Reliability based multiobjective optimization design procedure for PI controller tuning. [*]

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Abstract: In this work, we propose an hybrid multi-objective optimization design procedure for PI controller tuning. This procedure focuses on reliability-based optimization instances, where Montecarlo methods are used to evaluate quantitatively the performance degradation of a controller, due to unexpected or unmodeled dynamics. The procedure is evaluated on a non-linear Peltier process. The presented results validate the procedure and its usefulness for controller tuning.

Keywords: PI controller tuning, multiobjective optimization, multiobjective optimization design procedure, evolutionary algorithms.

1. INTRODUCTION

PI-PID controller remains as a reliable and practical control solution for several industrial processes. Owing to this, research for new tuning techniques is an ongoing research topic (see for example Åström and Hägglund (2001); Vilanova and Alfaro (2011)). Current research is heading to guarantee reasonable stability margins as well as a good overall performance for a wide variety of processes. One of the main advantages of PI-PID controllers is their ease of implementation as well as their tuning, giving a good trade-off between simplicity and cost to implement (Stewart and Samad (2011)).

New tuning techniques have been focused on fulfill several objectives and requirements, sometimes in conflict among them (Ang et al. (2005); Li et al. (2006)). Some tuning procedures are based on optimization statements (Ge et al. (2002); Toscano (2005); Goncalves et al. (2008); Åström et al. (1998); Panagopoulos et al. (2002); Reynoso-Meza et al. (2013b)) and recently, PI-PID controller tuning by means of multiobjective optimization (MO) procedures have been proposed (Tavakoli et al. (2007); Sánchez and Vilanova (2013a,b)). Such procedures are based on the so called Pareto front approximation, where all the solutions are Pareto optimal; meaning that there is no solution better in all objectives, but a set of solutions with different trade-offs.

Two common approaches are used to calculate the Pareto front approximation: on the one hand by means of deterministic algorithms and on the other hand by evolutionary multiobjective optimization (EMO). In the first case algorithms such the normalized normal constraint (NNC) developed by Messac et al. (2003) has shown to be useful for PI controller tuning (Sánchez and Vilanova (2013a,b)). The main advantage of this class of algorithms relies on their local convergence capabilities as well as their robustness to provide a good approximation of the Pareto front. However, they are highly sensitive to the initial solution required to run the optimization. Even if the objectives have been stated to guarantee convexity properties the constraints incorporated to achieve a good spread over the Pareto front approximation may modify the objective space. Also, some issues regarding computational burden and anchor points selection might appear as shown in Herrero et al. (2013). In the case of EMO, multiobjective evolutionary algorithms (MOEA's) have shown interesting properties to handle highly constrained and non-linear objective functions due to their flexibility and adaptability (Chai et al. (2013); Reynoso-Meza et al. (Accepted)); however two potential drawbacks are also known: (1) given their stochastic nature, their convergence can not be guaranteed and (2) the tuning of their own parameters could be a time consuming task and, if selected inappropriately, their performance could be deteriorated. Depending on the desirable characteristics related with the optimization problem at hand, one class of algorithms could be more desirable than the other. Regarding the desired Pareto front approximation, such desirable features will be related to convergence, diversity and preferences handling; regarding the multi-objective problem, they will be associated with constrained, multi-modal, robust, expensive, many-objectives, dynamic or reliability-based optimization instances.

In order to approximate a Pareto front based on an optimization reliability statement, it is important to measure the performance degradation (Beyer and Sendhoff (2007)). This could be caused by unmodeled dynamics or errors in the model used. Meaning that the designer has to keep in mind such discrepancies not only from the robustness point of view (ensuring controllability from a theoretical sense), but also the possible degradation of the objetives directly related with the model's use (ensuring viability from a practical sense). Thereby, a common approach to measure the performance degradation is by means of Montecarlo methods (*i.e.*, direct search through simulations). It is worth mentioning that running several simulations could affect the performance of deterministic algorithms because of computational burden; in the case of evolutionary algorithms it could affect their exploration capabilities and hence slow down the overall convergence of the algorithm. Therefore we present a hybrid approach between deterministic and evolutionary optimization techniques, to handle reliability based MO statements. With this approach, we expect solutions with a higher reliability, both in a theoretical and practical sense. The remains of this work is organized as follows: Section 2 briefly describes some ideas and concepts related to the multi-objective optimization; whilst in section 3 we present our proposal; in Section 4 an evaluation on a physical system is presented and discussed; finally, in Section 5 some concluding remarks are given.

