Preprints of the 19th World Congress
The International Federation of Automatic Control
Cape Town, South Africa. August 24-29, 2014

An On-Line Identification Method of Precision Positioning Stage

Ruili Dong*, Yonghong Tan+*, Xiang Chen, Ming Zheng****

*Shanghai Normal University, Shanghai 200234, China
(+ Corresponding Author,Tel: +86-21-57122955; e-mail: tany@shnu.edu.cn).
** University of Windsor, Windsor, ON, Canada N9B 3P4

Abstract: In this paper, an on-line identification method for a precision positioning stage with piezoactuator is proposed. In the proposed method, a sandwich model with embedded Duhem hysteresis submodel is utilized to describe the behavior of stage. In this modeling scheme, the nonlinear Duhem model is transformed to a pseudo linear model. Afterwards, all the parameters of the model are separated to constitute a linear combination of coefficients and nonlinear variables. Then, an extended recursive identification algorithm is proposed to estimate the corresponding parameters of the sandwich model. Finally, the experimental results of on-line identification of a stage with piezoactuator are illustrated.

1. INTRODUCTION

In precision mechatronic systems, such as atomic force microscopes, diamond turning machines and mask aligner (Zhang and Fang et. al., 2009; Huang and Lin, 2004; Lin and Yang, 2005), positioning stages with piezoactuator are usually used as actuating mechanisms since piezoactuator have high stiffness, nano-meter displacement resolution, large bandwidth, and fast frequency response. However, the existence of hysteresis in piezoactuator may have significant influence on the performance of stage. As hysteresis has the characteristic of non-smoothness and multi-valued mapping (Bhikkaji and Moheimani, 2008; Xu and Li, 2010, 2012; Janaideh, Rakheja and Su, 2011; Xie, Tan and Dong, 2013; Ge, 1996), moreover, it may cause unexpected positioning error and vibration which may deteriorate the performance of the stage. Usually, the influence of hysteresis should be compensated and model based compensation strategies are one of the option. For model based control or compensation methods (Xie, Tan and Dong, 2013; Ge, 1996), it is necessary to build the corresponding model with hysteretic behaviour to describe the positioning stage with piezoactuator.

For modelling of hysteresis, there have been some methods have proposed, e.g. Preisach models or modified Preisach models (Ge, 1996; Mayergoyz, 1991), Duhem models (Oh and Bernstein, 2005; Jayawardhana, Ouyang and Andrieu, 2012; Meurer, Qu, Jacobs, 2002), Bou-Wen models (Wang, Zhang and Mao, 2012), Prandtl-Ishlinskii (PI) models (Visintin, 1994; Janaideh, Rakheja and Su, 2011), and neural networks models based on expanded input space (Zhao and Tan, 2008; Dong, Tan and Chen, 2008). It is noticed that, in many precision mechatronics systems, a piezoactuator does not exist solitarily but is often connected with some other devices or equipments. In a precision positioning stage, the power supply to piezoactuator is usually provided by electronic amplifier with filtering circuit. On the other hand, the flexible hinge with load is driven by the piezoactuator. In the case of adaptive control, on-line identification only based on the input and output data of positioning stage is one of the key issues which should be investigated. In this situation, both input and output of the piezoactuator are not measurable directly. Moreover, the piezoactuator in the stage has the characteristics of multi-valued mapping, non-smoothness and rate-dependence. All those mentioned factors have become severe challenges to on-line identification for positioning stage with piezoactuator.

In this paper, a so called sandwich model is proposed to describe the positioning stage with piezoactuator. In this model, both amplifier with filtering circuit to supply power to the actuator and the flexible hinge with load are described by two linear dynamic submodels, respectively; while a hysteresis submodel is employed to describe the performance of piezoactuator sandwiched between the two linear dynamic submodels. For the identification of sandwich models Boutayeb and Darouach (1995), Tan and Godfrey (2002), Crama and Schoukens (2005), as well as Kibangou and Favier (2006) have proposed approaches for sandwich models embedded with smooth nonlinear functions but those methods can not be used directly for non-smooth sandwich models, especially for the case that the embedded nonlinear function is hysteresis which has multi-valued mapping and non-smoothness. Although Xie, Tan and Dong (2013) proposed a two-stage method for identification of sandwich models with hysteresis, the method can only be implemented off-line.

Therefore, in this paper, a recursive identification algorithm is proposed to identify the sandwich model to describe positioning stage with piezoactuator. The consideration of the rate-dependent property of hysteresis leads to the use of a Duhem model to describe the characteristic of piezoactuator with rate-dependent behaviour in the stage. Then, a simple transform based on key terms separation (Voros, 1995) is implemented to transform the Duhem function a pseudo linear function. Thus, we can obtain a linear combination between the coefficients and nonlinear variables of the model. Afterwards, an extended recursive identification algorithm (ERIA) is implemented to estimate the coefficients of the sandwich model on-line.

