Preprints of the 19th World Congress
The International Federation of Automatic Control
Cape Town, South Africa. August 24-29, 2014

Disturbance Decoupling with Stability in Continuous-Time Switched Linear Systems Under Dwell-Time Switching

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Abstract: This work deals with state feedback compensation of disturbance inputs in continuous-time switched linear systems, with the requirement that the closed-loop systems be exponentially stable under switching signals with a sufficiently large dwell-time. Constructive conditions for the problem to be solvable are shown, on the assumption that the given switched linear system has zero initial state. The effects of nonzero initial states are inspected. The theoretical background consists of both classic and novel ideas of the geometric approach, enhanced with notions specifically oriented to switched linear systems.

Keywords: switched linear systems; disturbance decoupling; exponential stability; dwell-time.

1. INTRODUCTION

In the last few decades, switched systems have effectively been employed in solving control problems that involve systems with different modes of operation: e.g., LQR optimal control (Balandat et al., 2012), $\mathcal{H}_2$ control (Mahmoud, 2009), $\mathcal{H}_\infty$ control (Deaecto et al., 2011), output regulation (Zattoni et al., 2013), model matching (Conte et al., 2014), and disturbance decoupling (Otsuka, 2010; Conte and Perdon, 2011; Zattoni and Marro, 2013) are typical synthesis problems recently formulated for switched systems. As to disturbance decoupling, the abovementioned papers are focused on the requirement that the closed-loop system be quadratically stable. In (Otsuka, 2010; Conte and Perdon, 2011), quadratic stability of the closed-loop system is sought for a suitable switching law. In (Zattoni and Marro, 2013), quadratic stability is requested for arbitrary switching signals. However, quadratic stability is quite a demanding specification. As is well-known (e.g., Lin and Antsaklis, 2009), quadratic stability under arbitrary switching is only a sufficient condition for asymptotic stability and could be rather restrictive. Moreover, it has also been shown that switched systems may not be asymptotically stable under arbitrary switching, but may enjoy this property for some classes of switching signals, satisfying specific constraints. In addition, restrictions on the switching signals may arise from physical constraints on the systems or may be inferred from some knowledge of the switching rules. For these reasons, in this work, we will investigate the problem of disturbance decoupling with exponential stability under restricted switching.

Notation: $\mathbb{R}$, $\mathbb{R}^+$, $\mathbb{Z}^+$, and $\mathbb{C}^-$ stand for the sets of real numbers, nonnegative real numbers, nonnegative integer numbers, and complex numbers with negative real part, respectively. Matrices and linear maps are denoted by upper-case letters, like A. The image, the kernel, and the spectrum of A are denoted by $\operatorname{im} A$, $\ker A$, and $\lambda(A)$, respectively. The transpose of A is denoted by $A^\top$. Vector spaces and subspaces are denoted by calligraphic letters, like $\mathcal{V}$. The quotient space of a subspace $\mathcal{V}$ over a subspace $\mathcal{W} \subseteq \mathcal{V}$ is denoted by $\mathcal{V}/\mathcal{W}$. The restriction of a linear map A to an A-invariant subspace $\mathcal{J}$ is denoted by $A|_\mathcal{J}$. The inverse image of a subspace $\mathcal{V}$ through a linear map B is denoted by $B^{-1}\mathcal{V}$. The symbol $\uplus$ denotes union with repetition count. The symbols I and O respectively stand for an identity matrix and a zero matrix with appropriate dimensions.

2. PROBLEM STATEMENT

Let $\Sigma_{\sigma(t)}$ be a continuous-time switched linear system defined by

$$\Sigma_{\sigma(t)} \equiv \begin{cases} \dot{x}(t) = A_{\sigma(t)}\, x(t) + B_{\sigma(t)}\, u(t) + H_{\sigma(t)}\, h(t), \\ e(t) = E_{\sigma(t)}\, x(t), \end{cases} \tag{1}$$

where $t \in \mathbb{R}^+$ is the time variable, $x \in \mathcal{X} = \mathbb{R}^n$ is the state, $u \in \mathbb{R}^p$ is the control input, $h \in \mathbb{R}^m$ is the disturbance input, and $e \in \mathbb{R}^q$ is the output, with $p, m, q \leq n$. Let the modes of $\Sigma_{\sigma(t)}$ be the linear time-invariant systems of the set $\{\Sigma_i,\ i \in \mathcal{I}\}$, where $\mathcal{I} = \{1, 2, \ldots, N\}$ and

$$\Sigma_i \equiv \begin{cases} \dot{x}(t) = A_i\, x(t) + B_i\, u(t) + H_i\, h(t), \\ e(t) = E_i\, x(t), \end{cases} \quad i \in \mathcal{I}, \tag{2}$$

with A_i, B_i, H_i, E_i constant real matrices of suitable dimensions. Let B_i, H_i, E_i be full-rank matrices. Let the sets of the admissible control input signals and of the admissible disturbance input signals be respectively defined as the sets of piecewise-continuous functions $u(t)$ and $h(t)$, with $t \in \mathbb{R}^+$, taking finite values in $\mathbb{R}^p$ and $\mathbb{R}^m$.

Let the switching signal $\sigma(t)$ be defined as a measurable and not a-priori known map $\sigma:\mathbb{R}^+\to\mathcal{I}$, $t\to i$, so that the active mode at the time $t\in\mathbb{R}^+$ is Σ_i, with $i=\sigma(t)$. The switching signal $\sigma(t)$ is assumed to be subject to time-domain restrictions as specified below. Let t_ℓ, with $\ell\in\mathbb{Z}^+$, be the sequence of the switching times. The positive real constant τ, defined as $\tau=\inf_{\ell\in\mathbb{Z}^+}\{t_{\ell+1}-t_\ell\}$, is assumed to be greater than or equal to a finite positive real constant τ_d. The set of all switching signals $\sigma(t)$ with τ no smaller than τ_d is denoted by $\mathscr{S}_{\tau_d}$ and the finite positive real constant τ_d is called dwell-time. Hence, the time-domain restriction on $\sigma(t)$ can be concisely expressed as $\sigma(t)\in\mathscr{S}_{\tau_d}$.