2. BACKGROUND ON MULTIOBJECTIVE OPTIMIZATION DESIGN

A multi-objective optimization problem (MOP) can be handled by performing a simultaneous optimization of all objectives. This implies the existence of a set of solutions, where no one is better than the others, but differ in the degree of performance between the objectives (Miettinen (1998)). This set of solutions will offer a higher degree of flexibility at the decision making stage. The role of the designer is to select the most preferable solution according to his (her) needs and preferences for a particular situation. A MOP, without loss of generality (since a maximization problem can be converted to a minimization problem), can be stated as follows:

$$\min_{\boldsymbol{\theta} \in \Re^n} \boldsymbol{J}(\boldsymbol{\theta}) = [J_1(\boldsymbol{\theta}), \dots, J_m(\boldsymbol{\theta})] \in \Re^m \quad (1)$$

Therefore a set of Pareto-optimal solutions is defined as the Pareto set $\boldsymbol{\Theta_P}$ and its projection into the objective space is known as the Pareto front $\boldsymbol{J_P}$. Where each solution in the Pareto front is said to be a non-dominated and Pareto-optimal solution. In general, it does not exist a unique solution because there is not a solution better than other in all the objectives.

MO techniques search for a discrete approximation $\boldsymbol{\Theta_P^*}$ of the Pareto set $\boldsymbol{\Theta_P}$ capable of generate a good quality description of the Pareto front $\boldsymbol{J_P^*}$. In this way, the decision maker (or simply the designer) can analyze the set and select the most preferable solution.

A general framework is required to successfully incorporate the MO approach into any engineering process. A multiobjective optimization engineering design (MOOD) methodology consists (at least) of three main steps: the MOP definition (objectives, decision variables and constraints), the MO process (optimizer selection) and the decision-making (DM) stage (analysis and selection of the calculated solutions).

Next, we will present a proposal for each step regarding reliability-based optimization statements for PI controller tuning.

3. MULTIOBJECTIVE OPTIMIZATION DESIGN PROPOSAL

Reliability based optimization instances concern to guarantee a given performance, considering the possibility (in the case of controller tuning) of unmodeled dynamics. Nevertheless, for this kind of analysis Montecarlo approaches are generally used, in order to evaluate the degradation of the objectives when uncertainties on the system exist. In this case, several simulations should be carried, increasing the computational burden of the cost function itself. Next, we will proposed a MOOD statement to face such MO instances.

3.1 Multiobjective problem definition

The process will be controlled with a proportional-integral controller (PI), whose output is

$$u(t) = K_p \left\{ r(t) - y(t) + \frac{1}{T_i} \int_0^\infty [r(\tau) - y(\tau)] \, d\tau \right\} \quad (2)$$

where K_p is the controller gain, T_i is the integral time constant, and $r(t)$ and $y(t)$ the reference and controlled variable measurement, respectively.

Perhaps the most basic MOP statement for PI controller tuning including performance's reliability as control goals could be represented as:

$$\min \boldsymbol{J}(k_p, T_i) = [J_1(k_p, T_i), J_2(k_p, T_i), J_3(k_p, T_i)] \quad (3)$$

$$J_1(k_p, T_i) = Performance(k_p, T_i)|_{G(s)} \quad (4)$$

$$J_2(k_p, T_i) = Robustness(k_p, T_i)|_{G(s)} \quad (5)$$

$$J_3(k_p, T_i) = \sigma \left(J_1(k_p, T_i)|_{G'(s)} \right) \quad (6)$$

Where $G(s)$ is the nominal model used as base case and $G'(s)$ any other model inside the possible set of models. Some common choices for $J_1(k_p, Ti)$ are $-ki = -\frac{k_p}{T_i}$, IAE and $ITAE$. These are recognized classical indexes to evaluate performance of control systems. For $J_2(k_p, T_i)$, indexes as the maximum value of the sensitivity function or total variation (TV) of the control action could be used; in the latter case, experiments in Sánchez and Vilanova (2013a) show that there is a correlation between the value of robustness and the total variation TV; we can therefore directly associate the robustness to the TV performance index and smoothness of control action. Finally $J_3(k_p, T_i)$ is the reliability based objective estimated by Montecarlo sampling approach.

3.2 Hybrid multiobjective optimization algorithm

As commented before, both deterministic and evolutionary algorithms have some drawbacks whilst leading with reliability based MO instances. Therefore, a hybrid approach (Algorithm I) using both could bring interesting alternatives and solutions. Here, we merge in a sequential manner both approaches for controller tuning, when a MOP as described by equations 3-6 is being solved.