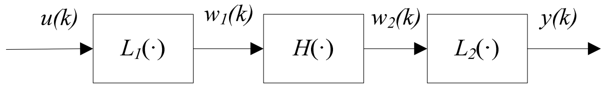

Fig.1. Structure of sandwich model with hysteresis

2. SANDWICH MODEL WITH HYSTERESIS

The architecture of the proposed sandwich model is shown in Fig.1. In Fig.1, $L_1(\cdot)$ and $L_2(\cdot)$ are input and output linear dynamic submodels, respectively; while $H(\cdot)$ is a hysteresis submodel. In this model, neither input $w_1(k)$ nor output $w_2(k)$ of $H(\cdot)$ can be measured, directly. The corresponding descriptions of the linear dynamic submodels are :

$L_1(\cdot)$:

$$w_1(k)=\sum_{j=1}^{n_{1b}} b_{1j}u(k-q_1-j)+b_{10}u(k-q_1)-\sum_{i=1}^{n_{1a}} a_{1i}w_1(k-i)\,, \quad (1)$$

and

$L_2(\cdot)$:

$$y(k)=\sum_{j=1}^{n_{2b}} b_{2j}w_2(k-q_2-j)+b_{20}w_2(k-q_2)-\sum_{i=1}^{n_{2a}} a_{2i}y(k-i)\,, \quad (2)$$

respectively, where q_1 and q_2 are the delays of $L_1(\cdot)$ and $L_2(\cdot)$; a_{1i} , a_{2i} , b_{1j} and b_{2j} are coefficients of linear dynamic submodels, n_{1a} , n_{1b} , n_{2a} and n_{2b} are the orders.

Assumption 1:

Both linear dynamic submodels $L_1(\cdot)$ and $L_2(\cdot)$ are subject to the following conditions

1) stable and minimum phase;

2) $L_1(\cdot)$ and $L_2(\cdot)$ are coprime.

For the uniqueness of the model, it is assumed that $b_{10}=1$ and $b_{20}=1$.

For the submodel $H(\cdot)$, the Duhem model (Zhao, 2003) is employed to describe the rate-dependent hysteresis, i.e.

$$\begin{aligned} w_2(k) &= w_2(k-1)+\alpha[f(w_1(k-1))-w_2(k-1)]\,|\Delta w_1(k)| \\ &\quad + g(w_1(k-1))\Delta w_1(k) \end{aligned}\,, \quad (3)$$

where $\Delta w_1(k)=w_1(k)\text{-}w_1(k-1)$, α is a positive bounded real number, both $f(\cdot)$ as well as $g(\cdot)$ are real-valued functions of $w_1(k)$. In (3), $f(\cdot)$ and $g(\cdot)$ are non-smooth nonlinear functions, and satisfy the following assumptions (Zhao, 2003) :

Assumption 2:

1) function $f(\cdot)$ is odd, monotone increasing and piece-wise continuously differentiable with a finite limit for its first order derivative at positive infinity;

2) function $g(\cdot)$ is even, piecewise continuous and at infinity of such a finite value that

$$\lim_{s\to\infty}\frac{df(s)}{ds}=\lim_{s\to\infty}g(s)\,; \quad (4)$$

3) functions $f(\cdot)$ and $g(\cdot)$ satisfy that

$$\frac{df(s)}{ds}\geq g(s)\,,\quad \forall s<\infty$$

and

$$\alpha e^{\alpha s}\int_s^{\infty}\left[\frac{df(\varsigma)}{d\varsigma}-g(\varsigma)\right]e^{-\alpha\varsigma}d\varsigma\leq g(s)\,,\quad \forall s<\infty\,.$$

Here, based on Assumption 2, a pseudo linearization transform is performed to select $f(\cdot)$ and $g(\cdot)$ as:

$$f(w_1(k))=f_0\,\mathrm{sgn}(w_1(k-1))+f_1w_1(k-1) \quad (5)$$

and

$$g(w_1(k))=g_0 \quad (6)$$

where f_0, f_1 and g_0 are the coefficients of $f(\cdot)$ and $g(\cdot)$, respectively, and

$$\mathrm{sgn}(w_1(k-1))=\begin{cases}1, & w_1(k-1)>0\\ 0, & w_1(k-1)=0\\ -1, & w_1(k-1)<0\end{cases}.$$

Substituting (5) and (6) into (3) leads to

$$\begin{aligned} w_2(k) &= w_2(k-1)+\alpha[f_0\,\mathrm{sgn}(w_1(k-1))+f_1w_1(k-1) \\ &\quad -w_2(k-1)]\,|\Delta w_1(k)|+g_0[\Delta w_1(k)] \end{aligned}\,. \quad (7)$$

Thus, Eqs. (1), (2) and (7) constitute the sandwich model with rate-dependent hysteresis.