Let $F_{\sigma(t)}$ denote a switched state feedback, associated with the set $\{F_i\in\mathbb{R}^{p\times n},\ i\in\mathcal{I}\}$. Hence, the closed-loop system is described by the continuous-time switched linear system

$$\hat{\Sigma}_{\sigma(t)}\equiv\begin{cases}\dot{x}(t)\,=\,(A_{\sigma(t)}+B_{\sigma(t)}\,F_{\sigma(t)})\,x(t)+H_{\sigma(t)}\,h(t),\\ e(t)\,=\,E_{\sigma(t)}\,x(t),\end{cases}\tag{3}$$

with the modes

$$\hat{\Sigma}_i\equiv\begin{cases}\dot{x}(t)\,=\,(A_i+B_i\,F_i)\,x(t)+H_i\,h(t),\\ e(t)\,=\,E_i\,x(t),\end{cases}\qquad i\in\mathcal{I}.\tag{4}$$

Let the following assumption hold:

$\mathcal{A}\,1$. $x(0)=0$.

Assumption $\mathcal{A}\,1$ is a standing assumption in perfect decoupling problems. However, as will be observed in Remark 37, if the initial state is different from zero, zero output can still be guaranteed, provided that the initial state belongs to a certain subspace, which will be determined precisely. Moreover, as will be pointed out in Remark 38, if the initial state is different from zero and does not belong to the abovementioned subspace, asymptotic decoupling can be achieved in place of perfect decoupling, provided that suitable stability conditions are satisfied.

The problem of disturbance decoupling, with the requirement that the closed-loop system be exponentially stable under dwell-time switching, is stated as follows.

Problem 1. Given the continuous-time switched linear system $\Sigma_{\sigma(t)}$, defined by (1), with the modes $\{\Sigma_i,\ i\in\mathcal{I}\}$, defined by (2), find a switched state feedback $F_{\sigma(t)}$, associated with the set $\{F_i,\ i\in\mathcal{I}\}$, such that, on Assumption $\mathcal{A}\,1$, the following requirements are satisfied:

$\mathcal{R}\,1$. the output $e(t)$ be equal to zero for all $t\in\mathbb{R}^+$, for any admissible disturbance $h(t)$, with $t\in\mathbb{R}^+$;

$\mathcal{R}\,2$. the system $\hat{\Sigma}_{\sigma(t)}$, defined by (3), with the modes $\{\hat{\Sigma}_i,\ i\in\mathcal{I}\}$, defined by (4), be exponentially stable over $\mathscr{S}_{\tau_d}$, for some finite positive real constant τ_d.

3. GEOMETRIC APPROACH FOR SWITCHED LINEAR SYSTEMS

The purpose of this section is to gather the notions of the geometric approach that will be used to solve Problem 1. For the reader's convenience, some basic concepts are reviewed (Basile and Marro, 1992; Wonham, 1985). Novel geometric objects, like the reachability subspaces constrained to the maximal robust controlled invariant subspace, and new geometric ideas, like those of internal and external exponential stabilizability of the maximal robust controlled invariant subspace under dwell-time switching, are also introduced.

The definitions and properties surveyed below refer to the continuous-time switched linear system $\Sigma_{\sigma(t)}$, defined by (1), with the modes $\{\Sigma_i,\ i\in\mathcal{I}\}$, defined by (2). Short notations for images and null spaces of input and output matrices, respectively, are used: $\mathcal{B}_i=\operatorname{im}B_i$, $\mathcal{H}_i=\operatorname{im}H_i$, and $\mathcal{E}_i=\ker E_i$, with $i\in\mathcal{I}$. The subspace $\mathcal{E}\subseteq\mathcal{X}$ is defined by $\mathcal{E}=\bigcap_{i\in\mathcal{I}}\mathcal{E}_i$. A subspace $\mathcal{J}\subseteq\mathcal{X}$ is said to be a robust A_i-invariant subspace if $A_i\,\mathcal{J}\subseteq\mathcal{J}$, for all $i\in\mathcal{I}$. A subspace $\mathcal{V}\subseteq\mathcal{X}$ is said to be a robust $(A_i,\mathcal{B}_i)$-controlled invariant subspace if $A_i\,\mathcal{V}\subseteq\mathcal{V}+\mathcal{B}_i$, for all $i\in\mathcal{I}$. A subspace $\mathcal{V}\subseteq\mathcal{X}$ is a robust $(A_i,\mathcal{B}_i)$-controlled invariant subspace if and only if there exists a set of linear maps $\{F_i,\ i\in\mathcal{I}\}$, such that $(A_i+B_i\,F_i)\,\mathcal{V}\subseteq\mathcal{V}$, for all $i\in\mathcal{I}$.

As was first shown in (Basile and Marro, 1987), the set of all robust $(A_i,\mathcal{B}_i)$-controlled invariant subspaces contained in a given subspace $\mathcal{E}$ is an upper semilattice, with the sum as binary operation and the inclusion as partial ordering relation. The maximum of the set of all robust $(A_i,\mathcal{B}_i)$-controlled invariant subspaces contained in the subspace $\mathcal{E}$ is called the maximal robust $(A_i,\mathcal{B}_i)$-controlled invariant subspace contained in $\mathcal{E}$ and is denoted by $\mathcal{V}_R^*$. A double-recursion algorithm for computing $\mathcal{V}_R^*$ was also given in (Basile and Marro, 1987, Algorithm 1).

The remainder of this section is split into two parts. Section 3.1 is aimed at introducing the notions of internal switched dynamics and internal exponential stabilizability under dwell-time switching of the maximal robust controlled invariant subspace. The purpose of Section 3.2 is introducing the notions of external switched dynamics and external exponential stabilizability under dwell-time switching of the same subspace.