We propose to merge two different tools: NNC algorithm (Messac et al. (2003)) and sp-MODE algorithm (Reynoso-Meza et al. (2010)).[1] Interested reader can refer to Appendix I and references therein for a brief description of the two algorithms.

Algorithm I:	*Hybrid NNC and spMODE algorithm.*
1 :	Determine the initial solution for NNC.
2 :	Compute anchor points for objectives $J_1(k_p, T_i)$ and $J_2(k_p, T_i)$.
3 :	Approximate the sub-Pareto front $\boldsymbol{J'}(k_p, T_u) = [J_1(k_p, T_i), J_2(k_p, T_i)]$ with NNC algorithm and the computed anchor points.
4 :	Use the Pareto set approximation $\boldsymbol{\Theta}'^*_p$, computed previously as initial population for sp-MODE algorithm.
5 :	Approximate Pareto front $\boldsymbol{J}(k_p, T_u) = [J_1(k_p, T_i), J_2(k_p, T_i), J_3(k_p, T_i)]$.
6 :	Perform DM step for controller selection.

The deterministic algorithm (NNC) is used in a bi-objective statement, where it will be less sensitive to the anchor solutions and will converge to the Pareto front; the evolutionary algorithm (sp-MODE) will use as initial population the locally Pareto-optimal approximation calculated by the deterministic approach in the 3-objective optimization instance. This would improve its exploitation capabilities and will reduce the numerical burden associated to the Montecarlo approach.

3.3 Multicriteria decision making

For decision making we propose the Nash (*NS*) criteria see (Figure 1). In order to explain this option, we introduce what can be called the disagreement point. If we think on both objectives independently, none of both would agree on this point as a common solution because it represents the worst situation. In addition, this selection can be improved with respect of both objectives. On that basis, the area of the rectangle defined by the points (NS, A, B) and the disagreement point provides a measure of the amount of solutions the NS point improves both objectives simultaneously. The *NS* is the one that maximizes such area, this denomination comes from identifying this point as the *Nash* Solution on a bargaining game among both objectives (Aumann and Hart (1994)). This approach provides a quite simple and direct approach for selecting one point from the Pareto front (Sánchez and Vilanova (2013b)).

Although a visual representation for two objectives is straightforward, for 3 objectives could be more complex. Here we are using level diagrams (Blasco et al. (2008))[2] as it represents a trade-off as commented in (Reynoso-Meza et al. (2013a)) among the properties for visualization stated by Lotov and Miettinen (2008): simplicity, persistence and completeness. Next, we will evaluate this approach on a physical system with such tools and ideas.

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[1] Both algorithms are available in Matlab Central at `http://www.mathworks.com/matlabcentral/fileexchange/38976` and `http://www.mathworks.com/matlabcentral/fileexchange/39215` respectively

[2] Tool available for Matlab at `http://www.mathworks.es/matlabcentral/fileexchange/24042`
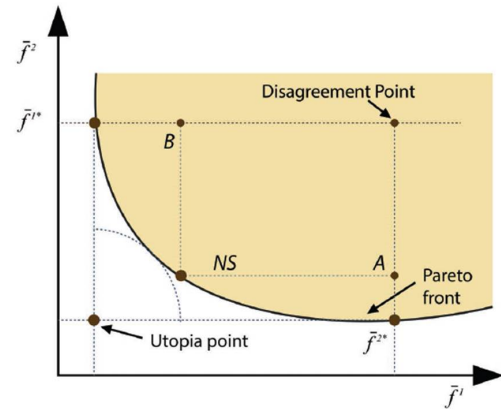

Fig. 1. The Nash solution for a bi-objective case.

4. TEST CASE

A Peltier cell (Figure 2) is selected to evaluate the above mentioned MOOD procedure. Such devices are based on the Peltier effect, which describes a cooling by means of thermoelectric processes. It is a heat pump where the manipulated variable is the current [%] whilst the controlled variable is the temperature [ºC] of the cold-face. This kind of processes has non-linear dynamics as it will be shown.