By considering the stability of Duhem model, the parameters of Duhem model should satisfy the following condition, i.e.

$$0<\alpha\,|\,w_1(k)-w_1(k-1)\,|<2\;. \quad (8)$$

3. IDENTIFICATION ALGORITHM

For convenience to separate the key terms and estimate the parameters of the model shown-above, the following incremental models are utilized:

$$\Delta w_1(k)=\sum_{j=1}^{n_{1b}} b_{1j}\Delta u(k-q_1-j)+\Delta u(k-q_1)-\sum_{i=1}^{n_{1a}} a_{1i}\Delta w_1(k-i) \quad (9)$$

$$\begin{aligned} \Delta w_2(k) &= \alpha[f_0\,\mathrm{sgn}(w_1(k-1))+f_1w_1(k-1) \\ &\quad -w_2(k-1)]\,|\Delta w_1(k)|+g_0\Delta w_1(k) \end{aligned}\,, \quad (10)$$

and

$$\begin{aligned} \Delta y(k) &= \sum_{j=1}^{n_{2b}} b_{2j}\Delta w_2(k-q_2-j)+\Delta w_2(k-q_2) \\ &\quad -\sum_{i=1}^{n_{2a}} a_{2i}\Delta y(k-i) \end{aligned}\,, \quad (11)$$

where $\Delta w_2(k)=w_2(k)-w_2(k-1)$; $\Delta y(k)=y(k)-y(k-1)$; $\Delta u(k)=u(k)-u(k-1)$.

Considering (1), and (9) -(11) based on the key term principle yields

$$
\begin{aligned}
\Delta y(k) &= \sum_{j=1}^{n_{2b}} b_{2j}\Delta w_2(k-q_2-j) - \alpha\,|\Delta w_1(k-q_2)|\,w_2(k-1-q_2) \\
&+ f_0\alpha\,|\Delta w_1(k-q_2)|\,\mathrm{sgn}(w_1(k-q_2-1)) \\
&+ f_1\alpha\,|\Delta w_1(k-q_2)|\,u(k-q_1-q_2-1) \\
&+ \sum_{j=1}^{n_{1b}} b_{1j}[f_1 u(k-q_1-j-q_2-1) + g_0\Delta u(k-q_1-j-q_2)] \\
&+ g_0\Delta u(k-q_1-q_2) - \sum_{i=1}^{n_{1a}} a_{1i}[f_1\alpha\,|\Delta w_1(k-q_2)|\,w_1(k-i-q_2-1) \\
&+ g_0\Delta w_1(k-i-q_2)] - \sum_{i=1}^{n_{2a}} a_{2i}\Delta y(k-i)
\end{aligned} \quad . \quad (12)
$$

Hence, Eq.(12) can be formed as:

$$\Delta y(k) = \mathbf{h}^T(k\,|\,\boldsymbol{\theta})\boldsymbol{\theta}, \quad (13)$$

where data vector $\mathbf{h}(k\,|\,\theta)$ and parameter vector $\boldsymbol{\theta}$ are defined as:

$$
\begin{aligned}
&\mathbf{h}^T(k\,|\,\boldsymbol{\theta}) = [\Delta w_2(k-q_2-1),\cdots,\Delta w_2(k-q_2-n_{2b}), \\
&-|\Delta w_1(k-q_2)|\,w_2(k-1-q_2),\ \alpha\,|\Delta w_1(k-q_2)|\,\mathrm{sgn}(w_1(k-q_2-1)), \\
&\alpha\,|\Delta w_1(k-q_2)|\,u(k-q_1-q_2), \{f_1 u(k-q_1-2-q_2) \\
&+g_0\Delta u(k-q_1-1-q_2)\},\cdots,\{f_1 u(k-q_1-n_{1b}-q_2-1)+ \\
&g_0\Delta u(k-q_1-n_{1b}-q_2)\},\Delta u(k-q_1-q_2), \\
&-\{f_1\alpha\,|\Delta w_1(k-q_2)|\,w_1(k-2-q_2)+g_0\Delta w_1(k-1-q_2)\},\cdots, \\
&-\{f_1\alpha\,|\Delta w_1(k-q_2)|\,w_1(k-n_{1a}-1-q_2) \\
&+g_0\Delta w_1(k-n_{1a}-q_2)\},-\Delta y(k-1),\cdots,-\Delta y(k-n_{2a})]^T
\end{aligned}
$$

and

$$
\begin{aligned}
\boldsymbol{\theta} = [&b_{21},\cdots,b_{2n_{2b}},\alpha,f_0,f_1,b_{11},\cdots,b_{1n_{1b}}, \\
&g_0,a_{11},\cdots,a_{1n_{1a}},a_{21},\cdots,a_{2n_{2a}}]^T
\end{aligned}
$$

respectively.

Thus, the parameters can be estimated based on the cost function as follows.

$$\hat{\boldsymbol{\theta}}(k) = \arg\min_{\boldsymbol{\theta}} \sum_{k=1}^{n} \Lambda(k)[\Delta y(k) - \hat{\mathbf{h}}^T(k\,|\,\hat{\boldsymbol{\theta}}(k-1))\hat{\boldsymbol{\theta}}(k-1)]^2 \ , \quad (14)$$

where $\Lambda(k) > 0$ is a weighted factor. Moreover, $\hat{\mathbf{h}}(k\,|\,\hat{\boldsymbol{\theta}}(k-1))$ and $\hat{\boldsymbol{\theta}}(k-1)$ are represented by

