admittance matrix. The power flow distribution of network can be obtained through this method.

Flow chart of the forward and backward substitution method is shown in Fig. 3. Specific calculation steps can refer to CHEN et al. (2010).

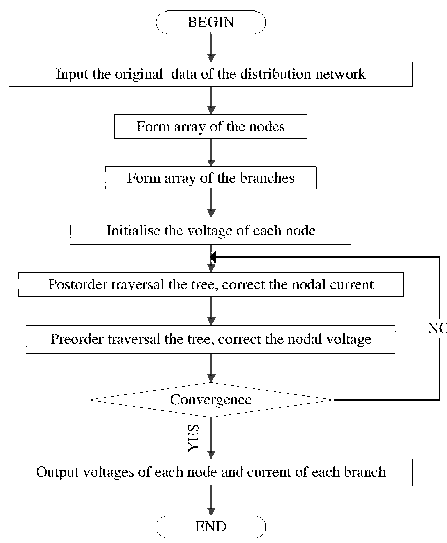


Fig. 3. Flow chart of the forward and backward substitution method

### 3.2 Steps of Approach

To solve the optimal problem, we use the approach with following steps:

- (1) For a given sequence of  $Q_{C0}$  at a node, the line loss for each value of  $Q_{C0}$  could be found. Then we can find the optimum  $Q_{Ci}$  to the problem at this node.
- (2) Choose another node to repeat step (1).
- (3) After all nodes except the root node being checked, we can compare the optimums at those nodes to obtain the optimum  $Q_C$  for the problem in the global scope.

## 4. EXAMPLE AND RESULTS

### 4.1 Example

The proposed approach is tested on IEEE 33 nodes radial distribution network which is shown in Fig. 1. The parameters of the network can refer to Baran and Wu (1989). The convergence condition for the power flow calculation is that voltage difference values between the consecutive rounds for all nodes are less than  $1.0 \times 10^{-6}$ kV. The root node voltage is fixed as  $U_0 = 12.66$ kV.

In this example, active power of each node changes with the typical daily load curve, and the line loss of the network can be calculated one hour by one hour. For convenience, we assume that each node has the same shape for the typical daily active power load curve and power factor. A typical daily load curve in summer Zhang (2009) depicted in Fig. 4 is adopted, where P.U.=1 means the actual active power is the maximum.

### 4.2 Discussions and Analysis

When the results are shown in the following figures. In the figures, the proportion refers to the capacitor percentage that is  $Q_C/Q_\varepsilon$ .

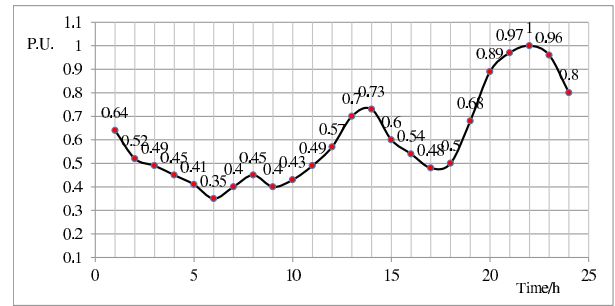


Fig. 4. A typical daily load curve in summer

The power factor is 0.85 and the maximum of compensation reactive power  $Q_\varepsilon$  in the network within a day is 2302.35021kVar. Fig. 5 shows the optimum for each node. From Fig. 5, we can see that the optimal proportions of reactive power compensation at different nodes are different and results in different line loss. With the topology structure of the radial distribution network and the optimization results investigated, some interesting information can be known. When the compensated node is near to the root node, the effect of compensation is not good. With the compensated node far away from the root node, the line loss decreases until reach the least at a node and the capacity of compensation decreases. Then, with the compensated node close to the ending node, the line loss increases while the capacity of compensation decreases. According to the parameters of the network, we can see that the resistance of the branch can influence the effective of the adjacent compensated nodes, but when the adjacent nodes are close and the resistance is small, their compensated effects are similar.