Successive step reference changes have been made in the range temperature $T_e = [-7.5ºC, 7.5ºC]$, in order to identify several first order plus dead time models (FOPDT) $P(s) = \frac{K}{Ts+1}e^{-Ls}$ where K is the process proportional gain, T the time constant and L the lag of the system. The resulting models are depicted in Figure 3. It is important to notice differences among models concerning K and T values, showing the non-linearity of the system.
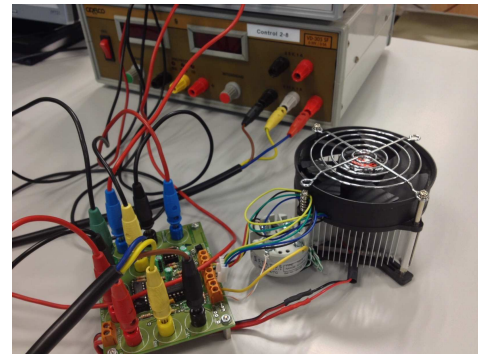

Fig. 2. Peltier cell set-up.

We are interested in tuning a controller to mainly work around $T_e = -5ºC$, but capable of achieving a good behavior in the range $T_e = [-7.5ºC, 7.5ºC]$. Therefore, as nominal model is selected the identified model at $-5ºC$:

$$P(s) = \frac{0.6404}{2.7395s + 1}e^{-0.2s} \qquad (7)$$
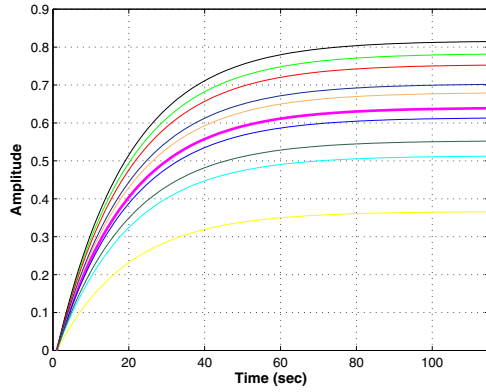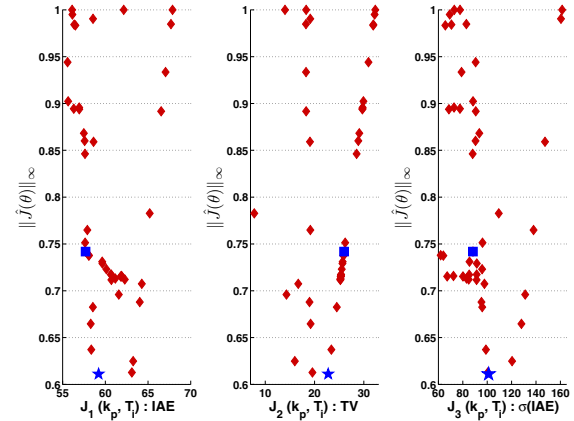
The objectives considered for controller tuning are:

Fig. 3. Step response for identified models of the Peltier cell at different temperatures.
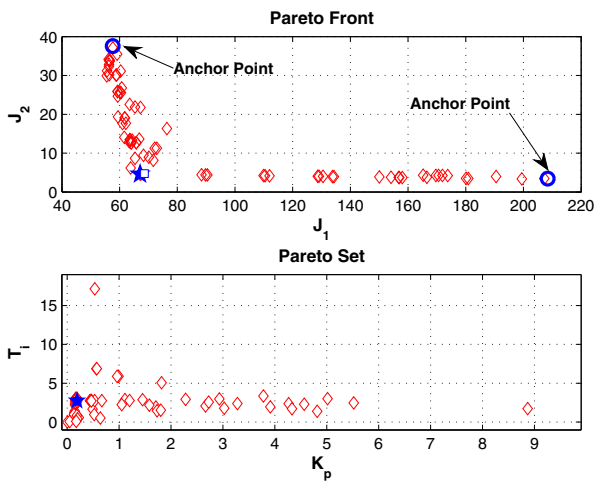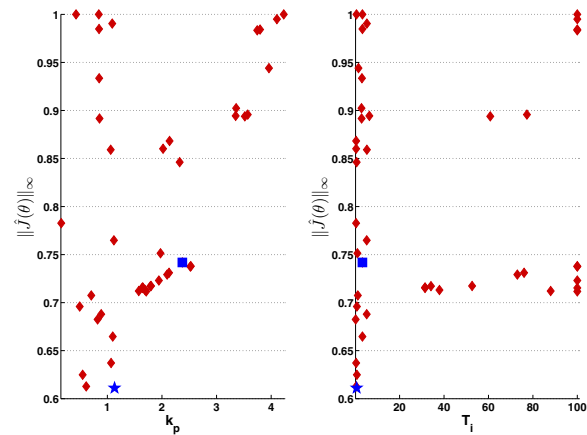