$$
\begin{aligned}
&\hat{\mathbf{h}}^T(k\,|\,\boldsymbol{\theta}(k-1)) = [\Delta\hat{w}_2(k-q_2-1),\cdots,\Delta\hat{w}_2(k-q_2-n_{2b}), \\
&-|\Delta\hat{w}_1(k-q_2)|\,\hat{w}_2(k-1-q_2),\ \hat{\alpha}(k-1)\,|\Delta\hat{w}_1(k-q_2)|\,\mathrm{sgn}(\hat{w}_1(k-q_2)), \\
&\hat{\alpha}(k-1)\,|\Delta\hat{w}_1(k-q_2)|\,u(k-q_1-q_2), \{\hat{f}_1(k-1)u(k-q_1-2-q_2) \\
&+\hat{g}_0(k-1)\Delta u(k-q_1-1-q_2)\},\cdots,\{\hat{f}_1(k-1)u(k-q_1-n_{1b}-1-q_2)+ \\
&\hat{g}_0(k-1)\Delta u(k-q_1-n_{1b}-q_2)\},\Delta u(k-q_1-q_2), \\
&-\{\hat{f}_1(k-1)\hat{\alpha}(k-1)\,|\Delta\hat{w}_1(k-q_2)|\,\hat{w}_1(k-2-q_2)+\hat{g}_0(k-1)\Delta\hat{w}_1(k-1-q_2)\},\cdots, \\
&-\{\hat{f}_1(k-1)\hat{\alpha}(k-1)\,|\Delta\hat{w}_1(k-q_2)|\,\hat{w}_1(k-n_{1a}-q_2-1) \\
&+\hat{g}_0(k-1)\Delta\hat{w}_1(k-n_{1a}-q_2)\},-\Delta y(k-1),\cdots,-\Delta y(k-n_{2a})]^T
\end{aligned}
$$

and

$$
\begin{aligned}
\hat{\boldsymbol{\theta}}(k-1) = [&\hat{b}_{21}(k-1),\cdots,\hat{b}_{2n_{2b}}(k-1),\hat{\alpha}(k-1),\hat{f}_0(k-1), \\
&\hat{f}_1(k-1),\hat{b}_{11}(k-1),\cdots,\hat{b}_{1n_{1b}}(k-1),\hat{g}_0(k-1), \\
&\hat{a}_{11}(k-1),\cdots,\hat{a}_{1n_{1a}}(k-1),\hat{a}_{21}(k-1),\cdots,\hat{a}_{2n_{2a}}(k-1)]^T
\end{aligned}
$$

Note that data vector $\hat{\mathbf{h}}^T(k\,|\,\hat{\boldsymbol{\theta}}(k-1))$ can be calculated based on the estimated parameters at the previous step. Therefore, $\hat{w}_1$, $\hat{w}_2$, $\Delta\hat{w}_1$ and $\Delta\hat{w}_2$ can be computed based on Eqs. (1)-(2) and (7)-(11) with the coefficient vector $\hat{\boldsymbol{\theta}}(k-1)$.

Based on what stated-above, an extended recursive algorithm is used to estimate the parameters of the model on-line, i.e.

$$e(k) = \Delta y(k) - \hat{\mathbf{h}}^T(k\,|\,\hat{\boldsymbol{\theta}}(k-1))\hat{\boldsymbol{\theta}}(k-1)\ , \quad (15)$$

$$\hat{\boldsymbol{\theta}}(k) = \hat{\boldsymbol{\theta}}(k-1) + \mathbf{K}(k)e(k)\ , \quad (16)$$

$$\mathbf{K}(k) = \frac{\mathbf{P}(k-1)\hat{\mathbf{h}}(k\,|\,\hat{\boldsymbol{\theta}}(k-1))}{\hat{\mathbf{h}}^T(k\,|\,\hat{\boldsymbol{\theta}}(k-1))\mathbf{P}(k-1)\hat{\mathbf{h}}(k\,|\,\hat{\boldsymbol{\theta}}(k-1)) + \mu(k)\Lambda^{-1}(k)} \quad (17)$$

$$
\begin{aligned}
\mathbf{P}(k) = &\frac{1}{\mu(k)}\cdot\left[\mathbf{I} - \mathbf{K}(k)\cdot\hat{\mathbf{h}}^T(k\,|\,\hat{\boldsymbol{\theta}}(k-1))\right]\cdot\mathbf{P}(k-1)\cdot[\mathbf{I} \\
&-\mathbf{K}(k)\cdot\hat{\mathbf{h}}^T(k\,|\,\hat{\boldsymbol{\theta}}(k-1))]^T + \mathbf{K}(k)\cdot\Lambda^{-1}(k)\cdot\mathbf{K}^T(k)
\end{aligned} \ , \quad (18)
$$

$$\Lambda^{-1}(k) = \Lambda^{-1}(k) + \rho(k)[e^2(k) - \Lambda^{-1}(k)]\ , \quad (19)$$

where $e(k)$, $\boldsymbol{K}(k)$, $\boldsymbol{P}(k)$ and $\Lambda^{-1}(k)$ are the modelling error, gain vector, covariance matrix and estimation value of the correlation of model error. Moreover, $\rho(k)\in(0,\ 1)$ and $\mu(k)\in(0,\ 1)$ are the convergence and forgetting factors, respectively; and $\mu(k) = \frac{\rho(k-1)}{\rho(k)}\left[1-\rho(k)\right]$ (Zhang, 2004).