3.1 Internal Switched Dynamics and Internal Exponential Stabilizability Under Dwell-Time Switching of the Maximal Robust Controlled Invariant Subspace

In this work, the notion of maximal robust controlled invariant subspace contained in a given subspace is referred to the modes of a switched linear systems. Hence, switched dynamics can be induced on that subspace and stabilizability issues can be raised for those dynamics. This section is centred on the definition of internal switched dynamics and the property of exponential stabilizability under dwell-time switching of such dynamics. The exponential stabilizability under dwell-time switching of the internal dynamics of $\mathcal{V}_R^*$ depends on the properties of the fixed internal dynamics of $\mathcal{V}_R^*$ with respect to each system of the set $\{\Sigma_i,\ i\in\mathcal{I}\}$. In order to analyze this aspect in detail, the reachability subspace constrained to $\mathcal{V}_R^*$ — henceforth denoted by $\mathcal{R}_{\mathcal{V}_R^*,i}$ — is introduced for each system Σ_i. Hence, the assignable and fixed internal dynamics of $\mathcal{V}_R^*$ with respect to Σ_i can easily be singled out, since the assignable internal dynamics of $\mathcal{V}_R^*$ with respect to Σ_i coincides with the internal dynamics of $\mathcal{R}_{\mathcal{V}_R^*,i}$. Based on this fact and on a sufficient condition for a switched linear dynamics to be exponentially stable under dwell-time switching (Morse, 1996), a sufficient condition for the internal switched dynamics of $\mathcal{V}_R^*$ to be exponentially stabilizable under dwell-time switching is given. It is worth mentioning that the approach based on the constrained reachability subspaces distinguishes this work from previous ones on disturbance decoupling in switched linear systems (e.g., Conte and Perdon, 2011), where a kind of left-invertibility assumption was made. For the sake of brevity, the following statements are presented without proof.

Standard mathematical arguments show that, for any $i \in \mathcal{I}$, the set of all $(A_i, \mathcal{V}_R^*)$-conditioned invariant subspaces containing $\mathcal{B}_i$ is a lower semilattice, with the intersection as binary operation and the inclusion as partial ordering relation. Hence, the following definition is well-posed.

Definition 2. The minimum of the set of all $(A_i, \mathcal{V}_R^*)$-conditioned invariant subspaces containing $\mathcal{B}_i$ is called the minimal $(A_i, \mathcal{V}_R^*)$-conditioned invariant subspace containing $\mathcal{B}_i$ and is denoted by $\mathcal{S}_{\mathcal{V}_R^*,i}$.

Since Definition 2 differs from that of the minimal $(A_i, \mathcal{E}_i)$-conditioned invariant subspace containing $\mathcal{B}_i$ in the subspace $\mathcal{E}_i$ being replaced with $\mathcal{V}_R^*$, the algorithm for computing $\mathcal{S}_{\mathcal{V}_R^*,i}$ can be derived from (Basile and Marro, 1992, Algorithm 4.1-1) by the consistent modification.

Algorithm 3. For any $i \in \mathcal{I}$, the subspace $\mathcal{S}_{\mathcal{V}_R^*,i}$ is the last term of the sequence $\mathcal{S}^0_{\mathcal{V}_R^*,i} = \mathcal{B}_i$, $\mathcal{S}^j_{\mathcal{V}_R^*,i} = A_i\,(\mathcal{S}^{j-1}_{\mathcal{V}_R^*,i} \cap \mathcal{V}_R^*) + \mathcal{B}_i$, with $j = 1, \ldots, k_i$, where $k_i < n$ is the least integer such that $\mathcal{S}^{k_i+1}_{\mathcal{V}_R^*,i} = \mathcal{S}^{k_i}_{\mathcal{V}_R^*,i}$.

Definition 4. Let $\mathcal{V} \subseteq \mathcal{V}_R^*$ be an $(A_i, \mathcal{B}_i)$-controlled invariant subspace for some $i \in \mathcal{I}$. The subspace $\mathcal{V}$ is said to be self-bounded with respect to $\mathcal{V}_R^*$ if $\mathcal{V} \supseteq \mathcal{V}_R^* \cap \mathcal{B}_i$.

Proposition 5. Let the linear map F_i be such that $(A_i + B_i\,F_i)\,\mathcal{V}_R^* \subseteq \mathcal{V}_R^*$ for some $i \in \mathcal{I}$. Then, $(A_i + B_i\,F_i)\,\mathcal{V} \subseteq \mathcal{V}$ holds for any $(A_i, \mathcal{B}_i)$-controlled invariant subspace $\mathcal{V} \subseteq \mathcal{V}_R^*$ self-bounded with respect to $\mathcal{V}_R^*$.

Proposition 6. Let $\mathcal{V}_1, \mathcal{V}_2 \subseteq \mathcal{V}_R^*$ be $(A_i, \mathcal{B}_i)$-controlled invariant subspaces self-bounded with respect to $\mathcal{V}_R^*$ for some $i \in \mathcal{I}$. Then, $\mathcal{V} = \mathcal{V}_1 \cap \mathcal{V}_2$ is an $(A_i, \mathcal{B}_i)$-controlled invariant subspace self-bounded with respect to $\mathcal{V}_R^*$.

Owing to Proposition 6, for any $i \in \mathcal{I}$, the set of all $(A_i, \mathcal{B}_i)$-controlled invariant subspaces self-bounded with respect to $\mathcal{V}_R^*$ is closed with respect to the intersection. Moreover, the set of all $(A_i, \mathcal{B}_i)$-controlled invariant subspaces contained in a given subspace is an upper semilattice, with the sum as binary operation and the inclusion as partial ordering relation. Hence, the set of all $(A_i, \mathcal{B}_i)$-controlled invariant subspaces self-bounded with respect to $\mathcal{V}_R^*$ is a lattice — henceforth denoted by $\Phi_{\mathcal{B}_i,\mathcal{V}_R^*}$ — with the sum and the intersection as binary operations and the inclusion as partial ordering relation. The maximum of $\Phi_{\mathcal{B}_i,\mathcal{V}_R^*}$ is $\mathcal{V}_R^*$, which is independent of $i \in \mathcal{I}$. The next theorem provides the minimum of $\Phi_{\mathcal{B}_i,\mathcal{V}_R^*}$, which depends on $i \in \mathcal{I}$, in general.

Theorem 7. For any $i \in \mathcal{I}$, the minimum of $\Phi_{\mathcal{B}_i,\mathcal{V}_R^*}$ is the subspace $\mathcal{S}_{\mathcal{V}_R^*,i} \cap \mathcal{V}_R^*$, denoted by $\mathcal{R}_{\mathcal{V}_R^*,i}$ and called the reachability subspace constrained to $\mathcal{V}_R^*$.