Fig. 4. Pareto front and Pareto set approximations calculated by NNC algorithm (line 1, 2 and 3 of the Algorithm I). Controller IS (square) and Controller NS-2D (star) are depicted.

$$J_1(k_p, T_i) = T_s \cdot \sum_{k=0}^{K} |e(k+1) - e(k)| \Bigg|_{P(s)} \qquad (8)$$

$$J_2(k_p, T_i) = \sum_{k=T_0}^{K} |u(k+1) - u(k)| \Bigg|_{P(s)} \qquad (9)$$

$$J_3(k_p, T_i) = \sigma \left(\sum_{k=0}^{K} |e(k+1) - e(k)| \Bigg|_{P'(s)} \right) \qquad (10)$$

Where $T_s = 200$ milliseconds is the sampling time, $J_1(k_p, T_i)$ is the IAE, $J_2(k_p, T_i)$ is the TV index and $J_3(k_p, T_i)$ is the variance of $J_1(k_p, T_i)$ when the controller is used in a random uniformly sampled model $P'(s)$ in the intervals $K = 0.6404 \pm 50\%$ and $T = 2.7395 \pm 30\%$. For this example, 51 models were used.

4.1 Multiobjective optimization and selection procedure

The preliminary bi-objective Pareto front $J_P'^*$ is approximated with the NNC algorithm (11,000 function evaluations approximately). As commented before, the NNC

(a) Pareto Front

(b) Pareto Set

Fig. 5. Pareto front and Pareto set approximations (∞-norm) calculated by sp-MODE algorithm (line 5 of Algorithm I). Controller NS-3D (star) and Controller LD-3D (square) are depicted.

algorithm is focuses on objectives $J_1(k_p, T_i)$ and $J_2(k_p, T_i)$. This approximation is depicted in Figure 4.

With such Pareto front, 2 solutions are selected for further evaluation: the initial solution employed in the optimization process $[k_p, Ti] = [0.19, 2.74]$ and the Nash-based $[k_p, Ti] = [0.1898, 2.6613]$ (IS and NS-2D respectively). In the execution using the sp-MODE (11,000 function evaluations) and the Pareto front approximation from NNC algorithm, pertinency is included in the algorithm to bound the new objective using the starting solution of the NNC algorithm. In order to avoid controllers with high degradation on the third objective, the maximum value $\overline{J_3}$ for $J_3(k_p, T_i)$ has been bound to $\overline{J_3} = J_3(0.19, 2.74)$. This approximation is depicted in Figure 5. Two solutions were selected: Nash-based $[k_p, T_i] = [0.5091, 0.4057]$ and a solution selected by analyzing the Pareto front using Level diagrams: $[k_p, T_i] = [2.3735, 3.2623]$ (NS-3D and LD-3D respectively). In the former selection, is interesting to notice that the Nash selection coincides with the solution with the lower ∞-norm. In the latter selection, it is expected to have a better performance on IAE at the expense of higher control effort.
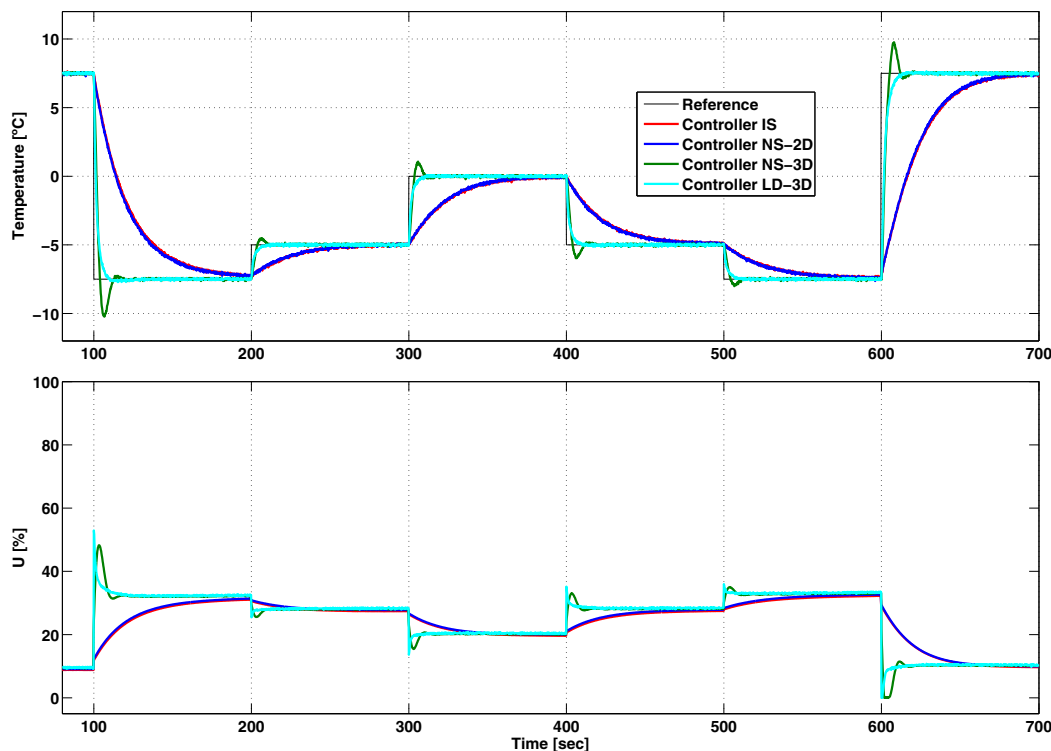
Fig. 6. Performance comparison on the physical process.