Lemma 1:
Suppose the input fed to the system is bounded. If Assumption 1 is held, for a given $M > 0$, we have $\hat{\mathbf{h}}^T(k\,|\,\hat{\boldsymbol{\theta}}(t-1))\hat{\mathbf{h}}(k\,|\,\hat{\boldsymbol{\theta}}(t-1)) \le M$.

Proof: As the input fed to the system is bounded and conditions 1)-3) in Assumption 1 are held, $\hat{w}_1(k\,|\,\hat{\boldsymbol{\theta}}(k-1))$ is also bounded. Based on (8), $\hat{w}_2(k\,|\,\hat{\boldsymbol{\theta}}(k-1))$ is bounded either. Thus, for the given $M > 0$, all the elements of $\hat{\mathbf{h}}(k\,|\,\hat{\boldsymbol{\theta}}(t-1))$ are bounded, and namely, $\hat{\mathbf{h}}^T(k\,|\,\hat{\boldsymbol{\theta}}(t-1))\hat{\mathbf{h}}(k\,|\,\hat{\boldsymbol{\theta}}(t-1)) \le M$.

Assume that the bounded input signal is selected to make the sandwich system with hysteresis to be fully excited in all the operation zones. That means the input excitation will cover all the equilibrium points of the system and satisfy the persistently exciting conditions at the equilibrium point of each operation zone. We have the following theorem:

Theorem 1:
Suppose input $u(k)$ is bounded and can fully excite all the operation zones of the sandwich system with hysteresis described by (1), (2), (7), and (9)-(11). Moreover, assume that the algorithm described by (15)-(19) is used to estimate the parameters of the system model. If $\rho(k)\mathbf{P}^{-1}(k)$ is a positive definite matrix; as well as

$$\lim_{k\to\infty} \lambda_{\max}[\mathbf{P}(k)] \to 0\ ; \quad (20)$$

$$\limsup_{k\to\infty} \frac{\lambda_{\max}[\mathbf{P}(k)]}{\lambda_{\min}[\mathbf{P}(k)]} < \infty\ ; \quad (21)$$

and

$$1 - \sqrt{\Delta_1(k)} \le \beta(k) \le 1 + \sqrt{\Delta_1(k)} \quad (22)$$

are held, it will lead to:

$$\lim_{k\to\infty} \hat{\boldsymbol{\theta}}(k) = \boldsymbol{\theta}$$

where $\boldsymbol{\theta}$ is the local equivalent truth value of the parameter vector of the system, and $\Delta_1(k) = 1 - \dfrac{\hat{\mathbf{h}}^T(k\,|\,\hat{\boldsymbol{\theta}}(k-1))\mathbf{P}(k-1)\hat{\mathbf{h}}(k\,|\,\hat{\boldsymbol{\theta}}(k-1))}{\hat{\mathbf{h}}^T(k\,|\,\hat{\boldsymbol{\theta}}(k-1))\mathbf{P}(k-1)\hat{\mathbf{h}}(k\,|\,\hat{\boldsymbol{\theta}}(k-1)) + \mu(k)\Lambda^{-1}(k)}$, $\beta(k)$ in (22) denotes the ratio between $\hat{\mathbf{h}}^T(k\,|\,\hat{\boldsymbol{\theta}}(k-1))(\boldsymbol{\theta} - \hat{\boldsymbol{\theta}}(k-1))$ and modelling error $e(k)$.

Proof: Omitted due to limited space.

Remark 1: In the recursive identification, the data vector $\hat{\mathbf{h}}^T(k\,|\,\hat{\boldsymbol{\theta}}(k-1))$ is calculated by the estimated parameter vector $\hat{\boldsymbol{\theta}}(k-1)$ at the previous step. $\beta(k)$ implies that the identification is affected by the estimated errors of the internal variables, and $\beta(k)$ will be equal to unity if the internal variables are equal to the truth values.

Remark 2: If the estimated parameters are far from the their corresponding truth values, the estimation error of $\hat{\mathbf{h}}^T(k\,|\,\hat{\boldsymbol{\theta}}(k-1))$ may be larger, and it may make $e(k)$ larger, which does harm to parameters convergence, and even leads to divergence of estimation for $\beta(k)$ can not satisfy the condition described by (22), especially at the initial period of identification. Hence, if the bounded input can fully excite all the operation zones of the system, the amplitude of the input should be selected as relatively small as possible.

4. EXPERIMENTAL RESULTS

In this section, both proposed non-smooth sandwich model and the extended recursive identification algorithm are applied to the identification of precision positioning stage with piezoactuator. The corresponding system architecture is shown in Fig. 2.

In the positioning system, the piezoactuator is employed to drive the load which is a flexible hinge with work platform, and a capacitive sensor is used to measure the displacement of the load. The variation range of the stage displacement is between $0\mu m$ and $10\mu m$. An amplifier with filtering circuit supplies voltage between $0v$ and $10v$ to the piezoactuator. In the experiment, the sampling frequency is chosen as $30KHz$. In this case, only the displacement of the load can be measured by the capacitive sensor, of course, the excitation signal generated by the computer to the amplifier can also be known.