Hence, Theorem 7 has introduced the reachability subspace constrained to $\mathcal{V}_R^*$ for each mode of the set $\{\Sigma_i,\, i \in \mathcal{I}\}$. The objective of the following statements is to show that, for any $i \in \mathcal{I}$, the internal dynamics of $\mathcal{R}_{\mathcal{V}_R^*,i}$ is assignable and matches the internal assignable dynamics of $\mathcal{V}_R^*$ with respect to Σ_i.

Lemma 8. Consider the continuous-time linear time-invariant systems of the set $\{\Sigma_i,\, i \in \mathcal{I}\}$ and the maximal robust $(A_i, \mathcal{B}_i)$-controlled invariant subspace contained in $\mathcal{E}$, $\mathcal{V}_R^*$. For any $i \in \mathcal{I}$, consider the minimal $(A_i, \mathcal{V}_R^*)$-conditioned invariant subspace containing $\mathcal{B}_i$, $\mathcal{S}_{\mathcal{V}_R^*,i}$, and the constrained reachability subspace $\mathcal{R}_{\mathcal{V}_R^*,i}$. Perform the state-space basis transformation $T_i' = [\,T'_{1,i}\;\; T'_{2,i}\;\; T'_{3,i}\;\; T'_{4,i}\,]$, with $\operatorname{im} T'_{1,i} = \mathcal{R}_{\mathcal{V}_R^*,i}$, $\operatorname{im}\,[\,T'_{1,i}\;\, T'_{2,i}\,] = \mathcal{V}_R^*$, $\operatorname{im}\,[\,T'_{1,i}\;\, T'_{3,i}\,] = \mathcal{S}_{\mathcal{V}_R^*,i}$, and the control-input-space basis transformation $U_i = [\,U_{1,i}\;\; U_{2,i}\,]$, with $\operatorname{im} U_{1,i} = B_i^{-1}\mathcal{V}_R^*$. Then, with respect to the new coordinates,

$$A_i' = (T_i')^{-1} A_i\, T_i' = \begin{bmatrix} A'_{11,i} & A'_{12,i} & A'_{13,i} & A'_{14,i} \\ O & A'_{22,i} & A'_{23,i} & A'_{24,i} \\ A'_{31,i} & A'_{32,i} & A'_{33,i} & A'_{34,i} \\ O & O & A'_{43,i} & A'_{44,i} \end{bmatrix}, \quad (5)$$

$$B_i' = (T_i')^{-1} B_i\, U_i = \begin{bmatrix} B'_{11,i} & B'_{12,i} \\ O & O \\ O & B'_{32,i} \\ O & O \end{bmatrix}, \quad (6)$$

$$H_i' = (T_i')^{-1} H_i = \begin{bmatrix} H'_{1,i} \\ H'_{2,i} \\ H'_{3,i} \\ H'_{4,i} \end{bmatrix}, \quad (7)$$

$$E_i' = E_i\, T_i' = \begin{bmatrix} O & O & E'_{3,i} & E'_{4,i} \end{bmatrix}. \quad (8)$$

Lemma 9. Consider the continuous-time linear time-invariant systems of the set $\{\Sigma_i,\, i \in \mathcal{I}\}$ and the maximal robust $(A_i, \mathcal{B}_i)$-controlled invariant subspace contained in $\mathcal{E}$, $\mathcal{V}_R^*$. Let the set $\{F_i,\, i \in \mathcal{I}\}$ be such that

$$(A_i + B_i\, F_i)\, \mathcal{V}_R^* \subseteq \mathcal{V}_R^*, \quad \forall\, i \in \mathcal{I}. \quad (9)$$

For any $i \in \mathcal{I}$, refer to the coordinates introduced in Lemma 8 and let

$$F_i' = U_i^{-1} F_i\, T_i' = \begin{bmatrix} F'_{11,i} & F'_{12,i} & F'_{13,i} & F'_{14,i} \\ F'_{21,i} & F'_{22,i} & F'_{23,i} & F'_{24,i} \end{bmatrix} \quad (10)$$

be partitioned accordingly. Then, with respect to the new coordinates, $A'_{F,i} = A_i' + B_i'\, F_i'$ has the structure shown in (11).

Definition 10. For any $i \in \mathcal{I}$, the restricted linear map $(A_i + B_i\, F_i)|_{\mathcal{R}_{\mathcal{V}_R^*,i}}$ is called the internal dynamics of $\mathcal{R}_{\mathcal{V}_R^*,i}$.

Definition 11. For any $i \in \mathcal{I}$, the restricted linear map $(A_i + B_i\, F_i)|_{\mathcal{V}_R^*}$ is called the internal dynamics of $\mathcal{V}_R^*$ with respect to the system Σ_i.

Remark 12. For any $i \in \mathcal{I}$, the restricted linear map $(A_i + B_i\, F_i)|_{\mathcal{R}_{\mathcal{V}_R^*,i}}$ is represented by the matrix

$$X'_{R,i} = A'_{11,i} + B'_{11,i}\, F'_{11,i} + B'_{12,i}\, F'_{21,i},$$

with respect to the coordinates introduced in Lemma 8.

Remark 13. For any $i \in \mathcal{I}$, the restricted linear map $(A_i + B_i\, F_i)|_{\mathcal{V}_R^*}$ is represented by the matrix

$$X'_{V,i} = \begin{bmatrix} X'_{R,i} & A'_{12,i} + B'_{11,i}\, F'_{12,i} + B'_{12,i}\, F'_{22,i} \\ O & A'_{22,i} \end{bmatrix},$$

with respect to the coordinates introduced in Lemma 8.

$$A'_{F,i} = A'_i + B'_i\, F'_i = \begin{bmatrix} A'_{11,i} + B'_{11,i}\, F'_{11,i} & A'_{12,i} + B'_{11,i}\, F'_{12,i} & A'_{13,i} + B'_{11,i}\, F'_{13,i} & A'_{14,i} + B'_{11,i}\, F'_{14,i} \\ +B'_{12,i}\, F'_{21,i} & +B'_{12,i}\, F'_{22,i} & +B'_{12,i}\, F'_{23,i} & +B'_{12,i}\, F'_{24,i} \\ O & A'_{22,i} & A'_{23,i} & A'_{24,i} \\ O & O & A'_{33,i} + B'_{32,i}\, F'_{23,i} & A'_{34,i} + B'_{32,i}\, F'_{24,i} \\ O & O & A'_{43,i} & A'_{44,i} \end{bmatrix}. \tag{11}$$

Proposition 14. For any $i \in \mathcal{I}$, the spectrum $\lambda((A_i + B_i\, F_i)|_{\mathcal{R}_{\mathcal{V}^*_R,i}})$ is assignable.