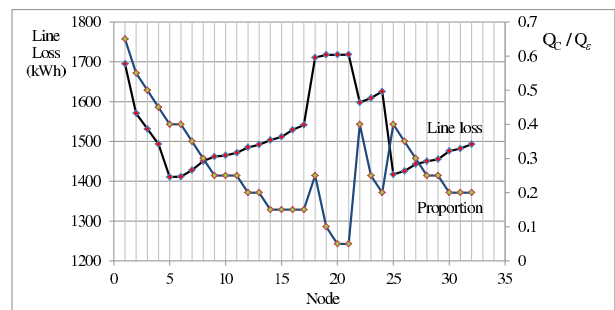


Fig. 5. Optimum at each node in the network

The results for some nodes with different proportions of compensation are shown in Fig. 6 and Fig. 7, from which it is seen that at a specific node, different compensation capacity generally results in different line loss, which is the motivation for the optimization of the compensation capacity. Meanwhile, we can see that even the same capacity of reactive power compensation for different compensated nodes, usually lead to different line loss of the network, which is the motivation for the optimization of the compensation allocation.

It is also seen that no matter what the compensation capacity is to the Node 1, the line loss changes little, while compensation capacity at any other nodes varies, the line loss has an obvious change. This probably because Node 1 is very close ( $R_{0,1}$  is very small) to the root node (the source node). The compensation reduces branch current



$I_{0,1}$ , which has little effect because of  $R_{0,1}$  in Equation (6) is very small. Meanwhile, it can be seen that the curves of line loss with respect to the compensation capacity is convex. Therefore, compensation capacity could be optimized such that the line loss can be minimized.

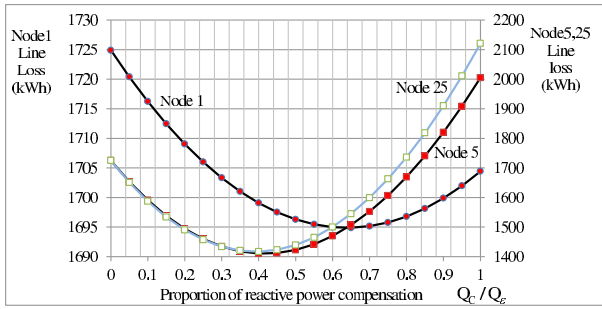


Fig. 6. Optimum result with power factor=0.85,  $Q_{\epsilon}=2302.35022$  kVar

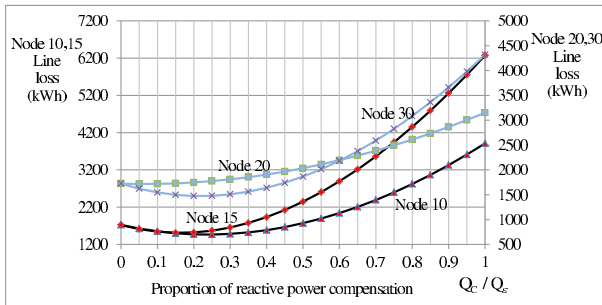


Fig. 7. Optimum result with power factor=0.85,  $Q_{\epsilon}=2302.35022$  kVar

We further investigate the optimization results in more scenarios, i.e., the scenarios with the active power and the power factor changed. In the rest of the paper, PLF represents active power load factor.

When  $P_{Loadj}$  is maintained and  $Q_{Loadj}$  is changed such that the power factor is 0.95, the optimization result is shown as Fig. 9, while  $P_{i,j}$  is 0.4 times of the baseline (i.e., PLF=0.4).

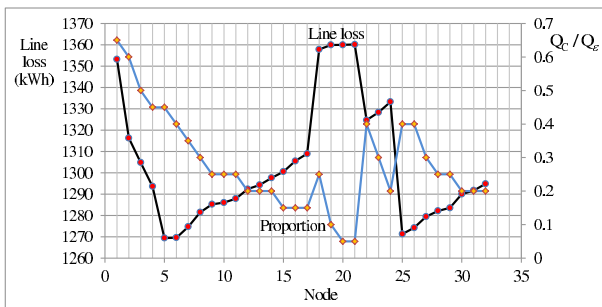


Fig. 8. Optimum result with power factor=0.95, PLF=1.0,  $Q_{\epsilon}=1221.06145$  kVar

Comparing Fig. 5 with Fig. 8 and Fig. 9, it is easy to find that the shapes of the two classes of optimization result curves are consistent, which implies that the relative effects of optimal compensation at different nodes are influenced by the topology structure of the distribution network.