In the identification, both amplifier + filtering circuit and the flexible hinge + load are described by linear dynamic submodels, respectively, whilst the Duhem model is employed to describe the behavior of the piezoactuator. Thus, the architecture of the corresponding sandwich model can be described as

$L_1(\cdot)$, for amplifier + filtering circuit:

$$w_1(k) = -a_{11}w_1(k-1) + b_{10}u(k-1)\,, \tag{23}$$

$H(\cdot)$, for piezoactuator:

$$\begin{aligned} w_2(k) = w_2(k-1) &+ \alpha[f_1 w_1(k-1) \\ &- w_2(k-1)]\,|\,\Delta w_1(k)\,| + g_0(\Delta w_1(k)) \end{aligned}, \tag{24}$$

$L_2(\cdot)$, for flexible hinge + load:

$$y(k) = -a_{21}y(k-1) - a_{22}y(k-2) + b_{20}w_2(k)\,. \tag{25}$$

where both b_{10} and b_{20} are set to unity for assuring the uniqueness of the identified model.

In the experiment, the excitation signal used to excite the stage is a modulated signal constituted by a PRBS sequence and a sinusoidal signal with attenuated amplitude and frequency. The amplitude of the PRBS sequence varies from $0.4v$ to $0.6v$. On the other hand, the maximum value of the frequency of the sine signal is 500Hz.

For on-line estimation, we set convergence factor $\rho(k) = (k+1)^{-9/13}$, and the initial value of covariance matrix as $\mathbf{P}(k) = 10^6 \times \mathbf{I}$. The initial values of the parameters are all set to 0.01. The up-bound of the index to stop iteration is set as $\varepsilon = 1\times 10^{-4}$.

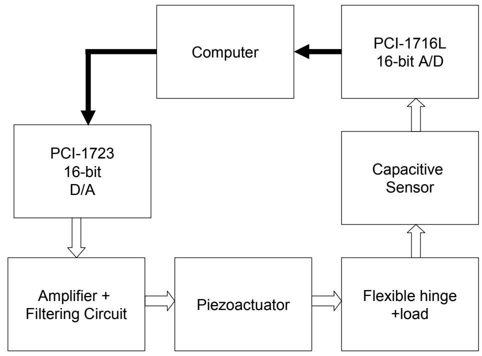

Fig. 2. Experimental setup of the positioning stage

After 400 steps, all the parameters converge. Fig.3 illustrates the convergence procedure of all the estimated parameters of the model to describe the positioning stage. Based on the estimated parameters, we obtain the correspondingly estimated sandwich model, i.e.

$$\hat{w}_1(k) = 0.529\hat{w}_1(k-1) + u(k-1)\,, \tag{26}$$

$$\begin{aligned} \hat{w}_2(k) = \hat{w}_2(k-1) &+ 0.8365[0.1354\hat{w}_1(k-1) \\ &- \hat{w}_2(k-1)]\,|\,\Delta\hat{w}_1(k)\,| + 0.0042\Delta\hat{w}_1(k) \end{aligned}, \tag{27}$$

and

$$\hat{y}(k) = 1.6049\hat{y}(k-1) - 0.9226\hat{y}(k-2) + \hat{w}_2(k)\,. \tag{28}$$

The corresponding model validation result is shown in Fig.4. The beginning output on the duration from 0.00 *sec* to 0.0017 *sec* is removed due to the large error induced by the influence of initial values. In Fig.4, it can be seen that the range of output variation is from $3.7\mu m$ to $6\mu m$, while the corresponding variation of modelling error is from $-0.059\mu m$ to $0.089\mu m$.

For comparison, a sandwich model with Prandtl-Ishlinskii (PI) hysteresis submodel is also used to describe the positioning stage. The structure of the model is shown as follows:

$$L_1(\cdot): w_1(k) = -a_{11}w_1(k-1) - a_{12}w_1(k-2) + b_{11}u(k-1) + b_{12}u(k-2) \quad (29)$$

$H(\cdot)$:

$$w_2(k) = \sum_{i=0}^{N} \mu_i \cdot p_i(k) = \sum_{i=0}^{N} \mu_i \cdot \{\max\{w_1(k) - r_i,\ \min[w_1(k) + r_i, p_i(k-1)]\}\} \quad (30)$$

$$L_2(\cdot): y(k) = -a_{21}y(k-1) - a_{22}y(k-2) + b_{21}w_2(k-1) + b_{22}w_2(k-2) \quad (31)$$

where N is the number of backlash operators in the PI hysteresis submodel, in this experiment, N=40; r_i is the threshold of backlash operator.

In the experiment, the signals used for model identification and validation are the same as what we used in the proposed method. In the identification, $\mathbf{P}(0) = 10^6 \times \mathbf{I}$ is the initial value of covariance matrix, and $\theta(0) = [0.001, \cdots, 0.001]^T$ is the initial value of the parameters vector. The index to stop identification is also set as $\varepsilon = 1 \times 10^{-4}$.