Proposition 15. For any $i \in \mathcal{I}$, the spectrum $\lambda((A_i + B_i\, F_i)|_{\mathcal{V}^*_R/\mathcal{R}_{\mathcal{V}^*_R,i}})$ is fixed.

Definition 16. The switched linear dynamics $(A_{\sigma(t)} + B_{\sigma(t)}\, F_{\sigma(t)})|_{\mathcal{V}^*_R}$, associated with the set of restrictions $\{(A_i + B_i\, F_i)|_{\mathcal{V}^*_R},\, i \in \mathcal{I}\}$, is called the internal switched dynamics of $\mathcal{V}^*_R$.

Definition 17. The subspace $\mathcal{V}^*_R$ is said to be internally exponentially stabilizable over $\mathscr{S}_{\tilde{\tau}_d}$, with $\tilde{\tau}_d$ denoting a finite positive real constant, if there exists a set $\{F_i,\, i \in \mathcal{I}\}$, such that $\mathcal{V}^*_R$ is a robust $(A_i + B_i\, F_i)$-invariant subspace and the switched dynamics $(A_{\sigma(t)} + B_{\sigma(t)}\, F_{\sigma(t)})|_{\mathcal{V}^*_R}$, associated with the set of restrictions $\{(A_i + B_i\, F_i)|_{\mathcal{V}^*_R},\, i \in \mathcal{I}\}$, is exponentially stable over $\mathscr{S}_{\tilde{\tau}_d}$.

Proposition 18. There exists a sufficiently large, finite positive real constant $\tilde{\tau}_d$ such that the subspace $\mathcal{V}^*_R$ is internally exponentially stabilizable over $\mathscr{S}_{\tilde{\tau}_d}$ if

$$\lambda(A'_{22,i}) \subset \mathbb{C}^-, \quad \forall\, i \in \mathcal{I}, \tag{12}$$

with $A'_{22,i}$ defined as in Lemma 8.

3.2 External Switched Dynamics and External Stabilizability Under Dwell-Time Switching of the Maximal Robust Controlled Invariant Subspace

This section deals with the definition of the external switched dynamics of the maximal robust controlled invariant subspace and the property of exponential stabilizability under dwell-time switching of such dynamics. The exponential stabilizability under dwell-time switching of the external switched dynamics of $\mathcal{V}^*_R$ depends on the properties of the fixed external dynamics of $\mathcal{V}^*_R$ with respect to each system of the set $\{\Sigma_i,\, i \in \mathcal{I}\}$. In particular, the assignable external dynamics of $\mathcal{V}^*_R$ with respect to the system Σ_i coincides with the dynamics induced on the quotient space $(\mathcal{V}^*_R + \mathcal{R}_i)\,/\,\mathcal{V}^*_R$, where $\mathcal{R}_i$ denotes the reachable subspace of the pair (A_i, B_i) or, equivalently, the minimal A_i-invariant subspace containing $\mathcal{B}_i$. Based on this fact and on the sufficient condition for a switched linear dynamics to be exponentially stable under dwell-time switching already exploited in the previous section (Morse, 1996), a sufficient condition for the external switched dynamics of $\mathcal{V}^*_R$ to be externally exponentially stabilizable under dwell-time switching is given. As in Section 3.1, the statements are presented without proof.

Lemma 19. Consider the continuous-time linear time-invariant systems of the set $\{\Sigma_i,\, i \in \mathcal{I}\}$ and the maximal robust $(A_i, \mathcal{B}_i)$-controlled invariant subspace contained in $\mathcal{E}$, $\mathcal{V}^*_R$. For any $i \in \mathcal{I}$, consider the reachable subspace $\mathcal{R}_i$. Perform the state-space basis transformation $T''_i = [\,T''_{1,i}\;\; T''_{2,i}\;\; T''_{3,i}\,]$, where $\operatorname{im} T''_{1,i} = \mathcal{V}^*_R$ and $\operatorname{im}\,[\,T''_{1,i}\; T''_{2,i}\,] = \mathcal{V}^*_R + \mathcal{R}_i$, and the control-input-space basis transformation $U_i = [\,U_{1,i}\;\, U_{2,i}\,]$, with $\operatorname{im} U_{1,i} = B_i^{-1}\mathcal{V}^*_R$. Then, with respect to new coordinates,

$$A''_i = (T''_i)^{-1} A_i\, T''_i = \begin{bmatrix} A''_{11,i} & A''_{12,i} & A''_{13,i} \\ A''_{21,i} & A''_{22,i} & A''_{23,i} \\ O & O & A''_{33,i} \end{bmatrix}, \tag{13}$$

$$B''_i = (T''_i)^{-1} B_i\, U_i = \begin{bmatrix} B''_{11,i} & B''_{12,i} \\ O & B''_{22,i} \\ O & O \end{bmatrix}, \tag{14}$$

$$H''_i = (T''_i)^{-1} H_i = \begin{bmatrix} H''_{1,i} \\ H''_{2,i} \\ H''_{3,i} \end{bmatrix}, \tag{15}$$

$$E''_i = E_i\, T''_i = \begin{bmatrix} O & E''_{2,i} & E''_{3,i} \end{bmatrix}. \tag{16}$$

Lemma 20. Consider the continuous-time linear time-invariant systems of the set $\{\Sigma_i,\, i \in \mathcal{I}\}$ and the maximal robust $(A_i, \mathcal{B}_i)$-controlled invariant subspace contained in $\mathcal{E}$, $\mathcal{V}^*_R$. Let the set $\{F_i,\, i \in \mathcal{I}\}$ be such that (9) holds. For any $i \in \mathcal{I}$, refer to the coordinates introduced in Lemma 19 and let

$$F''_i = U_i^{-1}\, F_i\, T''_i = \begin{bmatrix} F''_{11,i} & F''_{12,i} & F''_{13,i} \\ F''_{21,i} & F''_{22,i} & F''_{23,i} \end{bmatrix} \tag{17}$$

be partitioned accordingly. Then, with respect to new coordinates, $A''_{F,i} = A''_i + B''_i\, F''_i$ has the structure shown in (18).