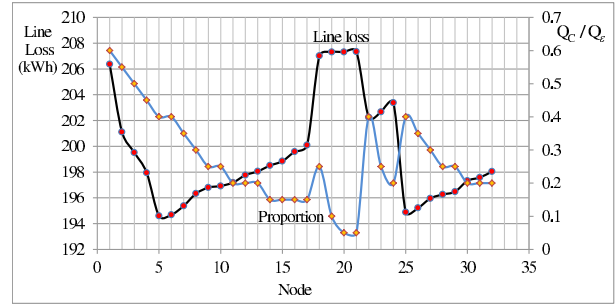


Fig. 9. Optimum result with power factor=0.95, PLF=0.4,  $Q_{\epsilon}=488.42458$  kVar

Comparing Fig. 5 with Fig. 8, it can be seen that the line loss of the network and compensation capacity decrease with the growth of power factor. This because the higher power factor is, the lower the reactive power  $Q_{\epsilon}$  is and then the smaller the branch currents of the network. The higher power factor results in smaller line loss and compensation capacity. So improving power factor can reduce the line loss effectively.

Comparing Fig. 9 with Fig. 8, it can be seen that when the power factor is the same, the line loss of the network and compensation capacity increase with the increase of the PLF. This because the higher PLF is, the more reactive power compensation is required.

Fig. 10 shows the optimization results while the power load factor is different and the power factor is the same. With different PLF,  $Q_{\epsilon}$  is different. In Fig. 10,  $Q_{\epsilon}$  is as follows:

- When PLF=0.4,  $Q_{\epsilon}=1516.02323$ kVar.
- When PLF=0.6,  $Q_{\epsilon}=2274.03485$ kVar.
- When PLF=0.8,  $Q_{\epsilon}=3032.04647$ 0kVar.
- When PLF=1.0,  $Q_{\epsilon}=3790.05808$ 8kVar.
- When PLF=1.2,  $Q_{\epsilon}=4548.06970$ 7kVar.

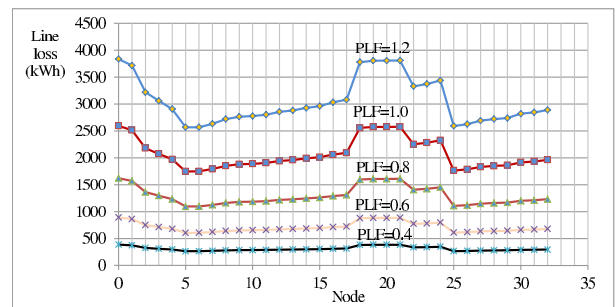


Fig. 10. Optimum results with power factor=0.7

Particularly, the line loss of Node 0 refers to the scenario that the compensation capacitor is installed at the bus node, implying the line loss is the result without compensation. It is obvious to see that the line loss grows with the increase of PLF in Fig. 10. But the optimum shapes of the curves in Fig. 10 are similar. Compare the curves in Fig. 10, it could be found that when PLF is low, the effects of the compensation at different nodes are not significant, this is because the optimum effect is not obvious compared with no compensation. When PLF is high, the differences of the compensation effects at different nodes are significant. So, with a high PLF, reactive power compensation can be optimized for sizing and allocating.

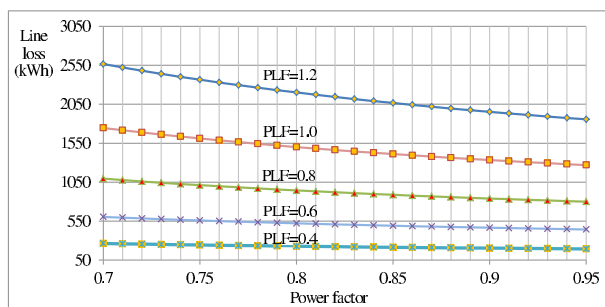


Fig. 11. Optimum results with compensating at Node 5

With different power factors, the optimum results are shown in Fig. 11. With the same PLF at Node 5, the optimum line loss of the distribution network varies with respect to the power factor. Obviously in Fig. 11, the line loss decreases with the increase of the power factor. From Fig. 11, we can see that when PLF is low, the optimized compensation effects are similar even with different power factors. However, when PLF is high, the differences of optimized compensation effects with different power factors are significant. Those are true for other nodes, too. Thus, the higher the PLF is, the more important the optimization of compensation.

## 5. CONCLUSIONS

This paper proposed an approach which is to find the optimal sizing and allocation of a fixed capacitor as the reactive power compensation device to minimize the line loss of a radial distribution network within a day. The approach is very simple and effective for practical engineering. The approach is tested on IEEE 33 nodes radial distribution network and the results are analyzed from some aspects which are compensation nodes, power factor, active power load factor and compensation proportion. The test illustrates the motivations and applicability of this study. The higher power load factor is and the lower the power factor is, the more important the optimization of reactive power compensation.

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