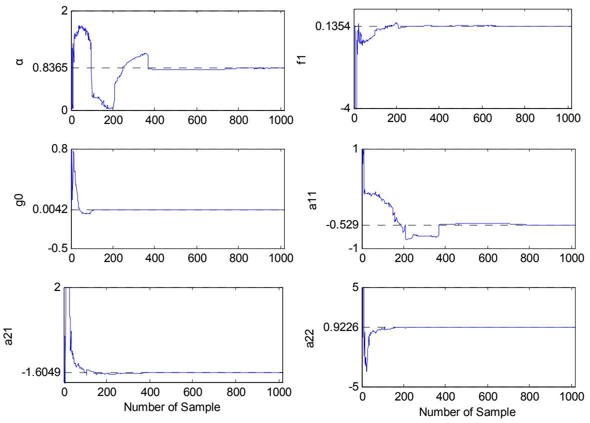

Fig.3. Convergence procedure of the estimated parameters (The proposed method)

After 2698 steps, all the estimated parameters almost converge. The corresponding convergent procedure of the parameters in linear submodels is shown in Fig.5. The convergence of the weights in the PI hysteresis submodel is omitted due to limited space. The corresponding model validation result is shown in Fig.6. From Fig.6, variation of modelling error is from $-0.162\mu m$ to $0.198\mu m$. From Figs.3 and 5, it is known that the proposed method has achieved faster convergence than that of the sandwich model with PI hysteresis submodel. From Figs.4 and 6, we also can see that the model validation error of the proposed modelling method is smaller than that of the sandwich model with PI hysteresis submodel. Moreover, the proposed method has simper model structure and less parameters needed to be estimated compared with the sandwich model with PI hysteresis submodel.

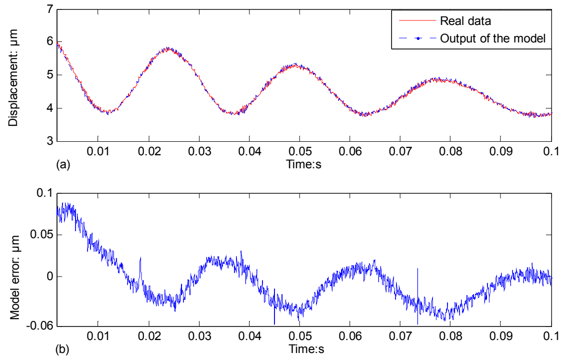

Fig.4. Model validation of the estimated sandwich model for positioning stage with piezoactuator

From the experiment, it is also known that the proposed sandwich model with hysteresis estimated by the ERIA is rather promising to describe the precision positioning stage with piezoactuator.

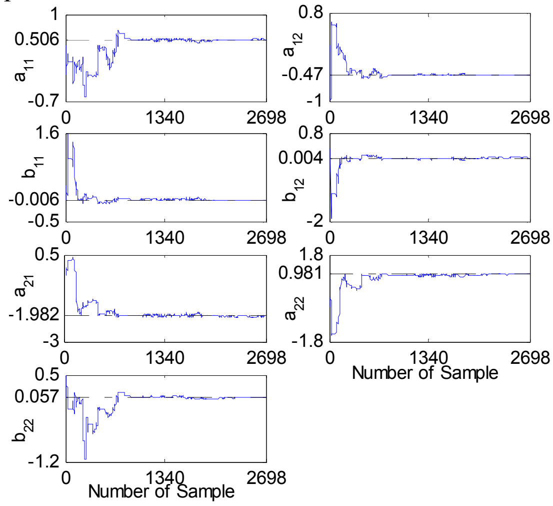

Fig.5. Convergence procedure of the estimated linear parameters of the stage (used by sandwich model with PI hysteresis submodel)

6. CONCLUSIONS

In this paper, a sandwich model with rate-dependent hysteresis is proposed to describe precision positioning stage with piezoactuator. In the modelling scheme, both amplifier + filtering circuit and flexible hinge + load are described by linear dynamic submodels, and the piezoactuator is described by a Duhem model. Based on a pseudo linearization transformation, the sandwich model can be formed as a linear combination of coefficients and the nonlinear variables. For on-line identification of the stage with non-smooth nonlinearity, an extended-recursive identification is developed to estimate the parameters of the non-smooth sandwich model with hysteresis on-line. Moreover, the convergence conditions of the identification algorithm are provided. The experiment on a positioning stage with piezoactuator has illustrated the promising potential of the proposed method to theoretical research and practical modelling of positioning stage.

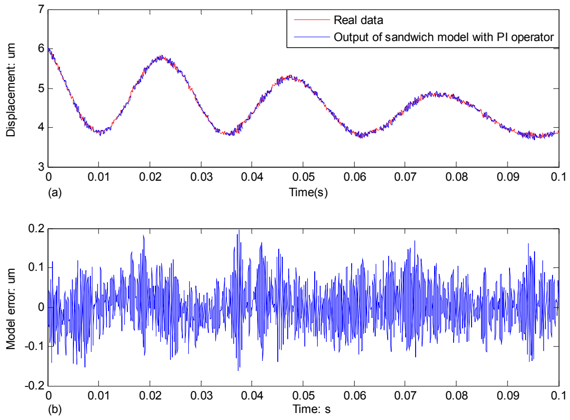

Fig.6. Model validation of the estimated sandwich model with PI hysteresis submodel for positioning stage with piezoactuator

ACKNOWLEDGMENTS

This research is partially supported by the Innovation Program of Shanghai Municipal Education Commission (Grant No. 13YZ056), the National Science Foundation of China (NSFC Grant Nos.: 61203108, 61371145 and 61171088).