Definition 21. For any $i \in \mathcal{I}$, the restricted linear map $(A_i + B_i\, F_i)|_{\mathcal{X}/\mathcal{V}^*_R}$ is called the external dynamics of $\mathcal{V}^*_R$ with respect to the system Σ_i.

Remark 22. For any $i \in \mathcal{I}$, the restricted linear map $(A_i + B_i\, F_i)|_{\mathcal{X}/\mathcal{V}^*_R}$ is represented by the matrix

$$X''_{V,i} = \begin{bmatrix} A''_{22,i} + B''_{22,i}\, F''_{22,i} & A''_{23,i} + B''_{22,i}\, F''_{23,i} \\ O & A''_{33,i} \end{bmatrix}$$

with respect to the coordinates introduced in Lemma 19.

Proposition 23. For any $i \in \mathcal{I}$, the spectrum $\lambda((A_i + B_i\, F_i)|_{(\mathcal{V}^*_R + \mathcal{R}_i)/\mathcal{V}^*_R})$ is assignable.

Proposition 24. For any $i \in \mathcal{I}$, the spectrum $\lambda((A_i + B_i\, F_i)|_{\mathcal{X}/(\mathcal{V}^*_R + \mathcal{R}_i)})$ is fixed.

Definition 25. The switched linear dynamics $(A_{\sigma(t)} + B_{\sigma(t)}\, F_{\sigma(t)})|_{\mathcal{X}/\mathcal{V}^*_R}$, associated with the set of restrictions $\{(A_i + B_i\, F_i)|_{\mathcal{X}/\mathcal{V}^*_R},\, i \in \mathcal{I}\}$, is called the external switched dynamics of $\mathcal{V}^*_R$.

Definition 26. The subspace $\mathcal{V}^*_R$ is said to be externally exponentially stabilizable over $\mathscr{S}_{\bar{\tau}_d}$, with $\bar{\tau}_d$ denoting a finite positive real constant, if there exists a set $\{F_i,\, i \in \mathcal{I}\}$, such that $\mathcal{V}^*_R$ is a robust $(A_i + B_i\, F_i)$-invariant subspace and the switched dynamics $(A_{\sigma(t)} + B_{\sigma(t)}\, F_{\sigma(t)})|_{\mathcal{X}/\mathcal{V}^*_R}$, associated

$$A''_{F,i} = A''_i + B''_i F''_i = \begin{bmatrix} A''_{11,i} + B''_{11,i} F''_{11,i} + B''_{12,i} F''_{21,i} & A''_{12,i} + B''_{11,i} F''_{12,i} + B''_{12,i} F''_{22,i} & A''_{13,i} + B''_{11,i} F''_{13,i} + B''_{12,i} F''_{23,i} \\ O & A''_{22,i} + B''_{22,i} F''_{22,i} & A''_{23,i} + B''_{22,i} F''_{23,i} \\ O & O & A''_{33,i} \end{bmatrix}. \quad (18)$$

with the set of restrictions $\{(A_i + B_i F_i)|_{\mathcal{X}/\mathcal{V}^*_R},\ i \in \mathcal{I}\}$, is exponentially stable over $\mathscr{S}_{\tilde{\tau}_d}$.

Proposition 27. There exists a sufficiently large, finite positive real constant $\tilde{\tau}_d$ such that the subspace $\mathcal{V}^*_R$ is externally exponentially stabilizable over $\mathscr{S}_{\tilde{\tau}_d}$ if

$$\lambda(A''_{33,i}) \subset \mathbb{C}^-, \quad \forall\, i \in \mathcal{I}, \tag{19}$$

with $A''_{33,i}$ defined as in Lemma 19.

4. PROBLEM SOLUTION

In this section, a constructive procedure for solving Problem 1 is presented. The following conditions will be shown to be sufficient for the design of the switched state feedback. The former condition is also proven to be necessary for structural decoupling (i.e., Problem 1 with the sole Requirement $\mathcal{R}\,1$). Arguments explaining why the latter conditions are not necessary for decoupling with stability are provided. Consider the continuous-time switched linear system $\Sigma_{\sigma(t)}$, with the modes $\{\Sigma_i,\ i \in \mathcal{I}\}$, the maximal robust $(A_i, \mathcal{B}_i)$-controlled invariant subspace contained in $\mathcal{E}$, $\mathcal{V}^*_R$, and the images $\mathcal{H}_i$ of the disturbance input matrices H_i, with $i \in \mathcal{I}$. The structural condition is

$\mathcal{C}\,1.$ $\mathcal{H}_i \subseteq \mathcal{V}^*_R, \quad \forall\, i \in \mathcal{I}.$

The stabilizability conditions are

$\mathcal{C}\,2.$ $\mathcal{V}^*_R$ is internally exponentially stabilizable over $\mathscr{S}_{\tau_d}$,
$\mathcal{C}\,3.$ $\mathcal{V}^*_R$ is externally exponentially stabilizable over $\mathscr{S}_{\tau_d}$,

where τ_d denotes a finite positive real constant.

In order to give Conditions $\mathcal{C}\,1$, $\mathcal{C}\,2$, and $\mathcal{C}\,3$ a characterization functional to the synthesis of the switched state feedback, a state-space basis transformation common to all the systems of the set $\{\Sigma_i,\ i \in \mathcal{I}\}$ is applied.