4.2 Discussions and insights

The performance of the selected controllers is shown in figure 6 and in Table 1. Whilst performance of controllers IS and NS-2D are similar it is interesting to notice differences between controllers NS-2D and NS-3D. Both of them have been selected using the same DM rule (Nash selection). Differences on the performance are due to the additional information used in the MOP statement minding degradation on IAE performance. Controller LD-3D selected from the LD visualization is consistent with the fact of improving IAE at expense of more control effort (TV).

Table 1. Controller performance on the physical process.

Controller	K_p	T_i	IAE	TV
IS	0.19	2.74	992.4	86.0
NS-2D	0.1898	2.6613	967.6	86.8
NS-3D	0.5091	0.4057	154.6	263.9
LD-3D	2.3735	3.2623	120.7	657.4

5. CONCLUSIONS

In this work we have presented a MOOD procedure involving a reliability based MOP statement for controller tuning. To improve results in the MO process a hybrid approach has been proposed, merging deterministic and evolutionary approaches to calculate the Pareto front approximation. The results presented validate the procedure as useful for PI tuning of non-linear systems. Future work will focus on PID controller tuning in more complex processes and the comparison with standard control techniques as gain scheduling.

ANNEX I: ALGORITHMS DESCRIPTION

Deterministic Optimization Approach

In the NNC method, the optimization problem is separated into several constrained single optimization problems. After series optimizations, a set of evenly distributed pareto solutions results see (Figure 7a). The NNC method incorporates a critical linear mapping of the design objectives. This mapping has the desirable property that the resulting performance of the method is entirely independent of the design objectives scales and in the ability to generate a well distributed set of Pareto points even in numerically demanding situations. The NNC method is presented here to solve a bi-objectives problem, but it can be generalized to *n-objectives*, for more details about the method see Messac et al. (2003).

Evolutionary Optimization Approach

The sp-MODE algorithm is an algorithm based on Differential Evolution Das and Suganthan (2010). As diversity improvement technique, it uses a spherical pruning technique (see Figure 7b). The basic idea of the spherical pruning is to analyze the proposed solutions in the current Pareto front approximation $\boldsymbol{J}_P^*$ by using normalized spherical coordinates from a reference solution. With such approach, it is possible to attain a good distribution along the Pareto front Reynoso-Meza et al. (2010). The algorithm selects one solution for each spherical sector, according to a given norm or measure. Such algorithm has been used with success for controller tuning purposes (Reynoso-Meza et al. (2013c)).

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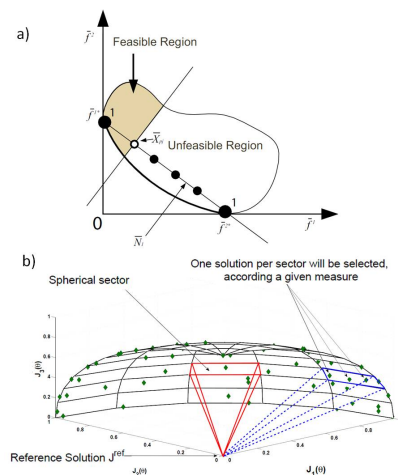
Fig. 7. a) NNC in the bi-objective case. b)Spherical relations on $\boldsymbol{J}_P^* \subset \Re^3$. For each spherical sector, the solution with the lowest norm will be selected.

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