REFERENCES

Zhang, Y., Y. Fang, X. Zhou, and X. Dong(2009). Image-based hysteresis modeling and compensation for an AFM piezo-scanner, *Asian Journal of Control*, **11(2)**, 166 -174.

Huang Y. and D. Lin (2004). Ultra-fine tracking control on piezoelectric actuated motion stage using piezoelectric hysteresis model, *Asian Journal of Control*, **6(2)**, 208-216.

Lin C. and S. Yang(2005). Modeling of a piezo-actuated positioning stage based on a hysteresis observer, *Asian Journal of Control*, **7(1)**, 73-80.

Bhikkaji B. and S. Moheimani (2008). Integral resonant control of a piezoelectric tube actuator for fast nanoscale positioning, *IEEE/ASME Trans. Mechatronics*, **13(5)**, 530–537.

Xu, Q. and Y. Li (2012). Model Predictive discrete-Time Sliding Mode Control of a Nanopositioning Piezostage without Modeling Hysteresis, *IEEE/ASME Trans. on Control Systems Technology* , **20(4)**, 983-994.

Xu, Q. and Y. Li(2010). Dahl model-based hysteresis compensation and precise positioning control of an XY parallel micromanipulator with piezoelectric actuation, *J. Dyn. Syst., Meas., Control*, **132(4)**, 425–437.

Janaideh, M., S. Rakheja, and C. Su (2011). An Analytical Generalized Prandtl-Ishlinskii Model Inversion for Hysteresis Compensation in Micropositioning Control, *IEEE/ASME Trans. on Mechatronics*, **16(4)**, 734-744.

Xie, Y., Y. Tan, and R. Dong (2013). Nonlinear Modeling and Decoupling Control of XY Micropositioning Stages with Piezoelectric Actuators, *IEEE/ASME Trans. on Mechatronics*,**18(3)**, 821-832.

Ge, P. (1996). *Modelling and control of hysteresis in piezoceramic actuator*, Ph.D. dissertation, Univ. Rhode Island, Kingston, RI, USA.

Mayergoyz, D.(1991). *Mathematical models of hysteresis*, Springer-Verlag, New York.

Oh, J. and D. Bernstein (2005). Semilinear Duhem model for rate-independent and rate-dependent hysteresis, *IEEE Trans. on Automatic Control*, **50(5)**, 631-645.

Jayawardhana, B., R. Ouyang, V. Andrieu(2012). Stability of systems with the Duhem hysteresis operator: The dissipativity approach, *Automatica*, **48(10)**, 2657-2662.

Meurer, T., J. Qu, and L.J. Jacobs (2002). Wave propagation in nonlinear and hysteretic media—a numerical study, *International Journal of Solids and Structures*, **39(21-22)**, 5585–5614.

Wang, Z., Z. Zhang and J. Mao (2012). Precision tracking control of piezoelectric actuator based on Bou-Wen hysteresis compensator, *Electronics letters*, **48(23)**, 1459-1460.

Visintin, A.(1994). *Differential Models of Hysteresis*. Springer-Verlag, New York.

Zhao, X. and Y. Tan (2008). Modeling hysteresis and its inverse model using neural networks based on expanded input space method, *IEEE Trans. on Control System Technology*, **16(3)**, 484–490.

Dong, R., Y. Tan, and H. Chen (2008). A neural networks based model for rate-dependent hysteresis for piezoceramic actuators, *Sensor and. Actuators: A*, **143(2)**, 370–376.

Boutayeb, M. and M. Darouach (1995). Recursive identification method for MISO Wiener- Hammerstein model, *IEEE Trans. on Automatic Control*, **40(2)**, 287-291.

Tan, A. and K. Godfrey (2002). Identification of Wiener-Hammerstein models using linear interpolation in frequency domain, *IEEE Trans. on Instrumentation and Measurement*, **51(3)**, 509-521.

Crama, P. and J. Schoukens (2005). Computing an initial estimate of a Wiener-Hammerstein system with a random phase multisine excitation, *IEEE Trans. on Instrumentation and Measurement*, **54(1)**, 117-122.

Kibangou, A. and G. Favier (2006). Wiener-Hammerstein systems modeling using diagonal Volterra kernels coefficients, *IEEE Signal Proc. Letters*, **13(6)**, 381-384

Tan, Y., R. Dong and R. Li (2009). Recursive identification of sandwich systems with dead zone and application", *IEEE Trans. on Control Systems Technology*, **17(4),** 945- 951.

Dong, R. and Y. Tan (2011). On-line identification algorithm and convergence analysis for sandwich systems with backlash. *International Journal of Control, Automation and Systems*. **9(3)**, 588-594.

Voros, J. (1995). Identification of nonlinear dynamic system using extended Hammerstein and Wiener models, *Control-Theory and Advanced Technology*, **10**, 1203-1212.

Zhao, X. (2003). *Modeling and control for hysteresis systems based on hysteretic operator*, Ph.D. dissertation, Shanghai Jiao Tong University, Shanghai, China.

Fang, C. (2004). *System Identification*, Qinghua University Press, Beijing, China.