Lemma 28. Consider the continuous-time linear time-invariant systems of the set $\{\Sigma_i,\ i \in \mathcal{I}\}$ and the maximal robust $(A_i, \mathcal{B}_i)$-controlled invariant subspace contained in $\mathcal{E}$, $\mathcal{V}^*_R$. Perform the state-space basis transformation $T = [\,T_1\ T_2\,]$, where $\operatorname{im} T_1 = \mathcal{V}^*_R$. Then, with respect to new coordinates,

$$A^*_i = T^{-1} A_i\, T = \begin{bmatrix} A^*_{11,i} & A^*_{12,i} \\ A^*_{21,i} & A^*_{22,i} \end{bmatrix}, \tag{20}$$

$$B^*_i = T^{-1} B_i = \begin{bmatrix} B^*_{1,i} \\ B^*_{2,i} \end{bmatrix}, \tag{21}$$

$$H^*_i = T^{-1} H_i = \begin{bmatrix} H^*_{1,i} \\ H^*_{2,i} \end{bmatrix}, \tag{22}$$

$$E^*_i = E_i\, T = \begin{bmatrix} O & E^*_{2,i} \end{bmatrix}, \tag{23}$$

for all $i \in \mathcal{I}$.

Lemma 29. Consider the continuous-time linear time-invariant systems of the set $\{\Sigma_i,\ i \in \mathcal{I}\}$ and the maximal robust $(A_i, \mathcal{B}_i)$-controlled invariant subspace contained in $\mathcal{E}$, $\mathcal{V}^*_R$. Let the set $\{F_i,\ i \in \mathcal{I}\}$ be such that $\mathcal{V}^*_R$ is a robust $(A_i + B_i F_i)$-invariant subspace. Refer to the coordinates introduced in Lemma 28 and let

$$F^*_i = F_i\, T = \begin{bmatrix} F^*_{1,i} & F^*_{2,i} \end{bmatrix}, \tag{24}$$

be partitioned accordingly, for all $i \in \mathcal{I}$. Then, with respect to new coordinates,

$$A^*_{F,i} = A^*_i + B^*_i F^*_i = \begin{bmatrix} A^*_{11,i} + B^*_{1,i} F^*_{1,i} & A^*_{12,i} + B^*_{1,i} F^*_{2,i} \\ O & A^*_{22,i} + B^*_{2,i} F^*_{2,i} \end{bmatrix}, \tag{25}$$

for all $i \in \mathcal{I}$.

Remark 30. The set of restricted linear maps $\{(A_i + B_i F_i)|_{\mathcal{V}^*_R},\ i \in \mathcal{I}\}$, associated with the restricted switched dynamics $(A_{\sigma(t)} + B_{\sigma(t)} F_{\sigma(t)})|_{\mathcal{V}^*_R}$, is represented by the set of matrices $\{A^*_{11,i} + B^*_{1,i} F^*_{1,i},\ i \in \mathcal{I}\}$, with respect to the coordinates introduced in Lemma 28.

Remark 31. The set of restricted linear maps $\{(A_i + B_i F_i)|_{\mathcal{X}/\mathcal{V}^*_R},\ i \in \mathcal{I}\}$, associated with the restricted switched dynamics $(A_{\sigma(t)} + B_{\sigma(t)} F_{\sigma(t)})|_{\mathcal{X}/\mathcal{V}^*_R}$, is represented by the set of matrices $\{A^*_{22,i} + B^*_{2,i} F^*_{2,i},\ i \in \mathcal{I}\}$, with respect to the coordinates introduced in Lemma 28.

Conditions $\mathcal{C}\,1$, $\mathcal{C}\,2$, and $\mathcal{C}\,3$ are expressed in coordinate-free terms. The following propositions provide respectively equivalent statements, referred to the coordinates introduced in Lemma 28.

Proposition 32. Consider the continuous-time linear time-invariant systems of the set $\{\Sigma_i,\ i \in \mathcal{I}\}$, the maximal robust $(A_i, \mathcal{B}_i)$-controlled invariant subspace contained in $\mathcal{E}$, $\mathcal{V}^*_R$, and the images $\mathcal{H}_i$ of the disturbance input matrices H_i, with $i \in \mathcal{I}$. Let $r = \dim \mathcal{V}^*_R$. Refer to the coordinates introduced in Lemma 28. Then, Condition $\mathcal{C}\,1$ holds if and only if matrices $\Lambda_i \in \mathbb{R}^{r \times m}$, with $i \in \mathcal{I}$, exist, such that

$$\begin{bmatrix} H^*_{1,i} \\ H^*_{2,i} \end{bmatrix} = \begin{bmatrix} \Lambda_i \\ O \end{bmatrix}, \quad \forall\, i \in \mathcal{I}. \tag{26}$$

Proposition 33. Consider the continuous-time switched linear system $\Sigma_{\sigma(t)}$, with the modes $\{\Sigma_i,\ i \in \mathcal{I}\}$, and the maximal robust $(A_i, \mathcal{B}_i)$-controlled invariant subspace contained in $\mathcal{E}$, $\mathcal{V}^*_R$. Refer to the coordinates introduced in Lemma 28 and let the set $\{F^*_i,\ i \in \mathcal{I}\}$ be partitioned as in Lemma 29. Condition $\mathcal{C}\,2$ holds if and only if there exists a set $\{F^*_{1,i},\ i \in \mathcal{I}\}$, such that

$$A^*_{21,i} + B^*_{2,i} F^*_{1,i} = 0, \quad \forall\, i \in \mathcal{I}, \tag{27}$$

and the switched dynamics $(A_{\sigma(t)} + B_{\sigma(t)} F_{\sigma(t)})|_{\mathcal{V}^*_R}$, associated with the set $\{A^*_{11,i} + B^*_{1,i} F^*_{1,i},\ i \in \mathcal{I}\}$, is exponentially stable over $\mathscr{S}_{\tau_d}$. Moreover, condition $\mathcal{C}\,3$ holds if and only if there exists a set $\{F^*_{2,i},\ i \in \mathcal{I}\}$, such that the switched dynamics $(A_{\sigma(t)} + B_{\sigma(t)} F_{\sigma(t)})|_{\mathcal{X}/\mathcal{V}^*_R}$, associated with the set $\{A^*_{22,i} + B^*_{2,i} F^*_{2,i},\ i \in \mathcal{I}\}$, is exponentially stable over $\mathscr{S}_{\tau_d}$.

Theorem 34. Consider the continuous-time switched linear system $\Sigma_{\sigma(t)}$, with the modes $\{\Sigma_i,\ i \in \mathcal{I}\}$. Let Assumption $\mathcal{A}\,1$ hold. Let Conditions $\mathcal{C}\,1$, $\mathcal{C}\,2$, and $\mathcal{C}\,3$ hold. Consider the switched closed-loop system $\hat{\Sigma}_{\sigma(t)}$, with the modes $\{\hat{\Sigma}_i,\ i\in\mathcal{I}\}$. Let the switched state feedback $F_{\sigma(t)}$ be chosen according to Proposition 33. Then, $\hat{\Sigma}_{\sigma(t)}$ satisfies the Requirements $\mathcal{R}\,1$ and $\mathcal{R}\,2$ of Problem 1.

Proof. First, it will be shown that Requirement $\mathcal{R}\,1$ of Problem 1 is met. Refer to the coordinates introduced in Lemma 28 and let the state $x(t)=[\,x_1(t)^\top\ \ x_2(t)^\top\,]^\top$, with $t\in\mathbb{R}^+$, be consistently partitioned. Hence, the modes $\{\hat{\Sigma}_i,\ i\in\mathcal{I}\}$ are described by

$$\hat{\Sigma}_i \equiv \left\{ \begin{aligned} \dot{x}_1(t) &= (A^*_{11,i} + B^*_{1,i}\,F^*_{1,i})\,x_1(t) + \\ &\quad\ (A^*_{12,i} + B^*_{1,i}\,F^*_{2,i})\,x_2(t) + \Lambda_i\,h(t), \\ \dot{x}_2(t) &= (A^*_{22,i} + B^*_{2,i}\,F^*_{2,i})\,x_2(t), \\ e(t) &= E^*_{2,i}\,x_2(t), \end{aligned} \right. \quad i\in\mathcal{I}, \tag{28}$$

where (20)–(24), (26), (27) have been taken into account. Assumption $\mathcal{A}\,1$ implies $x_1(0)=0$ and $x_2(0)=0$. Hence, $x_2(t)=0$, for all $t\in\mathbb{R}^+$, which implies $e(t)=0$, for all $t\in\mathbb{R}^+$, for any admissible disturbance $h(t)$, with $t\in\mathbb{R}^+$. In order to prove that Requirement $\mathcal{R}\,2$ is also met, note that (28) can be written as

$$\hat{\Sigma}_i \equiv \left\{ \begin{aligned} \dot{x}(t) &= A^*_{F,i}\,x(t) + H^*_i\,h(t), \\ e(t) &= E^*_i\,x(t), \end{aligned} \right. \quad i\in\mathcal{I}, \tag{29}$$

where $A^*_{F,i}$, H^*_i, E^*_i are respectively given by (25), (26), (23), with F^*_i determined according to Proposition 33, for all $i\in\mathcal{I}$. Hence, the switched dynamics $A_{F,\sigma(t)}$, associated with the set $\{A^*_{F,i},\ i\in\mathcal{I}\}$, is exponentially stable over $\mathscr{S}_{\tau_d}$ as a consequence of Proposition 33. $\square$

Remark 35. Condition $\mathcal{C}\,1$ is also necessary to solve the structural decoupling problem. Indeed, Condition $\mathcal{C}\,1$ is equivalent to the necessary and sufficient condition for structural decoupling given in (Otsuka, 2010, Theorem 3.1). Necessity of Condition $\mathcal{C}\,1$ hinges on the fact that if Condition $\mathcal{C}\,1$ is not met, no robust $(A_i,\mathcal{B}_i)$-controlled invariant subspace $\mathcal{V}_R$ contained in $\mathcal{E}$ exists, such that $\mathcal{H}_i\subseteq\mathcal{V}_R$, for all $i\in\mathcal{I}$, since the set of all robust $(A_i,\mathcal{B}_i)$-controlled invariant subspaces contained in $\mathcal{E}$ is an upper semilattice and $\mathcal{V}^*_R$ is the maximal robust $(A_i,\mathcal{B}_i)$-controlled invariant subspace contained in $\mathcal{E}$.

Remark 36. Conditions $\mathcal{C}\,2$ and $\mathcal{C}\,3$ are not necessary to solve Problem 1. In fact, if there exists a robust $(A_i,\mathcal{B}_i)$-controlled invariant subspace $\mathcal{V}_R\subset\mathcal{V}^*_R$, such that $\mathcal{H}_i\subseteq\mathcal{V}_R$, for all $i\in\mathcal{I}$, then internal and external exponential stabilizability of $\mathcal{V}_R$ over $\mathscr{S}_{\tau_d}$, with τ_d denoting a finite positive real constant, is sufficient to solve Problem 1, since the constructive procedure described in the proof of Theorem 34 can still be applied with $\mathcal{V}^*_R$ replaced by $\mathcal{V}_R$.

Remark 37. Equations (28) show that, if Assumption $\mathcal{A}\,1$ is not satisfied, zero output is still guaranteed, provided that the initial state belongs to the subspace $\mathcal{V}^*_R$. In fact, if the initial state belongs to $\mathcal{V}^*_R$, then $x_2(0)=0$. Hence, $x_2(t)=0$, for all $t\in\mathbb{R}^+$, which implies $e(t)=0$, for all $t\in\mathbb{R}^+$, for any admissible disturbance $h(t)$, with $t\in\mathbb{R}^+$.

Remark 38. Equations (28) show that, if Assumption $\mathcal{A}\,1$ is not satisfied and the initial state does not belong to $\mathcal{V}^*_R$, disturbance decoupling is achieved as the time goes to infinity. In fact, if the initial state is different from zero and does not belong to $\mathcal{V}^*_R$, then $x_2(0)\neq 0$. Hence, the component $x_2(t)$, with $t\in\mathbb{R}^+$, evolves according to the external switched dynamics $(A_{\sigma(t)} + B_{\sigma(t)}\,F_{\sigma(t)})|_{\mathcal{X}/\mathcal{V}^*_R}$, which is exponentially stable over $\mathscr{S}_{\tau_d}$. Hence, $x_2(t)$ goes to zero as t approaches infinity and so does $e(t)$.

5. CONCLUSIONS

A constructive procedure for the synthesis of a switched state feedback that achieves decoupling of inaccessible signals and exponential stability of the closed-loop system under dwell-time switching has been shown. The effects of a nonzero initial state have been considered. The methodological background consists of both classic and novel objects of the geometric approach, enhanced with notions specifically oriented to switched linear systems.

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